Mixed state quantum computers as Open Quantum Systems

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Delocalized Information

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Information requires a physical representation

Delocalized Information

- Information requires a physical representation
- Quantum information can lie delocalized across several physical systems

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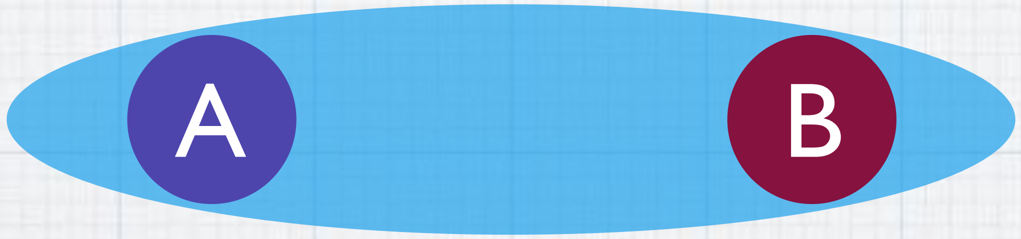

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$\rho_A = \rho_B = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Not just entanglement

$$\rho = \frac{1}{4}\big(|+\rangle\langle+| \otimes |0\rangle\langle 0| + |-\rangle\langle-| \otimes |1\rangle\langle 1|$$

$$+|0\rangle\langle 0| \otimes |+\rangle\langle+| + |1\rangle\langle 1| \otimes |-\rangle\langle-|\big)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$

Two quantum bits correlated in ways that two classical bits cannot be

Quantum computation

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Quantum computers can solve certain classes of computational problems exponentially faster than any known classical algorithm

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What are the quantum resources that enable this exponential speedup in quantum computers?

Proven results

For any quantum algorithm operating on pure states, we prove that the presence of multi-partite entanglement, with a number of parties that increases unboundedly with input size, is necessary if the quantum algorithm is to offer an exponential speed-up over classical computation...

Our results do not apply to quantum algorithms operating on mixed states in general and we discuss the suggestion that an exponential computational speed-up might be possible with mixed states in the total absence of entanglement.

Jozsa and Linden, Proc. Roy. Soc. 459, 2011 (2003)

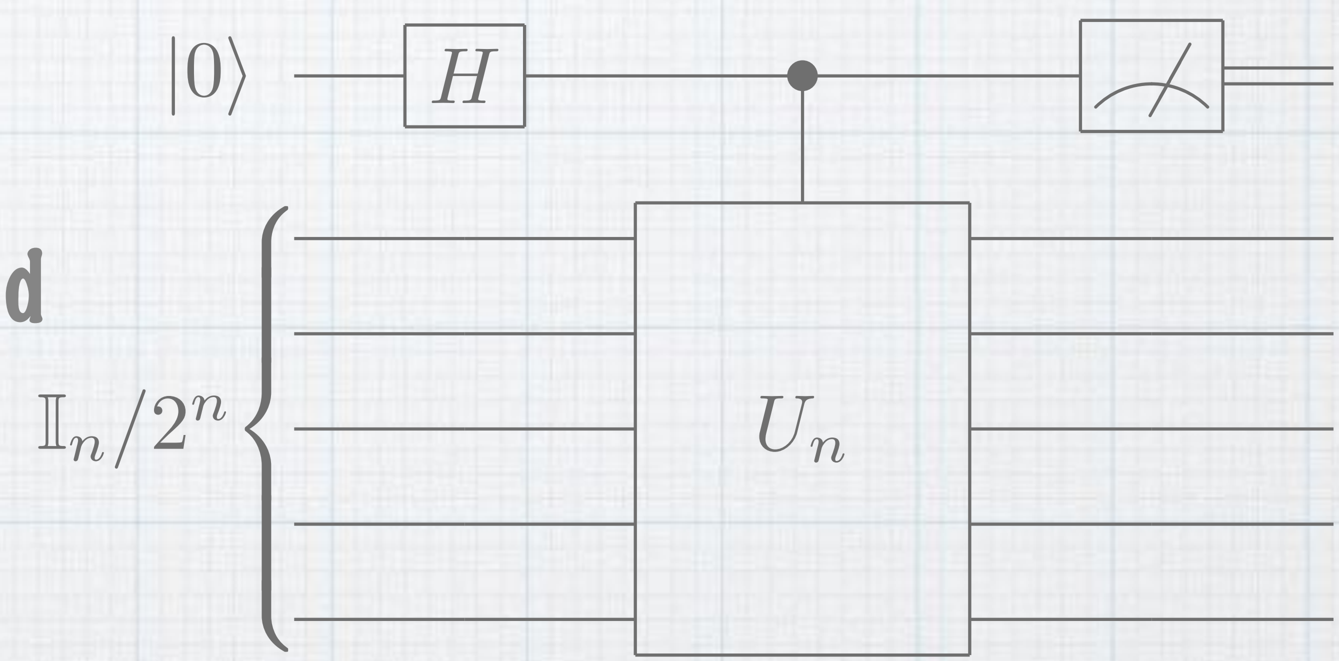
Mixed state quantum computing

* The role of entanglement (if any) in mixed state quantum computation is not known
* Even quantifying entanglement in mixed states is hard.
* Is there a more generic resource one can identify as the reason for exponential speedup in mixed state quantum computing?

The DQC1 Model

The DQC 1 model of quantum information processing consists of a single pure qubit and a collection of qubits in the completely mixed state:

$$\rho_{n+1} = \frac{1}{2^n}\begin{pmatrix} \mathbb{I}_n & U_n \\ U_n^\dagger & \mathbb{I}_n \end{pmatrix}$$



If the top qubit is measured

$$\langle \sigma_x \rangle = \mathrm{Re}\left[\mathrm{Tr} U_n\right]/2^n$$

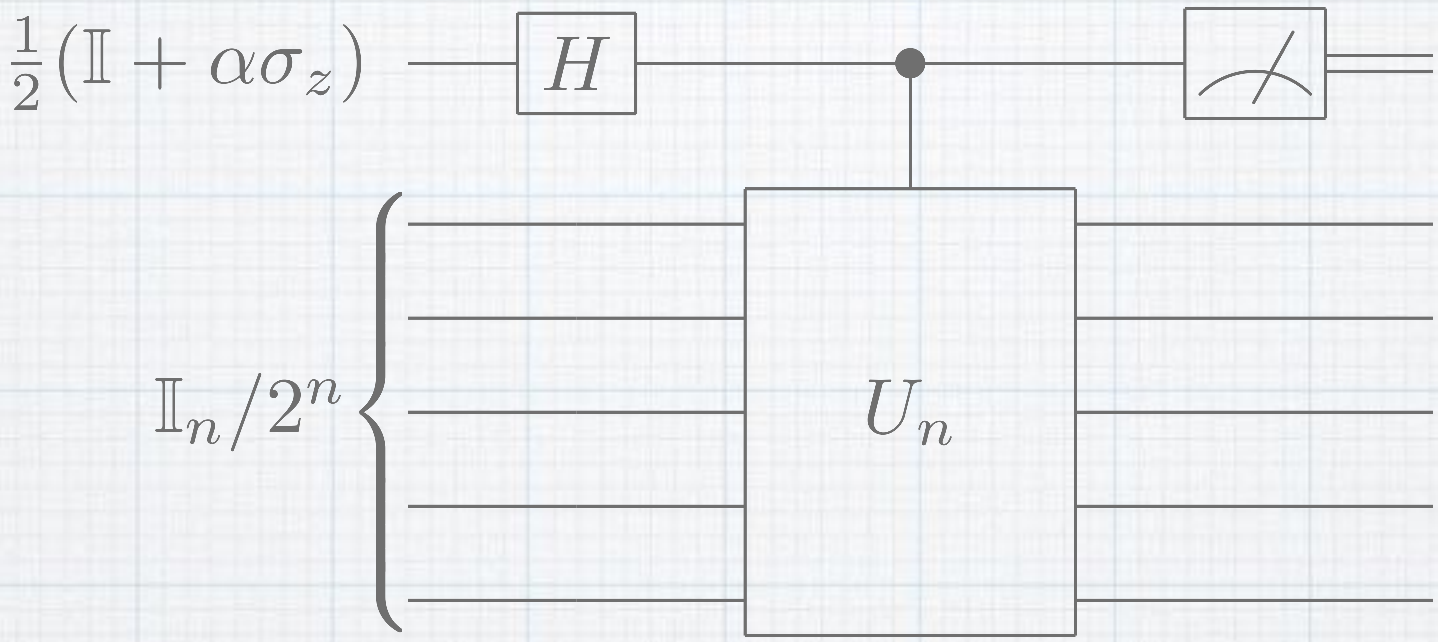
$$\langle \sigma_y \rangle = \mathrm{Im}\left[\mathrm{Tr} U_n\right]/2^n$$

The circuit can evaluate the normalized trace of the unitary efficiently provided the circuit can be implemented

No known efficient classical algorithm.

Knill and Laflamme, PRL, 81, 5672 (1998)

Only a tiny bit of purity



$$\rho_{n+1} = \frac{1}{2^{n+1}} \begin{pmatrix} \mathbb{I}_n & \alpha U_n \\ \alpha U_n^\dagger & \mathbb{I}_n \end{pmatrix}$$

The computational overhead grows as $1/\alpha^2$.

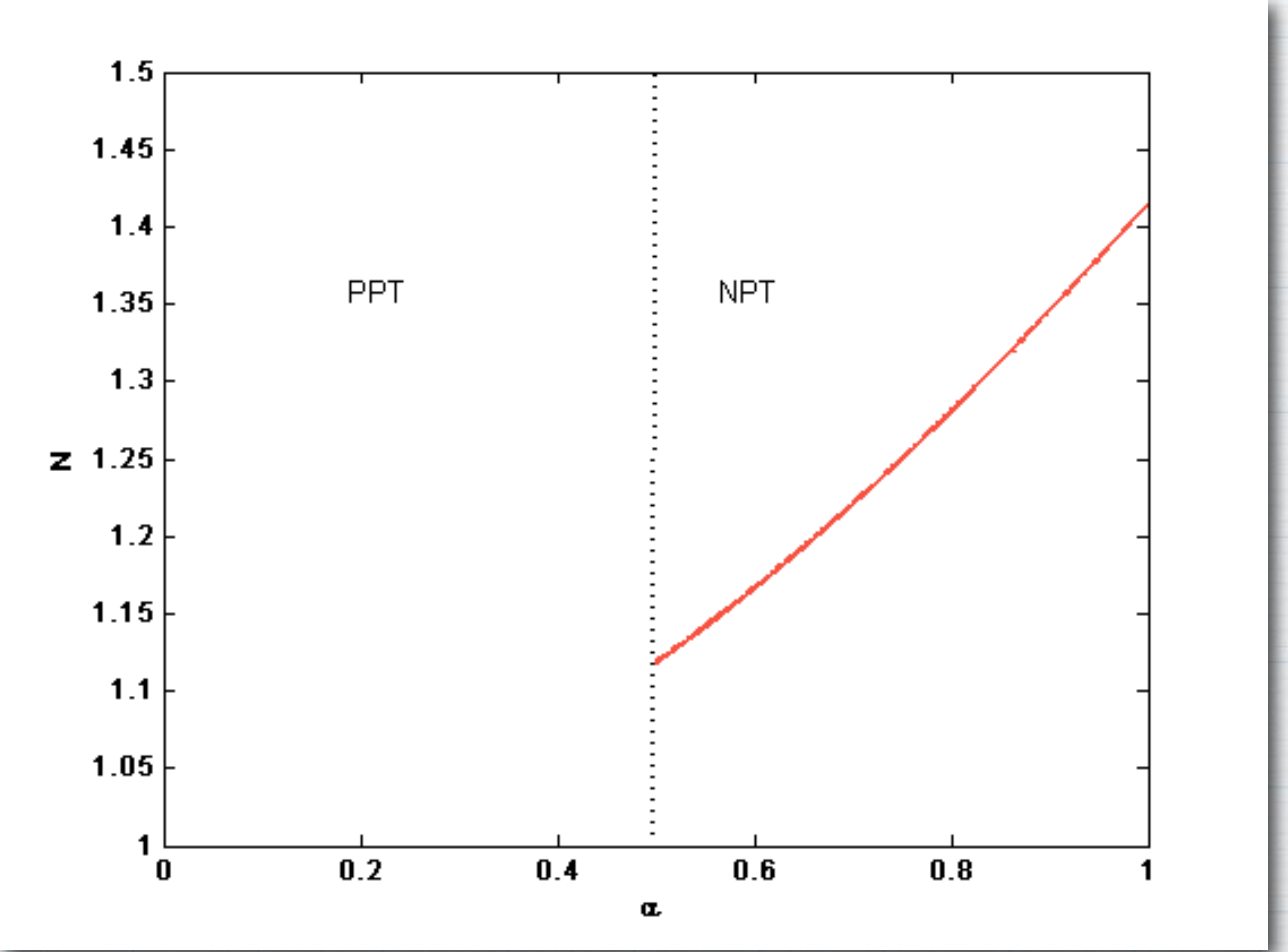
* The top qubit always remains separable from the bottom ones
* If the top qubit is traced out the remaining state is fully mixed

Bipartite Entanglement

* Entanglement between the pure qubit and the rest is zero
* For other bipartite splits, entanglement as quantified by the Peres negativity can be computed and bounds placed on it
* The negativity bound saturates to a small constant even though it could potentially grow as 2^n for n qubits.
* Asymptotically, the negativity is a vanishing fraction of the maximum possible negativity
* Multipartite entanglement may be present but no computable measure of such entanglement exists for the case of DQC1.

A. Datta, S. Flammia, C. M. Caves, PRA, 72, 042316, (2005)

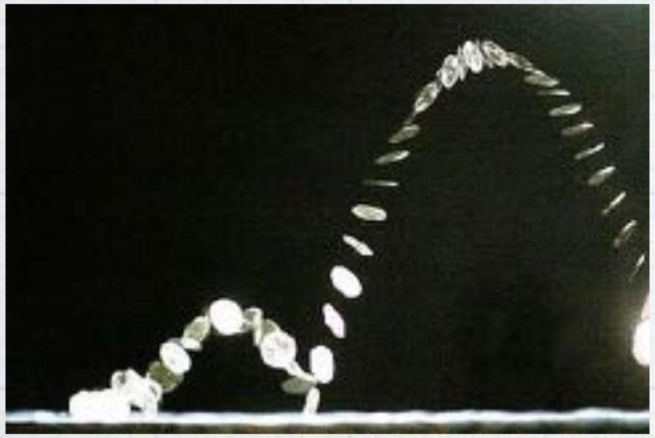
The negativity as a function of α



No detectable entanglement for α less than 1/2

A. Datta, S. Flammia, C. M. Caves, PRA, 72, 042316, (2005)

Correlations



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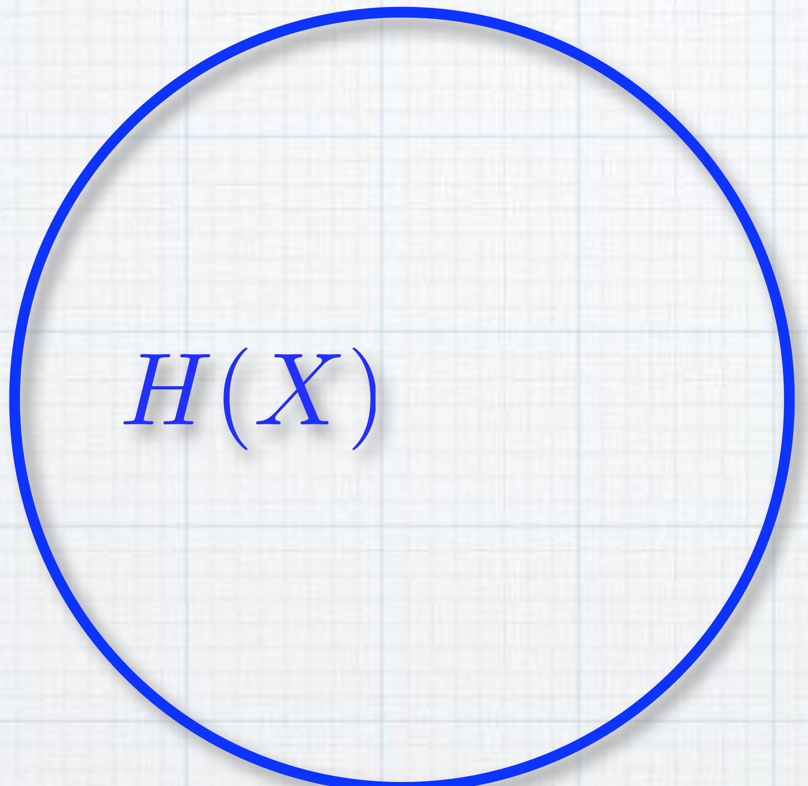

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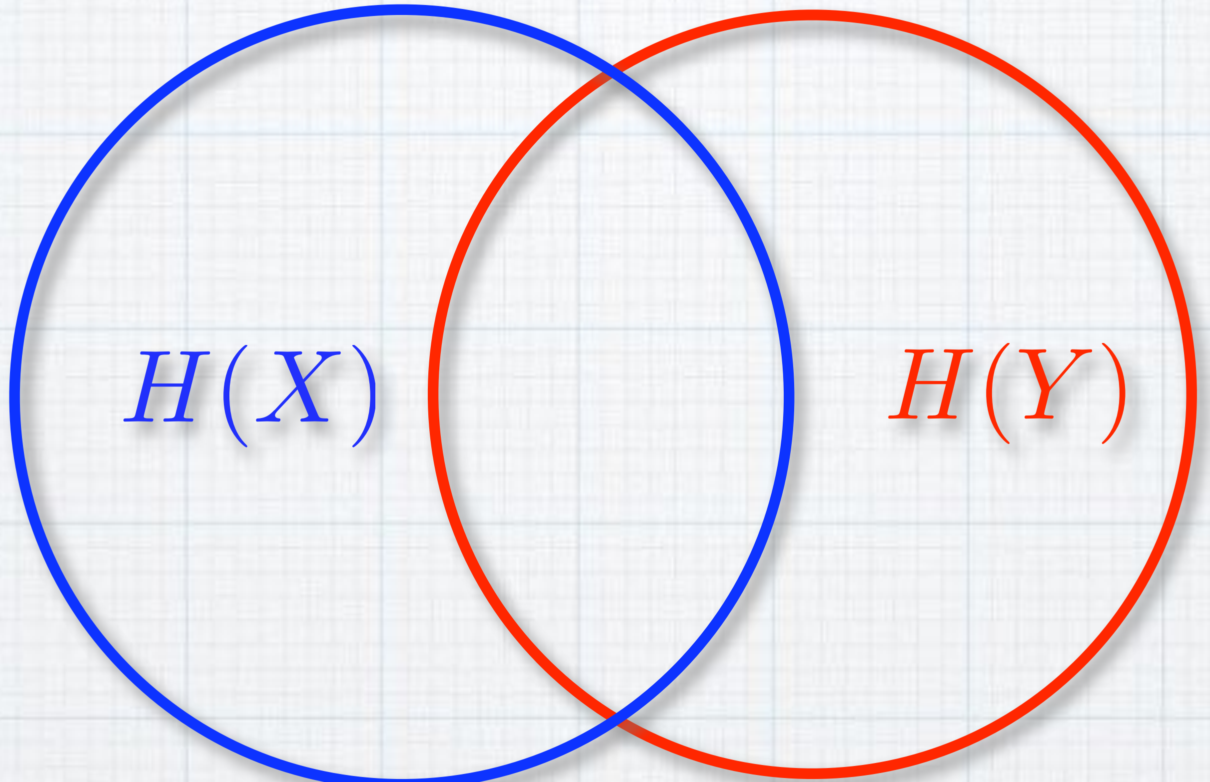
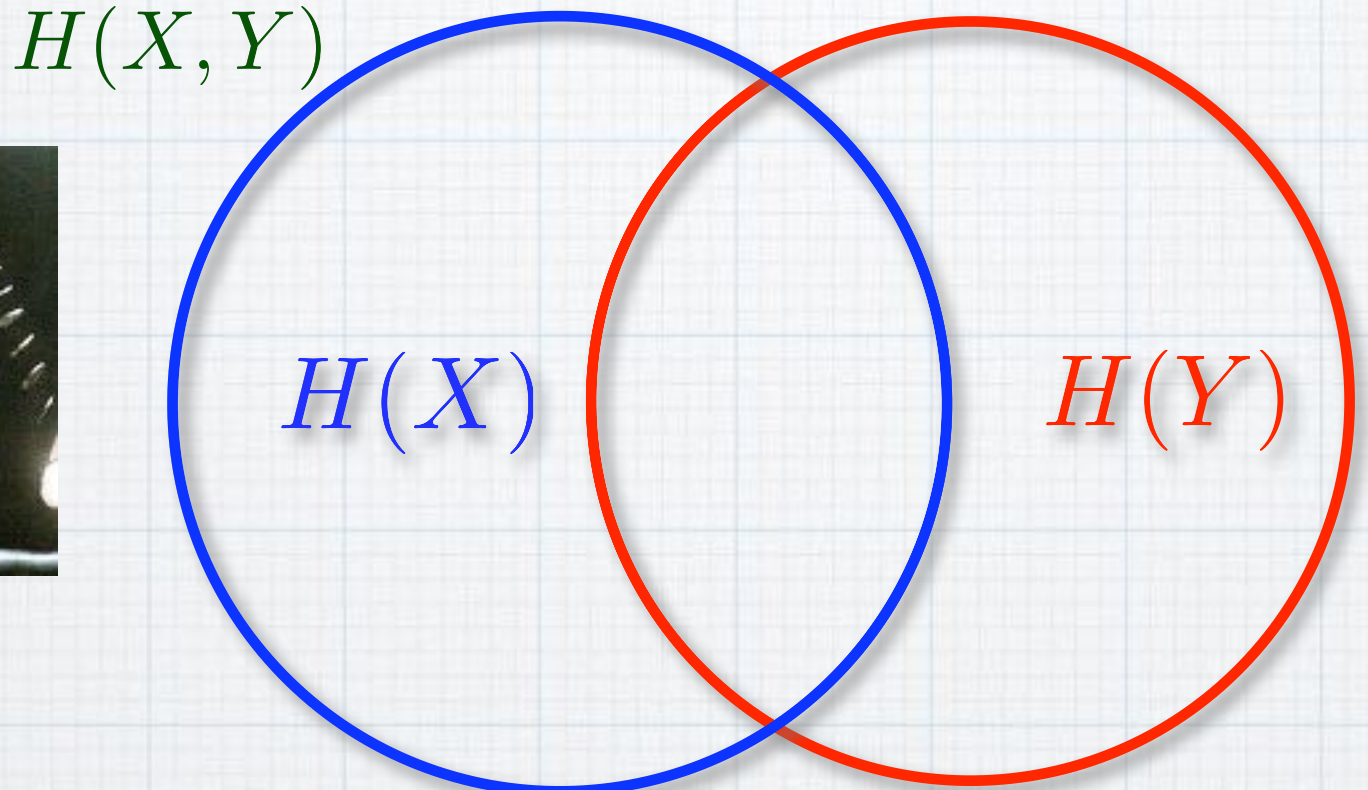


$$H(X) = -\sum_x p(x) \log p(x)$$

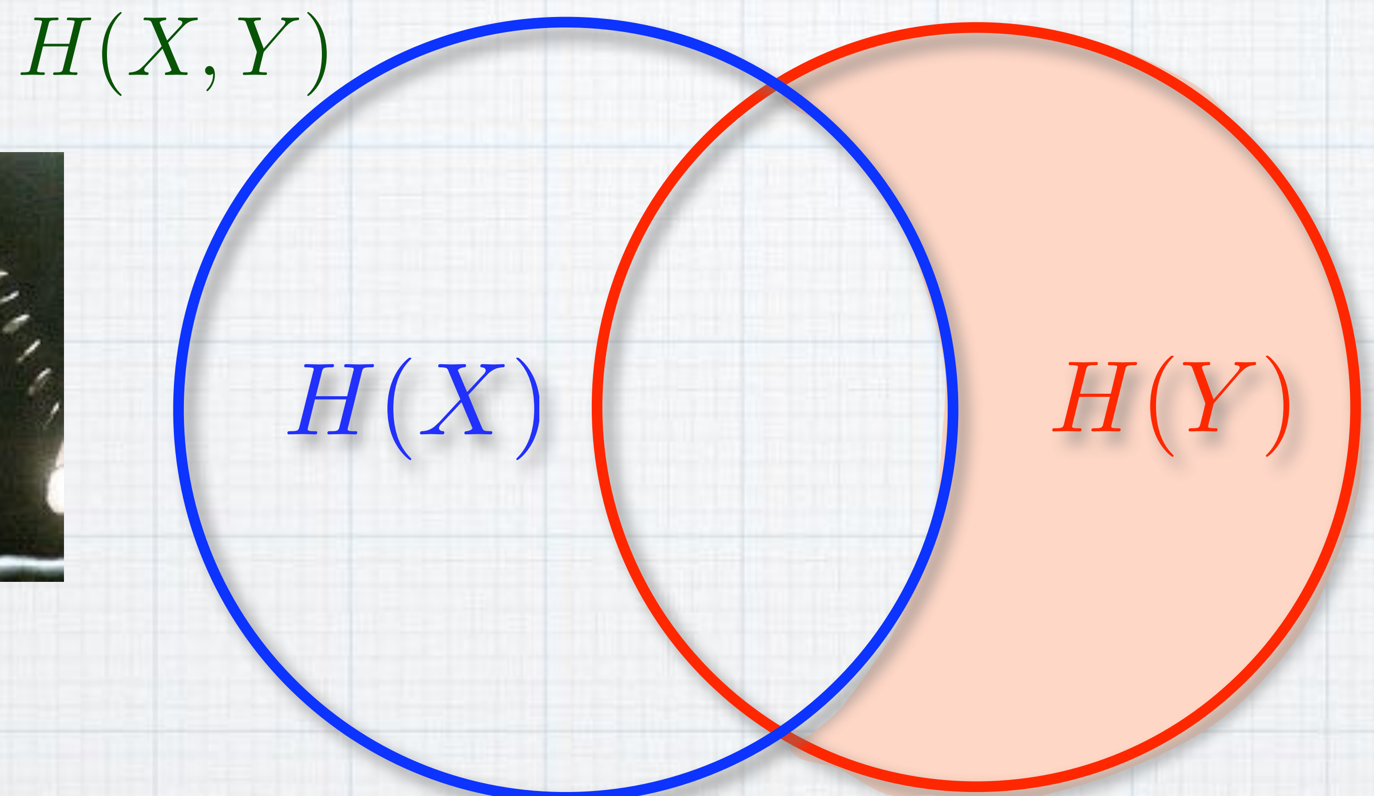
Correlations



$$H(X) = -\sum_x p(x) \log p(x)$$

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

Correlations



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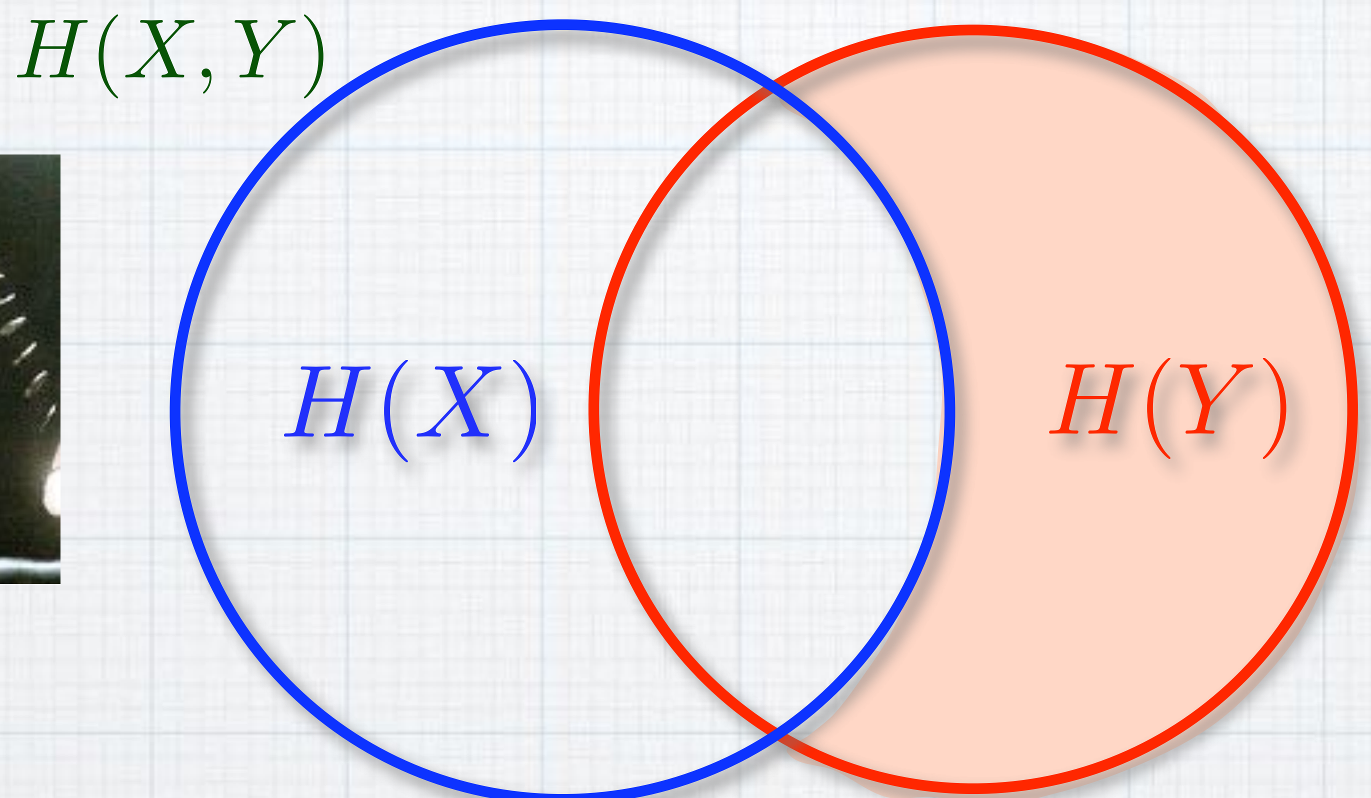

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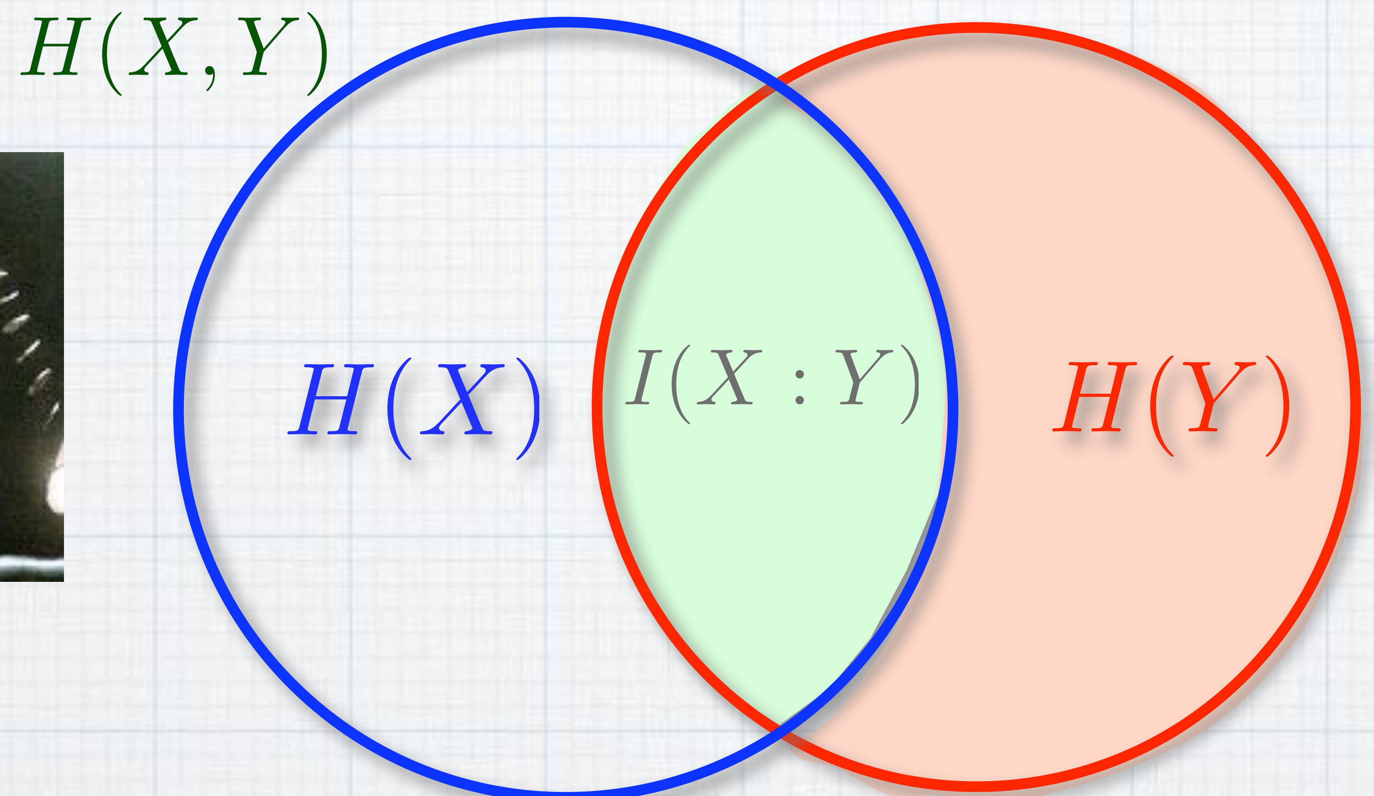
Correlations



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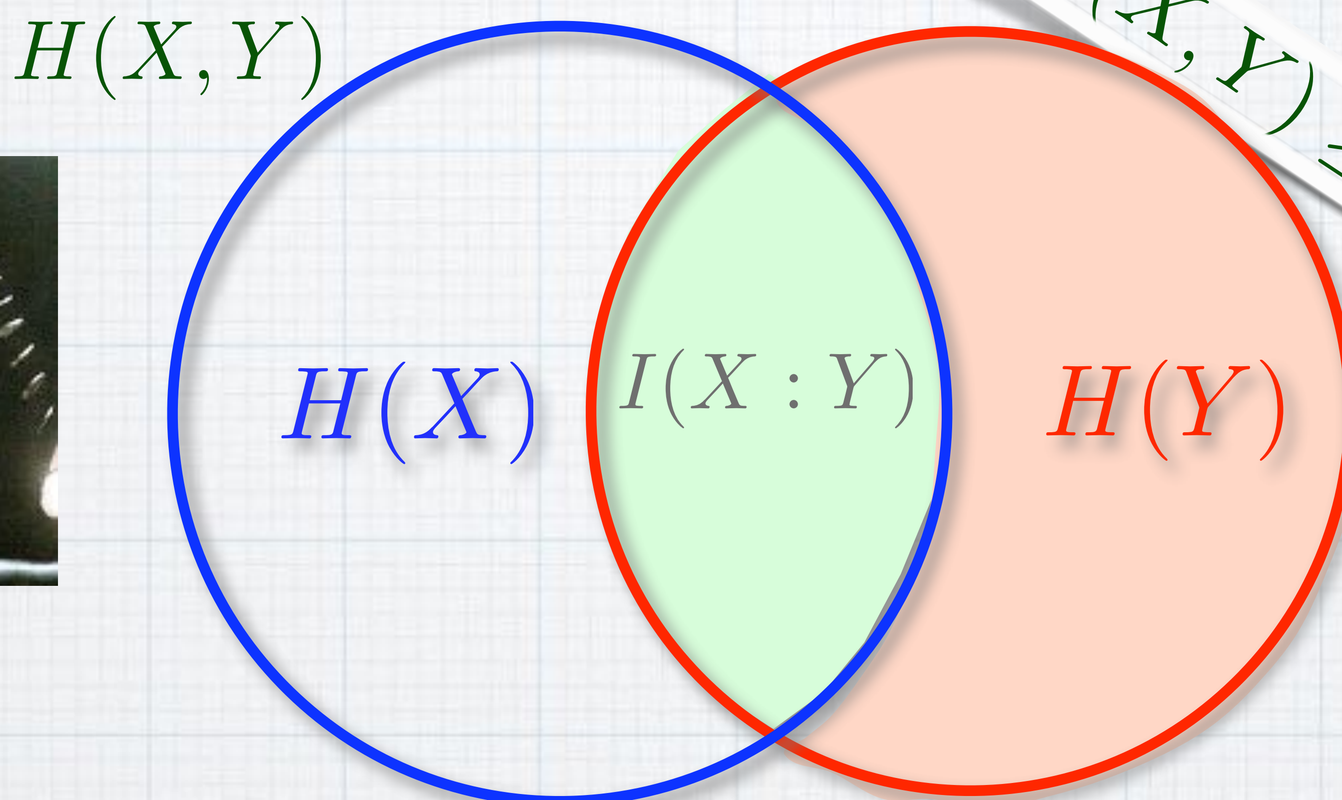
Correlations

$$H(X,Y) \geq \max\{H(X), H(Y)\}$$



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All correlations between quantum systems

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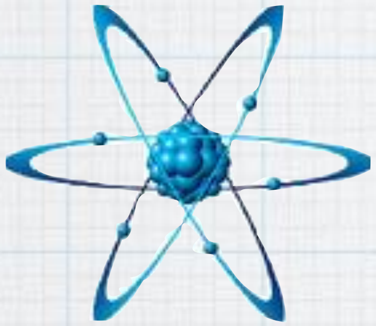

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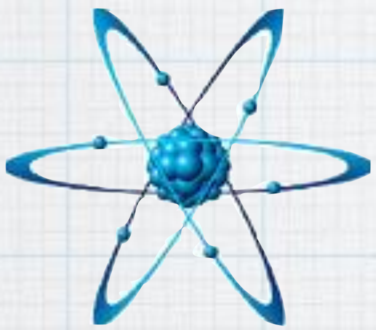

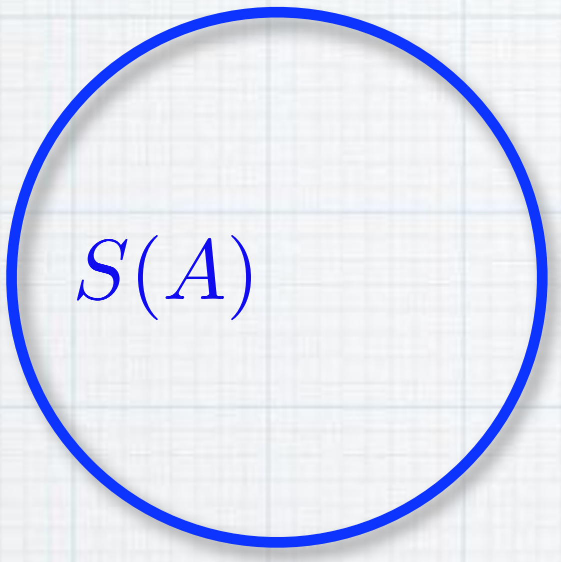

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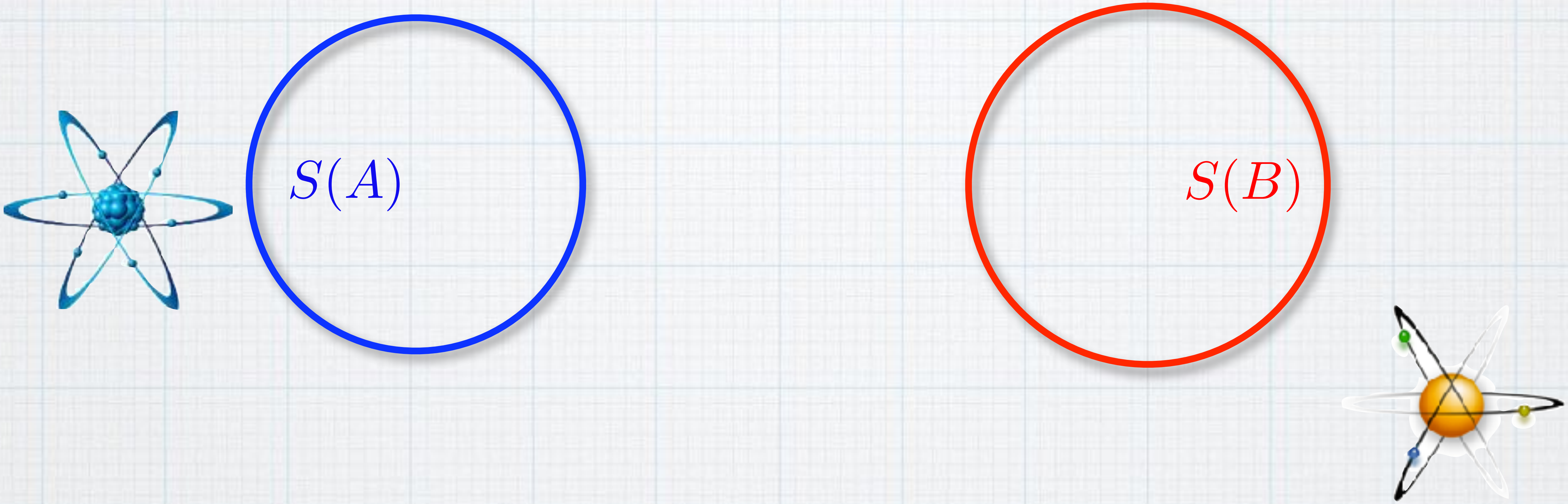

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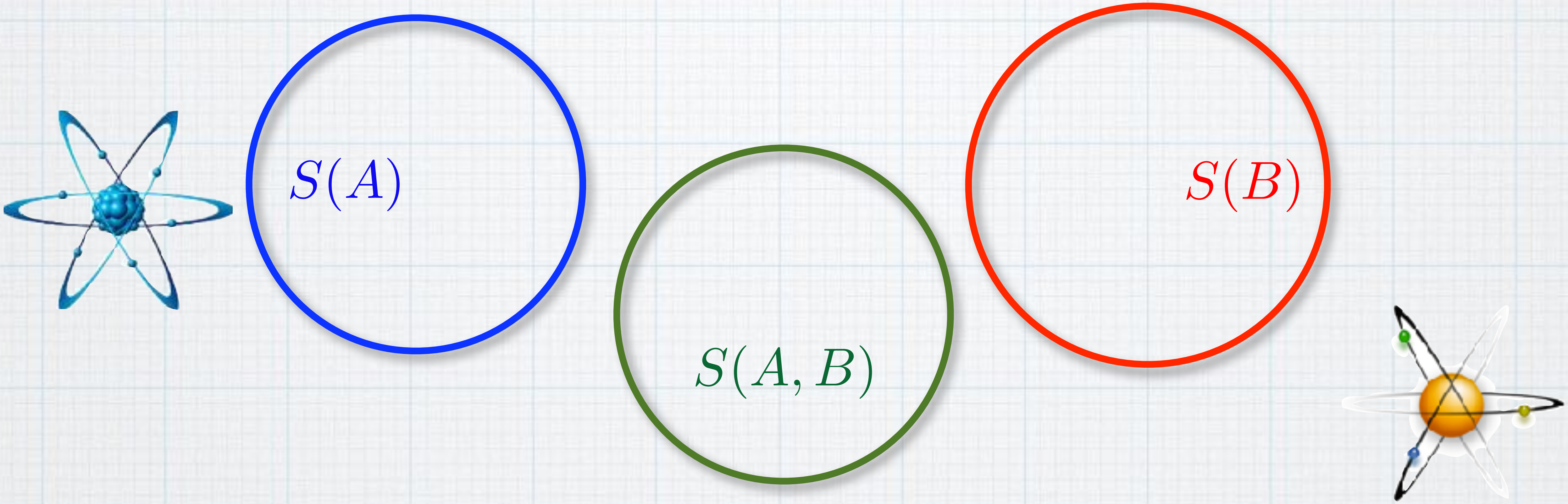

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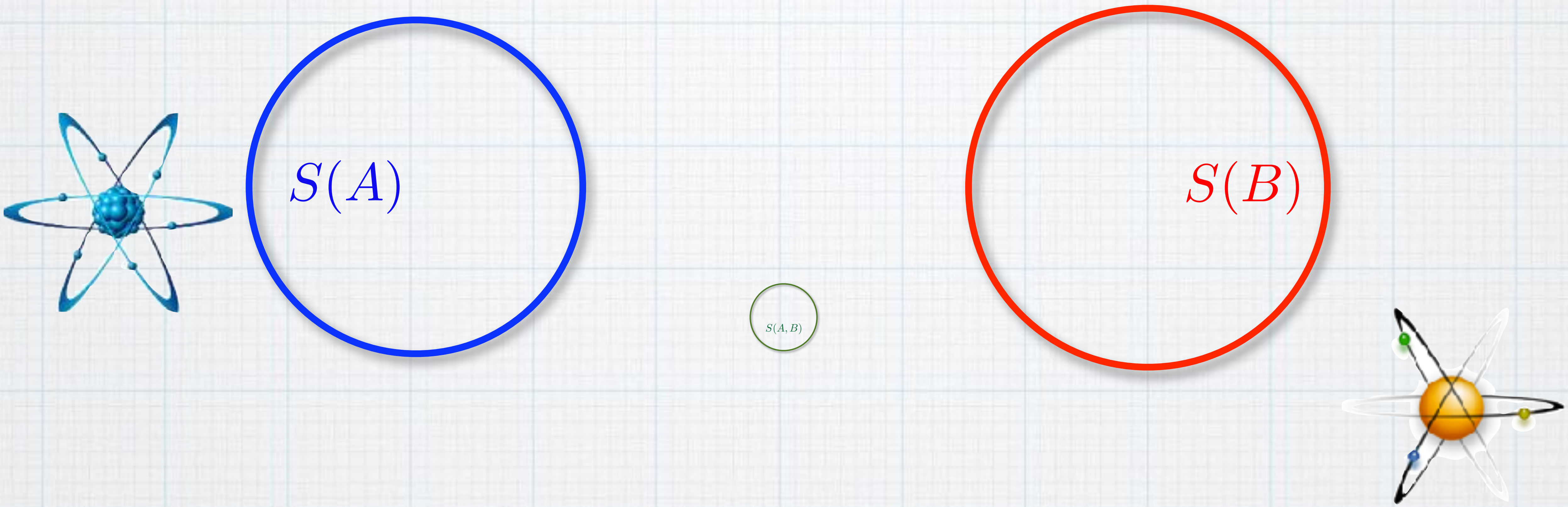

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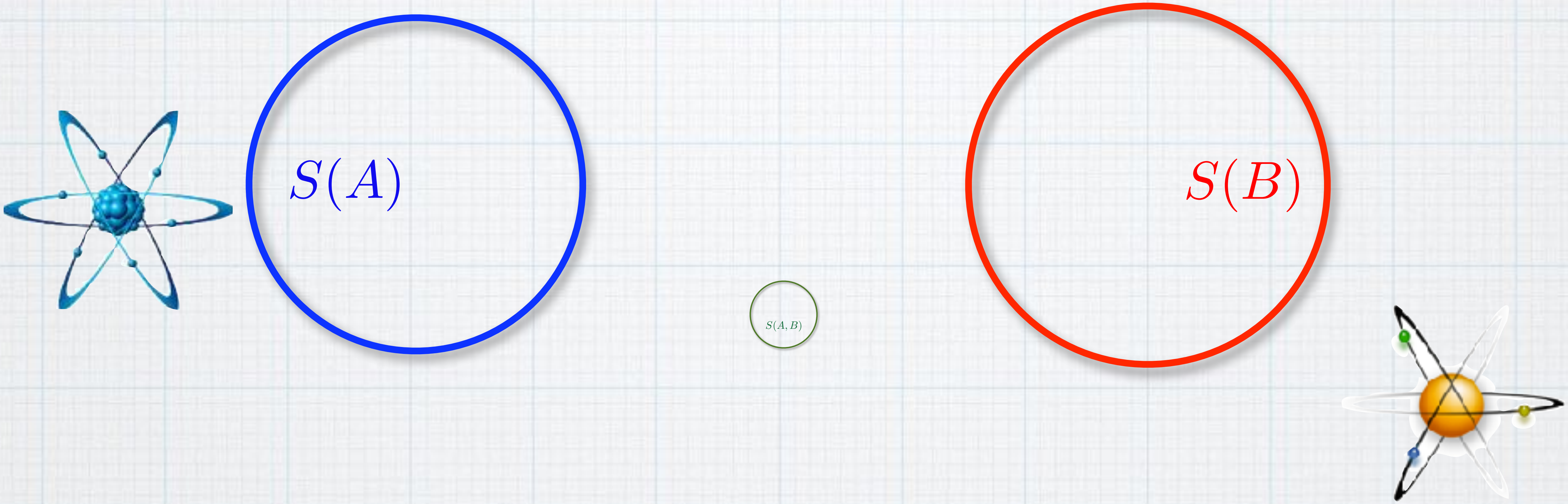

For maximally entangled pure states ignorance about the subsystems may be maximal while the global state is perfectly known

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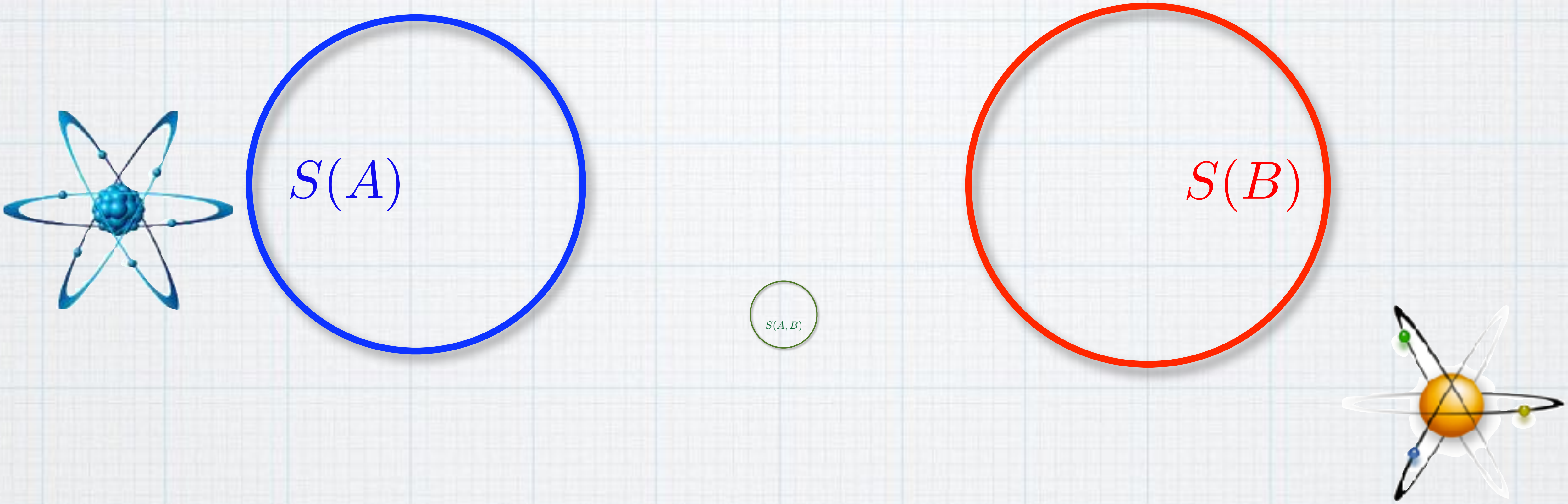


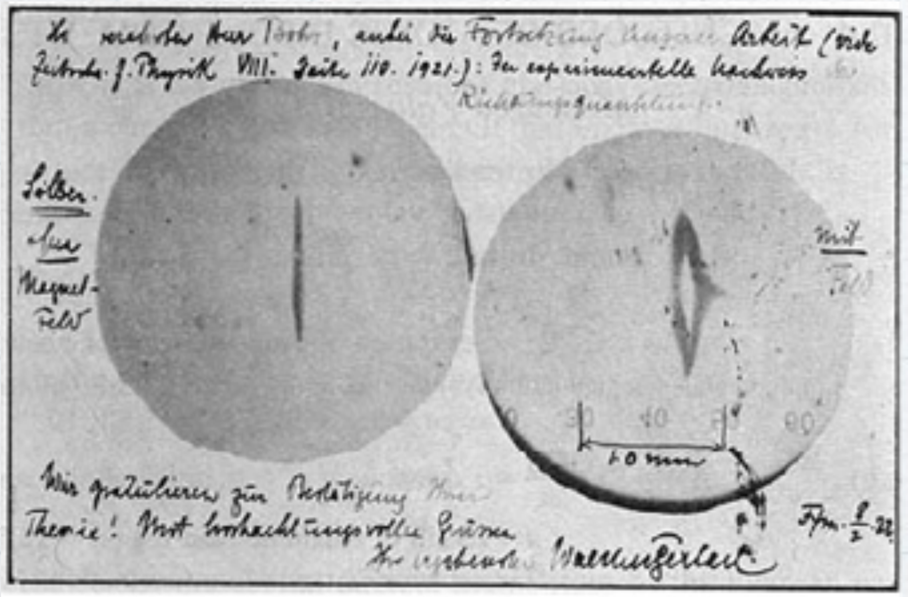
For maximally entangled pure states ignorance about the subsystems may be maximal while the global state is perfectly known

$$S(B:A) = S(\rho_B) + S(\rho_A) - S(\rho_{AB}), \quad S(\rho) = -\mathrm{Tr}\left[\rho \log \rho\right]$$

$$S(B|A) = S(\rho_{AB}) - S(\rho_A)$$

Measurements and Discord

To "know" a quantum system one has to do measurements and we start by thinking of projective measurements.



Gerlach's postcard, dated 8 February 1922, to Niels Bohr. It shows a photograph of the beam splitting, with the message, in translation: "Attached [is] the experimental proof of directional quantization. We congratulate [you] on the confirmation of your theory." (Physics Today December 2003)

$$\rho_{B|\Pi_k^A} = \frac{\Pi_k^A \rho_{AB} \Pi_k^A}{p_k}, \quad p_k = \mathrm{Tr}[\Pi_k^A \rho_{AB}]$$

$$\widetilde{J}(B:A) = S(\rho_B) - \sum_k p_k S(\rho_B|\Pi_k^A)$$

$$\mathcal{I}(B:A) \neq \widetilde{J}(B:A) \text{ in general}$$

$$\begin{aligned} \mathcal{D} &\equiv \mathcal{I}(B:A) - \mathcal{J}(B:A) \\ &= S(\rho_A) - S(\rho_{AB}) + \min_{\{\Pi_k^A\}} \sum_k p_k S(\rho_B|\Pi_k^A) \end{aligned}$$

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001)

Separability versus Discord

The notion of nonClassical correlation is fundamentally different from Werner's notion of separability

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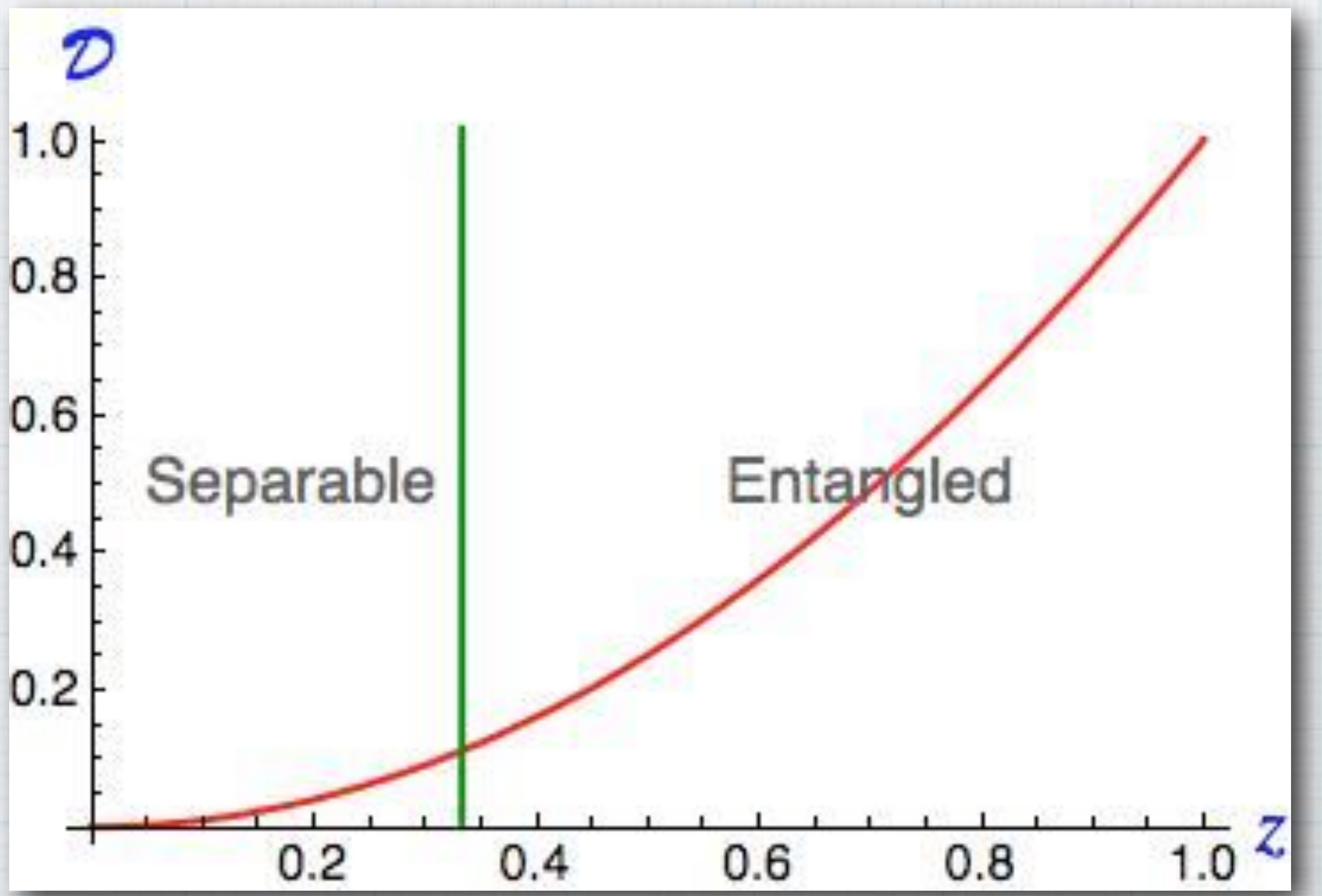
The notion of nonClassical correlation is fundamentally different from Werner's notion of separability

Werner state (separable for z < 1/3):

$$\rho_W = z|\Phi\rangle\langle\Phi| + \frac{1-z}{4}\mathbb{I}_4; \quad |\Phi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



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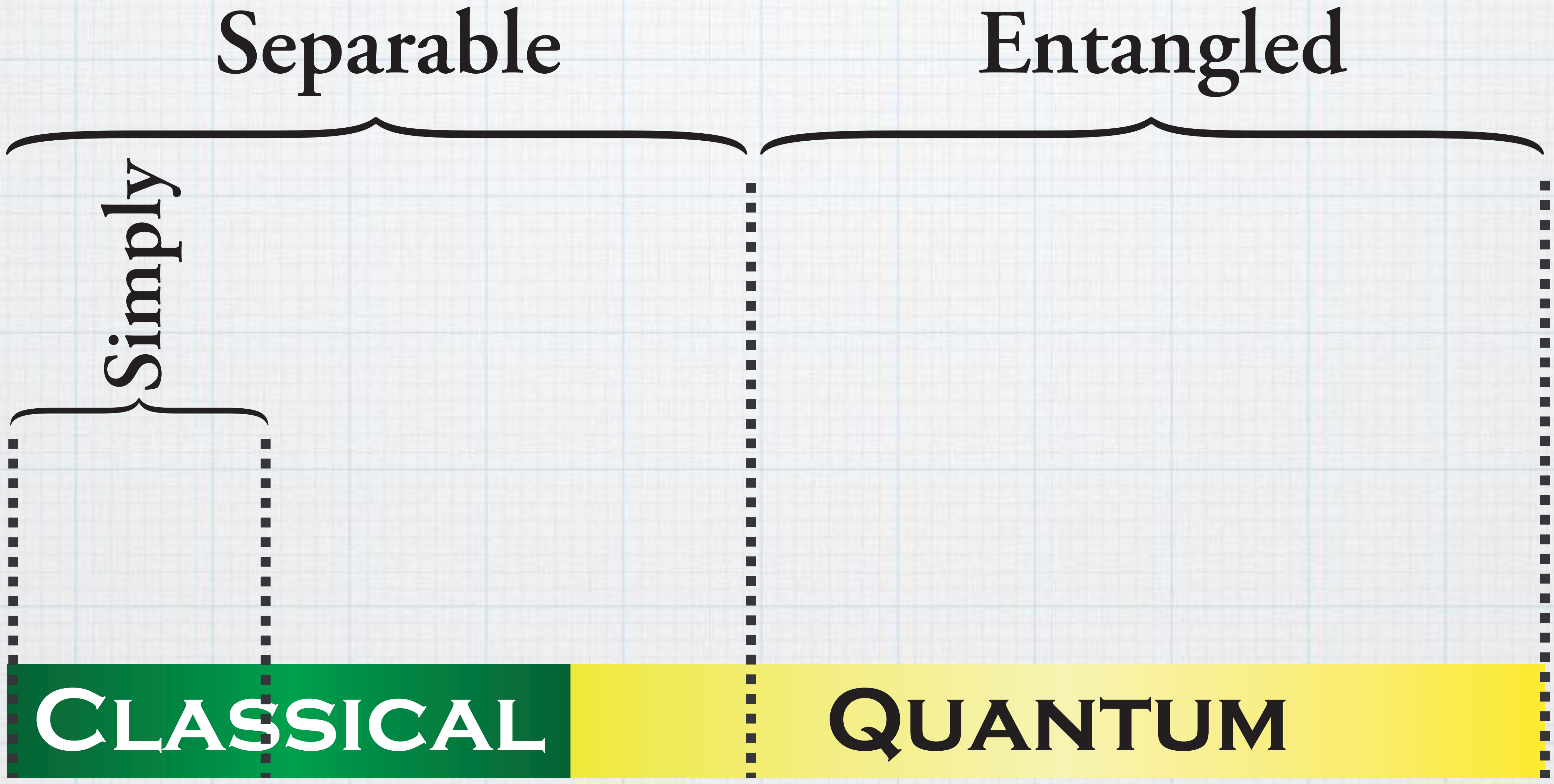


An example

A separable state with nonzero discord:

$$\rho = \frac{1}{4}\Big(|+\rangle\langle+| \otimes |0\rangle\langle 0| + |-\rangle\langle-| \otimes |1\rangle\langle 1|$$

$$+|0\rangle\langle 0| \otimes |+\rangle\langle+| + |1\rangle\langle 1| \otimes |-\rangle\langle-|\Big)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$

$$\mathcal{D}(\rho) = \frac{3}{4}\log_2\frac{4}{3} = 0.311$$

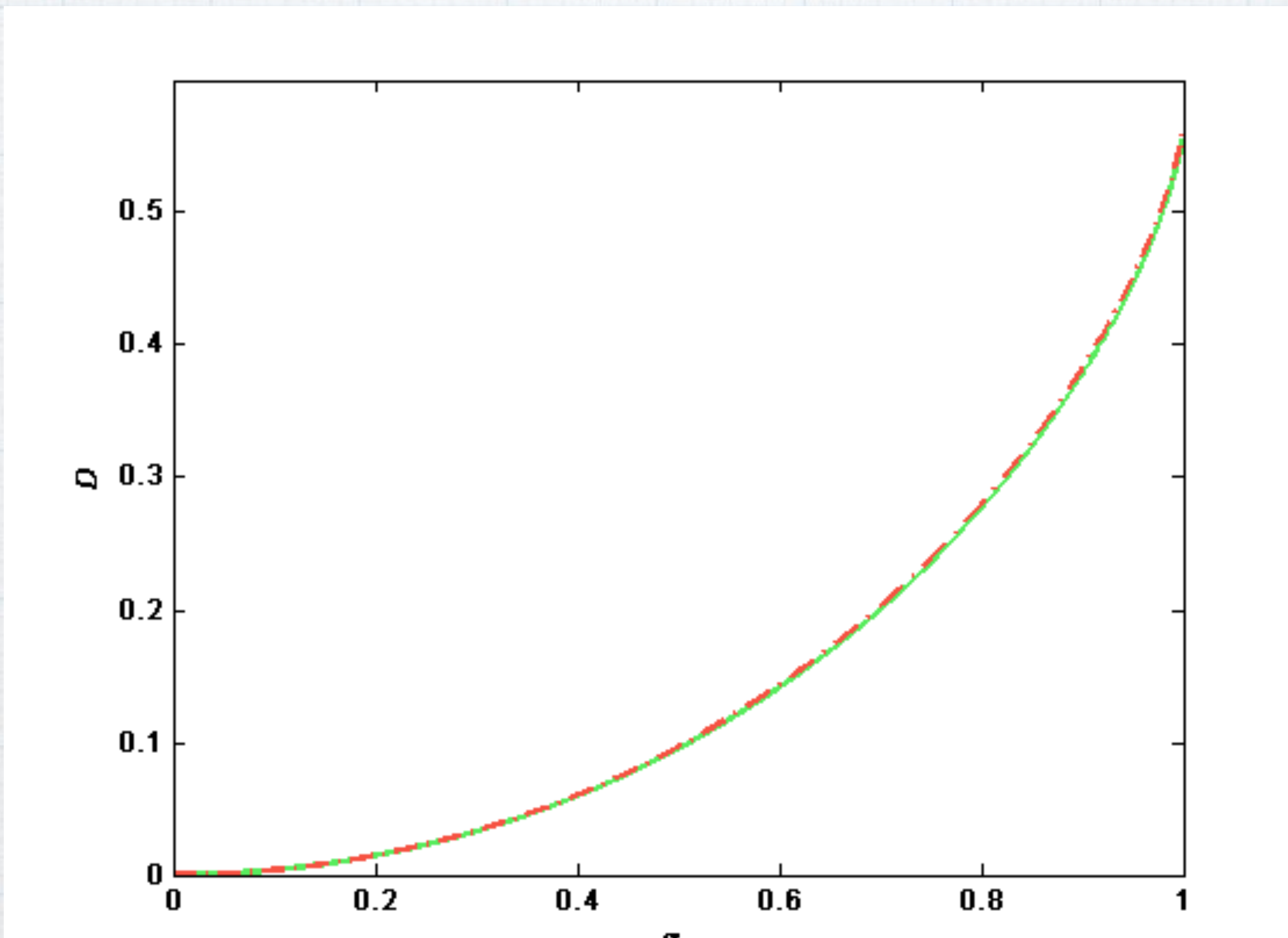
For pure states discord reduces to a measure of entanglement

$$\mathcal{D}(|\Psi\rangle\langle\Psi|) = S(\rho_X) - S(|\Psi\rangle\langle\Psi|) + \min_{\{\Pi_j^X\}} \sum_j p_j S(\rho_{Y|\Pi_j^X})$$

$$= S(\rho_X)$$

Discord in DQC1

$$\mathcal{D}_{\mathrm{DQC1}} = 1 + \frac{1}{2}\left[(1+\alpha)\log_2(1+\alpha) + (1-\alpha)\log(1-\alpha)\right]$$
$$- \log(1+\sqrt{1-2}) - (1-\sqrt{1-\alpha^2})\log e$$



Animesh Datta, Anil Shaji, Carlton M. Caves, PRL, **100**, 050502, (2008)

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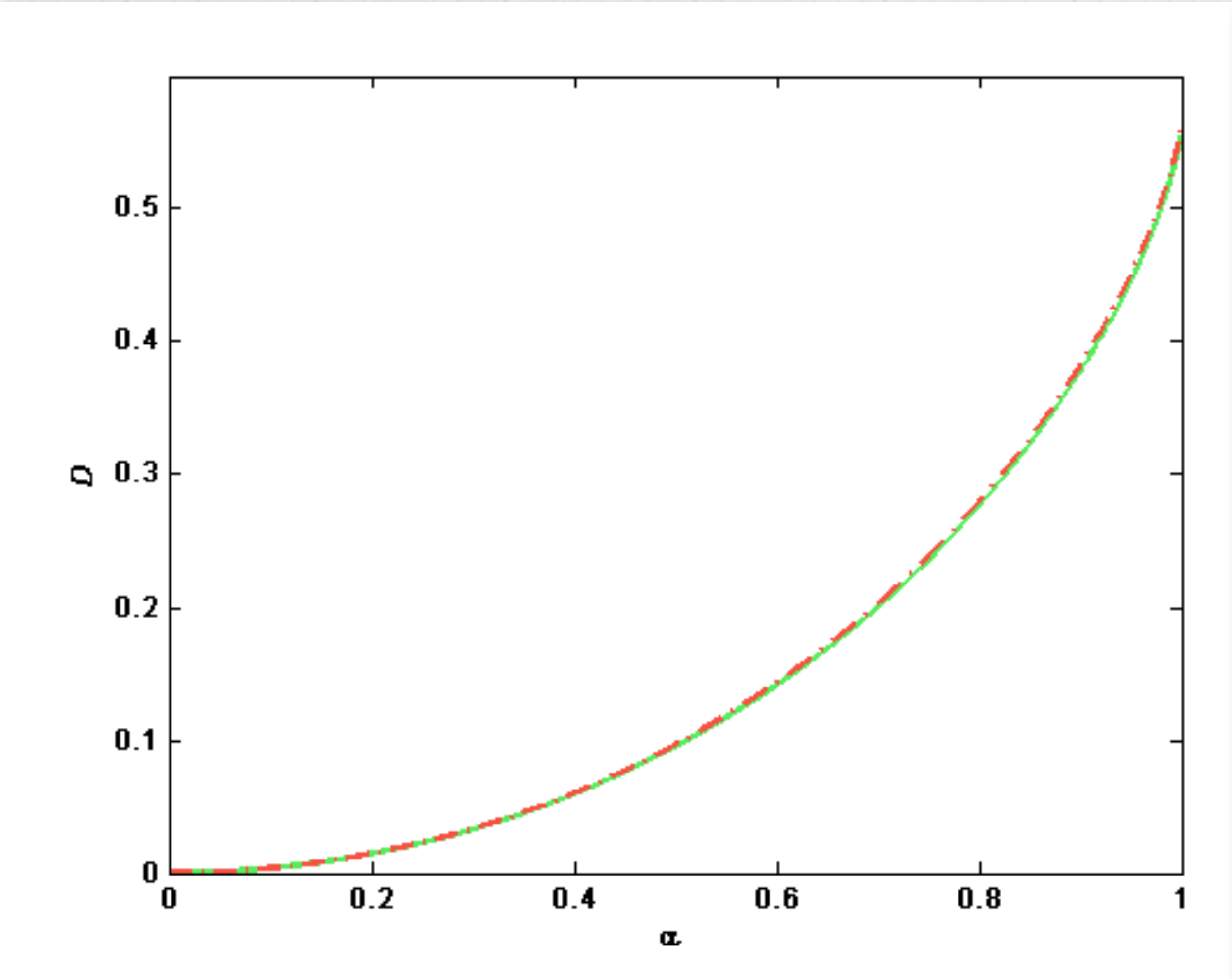

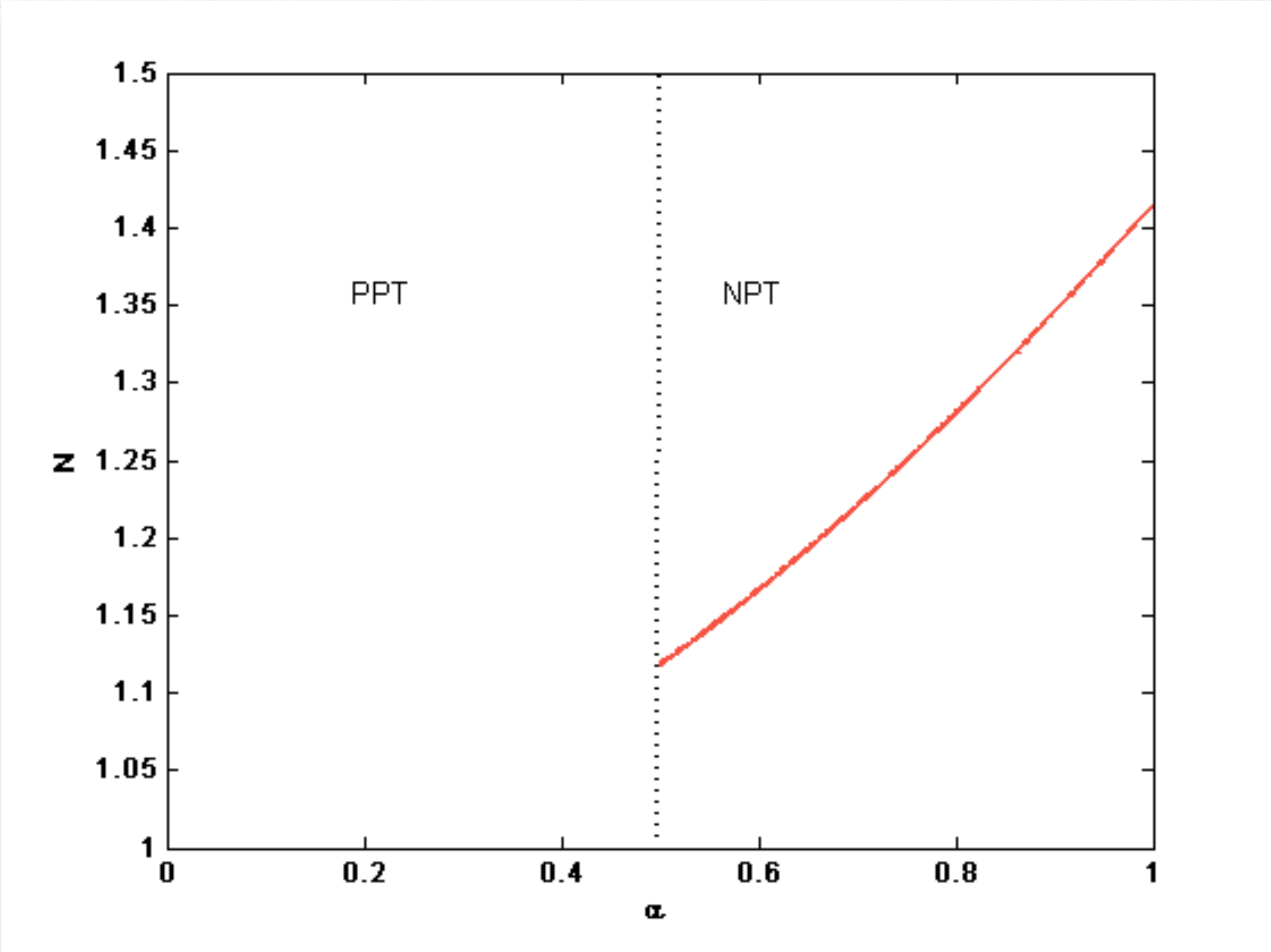

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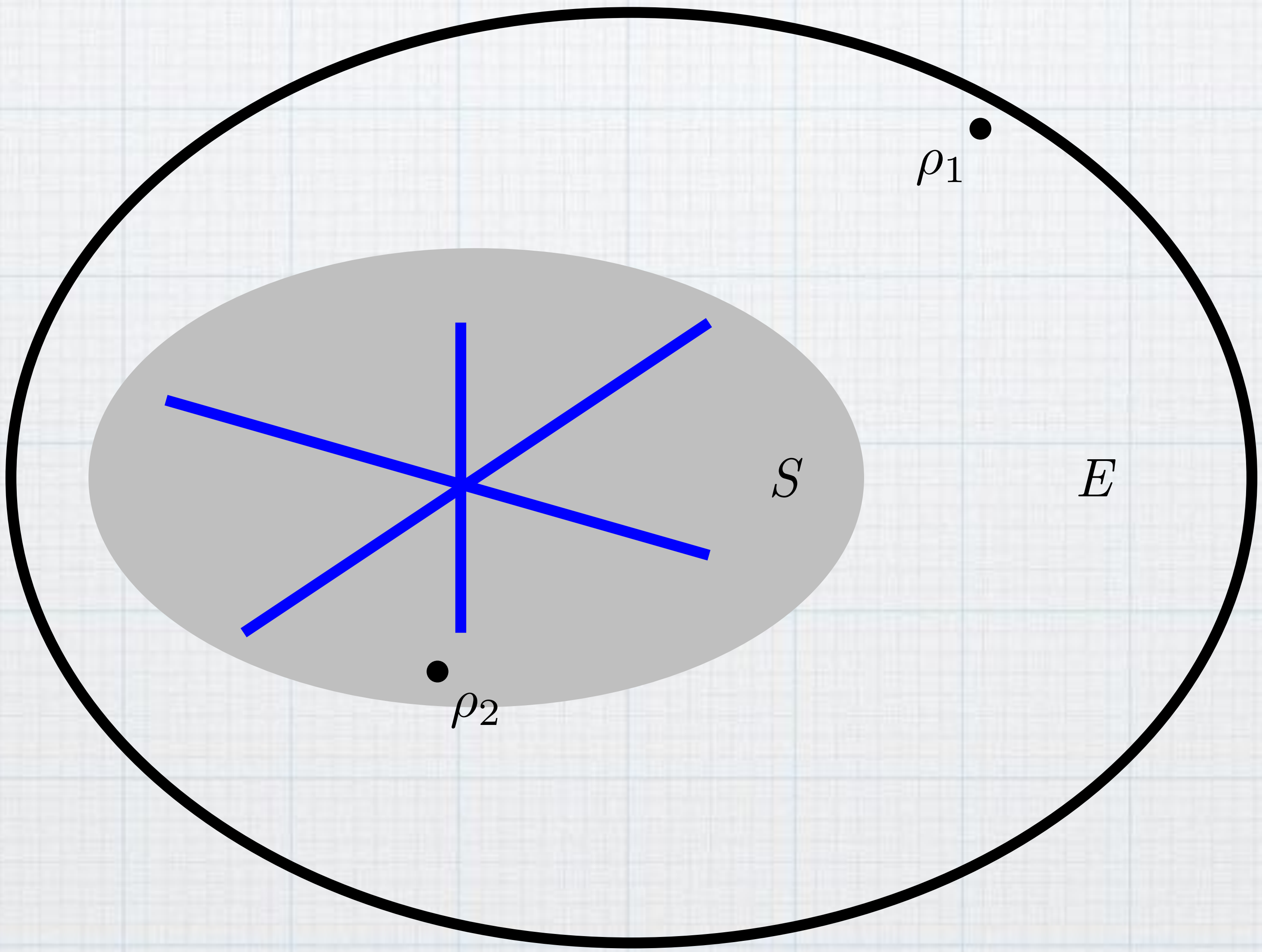
States with no discord

* Almost all states of a multipartite system - except for a set of measure zero - has non-zero discord.

* Quantum algorithms as shortcuts from input to output?



Partial results

* Concordant quantum computations can be simulated efficiently on a classical computer (Eastin, and Cable et al)
* DQC1 with zero discord producing unitary: Can it be simulated classically as well?
* Is discord in the QC a requirement of leveraging the entanglement that the mixed state has with the rest of the universe for the computational task?
* Is there a connection between global entanglement and discord of subsystems.

Global entanglement and discord

* Is discord in a multipartite mixed state a reflection of the the entanglement that the mixed subsystem has with the rest of the universe (purification)?
* We consider several multi-qubit systems in which global entanglement is present and compute the average subsystem discord in them

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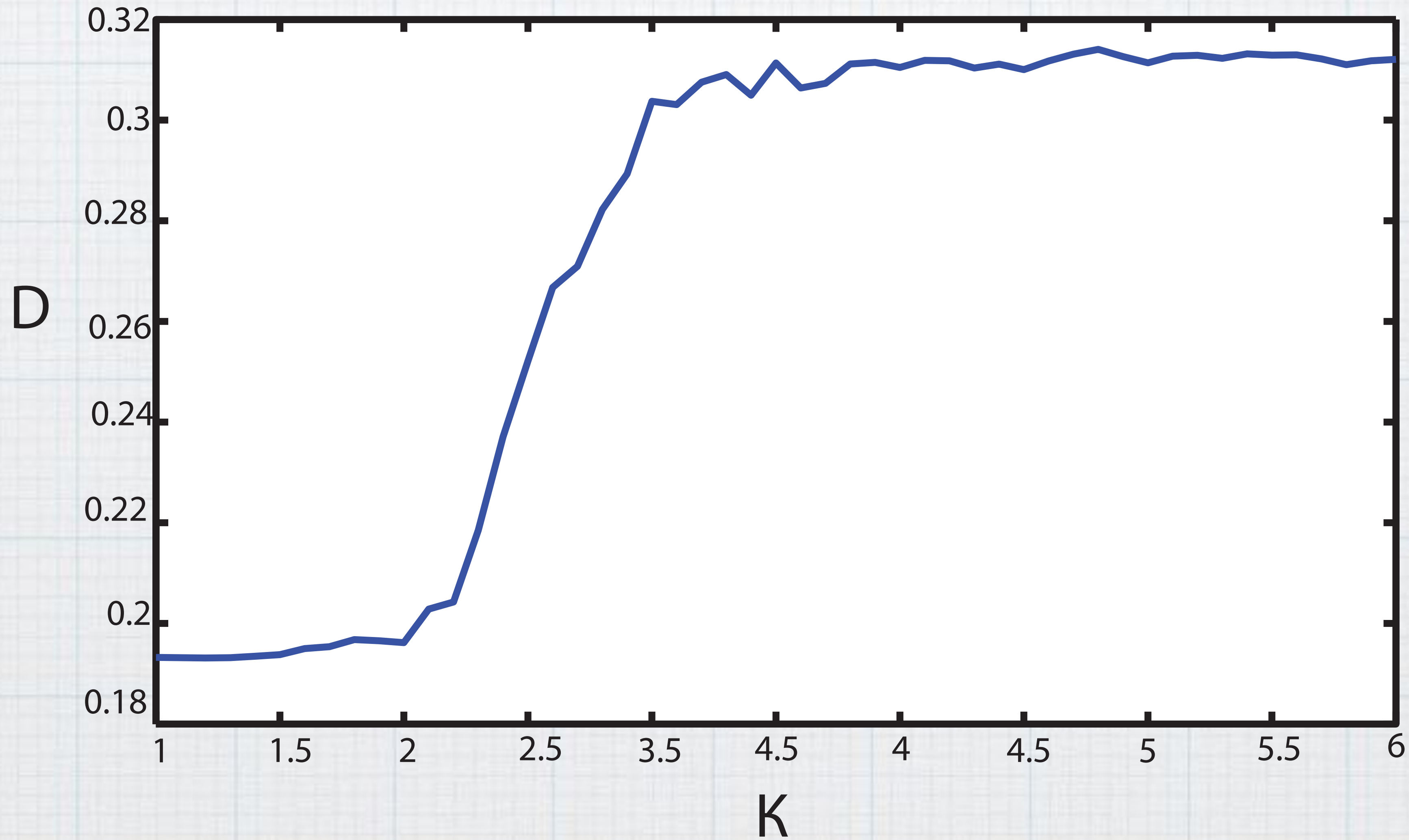
Quantum kicked top

$$H = \frac{\kappa}{2j\tau} J_z^2 + pJ_y \sum_{n=-\infty}^{\infty} \delta(t - n\tau)$$

Consider it as a system of N=2j qubits initially in a spin coherent state



V. Madhok et. al., "Signatures of chaos in the dynamics of quantum discord," Phys. Rev. E 91, 032906 (2015)

Global entanglement

We use the Generalized Geometric Measure of true multiparty entanglement to quantify the entanglement in the N qubit state

$$\varepsilon(|\psi\rangle) = 1 - \Lambda^2_{\max}(|\psi\rangle)$$

$$\Lambda_{\max} = \max|\langle\phi|\psi\rangle|$$

$|\psi\rangle$: An N-party pure state

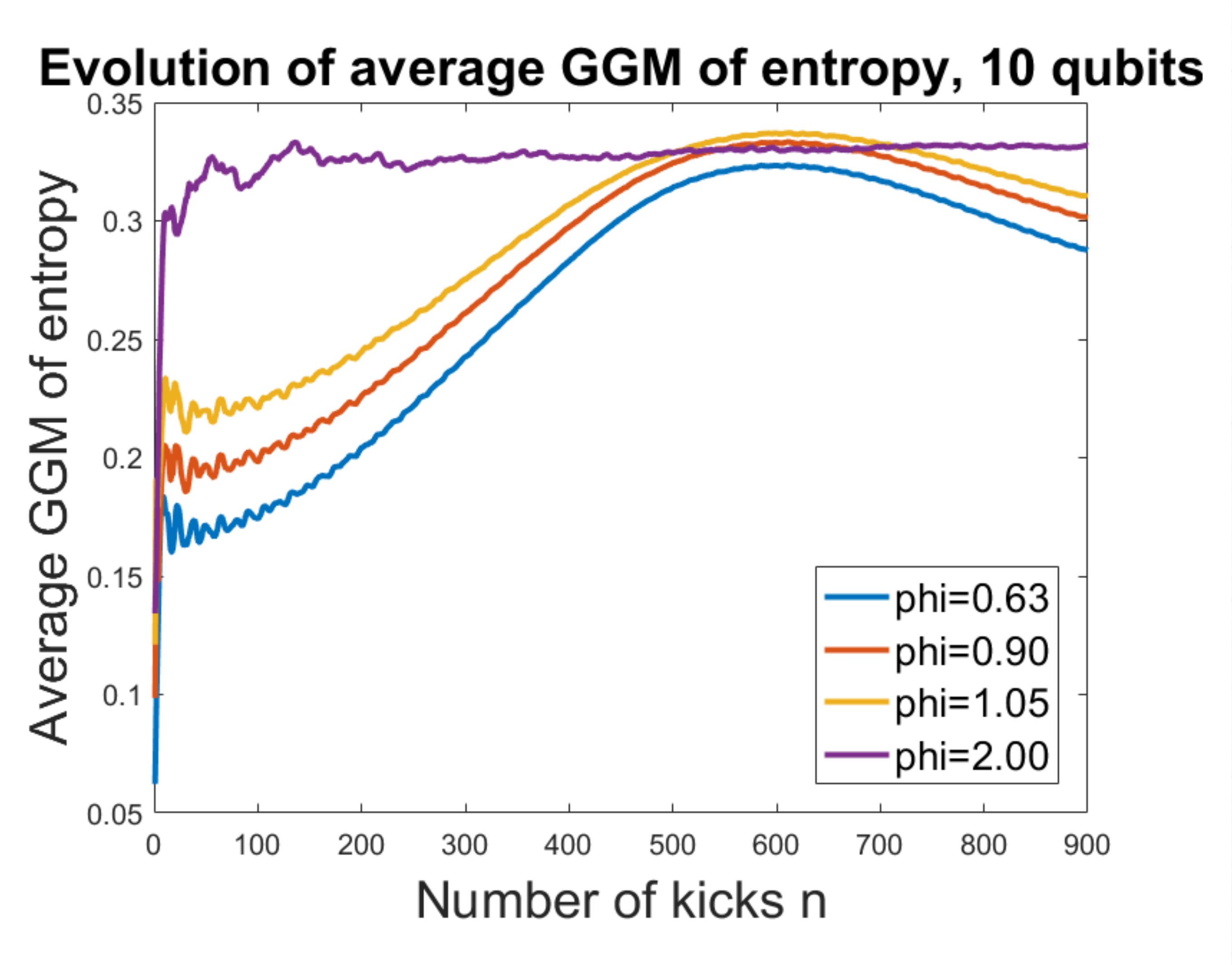
$|\phi\rangle$: N-party pure states that are not N-party entangled

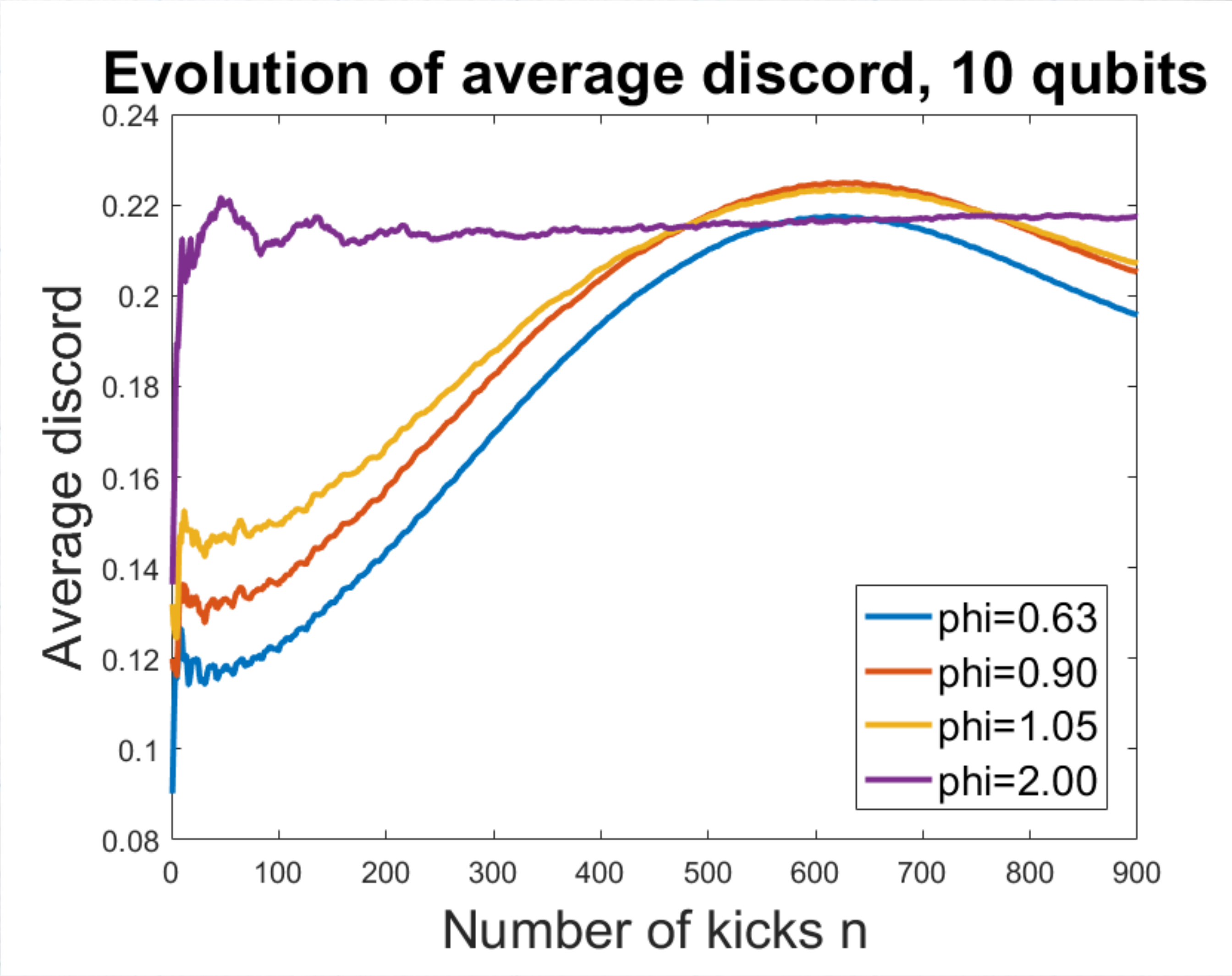
$$\varepsilon(|\psi\rangle) = 1 - \Lambda_{\max}\{\lambda_i\}$$

Eigenvalues of all possible density matrices obtained by tracing out 1 to N − 1 qubits

A. Sen(De) and U. Sen, Phys. Rev. A 81, 012308 (2010)

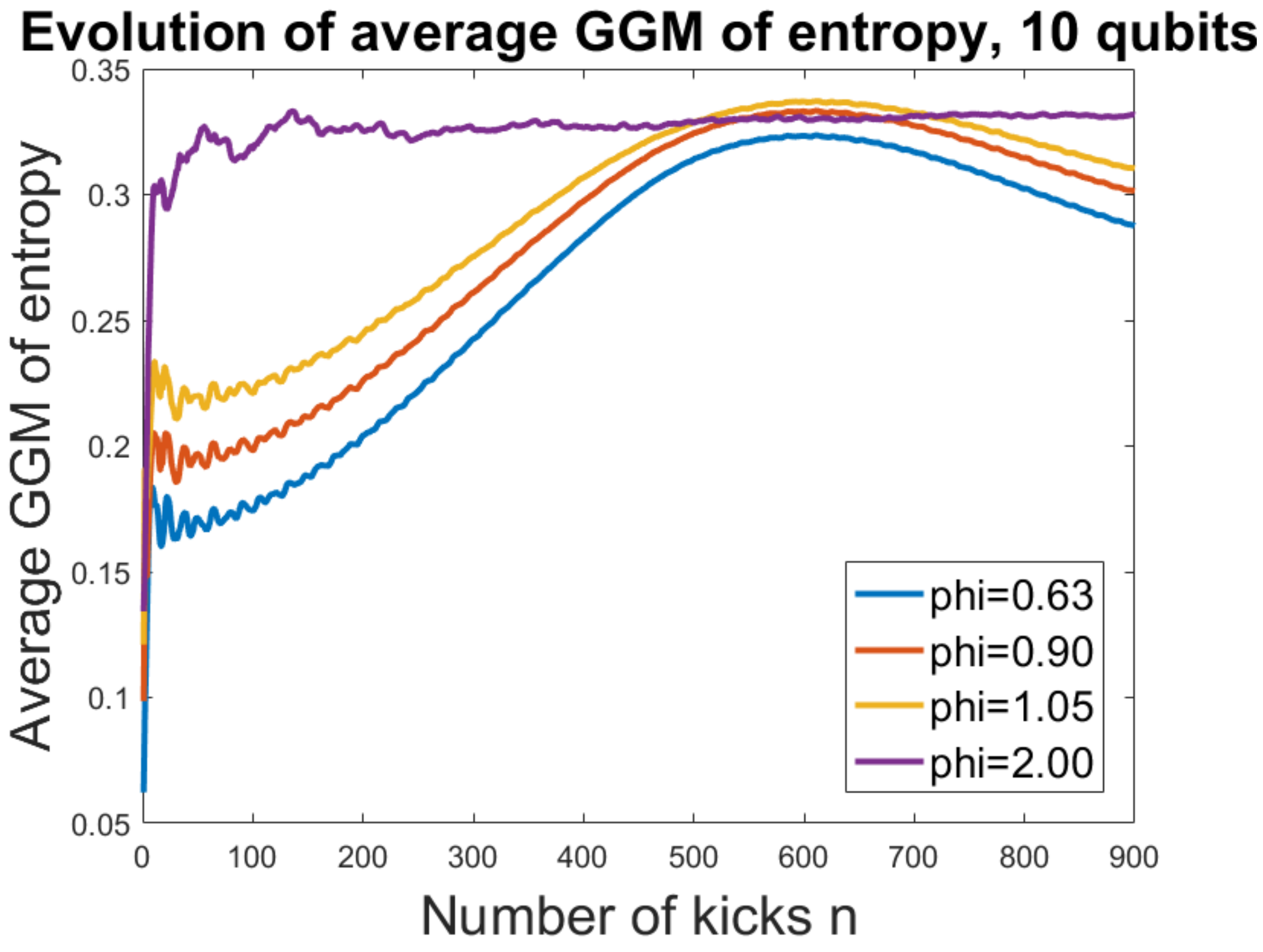
Quantum kicked top

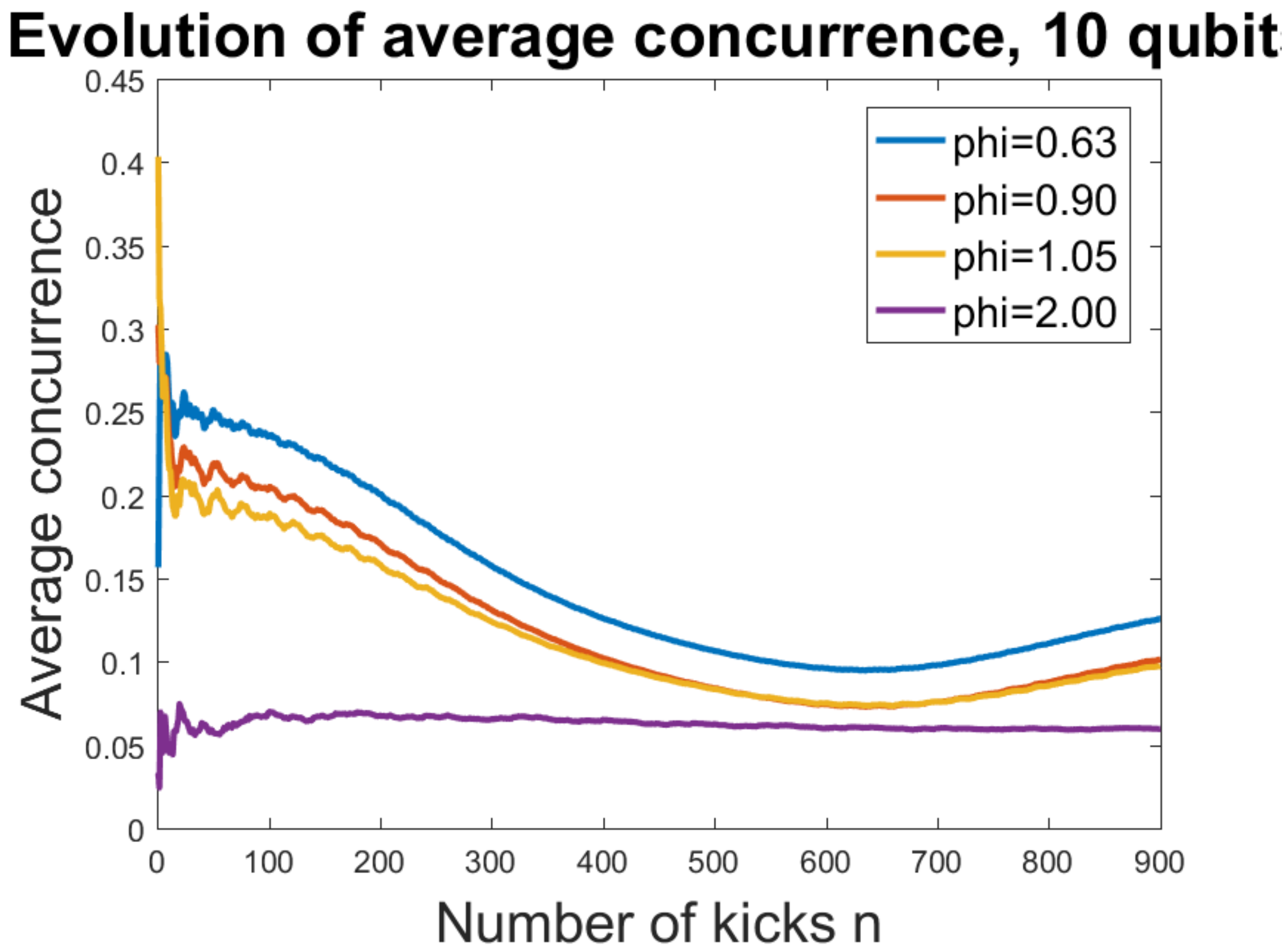




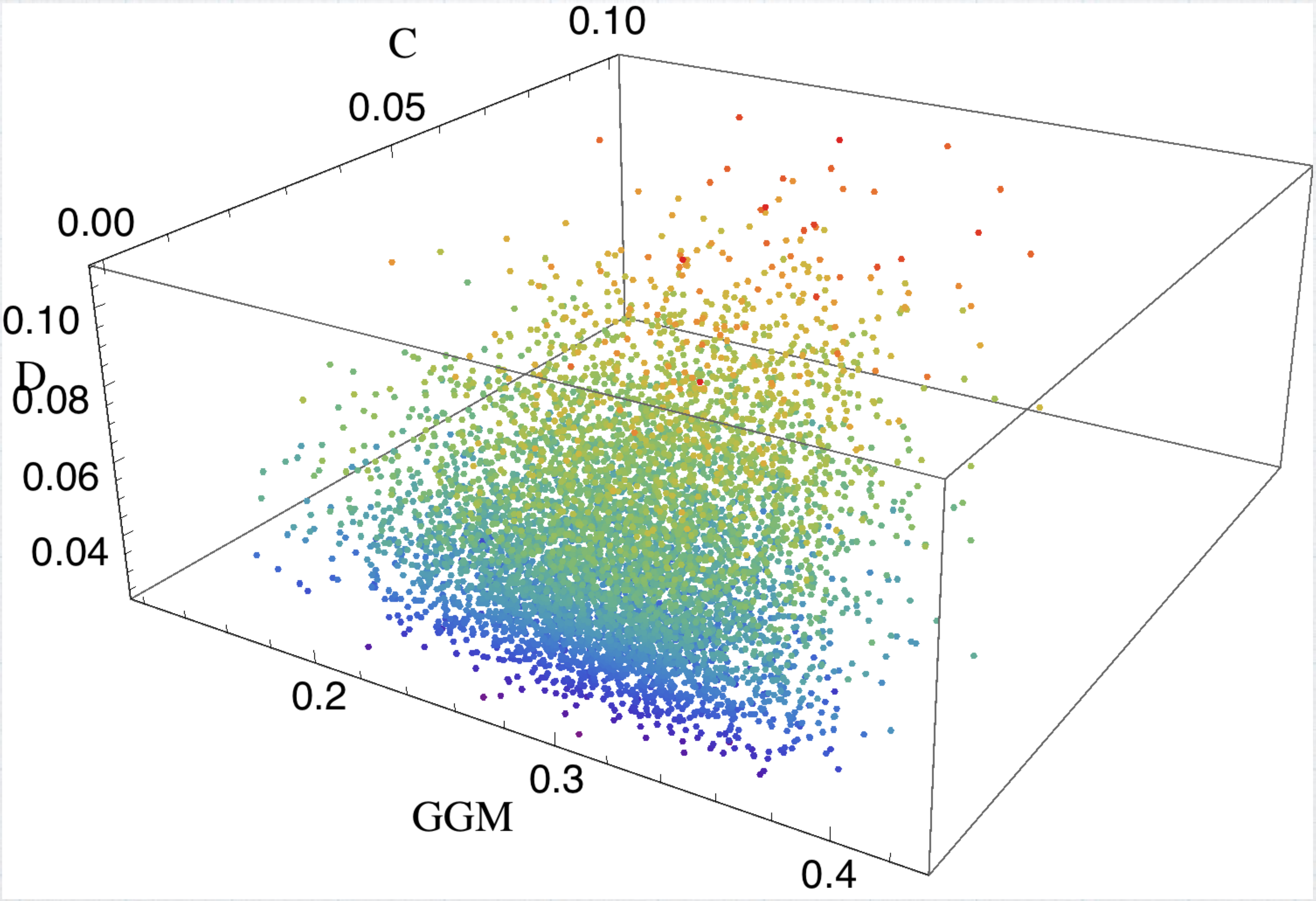
Ardra K S - Final year project at IISER TVM (Manuscript being prepared)

Quantum kicked top





Generic multi qubit states



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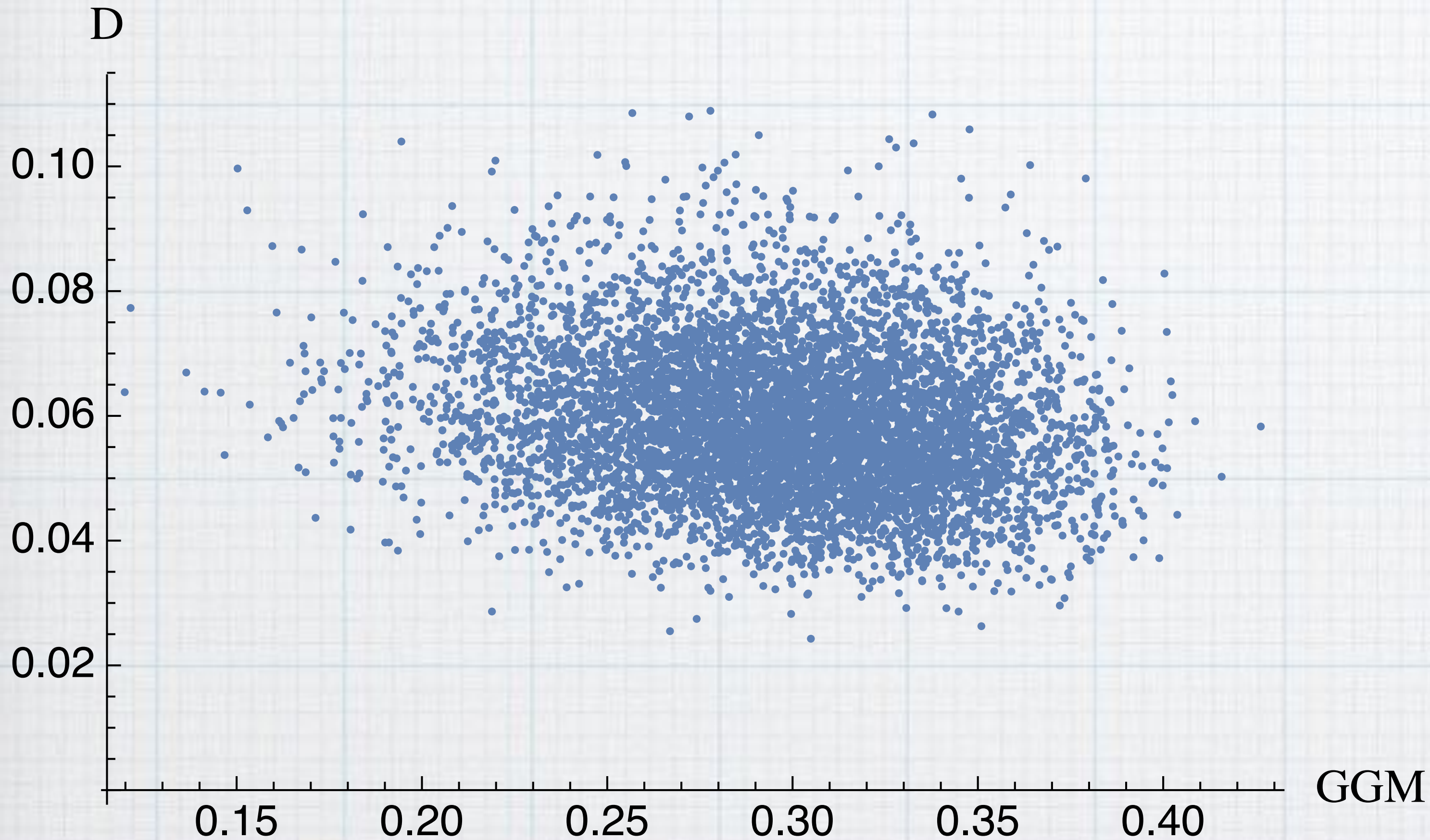
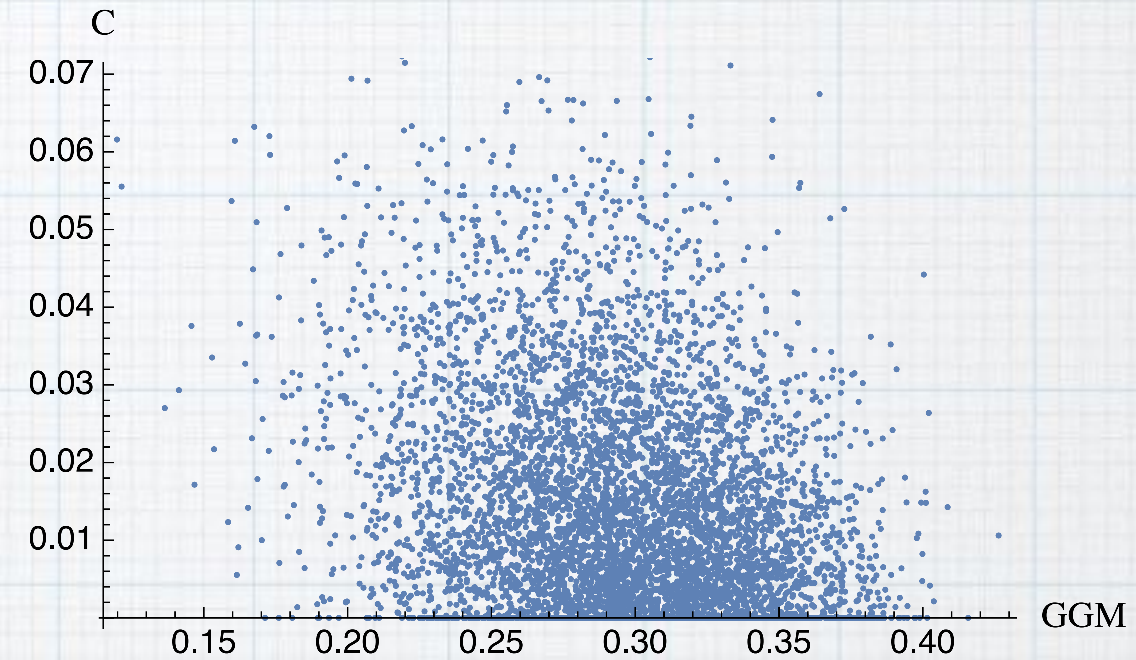
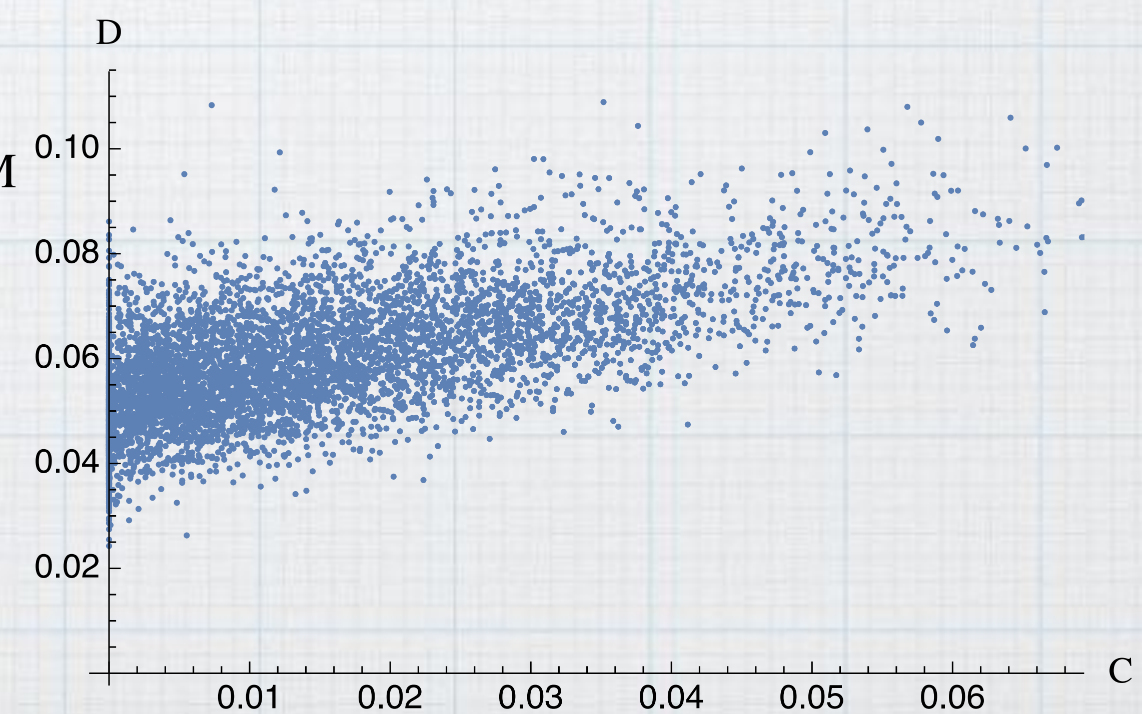
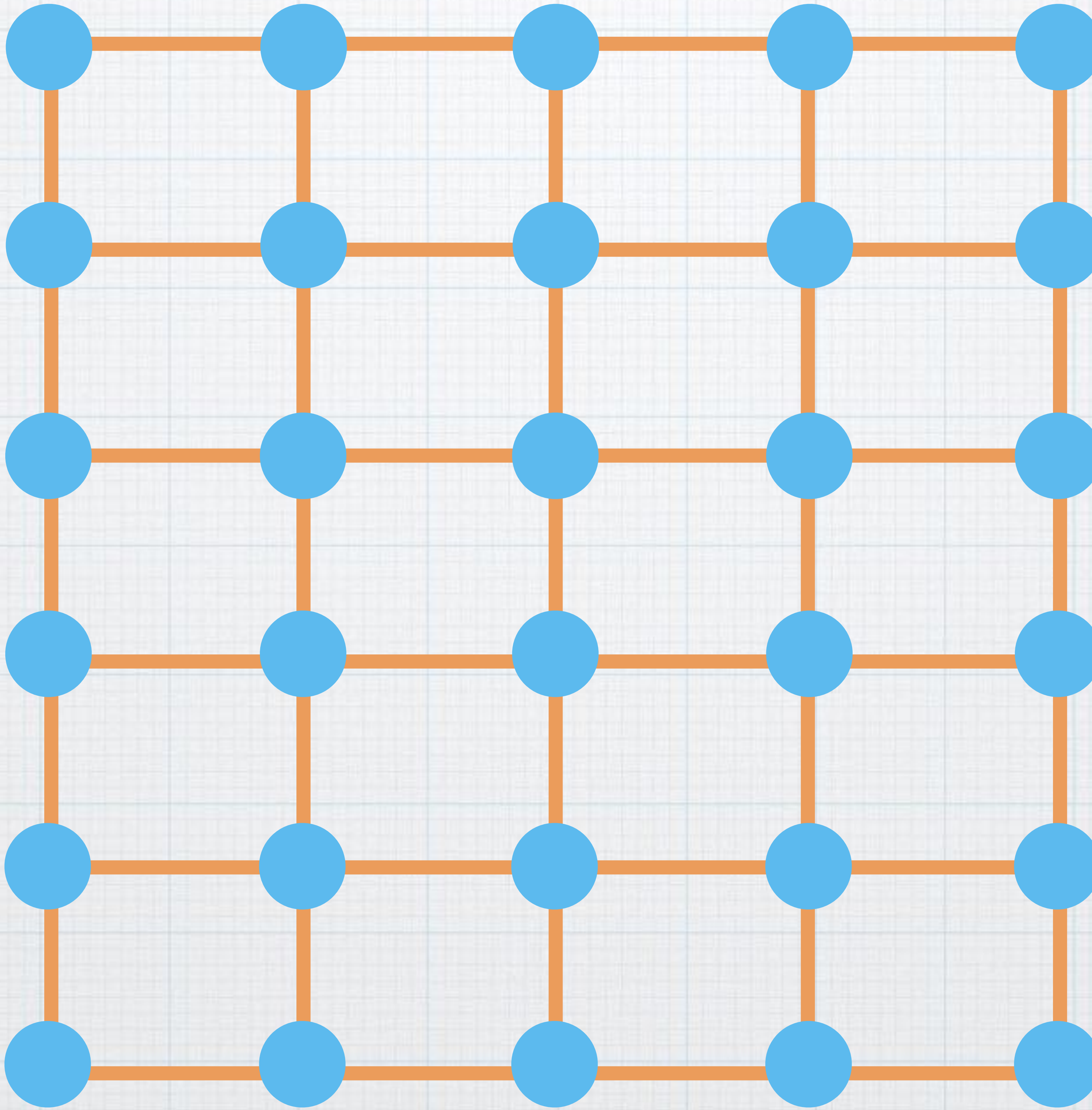






Cluster state

$$U(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\theta} \end{pmatrix}$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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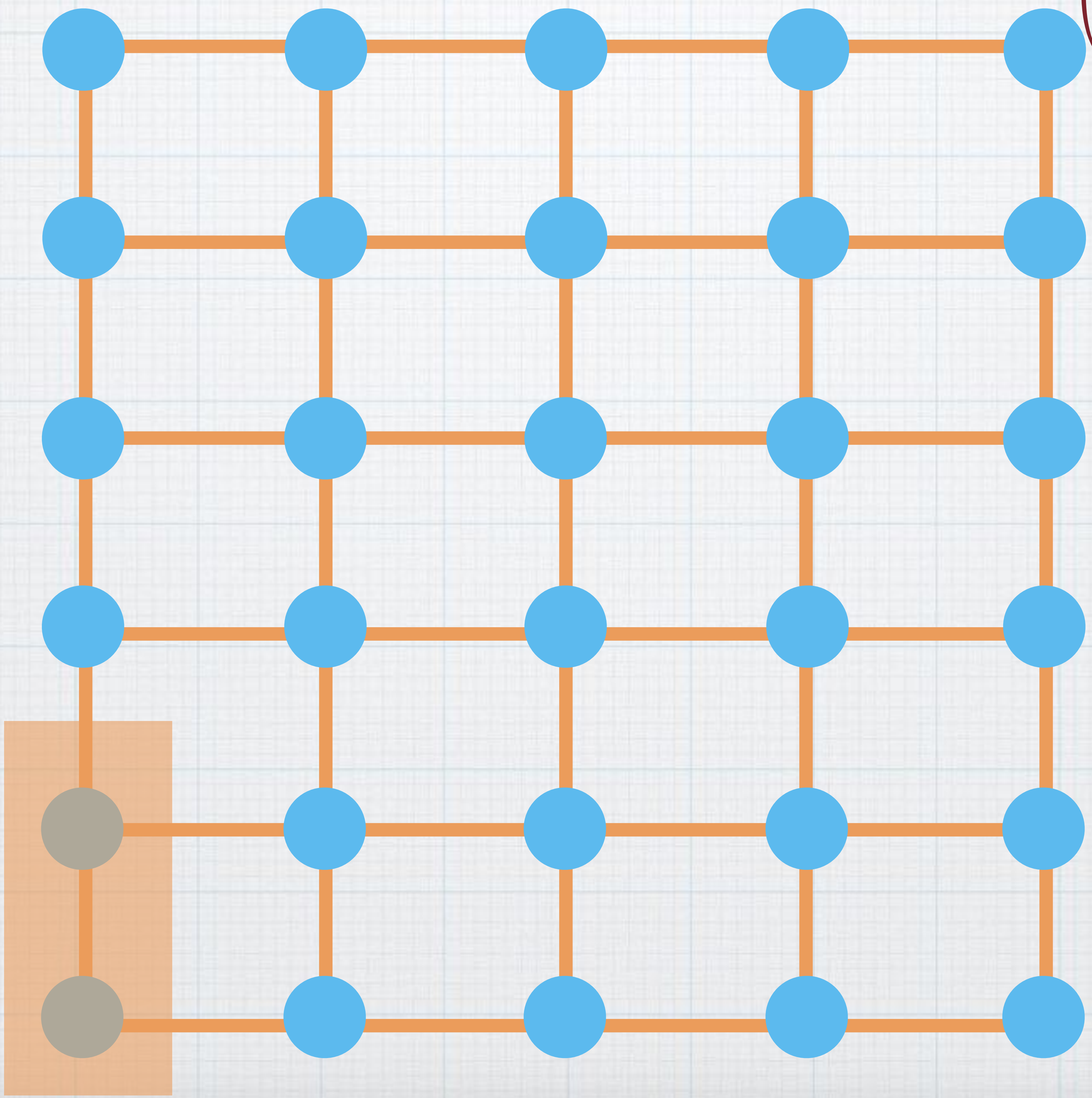

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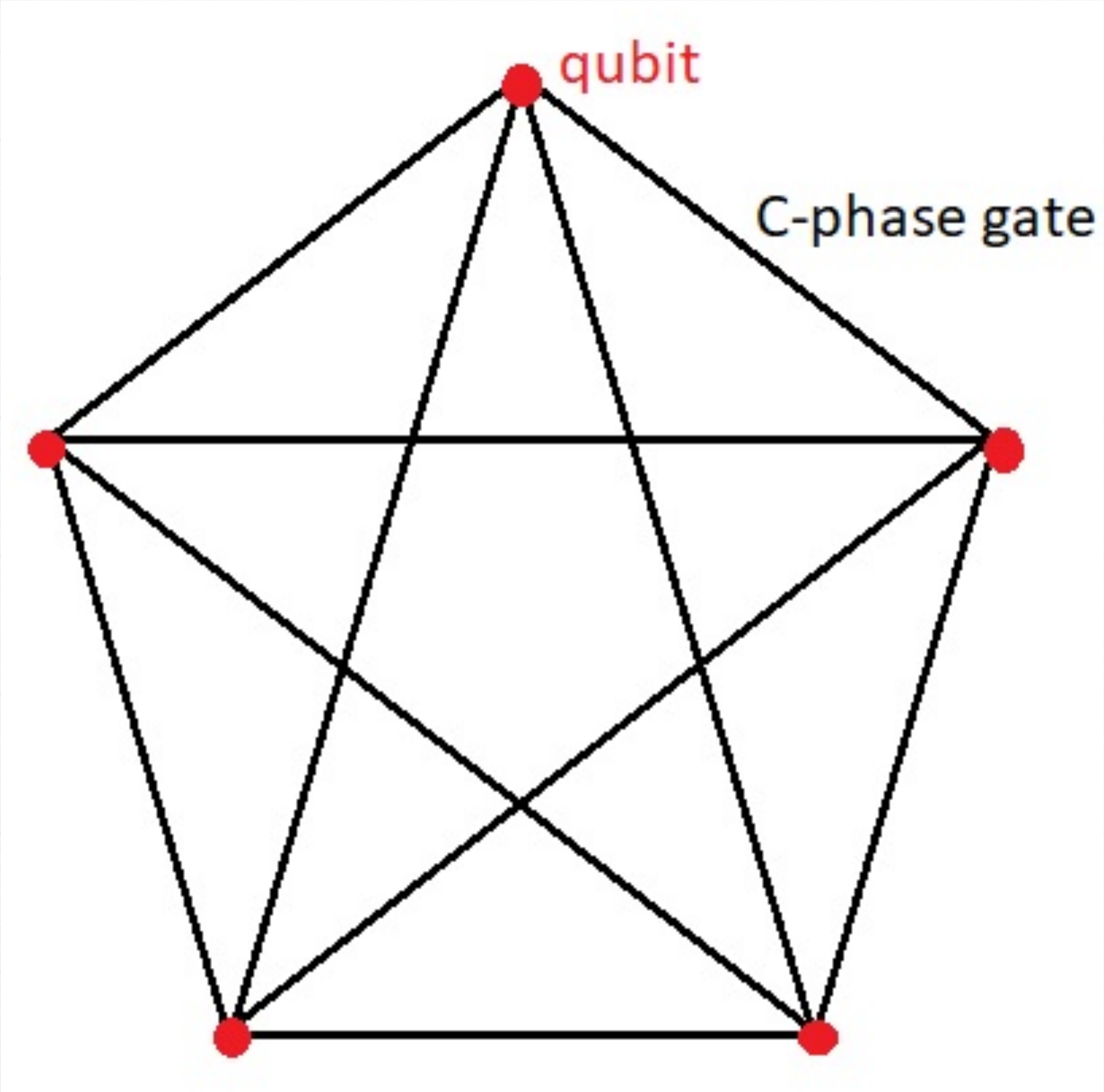

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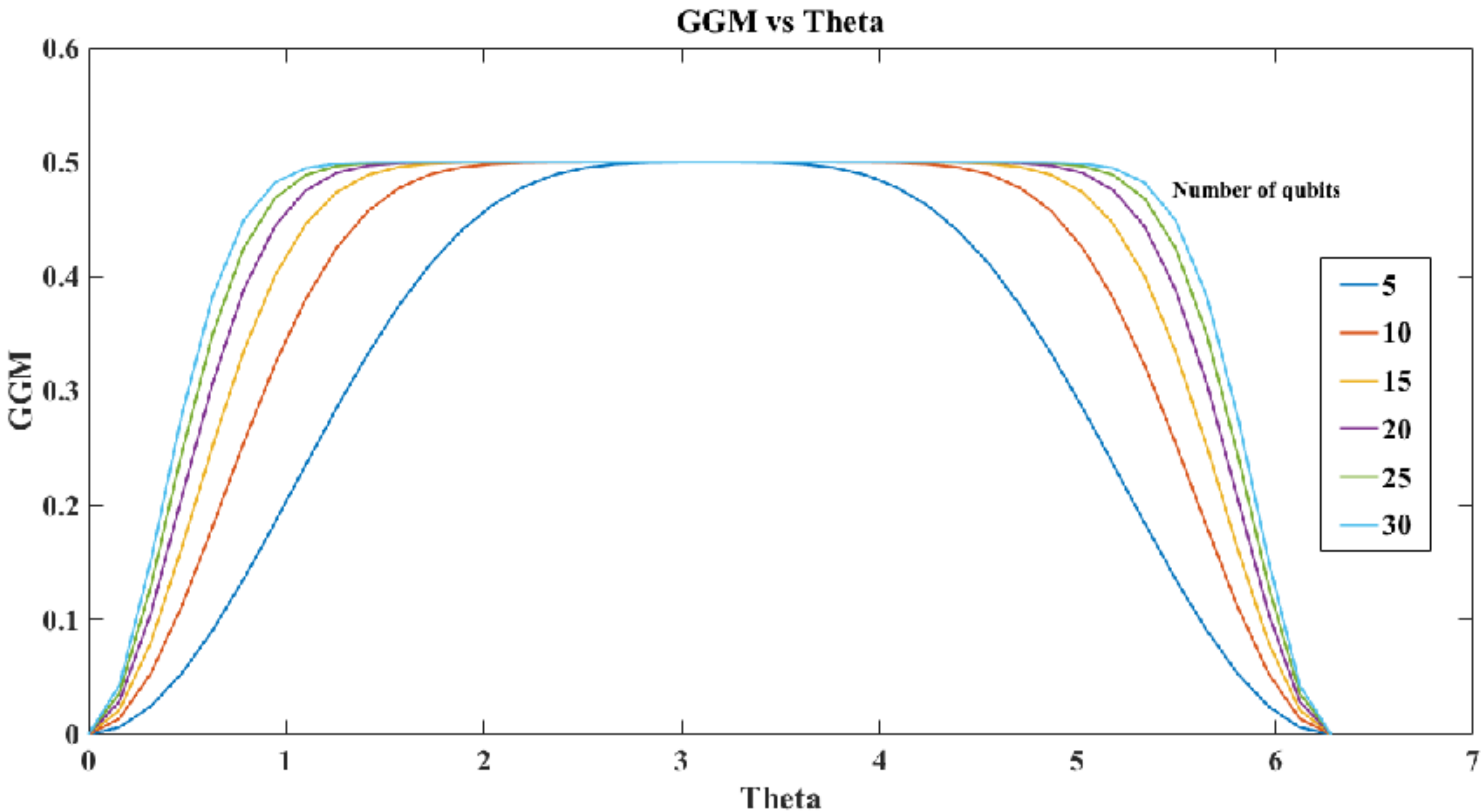

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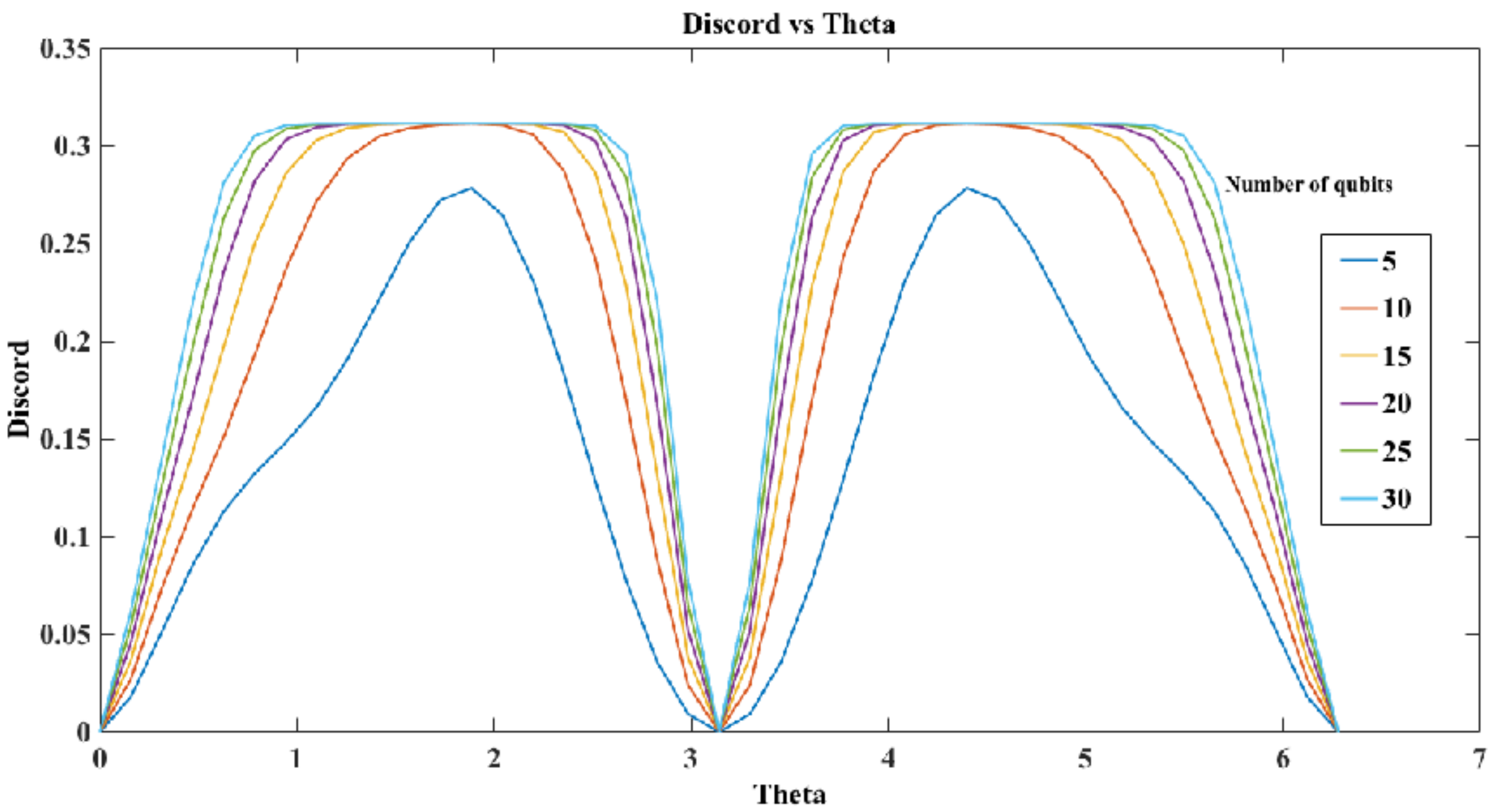

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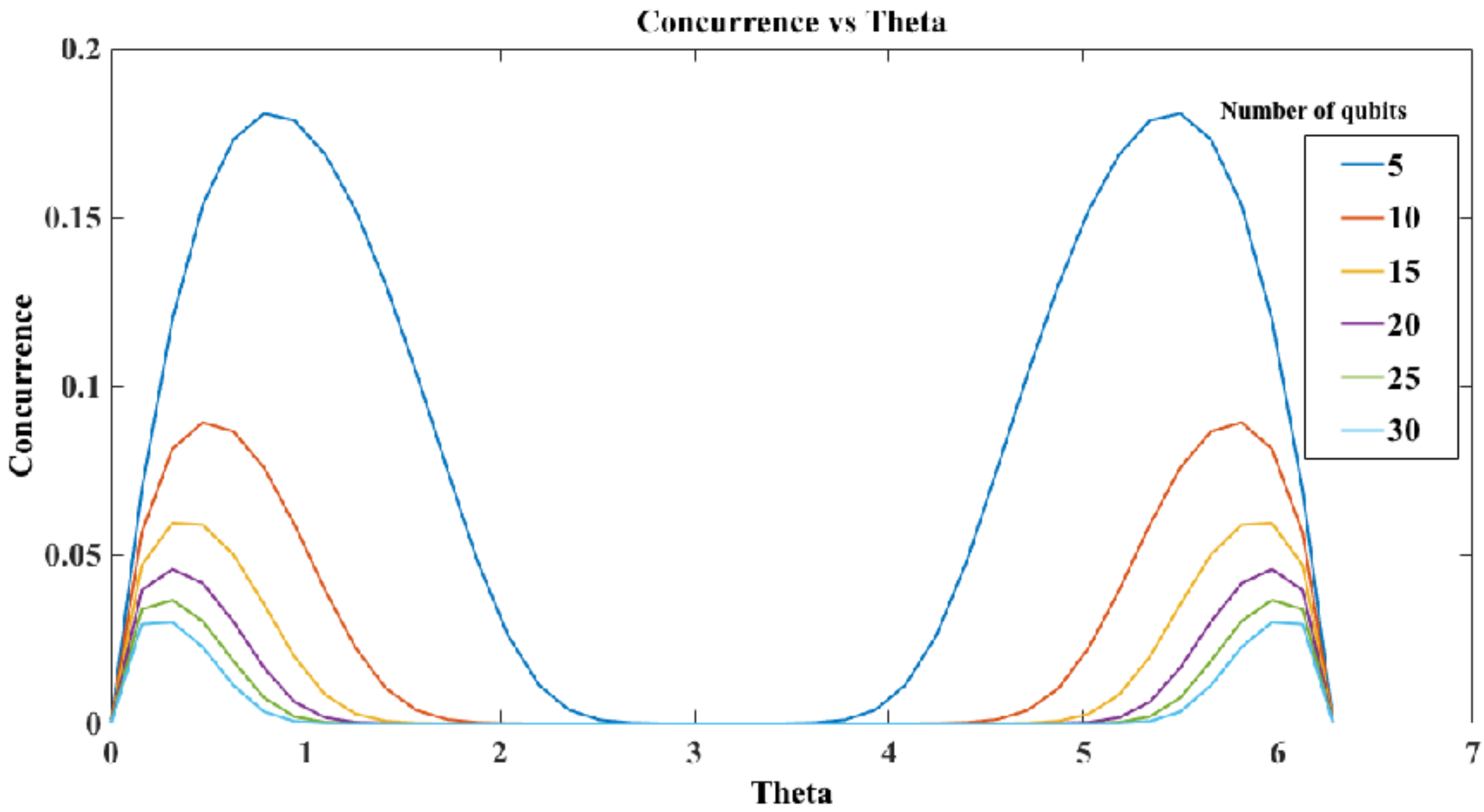

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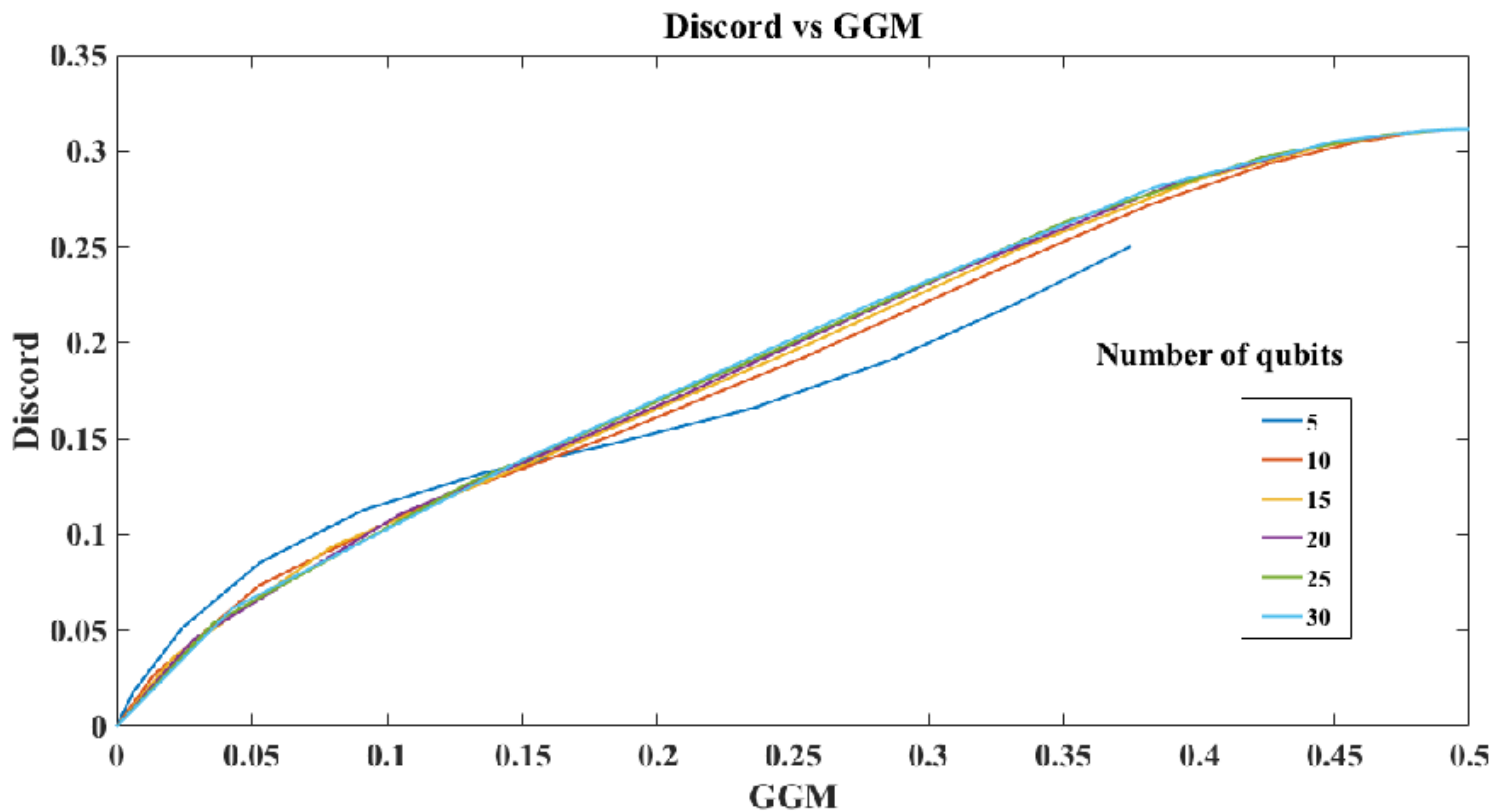
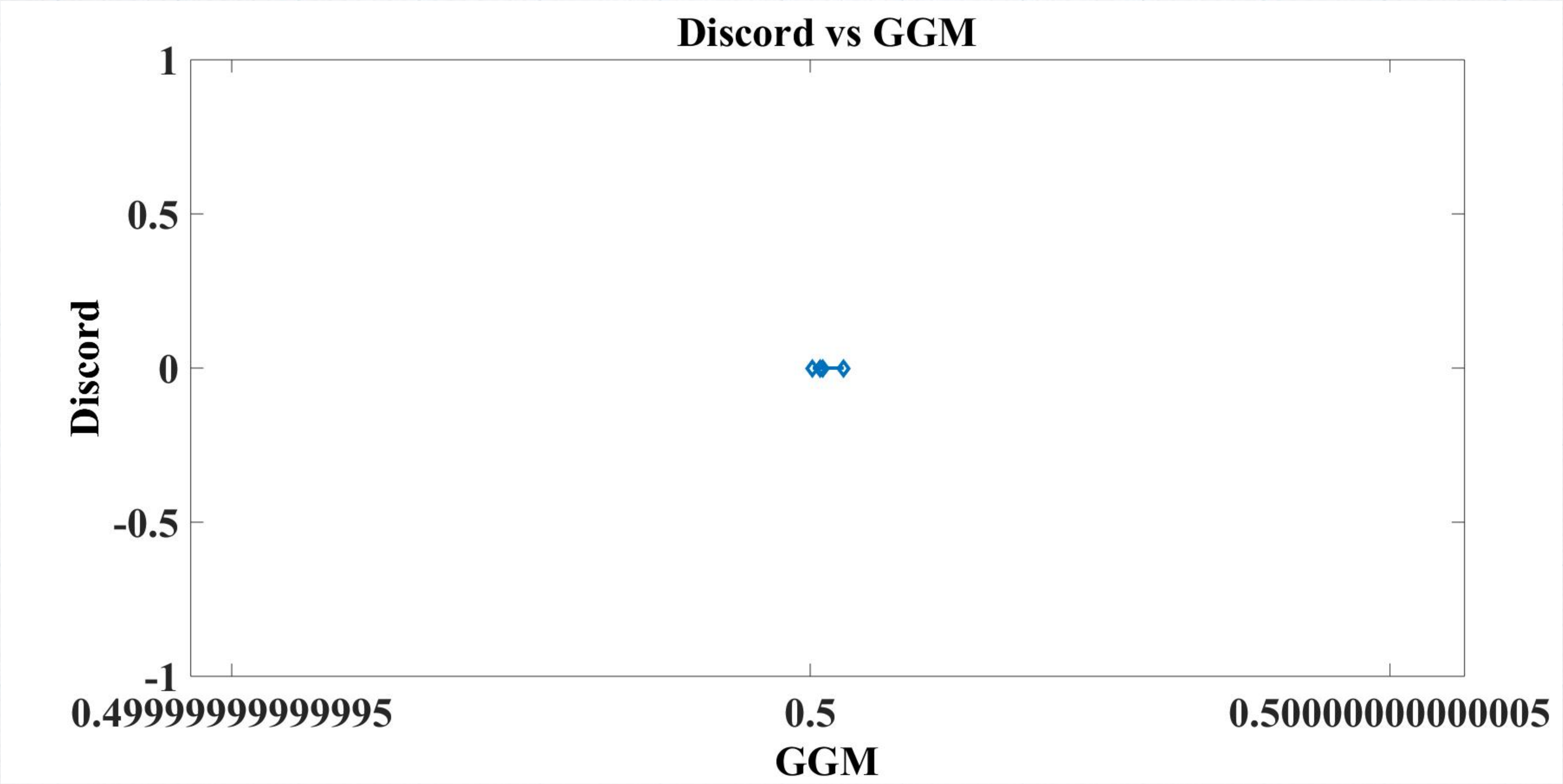


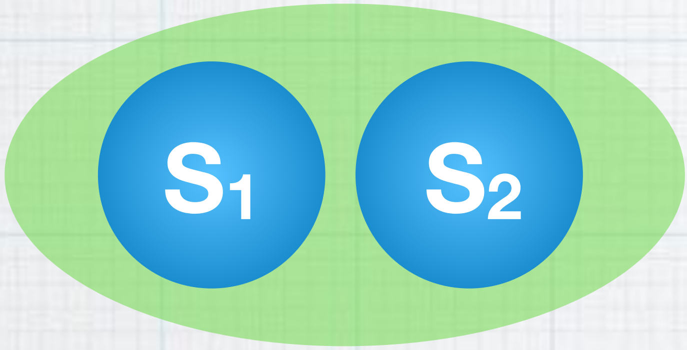
Stabilizer state

* A stabilizer state is constructed out of a grid of qubits by the action of Clifford group (Hadamard, Z, CNOT etc) on them
* Gottesmann and Knill showed that while the stabilizer states have high entanglement, they are not useful for universal quantum computation.
* The discord of two qubit subsystem for a global stabilizer state is identically zero.

Stabilizer state



Delocalized information in open quantum dynamics



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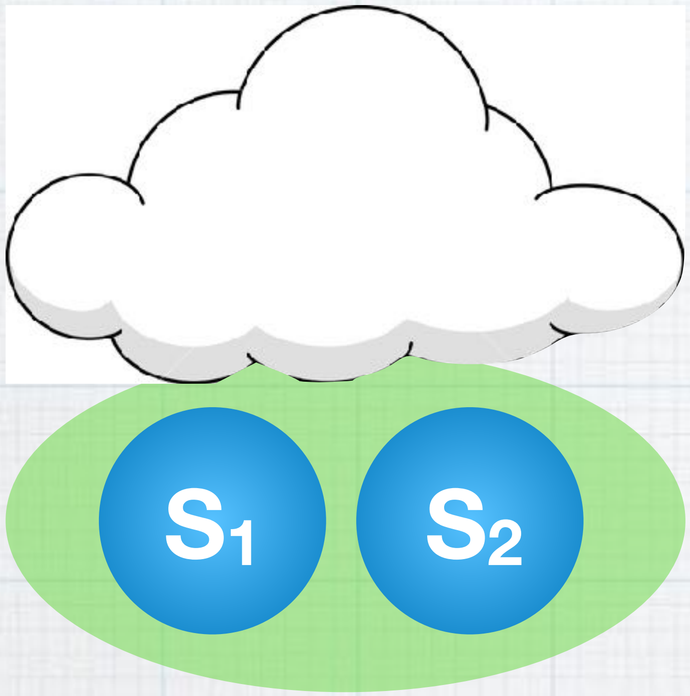
Delocalized information in open quantum dynamics



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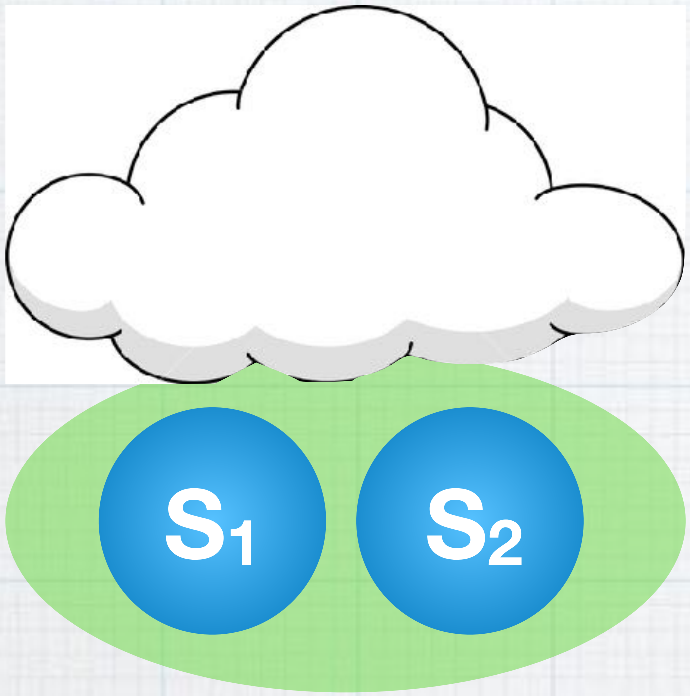
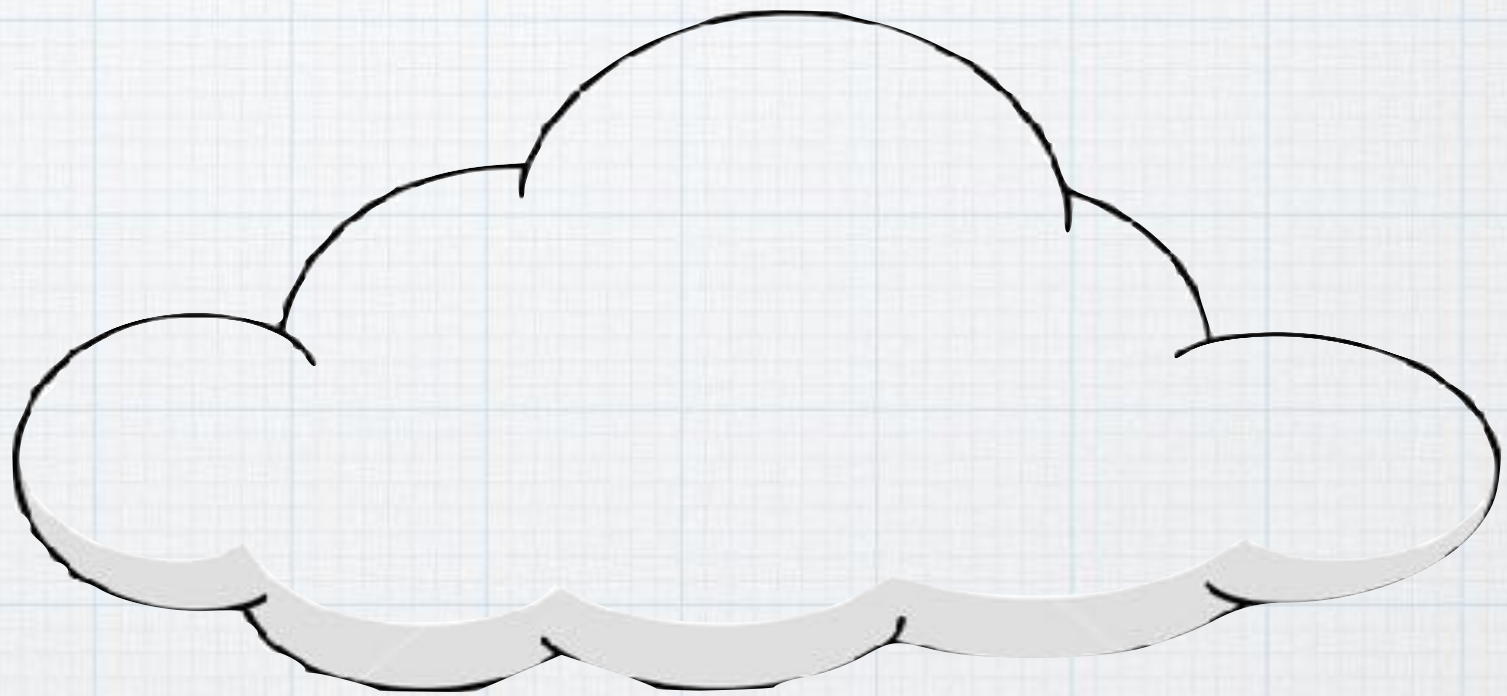
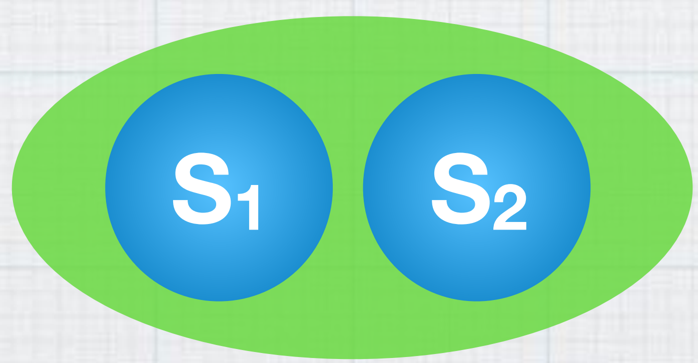
Delocalized information in open quantum dynamics







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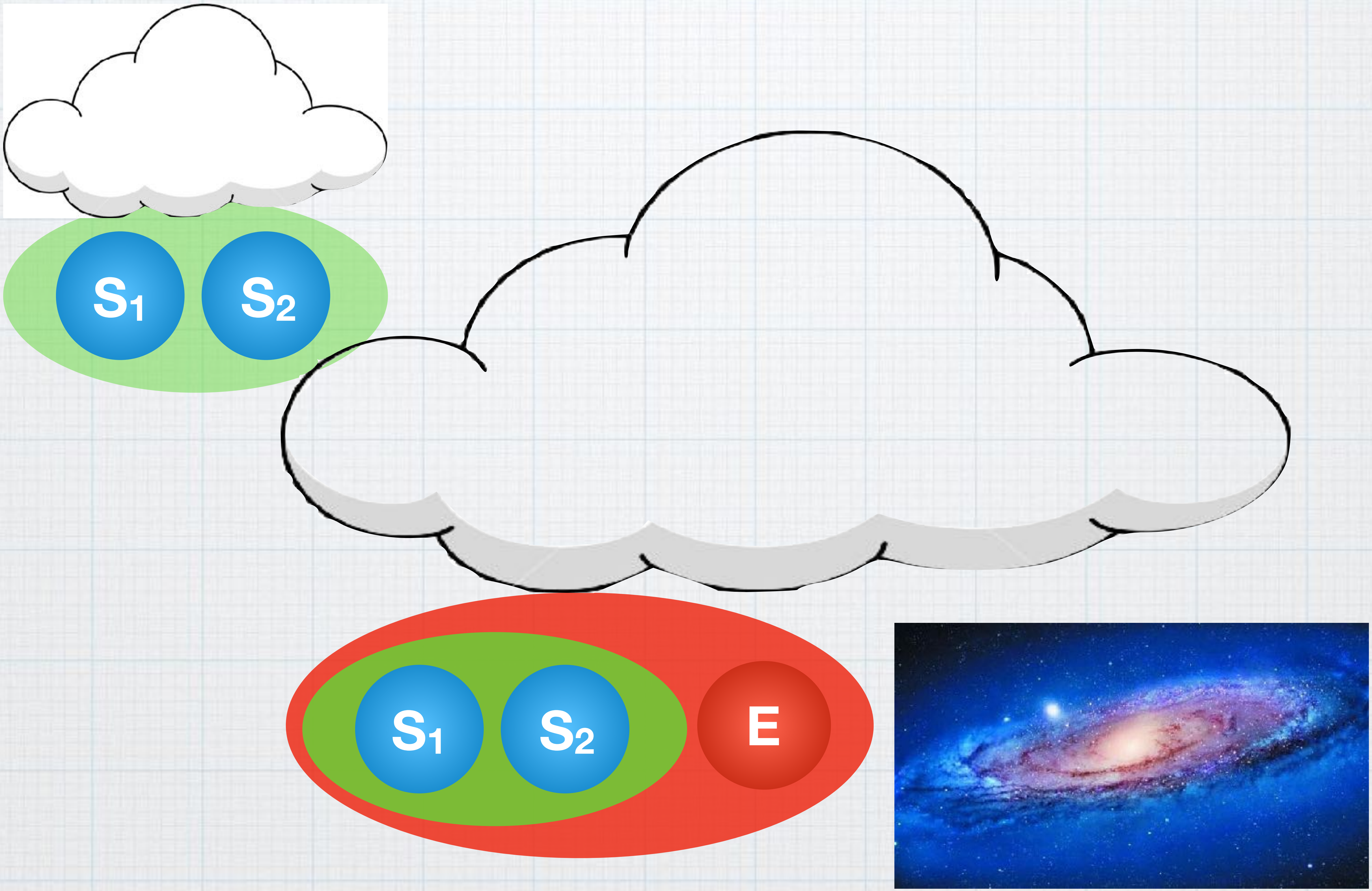

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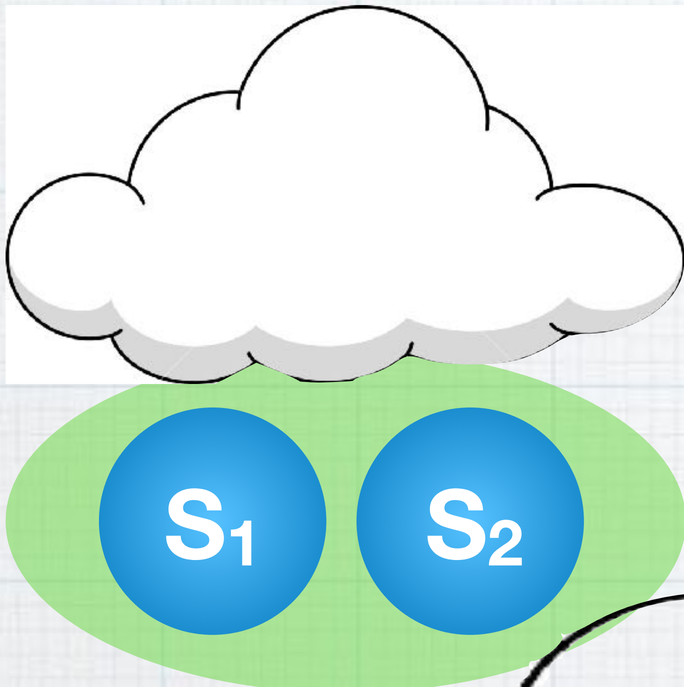
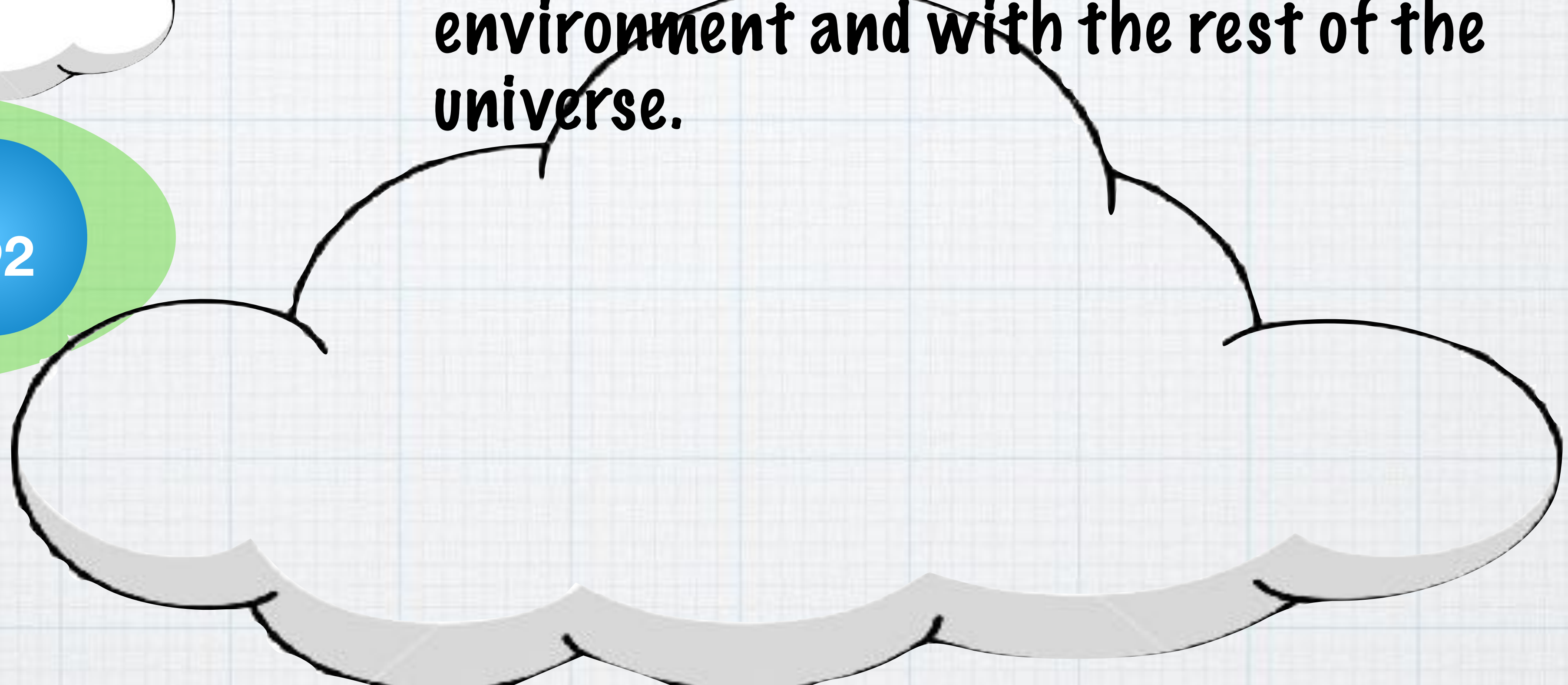
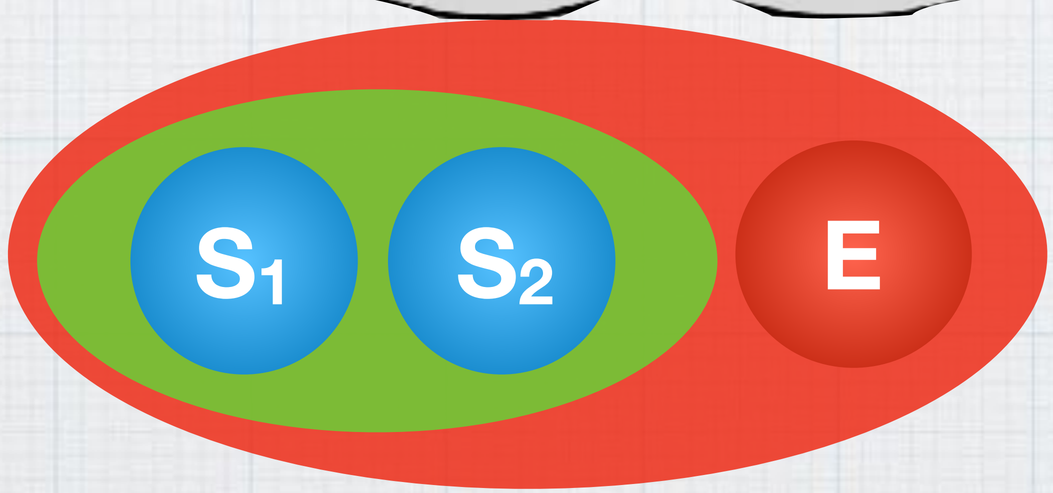
Delocalized information in open quantum dynamics



Nonclassical correlations between parts of a system as well as those between the system, among its parts, to its immediate environment and with the rest of the universe.





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Conclusion

* There is more to quantum information theory than entanglement
* NonClassical correlations present a more general resource which can be utilized in some situations.
* Can one prove a result like the Jozsa-Linden one for generic mixed state quantum computation and its effectiveness.
* What role does the rest of the universe have in determining and controlling quantum dynamics and whether this role can be put to good use?
* With advances in quantum technologies, many of these questions become experimentally accessible as well.


Thank You