Classification of three qubit states under local incoherent operations

Debasis Sarkar

[1]Department of Applied Mathematics
University of Calcutta

December 6, 2018

Collaborators: Prabir Kumar Dey, Dipayan Chakraborty,Nabendu Das, Indrani Chattopadhyay"

QIPA2018

Review on resource theory of Coherence

- Free states are incoherent states and free operations are incoherent operations.

Review on resource theory of Coherence

- Free states are incoherent states and free operations are incoherent operations.
- Incoherent states: Fix a specific basis $\{|\, i\rangle, i = 1, ..., d\}$. The set of incoherent states I are the all the states which are diagonal in this basis:

$$\rho = \sum_i p_i |\, i\rangle\langle i\,|$$

Review on resource theory of Coherence

- Free states are incoherent states and free operations are incoherent operations.
- Incoherent states: Fix a specific basis $\{|i\rangle, i = 1, ..., d\}$. The set of incoherent states I are the all the states which are diagonal in this basis:
$$\rho = \sum_i p_i |i\rangle\langle i|$$
- Definition of incoherent operations in resource theory of coherence is not unique. Two important incoherent operations are defined as:

Contd..

- • Incoherent operations: Incoherent operations [?] are characterized as the set of trace preserving completely positive maps admitting a set of Kraus operators $\{K_n\}$ such that $\sum_n K_n^\dagger K_n = I$ and, for all n and $\rho \in I$,

$$\frac{K_n \rho K_n^\dagger}{Tr[K_n \rho K_n^\dagger]} \in I$$

Contd..

- • Incoherent operations: Incoherent operations [?] are characterized as the set of trace preserving completely positive maps admitting a set of Kraus operators $\{K_n\}$ such that $\sum_n K_n^{\dagger} K_n = I$ and, for all n and $\rho \in I$,

$$\frac{K_n \rho K_n^{\dagger}}{Tr[K_n \rho K_n^{\dagger}]} \in I$$

- From the definition of incoherent operations it is clear that for any possible outcome coherence can never be generated from incoherent states via this operation, not even probabilistically.

Contd..

- • Incoherent operations: Incoherent operations [?] are characterized as the set of trace preserving completely positive maps admitting a set of Kraus operators $\{K_n\}$ such that $\sum_n K_n^\dagger K_n = I$ and, for all n and $\rho \in I$,
$$\frac{K_n \rho K_n^\dagger}{Tr[K_n \rho K_n^\dagger]} \in I$$
- From the definition of incoherent operations it is clear that for any possible outcome coherence can never be generated from incoherent states via this operation, not even probabilistically.
- Kraus operators representing Incoherent operations has the form $K_n = \sum_n c_n | f(n)\rangle\langle n |$, where $| f(n)\rangle$ is many to one function from basis set onto itself.

Contd..

- • Strictly incoherent operations: Strictly incoherent operations [**?**, **?**] are represented by set of completely positive, trace preserving maps having Kraus operator representation $\{K_n\}_n$ such that

$$K_n \triangle(\rho) K_n^\dagger = \triangle(K_n \rho K_n^\dagger) \;\; \forall n, \forall\ \rho$$

Contd..

- • Strictly incoherent operations: Strictly incoherent operations [**?**, **?**] are represented by set of completely positive, trace preserving maps having Kraus operator representation $\{K_n\}_n$ such that

$$K_n \triangle(\rho) K_n^\dagger = \triangle(K_n \rho K_n^\dagger) \ \ \forall n, \forall \ \rho$$

- Kraus operator representing strictly incoherent operations has the form $K_n = \sum_n c_n | \pi(n)\rangle\langle n |$, where $| \pi(n)\rangle$ is permutation from basis set onto itself.

Contd..

- • Strictly incoherent operations: Strictly incoherent operations [**?**, **?**] are represented by set of completely positive, trace preserving maps having Kraus operator representation $\{K_n\}_n$ such that

$$K_n \triangle (\rho) K_n^\dagger = \triangle(K_n \rho K_n^\dagger) \;\; \forall n, \forall \; \rho$$

- Kraus operator representing strictly incoherent operations has the form $K_n = \sum_n c_n | \pi(n) \rangle \langle n |$, where $| \pi(n) \rangle$ is permutation from basis set onto itself.
- There are several other kinds of incoherent operations, viz., MIO, PIO, DIO, FIO, GIO, SI, etc. They have many interesting properties.

Measure of Coherence

- • Measure of coherence: Any functional C mapping states to the non negative real numbers should satisfy following properties in order to be a proper coherence measure:

Measure of Coherence

- • Measure of coherence: Any functional C mapping states to the non negative real numbers should satisfy following properties in order to be a proper coherence measure:
- (C1)Nonnegativity:

$$C(\rho) \geq 0$$

in general, with equality if and only if ρ is incoherent

Measure of Coherence

- • Measure of coherence: Any functional C mapping states to the non negative real numbers should satisfy following properties in order to be a proper coherence measure:
- (C1)Nonnegativity:

  $$C(\rho) \geq 0$$

  in general, with equality if and only if ρ is incoherent
- (C2) Monotonicity: C does not increase under the action of incoherent operations, i.e.,

  $$C(\Lambda[\rho]) \leq C(\rho)$$

  for any incoherent operation Λ

Contd..

- (C3) Strong monotonicity: C does not increase on average under selective incoherent operations, i.e.,

$$\sum_i q_i C(\sigma_i) \leq C(\rho)$$

with probabilities $q_i = Tr[K_i \rho K_i^\dagger]$, post measurement states $\sigma_i = \frac{K_i \rho K_i^\dagger}{q_i}$, and incoherent Kraus operators K_i.

Contd..

- (C3) Strong monotonicity: C does not increase on average under selective incoherent operations, i.e.,

$$\sum_i q_i C(\sigma_i) \leq C(\rho)$$

  with probabilities $q_i = Tr[K_i \rho K_i^\dagger]$, post measurement states $\sigma_i = \frac{K_i \rho K_i^\dagger}{q_i}$, and incoherent Kraus operators K_i.
- (C4) Convexity: C is a convex function of the state, i.e.,

$$\sum_i p_i C(\rho_i) \geq C(\sum_i p_i \rho_i)$$

Contd..

- • Incoherent operations in multipartite case: The framework of local operation and classical operation is an important part of resource theory of entanglement as is used in various quantum information tasks like teleportation, state transformations. In the LOCC protocol, multiple parties who are spatially separated from each other are allowed to perform only local operations on their subsystems and they are allowed to communicate with each other via classical channel. It is very difficult to represent this protocol mathematically as LOCC operation can include arbitrary number of classical communication.

Contd..

- • Incoherent operations in multipartite case: The framework of local operation and classical operation is an important part of resource theory of entanglement as is used in various quantum information tasks like teleportation, state transformations. In the LOCC protocol, multiple parties who are spatially separated from each other are allowed to perform only local operations on their subsystems and they are allowed to communicate with each other via classical channel. It is very difficult to represent this protocol mathematically as LOCC operation can include arbitrary number of classical communication.
- In the above protocol if all the parties are allowed to perform only incoherent operations then corresponding protocol is known as local incoherent operation with classical communication [**?**, **?**]. Like LOCC it is also very difficult to represent whole LICC operation mathematically.

Contd..

- • Incoherent operations in multipartite case: The framework of local operation and classical operation is an important part of resource theory of entanglement as is used in various quantum information tasks like teleportation, state transformations. In the LOCC protocol, multiple parties who are spatially separated from each other are allowed to perform only local operations on their subsystems and they are allowed to communicate with each other via classical channel. It is very difficult to represent this protocol mathematically as LOCC operation can include arbitrary number of classical communication.
- In the above protocol if all the parties are allowed to perform only incoherent operations then corresponding protocol is known as local incoherent operation with classical communication [**?**, **?**]. Like LOCC it is also very difficult to represent whole LICC operation mathematically.
- Stochastic LICC (SLICC) operations describe state which can be interconverted by LICC non deterministically but with a non zero probability of success.

Necessary and sufficient condition for equivalance of two multipartite pure states under SLICC and LICC

- Before starting our result we shall first state some important results from multipartite entanglement.

Necessary and sufficient condition for equivalance of two multipartite pure states under SLICC and LICC

- Before starting our result we shall first state some important results from multipartite entanglement.
- • Lemma 1 :If the vectors $|\psi\rangle$, $|\phi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B \otimes .... \otimes \mathcal{H}_N$ are connected by a local operator as $|\phi\rangle = A \otimes B \otimes .... \otimes N|\psi\rangle$, then the local ranks satisfy $r(\rho_k^{\psi}) \geqslant r(\rho_k^{\phi})$, k=A,B,......,N

Necessary and sufficient condition for equivalance of two multipartite pure states under SLICC and LICC

- Before starting our result we shall first state some important results from multipartite entanglement.
- • Lemma 1 :If the vectors $|\psi\rangle$, $|\phi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B \otimes .... \otimes \mathcal{H}_N$ are connected by a local operator as $|\phi\rangle = A \otimes B \otimes .... \otimes N|\psi\rangle$, then the local ranks satisfy $r(\rho_k^{\psi}) \geqslant r(\rho_k^{\phi})$, k=A,B,......,N
- • Lemma 2: Two pure states of a multipartite system are equivalent under SLOCC if and only if they are related by a local, invertible operator.

Necessary and sufficient condition for equivalance of two multipartite pure states under SLICC and LICC

- Before starting our result we shall first state some important results from multipartite entanglement.
- • Lemma 1 :If the vectors $|\psi\rangle, |\phi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B \otimes \ldots\ldots\ldots \otimes \mathcal{H}_N$ are connected by a local operator as $|\phi\rangle = A \otimes B \otimes \ldots\ldots\ldots \otimes N|\psi\rangle$, then the local ranks satisfy $r(\rho_k^{\psi}) \geqslant r(\rho_k^{\phi})$, k=A,B,......,N
- • Lemma 2: Two pure states of a multipartite system are equivalent under SLOCC if and only if they are related by a local, invertible operator.
- Above lemma gives necessary and sufficient condition for equivalence of two multipartite states under SLOCC. Now we shall use this lemma to prove necessary and sufficient condition for equivalence of two multipartite states under SLICC. The proof is very similar to above lemma.

Contd...

- • Lemma 3: Two pure states of a multipartite system are equivalent under SLICC iff they are related by local, strictly incoherent operators $A, B, ..., N$ such that

$$|\phi\rangle = A \otimes B \otimes ... \otimes N|\psi\rangle$$

Contd...

- • Lemma 3: Two pure states of a multipartite system are equivalent under SLICC iff they are related by local, strictly incoherent operators $A, B, ..., N$ such that

$$|\phi\rangle = A \otimes B \otimes ... \otimes N|\psi\rangle$$

- Lemma 4: Two pure states $|\psi\rangle$ and $|\phi\rangle$ of a multipartite system are equivalent under LICC iff they are related by local, incoherent, invertible and unitary operators $A, B, ..., N$ such that

$$|\phi\rangle = A \otimes B \otimes ... \otimes N|\psi\rangle$$

Contd...

- • Lemma 3: Two pure states of a multipartite system are equivalent under SLICC iff they are related by local, strictly incoherent operators $A, B, ..., N$ such that

$$|\phi\rangle = A \otimes B \otimes ... \otimes N|\psi\rangle$$

- Lemma 4: Two pure states $|\psi\rangle$ and $|\phi\rangle$ of a multipartite system are equivalent under LICC iff they are related by local, incoherent, invertible and unitary operators $A, B, ..., N$ such that

$$|\phi\rangle = A \otimes B \otimes ... \otimes N|\psi\rangle$$

- The proofs are little bit involved, we are skipping that part.

Contd...

- Now our aim is to classify pure three qubit state with respect to SLICC using above lemma. For example for equivalence of two pure three qubit states $|\psi\rangle$ and $|\phi\rangle$ under SLICC we have to choose three local, strictly incoherent operators A, B, C as follows:

Contd...

- Now our aim is to classify pure three qubit state with respect to SLICC using above lemma. For example for equivalence of two pure three qubit states $|\psi\rangle$ and $|\phi\rangle$ under SLICC we have to choose three local, strictly incoherent operators A, B, C as follows:
- $$1)\ \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$
$$2)\ \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
$$3)\ \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$
$$4)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$
$$5)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Contd...

- $$6)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
  $$7)\ \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
  $$8)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Contd...

- $$6)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
$$7)\ \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
$$8)\ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$
- If we can relate two pure three qubit states $|\psi\rangle$ and $|\phi\rangle$ by $|\phi\rangle = A \otimes B \otimes C|\psi\rangle$ with A, B, C in any of the above combinations [(1)-(8)] then we can say that two states will lie in same SLICC class.

Condition for SLICC equivalence of two states after changing coefficients of states

- We consider the state where number of product terms 8, if we can convert $|\psi\rangle = a|000\rangle+b|001\rangle+c|010\rangle+d|011\rangle+e|100\rangle+f|101\rangle+g|110\rangle+h|111\rangle$ to $|\phi\rangle = a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$ under SLICC then according to above condition there exists SIO operators A,,B,C such that $|\phi\rangle = A \otimes B \otimes C|\psi\rangle$. The required form of A,B,C have been mentioned above.

Condition for SLICC equivalence of two states after changing coefficients of states

- We consider the state where number of product terms 8, if we can convert $|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ to $|\phi\rangle = a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$ under SLICC then according to above condition there exists SIO operators A,,B,C such that $|\phi\rangle = A \otimes B \otimes C|\psi\rangle$. The required form of A,B,C have been mentioned above.
- (1) If we convert $|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ to $|\phi\rangle = a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$ by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Then we have $\frac{a'd'}{b'c'} = \frac{ad}{bc}$, $\frac{a'f'}{b'e'} = \frac{af}{be}$, $\frac{a'h'}{b'g'} = \frac{ah}{bg}$, $\frac{a'g'}{c'e'} = \frac{ag}{ce}$, $\frac{a'h'}{c'f'} = \frac{ah}{cf}$, $\frac{a'h'}{d'c'} = \frac{ah}{dc}$

Contd..

(2) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Contd..

(2) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Then we have $\frac{b'c'}{a'd'} = \frac{ad}{bc}$, $\frac{b'e'}{a'f'} = \frac{af}{be}$, $\frac{b'g'}{d'e'} = \frac{ah}{cf}$, $\frac{b'g'}{a'h'} = \frac{ah}{bg}$, $\frac{b'h'}{d'f'} = \frac{ag}{ce}$, $\frac{b'g'}{c'f'} = \frac{ah}{ed}$

Contd..

3) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Contd..

3) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Then we have $\frac{e'h'}{g'f'} = \frac{ad}{bc}$, $\frac{b'e'}{a'f'} = \frac{af}{be}$, $\frac{d'e'}{c'f'} = \frac{ah}{bg}$, $\frac{c'e'}{a'g'} = \frac{ag}{ce}$, $\frac{d'e'}{b'g'} = \frac{ah}{cf}$,
$\frac{d'e'}{a'h'} = \frac{ah}{ed}$

Contd..

4) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Contd..

4) If we convert $|\psi\rangle =$ $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ to $|\phi\rangle =$ $a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$ by

$$\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Then we have $\frac{b'c'}{a'd'} = \frac{ad}{bc}$, $\frac{c'h'}{d'g'} = \frac{af}{be}$, $\frac{c'f'}{d'e'} = \frac{ah}{bg}$, $\frac{c'e'}{a'g'} = \frac{ag}{ce}$, $\frac{c'f'}{a'h'} = \frac{ah}{cf}$, $\frac{c'f'}{b'g'} = \frac{ah}{de}$

Contd..

5) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Contd..

5) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Then we have $\frac{a'h'}{b'g'} = \frac{ah}{bg}$, $\frac{a'h'}{c'f'} = \frac{ah}{cf}$, $\frac{a'h'}{d'e'} = \frac{ah}{de}$, $\frac{a'd'}{b'c'} = \frac{eh}{dg}$, $\frac{a'f'}{b'e'} = \frac{ch}{dg}$, $\frac{a'g'}{c'e'} = \frac{bh}{df}$

Contd..

6) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Contd..

6) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

Then we have $\frac{a'h'}{b'g'} = \frac{bg}{ah}$, $\frac{b'c'}{a'd'} = \frac{eh}{gf}$, $\frac{a'h'}{c'f'} = \frac{bg}{de}$, $\frac{a'h'}{d'e'} = \frac{bg}{cf}$, $\frac{b'e'}{a'f'} = \frac{ch}{dg}$, $\frac{a'g'}{c'e'} = \frac{ag}{ce}$

Contd..

7) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Contd..

7) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & b_1 \\ b_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Then we have $\frac{a'd'}{b'c'} = \frac{ad}{bc}$, $\frac{c'f'}{a'h'} = \frac{bg}{ae}$, $\frac{c'f'}{b'g'} = \frac{bg}{cf}$, $\frac{b'f'}{a'e'} = \frac{cg}{dh}$, $\frac{c'e'}{a'g'} = \frac{bh}{df}$, $\frac{d'e'}{a'h'} = \frac{ah}{de}$

Contd..

8) If we convert $|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ to $|\phi\rangle = a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$ by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Contd..

8) If we convert $|\psi\rangle =$
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$
to $|\phi\rangle =$
$a'|000\rangle + b'|001\rangle + c'|010\rangle + d'|011\rangle + e'|100\rangle + f'|101\rangle + g'|110\rangle + h'|111\rangle$
by

$$\begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \otimes \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \otimes \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix}$$

Then we have $\frac{b'c'}{a'd'} = \frac{eh}{fg}$, $\frac{c'e'}{a'g'} = \frac{bh}{af}$, $\frac{a'f'}{b'e'} = \frac{af}{be}$, $\frac{b'g'}{a'h'} = \frac{de}{cf}$, $\frac{c'f'}{a'h'} = \frac{ah}{cf}$, $\frac{d'e'}{a'h'} = \frac{bg}{cf}$

CLASSIFICATION OF PURE THREE QUBIT STATES

Now we shall give classification of pure three qubit states with respect to SLICC. It will provide us all SLICC inequivalent classes of pure three qubit states.

- Number of product terms in the state is 1

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
1	$\|000\rangle$	

Contd..

- Number of product terms in the state is 2

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
2a	$a\|000\rangle + b\|001\rangle$	
2b	$a\|000\rangle + b\|010\rangle$	
2c	$a\|000\rangle + b\|011\rangle$	
2d	$a\|000\rangle + b\|100\rangle$	
2e	$a\|000\rangle + b\|101\rangle$	
2f	$a\|000\rangle + b\|110\rangle$	
2g	$a\|000\rangle + b\|111\rangle$	

Contd..

- Number of product terms in the state is 3

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
3a	$a\|000\rangle + b\|001\rangle + c\|010\rangle$	
3b	$a\|000\rangle + b\|001\rangle + c\|100\rangle$	
3c	$a\|000\rangle + b\|001\rangle + c\|110\rangle$	
3d	$a\|000\rangle + b\|011\rangle + c\|100\rangle$	
3e	$a\|000\rangle + b\|011\rangle + c\|101\rangle$	
3f	$a\|000\rangle + b\|010\rangle + c\|100\rangle$	
3g	$a\|000\rangle + b\|010\rangle + c\|101\rangle$	

Contd..

- Number of product terms in the state is 4

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
4a	$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|011\rangle$	$\Delta_1' = \Delta_1$ or $\Delta_1 = \frac{1}{\Delta_1'}$
4b	$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|100\rangle$	
4c	$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|101\rangle$	
4d	$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|110\rangle$	
4e	$a\|000\rangle + b\|001\rangle + c\|010\rangle + d\|111\rangle$	
4f	$a\|000\rangle + b\|001\rangle + c\|100\rangle + d\|101\rangle$	

Contd..

4g	$a\|000\rangle + b\|001\rangle + c\|100\rangle$ $+d\|110\rangle$	
4h	$a\|000\rangle + b\|001\rangle + c\|100\rangle$ $+d\|111\rangle$	
4i	$a\|000\rangle + b\|001\rangle + c\|110\rangle$ $+d\|111\rangle$	$\Delta_1' = \Delta_1$ or $\Delta_1 = \frac{1}{\Delta_1'}$
4j	$a\|000\rangle + b\|010\rangle + c\|100\rangle$ $+d\|110\rangle$	
4k	$a\|000\rangle + b\|010\rangle + c\|100\rangle$ $+d\|111\rangle$	
4l	$a\|000\rangle + b\|010\rangle + c\|101\rangle$ $+d\|111\rangle$	$\Delta_1' = \Delta_1$ or $\Delta_1 = \frac{1}{\Delta_1'}$
4m	$a\|000\rangle + b\|011\rangle + c\|100\rangle$ $+d\|111\rangle$	$\Delta_1' = \Delta_1$ or $\Delta_1 = \frac{1}{\Delta_1'}$
4n	$a\|000\rangle + b\|011\rangle + c\|101\rangle$ $+d\|110\rangle$	

Contd..

- Number of product terms in the state is 5

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
5a	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|011\rangle + e\|100\rangle$	$\Delta_1' = \Delta_1$
5b	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|100\rangle + e\|101\rangle$	$\Delta_5' = \Delta_5$
5c	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|100\rangle + e\|110\rangle$	$\Delta_9' = \Delta_9$
5d	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|100\rangle + e\|111\rangle$	$\Delta_{11}' = \Delta_{11}$

Contd..

5e	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|101\rangle + e\|110\rangle$	$\Delta'_{10} = \Delta_{10}$
5f	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|101\rangle + e\|111\rangle$	$\Delta'_{9} = \Delta_{9}$
5g	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|110\rangle + e\|111\rangle$	$\Delta'_{5} = \Delta_{5}$

Contd..

- Number of product terms in the state is 6

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
6a	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|011\rangle + e\|100\rangle + f\|101\rangle$	$\Delta'_1 = \Delta_1$ *and* $\Delta'_2 = \Delta_2$ or $\Delta'_1 = 1/\Delta_1$ *or* $\Delta'_2 = 1/\Delta_2$
6b	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|011\rangle + e\|100\rangle + f\|110\rangle$	$\Delta'_1 = \Delta_1$ *and* $\Delta'_3 = \Delta_3$ or $\Delta'_1 = 1/\Delta_1$ *or* $\Delta'_3 = 1/\Delta_3$
6c	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|011\rangle + e\|100\rangle + f\|111\rangle$	$\Delta'_1 = \Delta_1$ *and* $\Delta'_4 = \Delta_4$ or $\Delta'_1 = 1/\Delta_1$ *or* $\Delta'_4 = 1/\Delta_4$

Contd..

6d	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|100\rangle + e\|101\rangle + f\|110\rangle$	$\Delta_5' = \Delta_5$ *and* $\Delta_6' = \Delta_6$ or $\Delta_5' = 1/\Delta_5$ *or* $\Delta_6' = 1/\Delta_6$
6e	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|100\rangle + e\|101\rangle + f\|111\rangle$	$\Delta_5' = \Delta_5$ *and* $\Delta_3' = \Delta_3$ or $\Delta_5' = \Delta_5$ *or* $\Delta_3' = 1/\Delta_3$
6f	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|101\rangle + e\|110\rangle + f\|111\rangle$	$\Delta_2' = \Delta_2$ *and* $\Delta_6' = \Delta_6$
6g	$a\|000\rangle + b\|001\rangle + c\|011\rangle$ $+d\|101\rangle + e\|110\rangle + f\|111\rangle$	$\Delta_2' = \Delta_2$ *and* $\Delta_7' = \Delta_7$ or $\Delta_2' = 1/\Delta_2$ *or* $\Delta_7' = \Delta_7$

Contd..

- Number of product terms in the state is 7

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
7	$a\vert 000\rangle + b\vert 001\rangle + c\vert 010\rangle$ $+d\vert 011\rangle + e\vert 100\rangle + f\vert 101\rangle$ $+g\vert 110\rangle$	$\Delta_1' = \Delta_1, \Delta_2' = \Delta_2$ and $\Delta_8' = \Delta_8$

Contd..

- Number of product terms in the state is 8

No.	SLICC inequivalet states under same no of product terms in the state	condition under which there exists infinite number of inequivalent classes
8	$a\|000\rangle + b\|001\rangle + c\|010\rangle$ $+d\|011\rangle + e\|100\rangle + f\|101\rangle$ $+g\|110\rangle + h\|111\rangle$	mentioned in previous section

In the above tabular format we shall use following notations $\Delta_1 = \frac{ad}{bc}$, $\Delta_2 = \frac{af}{be}$, $\Delta_3 = \frac{af}{ce}$, $\Delta_4 = \frac{af}{de}$, $\Delta_5 = \frac{ae}{bd}$, $\Delta_6 = \frac{af}{dc}$, $\Delta_7 = \frac{bf}{cd}$, $\Delta_8 = \frac{ag}{ce}$, $\Delta_9 = \frac{ae}{cd}$, $\Delta_{10} = \frac{be}{cd}$, $\Delta_{11} = \frac{a^2e}{bcd}$, $\Delta_1' = \frac{a'd'}{b'c'}$ and similarly for the others .

observations from table

1. From the tabular form given in the previous section we can summarise classification of pure three qubit states with respect to SLICC in following way

   • **Classification under separable class**

   First we consider states from the tabular form which lie under separable states ($|\psi\rangle^{ABC} = |\phi\rangle^{A} \otimes |\eta\rangle^{B} \otimes |\tau\rangle^{C}$), i.e., states for which all reduced density matrices are pure state. This states can be again subclassified under following ways.

observations from table

1. From the tabular form given in the previous section we can summarise classification of pure three qubit states with respect to SLICC in following way
   - **Classification under separable class**

   First we consider states from the tabular form which lie under separable states ($|\psi\rangle^{ABC} = |\phi\rangle^{A} \otimes |\eta\rangle^{B} \otimes |\tau\rangle^{C}$), i.e., states for which all reduced density matrices are pure state. This states can be again subclassified under following ways.
2. - **Three single qubit reduced density matrices are pure incoherent state**

   $|\phi\rangle^{A}$, $|\eta\rangle^{B}$ and $|\tau\rangle^{C}$ are incoherent: $|000\rangle$ and its SLICC equivalent states

Contd..

1. • **Two single qubit reduced density matrices are incoherent and another is coherent**
(a) $|\phi\rangle^A$, $|\eta\rangle^B$ are incoherent and $|\tau\rangle^C$ is coherent:
$a|000\rangle + b|001\rangle$ and its SLICC equivalent states
(b) $|\eta\rangle^B$, $|\tau\rangle^C$ are incoherent and $|\phi\rangle^A$ is coherent:
$a|000\rangle + b|100\rangle$ and its SLICC equivalent states
(c) $|\phi\rangle^A$, $|\tau\rangle^C$ are incoherent and $|\eta\rangle^B$ is coherent:
$a|000\rangle + b|010\rangle$ and its SLICC equivalent states

Contd..

1. • **Two single qubit reduced density matrices are incoherent and another is coherent**

(a) $|\phi\rangle^A$, $|\eta\rangle^B$ are incoherent and $|\tau\rangle^C$ is coherent:
$a|000\rangle + b|001\rangle$ and its SLICC equivalent states
(b) $|\eta\rangle^B$, $|\tau\rangle^C$ are incoherent and $|\phi\rangle^A$ is coherent:
$a|000\rangle + b|100\rangle$ and its SLICC equivalent states
(c) $|\phi\rangle^A$, $|\tau\rangle^C$ are incoherent and $|\eta\rangle^B$ is coherent:
$a|000\rangle + b|010\rangle$ and its SLICC equivalent states

• **One single qubit reduced density matrices is incoherent and others are coherent**

(a) $|\phi\rangle^A$, $|\eta\rangle^B$ are coherent and $|\tau\rangle^C$ is incoherent:
$a|000\rangle + b|010\rangle + c|100\rangle + d|110\rangle$(with ad=bc) and its SLICC equivalent states.
(b) $|\eta\rangle^B$, $|\tau\rangle^C$ are coherent and $|\phi\rangle^A$ is incoherent:
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle$(with ad=bc) and its SLICC equivalent states.
(c) $|\phi\rangle^A$, $|\tau\rangle^C$ are coherent and $|\eta\rangle_B$ is incoherent:
$a|000\rangle + b|001\rangle + c|100\rangle + d|101\rangle$(with ad=bc) and its SLICC equivalent states.

Contd..

- **All reduced density matrices are coherent**

$|\phi\rangle_A$, $|\eta\rangle_B$ and $|\tau\rangle_C$ are coherent:
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$(with $\frac{ad}{bc} = \frac{af}{be} = \frac{ah}{bg} = \frac{ag}{ce} = \frac{ah}{cf} = \frac{bh}{df} = \frac{bg}{de} = \frac{ah}{de} = 1$) and its SLICC equivalent states.

Contd..

- **All reduced density matrices are coherent**

$|\phi\rangle_A$, $|\eta\rangle_B$ and $|\tau\rangle_C$ are coherent: $a|000\rangle+b|001\rangle+c|010\rangle+d|011\rangle+e|100\rangle+f|101\rangle+g|110\rangle+h|111\rangle$(with $\frac{ad}{bc}=\frac{af}{be}=\frac{ah}{bg}=\frac{ag}{ce}=\frac{ah}{cf}=\frac{bh}{df}=\frac{bg}{de}=\frac{ah}{de}=1$) and its SLICC equivalent states.

- **Classification under biseparable class**

Now we consider states from the tabular form which lie under biseparable class. This can be again sub classified in following way

- **One single qubit reduced density matrices is pure incoherent and others are mixed incoherent**

(a) Reduced density matrices of system C is pure incoherent:
$a|000\rangle + b|110\rangle$ and its SLICC equivalent states
(b) Reduced density matrices of system B is pure incoherent:
$a|000\rangle + b|101\rangle$ and its SLICC equivalent states
(c) Reduced density matrices of system A is pure incoherent:
$a|000\rangle + b|011\rangle$ and its SLICC equivalent states

Contd..

• **One single qubit reduced density matrices is pure incoherent and others are mixed coherent**

(a) Reduced density matrices of system A is pure incoherent: $a|000\rangle + b|001\rangle + c|010\rangle$, $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle$ (with $ad \neq bc$) and their SLICC equivalent states.

(b) Reduced density matrices of system B is pure incoherent: $a|000\rangle + b|001\rangle + c|100\rangle$, $a|000\rangle + b|001\rangle + c|100\rangle + d|101\rangle$ (with $ad \neq bc$) and their SLICC equivalent states

(c) Reduced density matrices of system A is pure incoherent: $a|000\rangle + b|010\rangle + c|100\rangle$, $a|000\rangle + b|010\rangle + c|100\rangle + d|110\rangle$ (with $ad \neq bc$) and their SLICC equivalent states

Contd..

• **One single qubit reduced density matrices is pure incoherent and others are mixed coherent**

(a) Reduced density matrices of system A is pure incoherent:
$a|000\rangle + b|001\rangle + c|010\rangle$, $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle$ (with $ad \neq bc$) and their SLICC equivalent states.

(b) Reduced density matrices of system B is pure incoherent:
$a|000\rangle + b|001\rangle + c|100\rangle$, $a|000\rangle + b|001\rangle + c|100\rangle + d|101\rangle$ (with $ad \neq bc$) and their SLICC equivalent states

(c) Reduced density matrices of system A is pure incoherent:
$a|000\rangle + b|010\rangle + c|100\rangle$, $a|000\rangle + b|010\rangle + c|100\rangle + d|110\rangle$ (with $ad \neq bc$) and their SLICC equivalent states

• **Classification under genuine tripartite entangled class**

Now we shall consider states from the tabular form given in previous section such that they lie in genuine tripartite entangled class, i.e., GHZ or W class. In this case all reduced density matrices are mixed states. This can be again sub classified in following way.

(a) **All reduced density matrices are mixed incoherent states**:
$a|000\rangle + b|111\rangle$, $a|000\rangle + b|011\rangle + c|101\rangle$, $a|000\rangle + b|011\rangle + c|101\rangle + d|110\rangle$ and their SLICC equivalent states.

Contd..

(b) **Two single qubit reduced density matrices are mixed incoherent states and another is mixed coherent**

• Reduced density matrices of system C is mixed coherent:
$a|000\rangle + b|001\rangle + c|110\rangle$, $a|000\rangle + b|001\rangle + c|110\rangle + d|111\rangle$ and their SLICC equivalent states.

• Reduced density matrices of system A is mixed coherent:
$a|000\rangle + b|011\rangle + c|100\rangle$, $a|000\rangle + b|011\rangle + c|100\rangle + d|111\rangle$ and their SLICC equivalent states.

• Reduced density matrices of system B is mixed coherent:
$a|000\rangle + b|010\rangle + c|101\rangle$, $a|000\rangle + b|010\rangle + c|101\rangle$ $+d|111\rangle$ and their SLICC equivalent states.

Contd..

(b) **Two single qubit reduced density matrices are mixed incoherent states and another is mixed coherent**

- Reduced density matrices of system C is mixed coherent: $a|000\rangle + b|001\rangle + c|110\rangle$, $a|000\rangle + b|001\rangle + c|110\rangle + d|111\rangle$ and their SLICC equivalent states.
- Reduced density matrices of system A is mixed coherent: $a|000\rangle + b|011\rangle + c|100\rangle$, $a|000\rangle + b|011\rangle + c|100\rangle + d|111\rangle$ and their SLICC equivalent states.
- Reduced density matrices of system B is mixed coherent: $a|000\rangle + b|010\rangle + c|101\rangle$, $a|000\rangle + b|010\rangle + c|101\rangle$ $+d|111\rangle$ and their SLICC equivalent states.

(c) **Two single qubit reduced density matrices are mixed coherent states and another one is mixed incoherent**

- Reduced density matrices of system A is mixed incoherent: $a|000\rangle + b|001\rangle + c|010\rangle + d|111\rangle$ and it's SLICC equivalent states.
- Reduced density matrices of system B is mixed incoherent: $a|000\rangle + b|001\rangle + c|100\rangle + d|111\rangle$ and it's SLICC equivalent states.
- Reduced density matrices of system C is mixed incoherent: $a|000\rangle + b|010\rangle + c|100\rangle d|111\rangle$ and it's SLICC equivalent states.

Contd..

(d) **All single qubit reduced density matrices are coherent**:

$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle$, $a|000\rangle + b|001\rangle + c|010\rangle + d|101\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|110\rangle$, $a|000\rangle + b|001\rangle + c|100\rangle + d|110\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|101\rangle + e|110\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|101\rangle + e|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|110\rangle + e|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|101\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|110\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|101\rangle + f|111\rangle$,
$a|000\rangle + b|001\rangle + c|011\rangle + d|101\rangle + e|110\rangle + f|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|101\rangle + e|110\rangle + f|111\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|110\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|101\rangle + f|110\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101 + g|110\rangle\rangle$,
$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101 + g|110\rangle\rangle + h|111\rangle$

References

- 1 M.A Nielsen and I.L. Chuang, Quantum Computation and Quantum Information, (Cambridge University Press, Cambridge, UK, 2000).
- 2 R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865-942 (2009).
- 3 B.Coecke, T.Fritz and R. W. Spekkens, Information and computation **250**,59(2016).
- 4 C. H. Bennett and D. P. Divincenzo, Nature (London) **404**, 247-255 (2000).
- 5 R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188-5191 (2001).
- 6 S.-S. Li, G.-L. Long, F.-S. Bai, S.-L. Feng, and H.-Z. Zheng, Proc. Natl. Acad. Sci. U.S.A. **98**(21), 11847 (2001).
- 7 T. Baumgratz, M. Cramer, and M. B. Plenio, Physical Review Letters **113**, 140401 (2014).
- 8 H.-L. Shi, X.-H. Wang, S.-Y. Liu, W.-L. Yang, Z.-Y. Yang,and H. Fan, Scientific Reports **7**, 14806 (2017).

References

- 9 A. Streltsov, S. Rana, P. Boes, and J. Eisert, Physical Review Letters **119**, 140402 (2017).
- 10 A. Streltsov, U. Singh, H. S. Dhar, M. N. Bera, and G. Adesso, Physical Review Letters **115**, 020403 (2015).
- 11 E. Chitambar and M.-H. Hsieh, Physical Review Letters **117**, 020402 (2016).
- 12 A. Streltsov, S. Rana, M. N. Bera, and M. Lewenstein,Physical Review X **7**, 011024 (2017).
- 13 S. Du, Z. Bai, and Y. Guo, Physical Review A **91**, 052120(2015).
- 14 L.-H. Shao, Z. Xi, H. Fan, and Y. Li, Physical Review A **91**,042120 (2015).
- 15 Z. Xi, Y. Li, and H. Fan, Scientific Reports **5**, 10922 (2015).
- 16 M.-L. Hu, S.-Q. Shen, and H. Fan, Physical Review A **96**,052309 (2017).

References

- 17 A. Winter and D. Yang, Physical Review Letters **116**,120404 (2016).
- 18 E. Chitambar and G. Gour, Physical Review Letters **117**,030401 (2016).
- 20 M. Lostaglio, D. Jennings, and T. Rudolph, Nature Communications **6**, 6383 (2015).
- 21 O. Karlstrum, H. Linke, G. Karlstrum, and A. Wacker, Physical Review B **84**, 113415 (2011).
- 22 E. Gauger, E. Rieper, J. J. L. Morton, S. C. Benjamin, V. Vedral, Phys. Rev. Lett. **106**, 040503 (2011).
- 23 W. Dur, G. Vidal, and J. I. Cirac, Physical Review A **62**,062314 (2000).

THANK YOU