

Resource theories of nonclassicality and thermodynamics in bosonic quantum systems

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Meeting on Quantum Information Processing and Applications
Harish-Chandra Research Institute, Allahabad
7 December, 2018

Imperial College
London



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Operational
limitations

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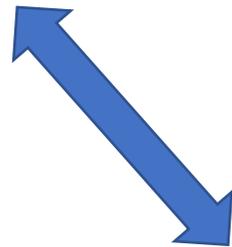
Resource states
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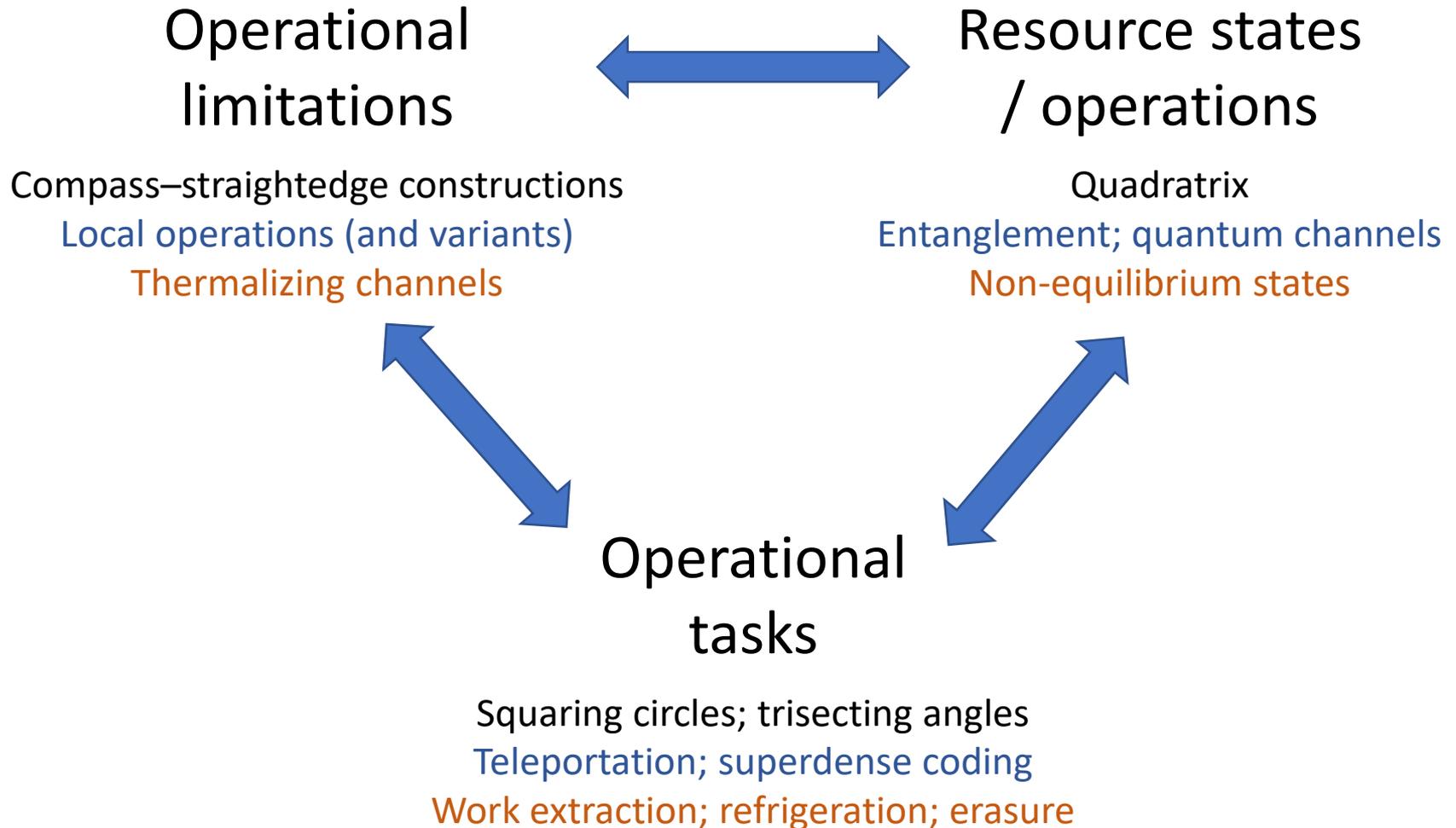
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Resource theories for continuous-variable systems

Resources: Non-Gaussianity • Non-classicality • Entanglement • Thermal non-equilibrium

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- Difficult to understand entanglement beyond Gaussian states
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Recent work (2017–18):

- Tan *et al.*, **Quantifying the Coherence between Coherent States**. PRL 119, 190405
- Theurer *et al.*, **Resource Theory of Superposition**. PRL 119, 230401
- Lami *et al.*, **Gaussian quantum resource theories**. arXiv:1801.05450
- Zhuang *et al.*, **Resource theory of non-Gaussian operations**. PRA 97, 052317
- Takagi and Zhuang, **Convex resource theory of non-Gaussianity**. PRA 97, 062337
- Kwon *et al.*, **Nonclassicality of Light as a Quantifiable Resource for Quantum Metrology**. arXiv:1804.09355
- Albarelli *et al.*, **Resource theory of quantum non-Gaussianity and Wigner negativity**. PRA 98, 052350 (Alessandro Ferraro’s talk at QIPA 2018)

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- But $|\alpha\rangle$ basis is **non-orthogonal, overcomplete!**

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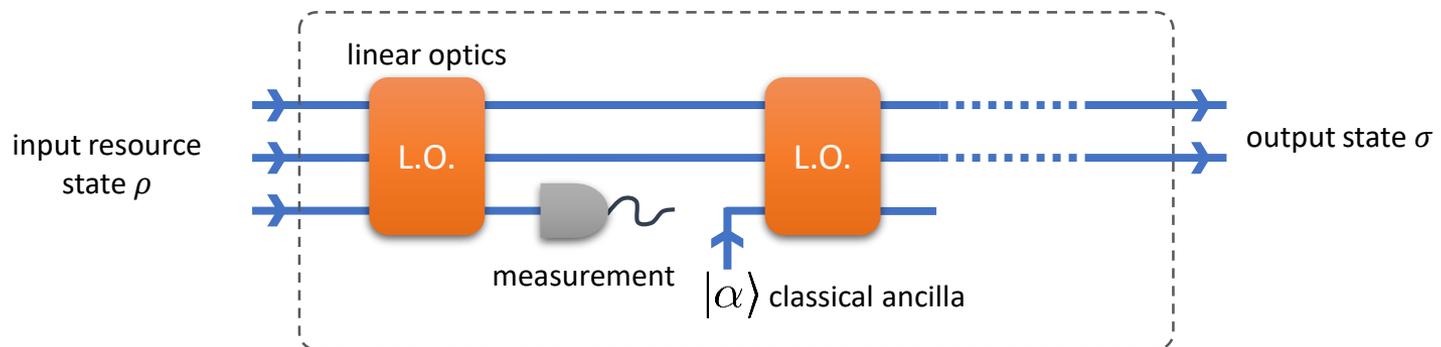
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Linear-optic networks, classical ancillas, destructive measurements (and feed-forward)



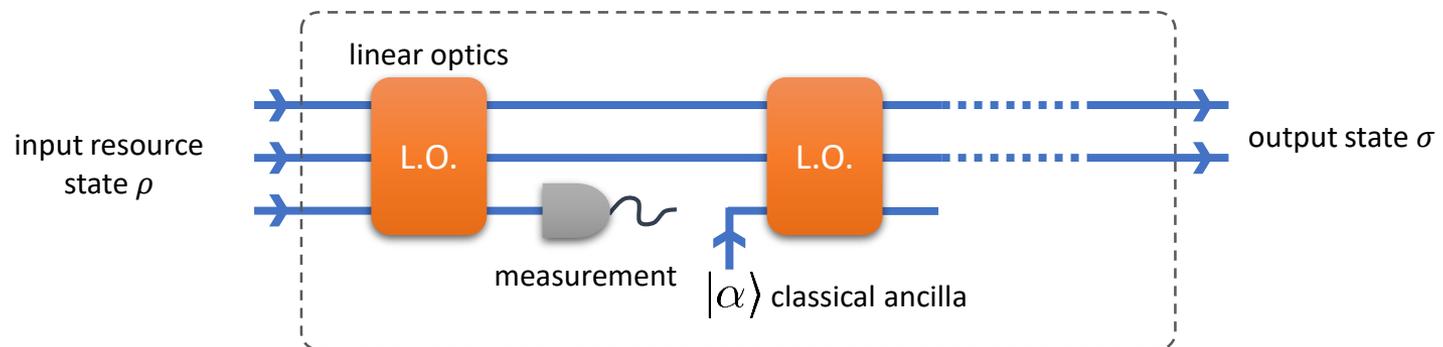
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Can consider different numbers of measurement rounds

For n measurement rounds, the class of operations is denoted \mathcal{P}_n

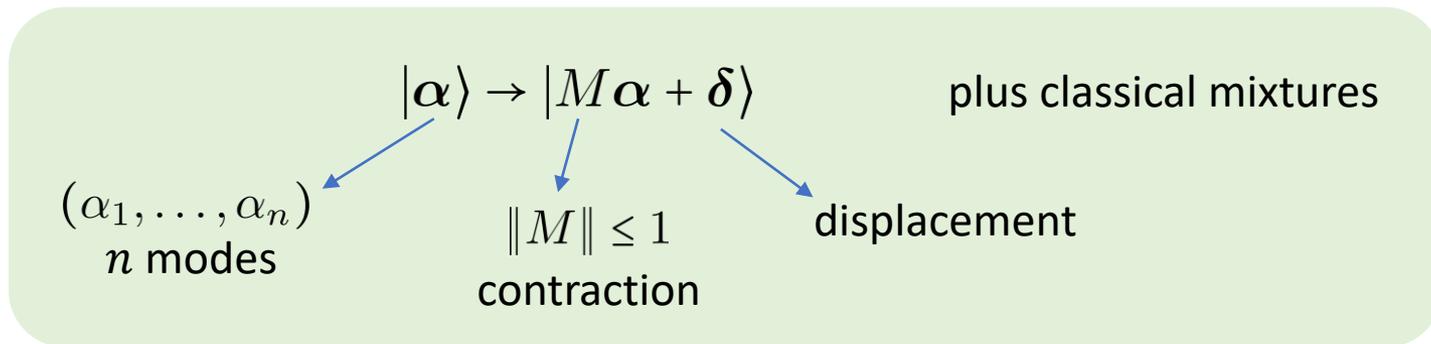
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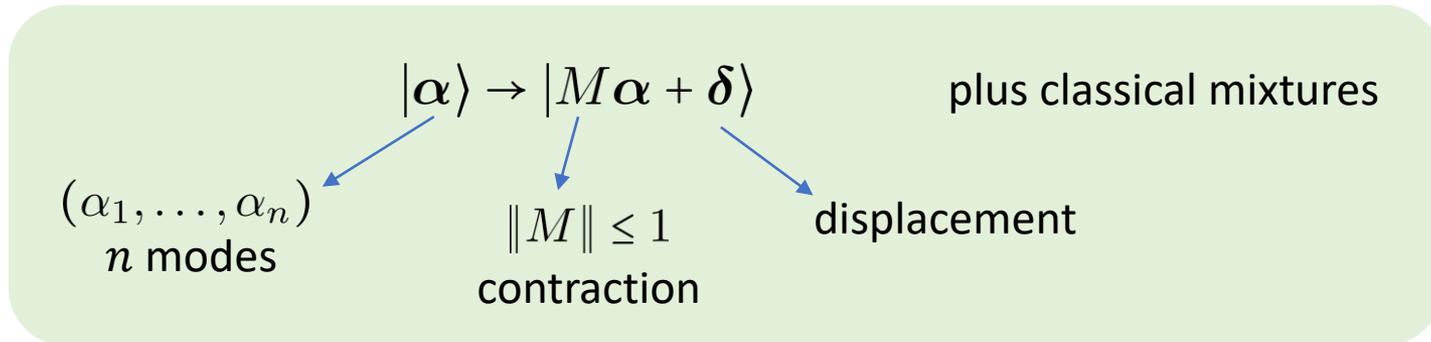
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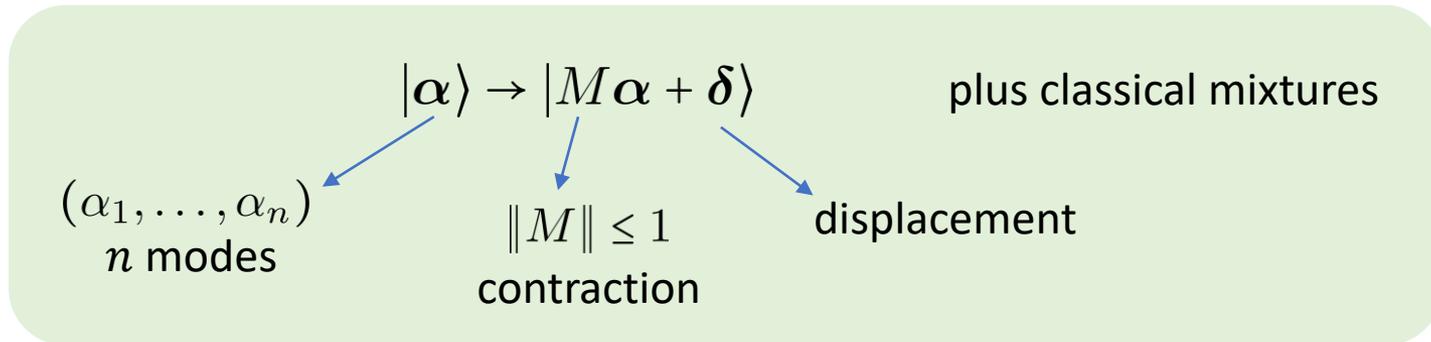


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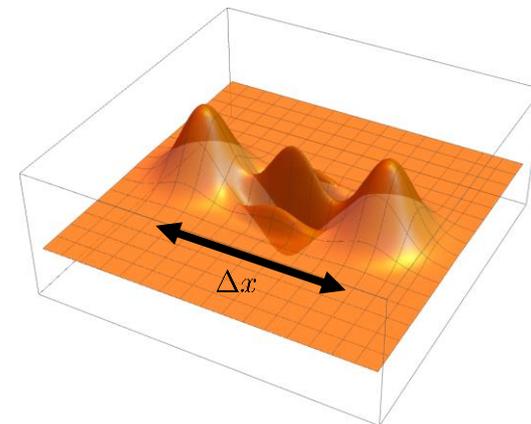


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is possible (with any success probability) only for $|g| \leq 1$
- Well-known measures of nonclassicality fail to reflect this!
[Hillery, PRA 35, 725 (1987); Lee, PRA 44, R2775 (1991)]

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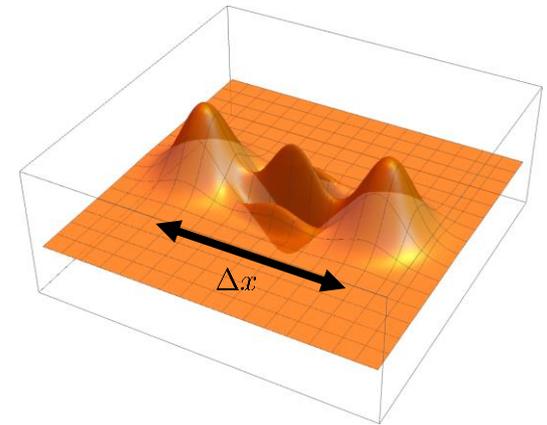
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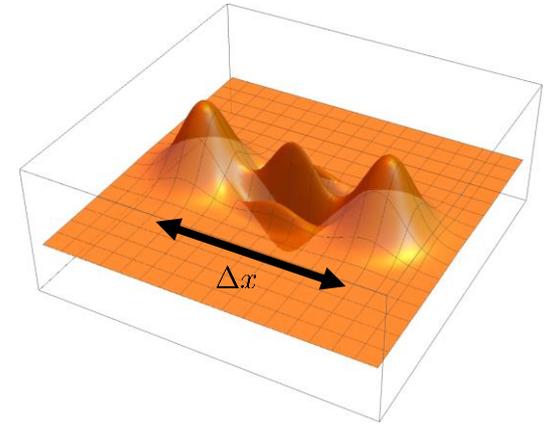
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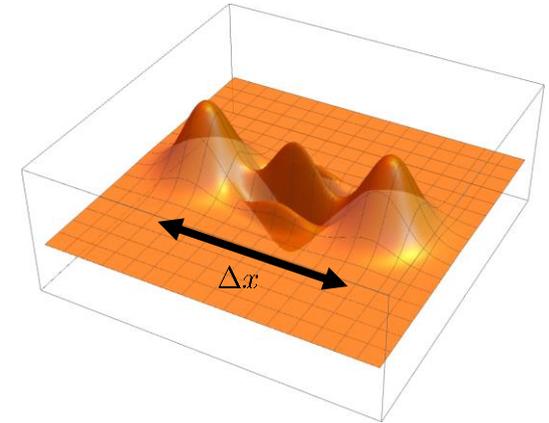
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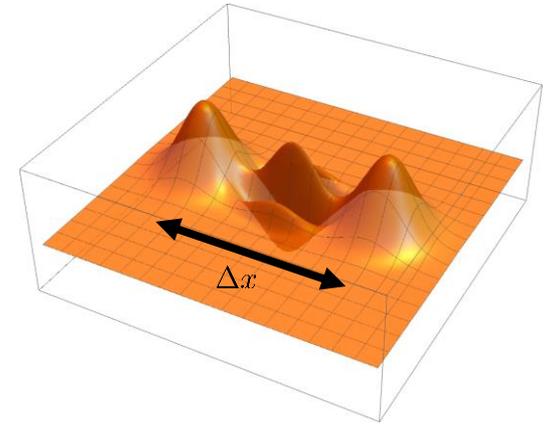
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E.g. there is a protocol for “growing” cat states: $(|\alpha\rangle + |-\alpha\rangle)^{\otimes 2} \mapsto |\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle$

Applying this result: success probability $\leq \frac{1}{2}$

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- Crucially, also a monotone under \mathcal{P}_n

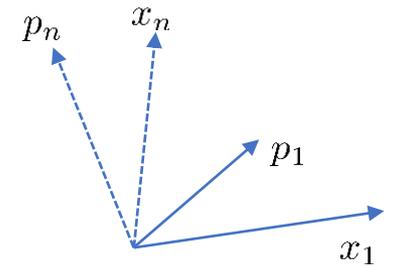
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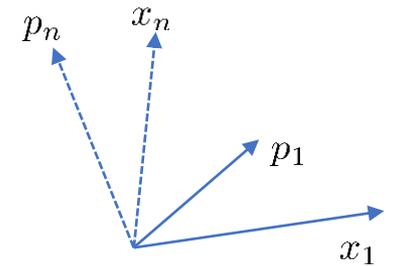


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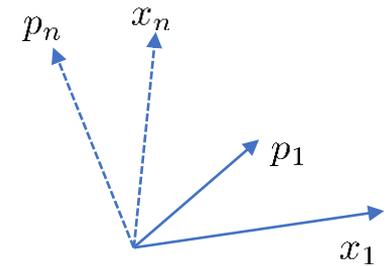
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Under \mathcal{P}_0 (i.e., without measurements), this concentration of utility for parameter estimation is impossible.



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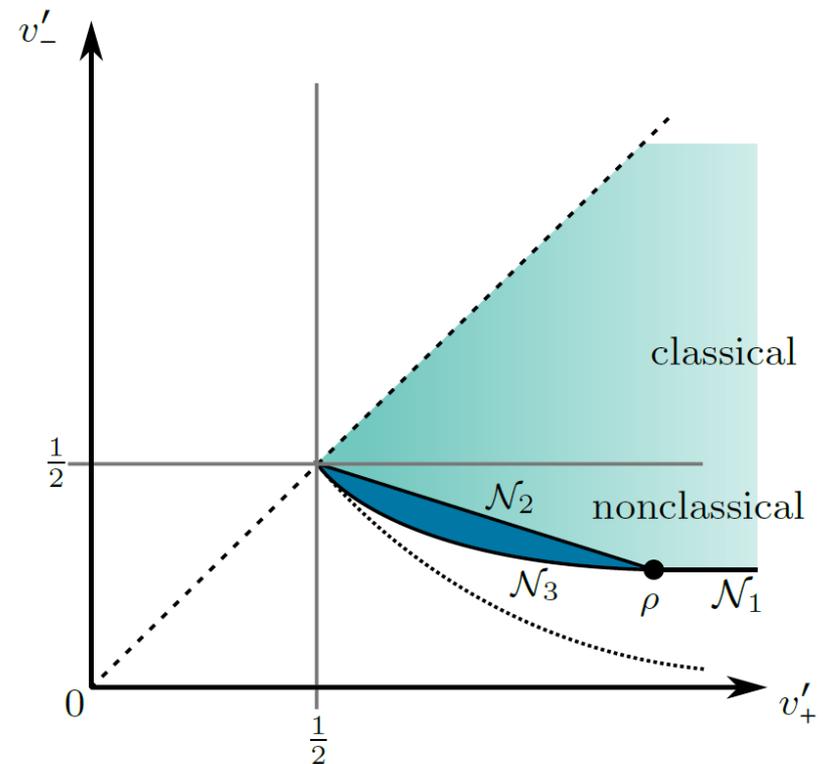
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- Completely characterised by covariance matrix
- 1-mode case: is ρ more nonclassical than σ if it is more squeezed?
- Surprisingly, reduction in squeezing is necessary for a \mathcal{P}_n transformation but not sufficient!

A second constraint is needed:

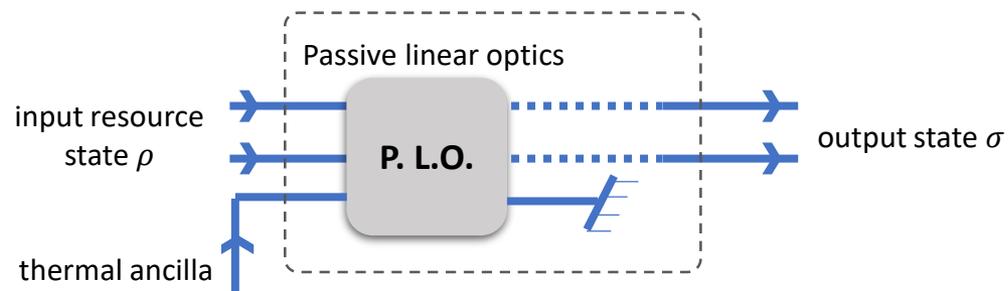
There is a trade-off between removing noise and maintaining squeezing



Resource theory of thermal non-equilibrium

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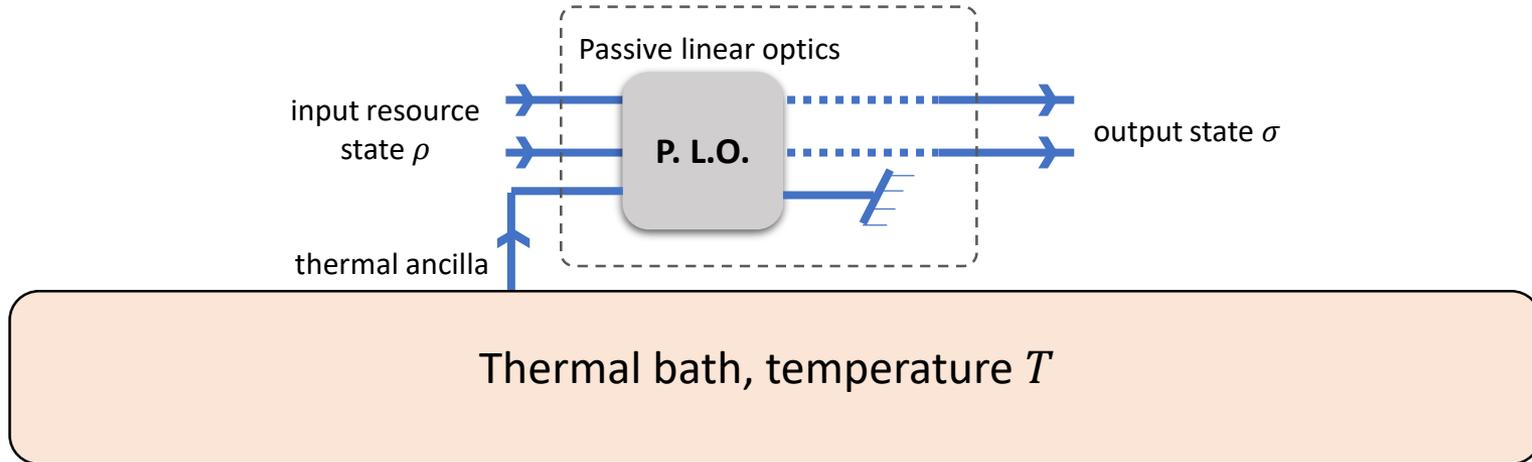
Gaussian thermal operations (GTO)



Thermal bath, temperature T

Resource theory of thermal non-equilibrium

Gaussian thermal operations (GTO)



- Adaptation of finite-dimensional thermal operations framework*
- Passive linear optics: built-in First Law of Thermodynamics
- Gaussian dilations, but all ancillary states thermal at T
- More restrictive than operations in nonclassicality resource theory

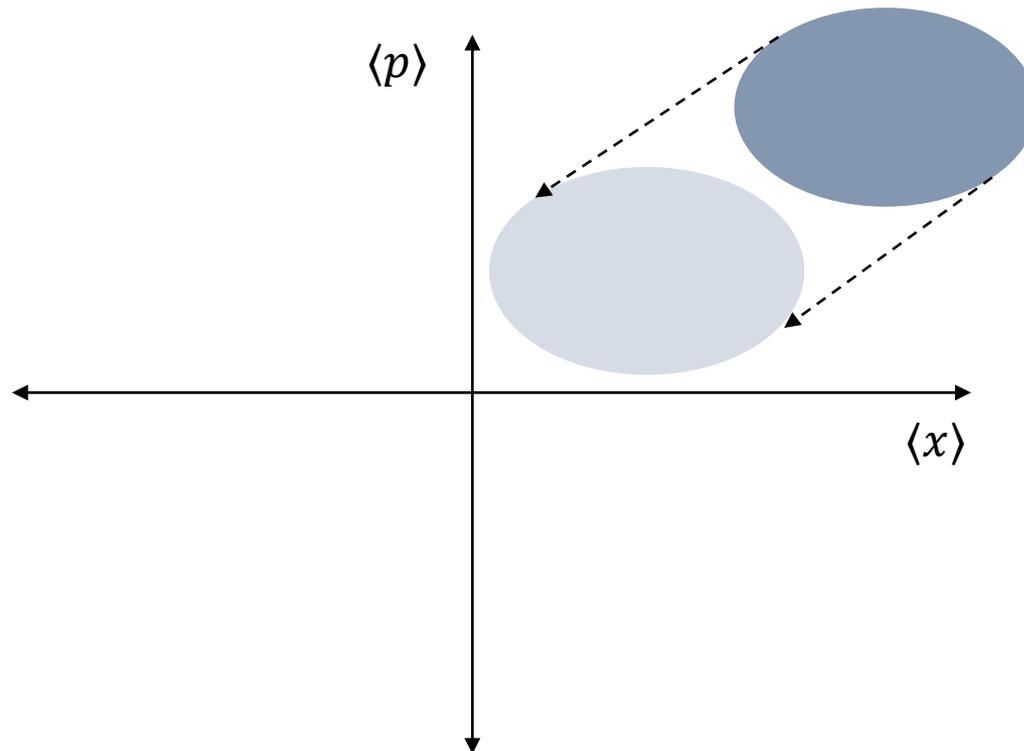
*Brandão *et al.*, **Resource Theory of Quantum States Out of Thermal Equilibrium.**

PRL 111, 250404.

Laws of Gaussian thermodynamics

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1. Contraction of phase-space displacement: Signal loss



Laws of Gaussian thermodynamics

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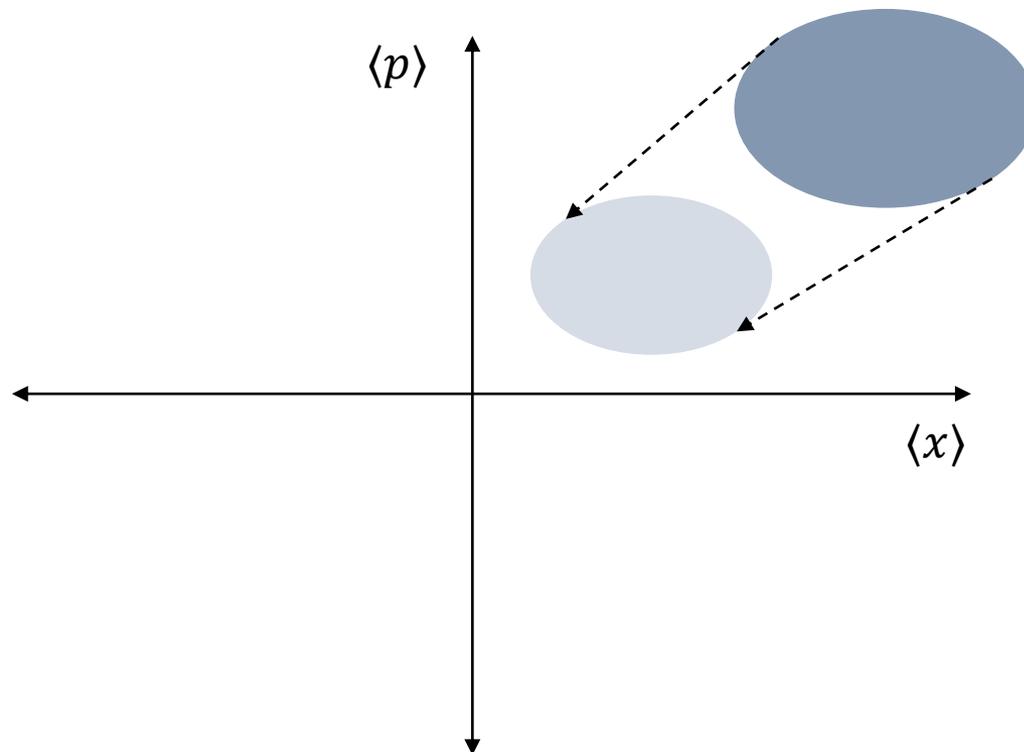


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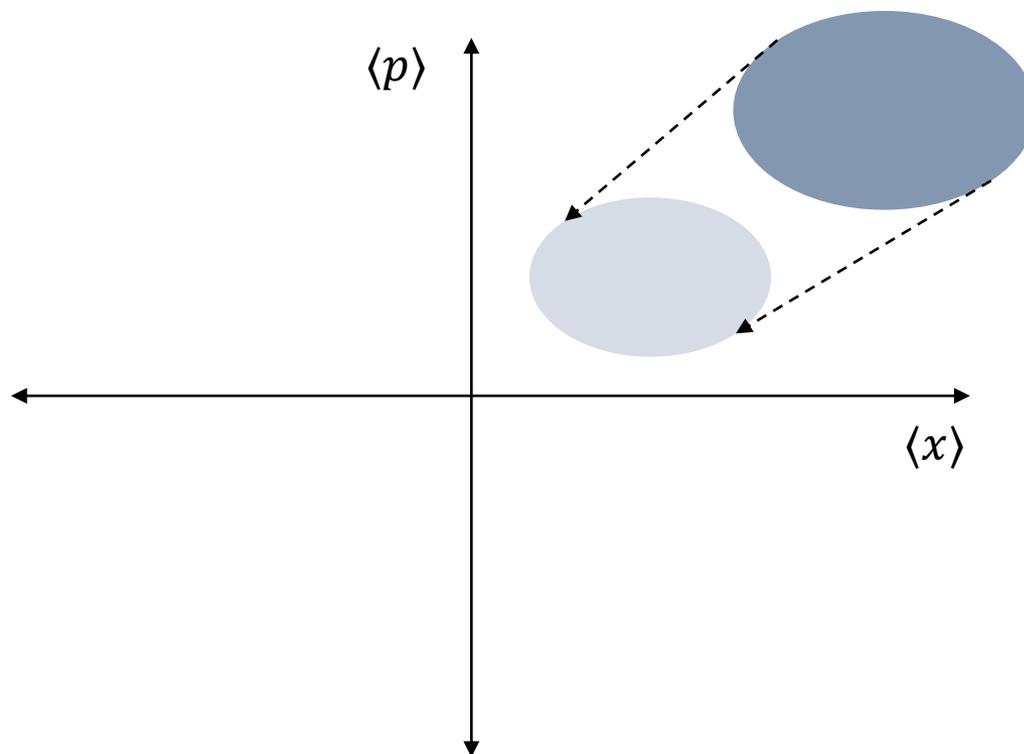
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4. More to come...

- Laws of thermodynamics on higher-order quadrature moments
- Quantifying the work cost of squeezing, displacement, etc.
- Gaussian thermal engines
- Beyond Gaussian: more general energy-conserving interactions
- Unified understanding of CV resources

ধন্যবাদ

Presentation based on

- [1] Benjamin Yadin, Felix C. Binder, Jayne Thompson, VN, Mile Gu, and M. S. Kim. **Operational Resource Theory of Continuous-Variable Nonclassicality**. Phys. Rev. X 8, 041038 (2018)
- [2] VN, Felix C. Binder, Jayne Thompson, Benjamin Yadin, Syed M. Assad, and Mile Gu. In preparation.

(Independent related work: Kwon et al., arXiv:1804.09355)