# Unbounded violation of quantum steering inequalities

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1.Unbounded violation of quantum steering inequalities (PRL 115, 170401 (2015))

2.Quantum steering inequality with tolerance for measurement-setting-errors: experimentally feasible signature of unbounded violation (PRL 118 (2), 020402 (2016))

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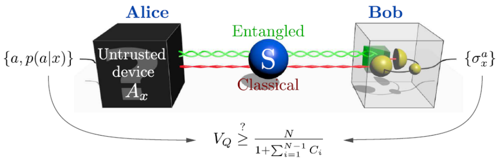
# Outline

- The concept of quantum steering was introduced by Schrödinger in 1935 as a generalization of EPR paradox.




Alice and Bob have locally access to subsystems of a bipartite system described by a quantum state $\rho$. Alice chooses one of her settings $x \in \{1, \ldots, N\}$, measures a nondegenerate observable $A_x$ with eigenvectors $\{\varphi_x^a\}$ and receives a result $a \in \{1, \ldots, d\}$ with probability $p(a|x) = \mathrm{Tr}\{(|\varphi_x^a\rangle \langle \varphi_x^a| \otimes I)\rho\}$. Only after Alice has collected the result $a$, the following conditional state

$$\sigma_x^a = \mathrm{Tr}_A\{(|\varphi_x^a\rangle \langle \varphi_x^a| \otimes I)\rho\} \tag{1}$$

was "created at a distance" at Bob's location.

## The steering scheme

- Violation of steering inequalities has been confirmed in numerous experimental demonstrations involving a single photon, a two-photon singlet or a Werner state
- Since quantum steering can be formulated as a quantum-information task where the classical measurements simulate an untrusted device, it has been extended to a multipartite scenario useful for semi-device-independent entanglement certification in quantum networks

- $$\exists_{\rho_{spl}} \nrightarrow \text{Steering (i.e. admits LHS model)}$$
- $$\begin{aligned} \exists_{\rho_{spl}} \quad & \rightarrow \quad \text{Steering (i.e. does not admit LHS model)} \\ & \nrightarrow \quad \text{Bell nonlocality ( admits LHV model)} \end{aligned}$$
- $$\begin{aligned} \text{LHS} \quad & \rightarrow \quad \text{LHV} \\ \text{LHV} \quad & \nrightarrow \quad \text{LHS} \end{aligned}$$
- $$\begin{aligned} \exists_{\rho_{spl}} \quad \rightarrow \quad & \text{Steerable from A } \rightarrow \text{B} \\ & \text{B} \nrightarrow \text{A} \end{aligned}$$

# Outline

1. The concept of quantum steering
   - The steering scheme
   - Why is it important?
   - Comparison of LHS and LHV models
2. Mathematical formulation of quantum steering problem
   - Mathematical setup
   - Known results
3. How to solve the problem
   - Theorem
   - Compare with knowing results
   - Application
4. Conclusions

We consider the following steering scenario (Pussey 2013)

- Suppose there are two observers: Alice and Bob.
- Alice can choose among $n$ different measurement settings labeled by $x = 1, \ldots, n$. Each of them can result in one of $m$ outcomes, labeled by $a = 1, \ldots, m$.
- $\mathcal{H}_B$ is the local Hilbert space for Bob, $\dim \mathcal{H}_B = d$.
- The available data are the steered states:

$$\sigma_x^a \in \mathcal{B}(\mathcal{H}_B), \quad x = 1, \ldots, n, \;\; a = 1, \ldots, m$$

  - Positivity: $\sigma_x^a \geq 0$
  - Non-signaling: $\mathrm{Tr}\left(\sum_{a=1}^{m} \sigma_x^a\right) = 1$ for any $x$
- The set $\sigma = \{\sigma_x^a : \; x = 1, \ldots, n, \; a = 1, \ldots, m\}$ is called an assemblage. The set of all assemblages is denoted by $\mathcal{Q}$.

- Schrödinger 1936; Hughston, Jozsa, Wootters 1993:
  Any assemblage $\sigma$ has a *quantum realization*, i.e. it can be generated remotely, by performing measurements on a subsystem of bipartite quantum states.
- More precisely: for any $\sigma$ there are
  - a Hilbert space $\mathcal{H}_A$,
  - a density matrix $\rho \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$
  - a POVM measurements $\{E_x^a : \ a = 1, \ldots, m\}$ on $\mathcal{H}_A$ for $x = 1, \ldots, n$

  such that

$$\sigma_x^a = \mathrm{Tr}_A \left( (E_x^a \otimes \mathbb{1}) \rho \right)$$

- The assemblage has a *local hidden state (LHS)* model, if there are:
  - a finite set of indices $\Lambda$,
  - nonnegative coefficients $q_\lambda$ such that $\sum_\lambda q_\lambda = 1$,
  - density matrices $\sigma_\lambda$ in $\mathcal{B}(\mathcal{H}_B)$ for $\lambda \in \Lambda$,
  - probability distributions $\{p_\lambda(a|x)\}_a$ for every $x$ and $\lambda$

  such that

  $$\sigma_x^a = \sum_{\lambda \in \Lambda} q_\lambda p_\lambda(a|x) \sigma_\lambda,$$

  for every $x, a$. We denote the set of LHS assemblages by $\mathcal{L}$.

- Suppose we make a choice of measurement $x$ to perform on Alice system ($A$), and obtain an outcome $a$. Denote the steered state for Bob system ($B$) by $\sigma_x^a$. Then if the measurement on $A$ simply reveals information about parameter $\lambda$ that determines which state $\sigma_\lambda$ applies to $B$, then we have:

  $$\sigma_x^a = \sum_{\lambda \in \Lambda} q_\lambda p_\lambda(a|x) \sigma_\lambda, \tag{2}$$

  where $p_\lambda(a|x)$ is the probability of obtaining the outcome $a$ from measurement $x$ when the parameter is $\lambda$.

- We can use **steering inequalities** to study the difference between the two sets $\mathcal{L}$ and $\mathcal{Q}$. (Cavalcanti at al. 2009)
- Let $F : \mathcal{Q} \mapsto \mathbb{R}$ be a function.

$$S_{\mathrm{LHS}}(F) = \sup\{F(\sigma) : \ \sigma \in \mathcal{L}\}.$$

  **Steering inequality:** $F \leq S_{\mathrm{LHS}}$.
- Let

$$S_{\mathcal{Q}}(F) = \sup\{F(\sigma) : \ \sigma \in \mathcal{Q}\}$$

  If $S_{\mathcal{Q}}(F) > S_{\mathrm{LHS}}(F)$ then we say that the steering inequality is nontrivial, i.e. it can be violated by some entangled states (Pusey 2013)

- We will consider only linear functions $F$. Namely, $F$ is a steering functional if it is of the form

$$F(\sigma) = \mathrm{Tr}\left(\sum_{x,a} F_x^a \sigma_x^a\right)$$

for some set $\{F_x^a : \; x = 1, \dots, n, \; a = 1, \dots, m\}$ of real $d \times d$ real matrices.

- Quantum violation of $F$:

$$V(F) = \frac{S_Q(F)}{S_{LHS}(F)}.$$

We say that it is unbounded if the ratio $V$ between the quantum and classical value of the steering functional is an increasing function of some experimental parameters, for example of the amount of entanglement in $\rho$ or of a number and characteristics of the measured observables.

- A steering functional with large violation, will tell us the sets $\mathcal{L}$ and $\mathcal{Q}$ are prominently different.
- For given Bell or steering functional, it is difficult to calculate its violation.
- Operator space approach (MHY 2015r):
  For $n = m = d$ consider

$$F_x^a = \frac{1}{d}\sum_{k=1}^{d} \epsilon_{x,a}^k \, |1\rangle\langle k| , \quad x, a = 1, \cdots, d,$$

  where $\epsilon_{x,a}^k, x, a, k = 1, \cdots, d$ are independent Bernoulli random variables. Then

$$V(F) = O\left(\sqrt{\frac{d}{\log d}}\right) \quad \text{with high probability.}$$

- Algebraic approach (RHMY 2015)
  Let us study a steering functional constructed by means of mutually unbiased bases (MUBs). Let $M_1 = \{|\phi_1^a\rangle : a = 1, \ldots, d\}$ and $M_2 = \{|\phi_2^a\rangle : a = 1, \ldots, d\}$ be orthonormal bases in the $d-$dimensional Hilbert space. Then they are said to be mutually unbiased if $\left|\langle\phi_1^a|\phi_2^b\rangle\right| = \frac{1}{\sqrt{d}}$ for all $a$, $b = 1, \ldots, d$. A set $M = \{M_x : x = 1, \ldots, n\}$ of orthonormal bases of $\mathbb{C}^d$ is said to be a set of mutually unbiased bases (MUBs), if $M_x$ and $M_y$ are mutually unbiased for every $x \neq y$.

$$V(F) \geq \frac{n\sqrt{d}}{n+1+\sqrt{d}}. \tag{3}$$

  If the dimension $d$ is an integer power of a prime number, then we can always find $d+1$ MUBs. In this case $n = d+1$; hence, we can find a steering functional $F$, with violation $\Omega(\sqrt{d})$.

- Clifford observables $A_x$ (RHMY 2015)

$$F_x^1 = \frac{1}{2}A_x, \quad F_x^2 = -\frac{1}{2}A_x, \quad x = 1, \cdots, n. \tag{4}$$

it has been shown

$$V(F) \geq \sqrt{\frac{n}{2}}. \tag{5}$$

- SDP (SK 2016) They have shown, that the problem of calculate of steering inequality can be express as an instance of a semidefinite program, and using the duality theory they found upper bound of $S_{SLH}$:

$$\alpha = \max_{x,x'>a,a'} \sqrt{\mathrm{tr}(\Pi_{a|x}\Pi_{a'|x'}} = \cos(\theta)$$

i.e the maximal inner product between any two measurement.

$$S_{SLH} \leq 1 + (n-1)\cos(\theta)$$

Thus, any assemblage which obtains a value greater that this value demonstrates steering.

# Outline

*Theorem. Given a quantum steering scenario involving* $x \in \{1, \ldots, N\}$ *settings,* $a \in \{1, \ldots, d\}$ *outcomes, and a set of N orthonormal eingenbases* $\{\phi_x^a\}$ *defining the receiver's (Bob's) measurements, the LHS steering functional is bounded from above*

$$S_{LHS} \leq 1 + \sum_{i=1}^{N-1} C_i, \tag{6}$$

*where* $C_i = \max_x C_{x\,N+x-i}$ *and* $C_{xy} = \max_{a,b} |\langle \phi_x^a | \phi_y^b \rangle|$ *for* $x, y \in 1, \ldots, N$ *is defined as in the Maassen–Uffink uncertainty relations. This implies:*

$$V_Q \geq \frac{N}{1 + \sum_{i=1}^{N-1} C_i}. \tag{7}$$

*In particular, a weaker bound can be derived:*

$$V_Q \geq \frac{N}{1 + (N-1)C} \tag{8}$$

*with* $C = \max_i C_i = \max_{x \neq y} C_{xy}$.

First, we will compute the quantum value $S_Q(F)$ for the steering functional $F$ defined as a set $F = \{|\phi_x^a\rangle\langle\phi_x^a| : a = 1, \dots, d,\ x = 1, \dots, N\}$. In our case it is enough to show that $S_Q(F) \leq N$, where $N$ is the number of bases

$$
\begin{aligned}
S(F) = \mathrm{Tr}\left\{\sum_{x=1}^{N}\sum_{a=1}^{d} |\phi_x^a\rangle\langle\phi_x^a| \sigma_x^a\right\} &\leq \mathrm{Tr}\left\{\sum_{x=1}^{N}\sum_{a=1}^{d} |\phi_x^a\rangle\langle\phi_x^a| \sum_{a'} \sigma_x^{a'}\right\} \\
&= \mathrm{Tr}\left\{\sum_{x=1}^{N}\sum_{a=1}^{d} |\phi_x^a\rangle\langle\phi_x^a| \rho_x\right\} \qquad (9) \\
&= \sum_{x=1}^{N}\underbrace{\sum_{a=1}^{d} p_x(a|x)}_{=1} = N
\end{aligned}
$$

$$\Downarrow$$

$$S_Q \leq N. \qquad (10)$$

On the other hand, let us choose the assemblage of the form $\sigma_x^a = \frac{1}{d}|\phi_x^a\rangle\langle\phi_x^a|$. By direct calculations one can obtain $S(F) = n$, what means that

$$S_Q(F) \geq n. \tag{11}$$

Comparing these results we get

$$S_Q(F) = n. \tag{12}$$

Second, we will describe a general method of computation of the classical bound $S_{\mathrm{LHS}}\left(|\phi_x^a\rangle\langle\phi_x^a|\right)$. Let $\sigma_x^a = \sum_\lambda q_\lambda\, p_\lambda(x|a)\,\sigma_\lambda$. Then, the following inequality holds

$$\begin{aligned} \mathrm{Tr}\left\{\sum_{x=1}^{N}\sum_{a=1}^{d}|\phi_x^a\rangle\langle\phi_x^a|\,\sigma_x^a\right\} &= \sum_{x=1}^{N}\sum_{a=1}^{d}\mathrm{Tr}\left\{|\phi_x^a\rangle\langle\phi_x^a|\sum_\lambda q_\lambda\, p_\lambda(a|x)\,\sigma_\lambda\right\} \\ &= \sum_\lambda q_\lambda \sum_{x=1}^{N}\sum_{a=1}^{d}\mathrm{Tr}\{|\phi_x^a\rangle\langle\phi_x^a|\,p_\lambda(a|x)\,\sigma_\lambda\} \\ &\leq \sup_\lambda \left\|\sum_{x=1}^{N}\sum_{a=1}^{d}|\psi_{x,\lambda}^a\rangle\langle\psi_{x,\lambda}^a|\right\|, \end{aligned} \tag{13}$$

where $|\psi_{x,\lambda}^a\rangle = \sqrt{p_\lambda(a|x)}\,|\phi_x^a\rangle$. For any $\lambda$, let $G_\lambda = \sum_{x,y=1}^{N}\sum_{a,b=1}^{d}\langle\psi_{x,\lambda}^a|\psi_{y,\lambda}^b\rangle\,|x\rangle\langle y|\otimes|a\rangle\langle b|$. Using the purification of $\sum_{x=1}^{N}\sum_{a=1}^{d}|\psi_{x,\lambda}^a\rangle\langle\psi_{x,\lambda}^a|$ and its Schmidt decomposition, we can show that

$$\left\|\sum_{x=1}^{N}\sum_{a=1}^{d}|\psi_{x,\lambda}^a\rangle\langle\psi_{x,\lambda}^a|\right\| = \|G_\lambda\|\,. \tag{14}$$

In further considerations we will omit the index $\lambda$. Let us define the shift operator $S : \mathbb{C}^N \to \mathbb{C}^N$ acts on the bases vectors in the following way:

$$S\left|k\right\rangle = \left|k+1\right\rangle \mod N, \tag{15}$$

and observe that $\sum_{i=1}^N S^i = \mathbb{I}$ – every element of $\mathbb{I}$ is equal to 1. We decompose $G$ in the following way

$$G = \sum_{x,y=1}^{N} \left|x\right\rangle\left\langle y\right| \otimes G_{xy} = \sum_{i=1}^{N} A_i, \tag{16}$$

where

$$A_i = \sum_{(x,y)\in\mathcal{S}_i} \left|x\right\rangle\left\langle y\right| \otimes G_{xy}, \tag{17}$$

and the set $\mathcal{S}_i = \left\{(x,y) : S^i_{xy} = 1\right\}$. Next, we use the following fact

$$\left\|G\right\| \leq \sum_{i=1}^{N} \left\|A_i\right\|. \tag{18}$$

Hence, in order to estimate the norm of $G$ we have to estimate the norm of $A_i$. This is just the maximal singular value of $A_i$ or, equivalently, the maximal eigenvalue of $A_i A_i^\dagger$, squared.

Since this operator is block diagonal ($S^i$ are permutation operators), we have to calculate the maximal singular value of $G_{xy}$, taking into account the proper index of $i$. To this end, let us estimate this singular value of $G_{xy}$, which possess the following general form

$$G_{xy} = \sum_{a,b=1}^{d} \alpha_{xy}^{ab} e^{i\psi_{xy}^{ab}} \sqrt{p(a|x)\, p(b|y)}\, |a\rangle \langle b|\,, \tag{19}$$

where $\alpha_{xy}^{ab} e^{i\psi_{xy}^{ab}} = \langle \phi_x^a | \phi_y^b \rangle$ and $\alpha_{xy}^{ab} = |\langle \phi_x^a | \phi_y^b \rangle|$ while $\psi_{xy}^{ab}$ are phases for given indices $a$, $b$, $x$ and $y$. This results in

$$\begin{aligned} G_{xy}\, G_{xy}^{\dagger} &= \sum_{a,b,a',b'=1}^{d} \alpha_{xy}^{ab} \alpha_{xy}^{a'b'} e^{i\left(\psi_{xy}^{ab} - \psi_{xy}^{a'b'}\right)} \sqrt{p(a|x)\, p(b|y)\, p(a'|x)\, p(b'|y)}\, |a\rangle \langle b|\, |b'\rangle \langle a'| \\ &= \sum_{a,b,a'=1}^{d} \alpha_{xy}^{ab} \alpha_{xy}^{a'b} e^{i\left(\psi_{xy}^{ab} - \psi_{xy}^{a'b}\right)} p(b|y) \sqrt{p(a|x)\, p(a'|x)}\, |a\rangle \langle a'|\,. \end{aligned} \tag{20}$$

Here we use the fact that $G_{xy}G_{xy}^{\dagger} \geq 0$. This means that $\mathrm{Tr}\left\{G_{xy}G_{xy}^{\dagger}\right\} = \sum_{j=1}^{d} \lambda_{xy}^{j}$, where $\lambda_{xy}^{j}$ are eigenvalues of $G_{xy}G_{xy}^{\dagger}$. Let us denote the maximal eigenvalue as $\lambda_{xy}^{\max} = \max_j\left\{\lambda_{xy}^{j}\right\}$. From (20) we obtain the maximal singular value of $G_{xy}$, $\sigma_{xy}^{\max}$

$$\begin{aligned}
\left(\sigma_{xy}^{\max}\right)^2 = \lambda_{xy}^{\max} \leq \sum_{i=1}^{d} \lambda_{xy}^{i} = \mathrm{Tr}\left\{G_{xy}G_{xy}^{\dagger}\right\} \\
= \mathrm{Tr}\left\{\sum_{a,b,a'=1}^{d} \alpha_{xy}^{ab}\alpha_{xy}^{a'b} e^{i\left(\psi_{xy}^{ab}-\psi_{xy}^{a'b}\right)} p_y^b \sqrt{p_x^a p_x^{a'}}\, |a\rangle\langle a'|\right\} \\
= \sum_{a,b=1}^{d} \left(\alpha_{xy}^{ab}\right)^2 p_y^b p_x^a \leq \sum_{a,b=1}^{d} \left(\alpha_{xy}^{\max}\right)^2 p_y^b p_x^a = \left(\alpha_{xy}^{\max}\right)^2.
\end{aligned} \tag{21}$$

Therefore, in order to estimate the norm of $G$ we must calculate the maximal absolute value of the overlap between vectors $\alpha_{xy}^{\max}$ of the basis given by the number $x, y$. Then, the norm

$$\|G\| \leq \sum_{i=1}^{N} C_i, \tag{22}$$

where $C_i = \alpha_i^{\max} = \max_{x,y}\{\alpha_{xy}^{max} : (x,y) \in \mathcal{S}_i\}$ ($C_i$ is just Mussen-Uffink value for each $i$ ). Let us observe that for $x = y$ it is just identity transformation between these two bases (it corresponds to the case $i = N$) hence $\alpha_N^{\max} = 1$ and

$$S_{\text{LHS}} \leq \|G\| \leq \sum_{i=1}^{N} C_i \leq 1 + \sum_{i=1}^{N-1} C_i. \tag{23}$$

Finally, the violation of the steering inequality

$$V_{\text{Q}} \geq \frac{N}{1 + \sum_{i=1}^{N-1} C_i}, \tag{24}$$

- Random functional
- MUB's
- Clifford observables
- The case of Cavalcanti and Skrzypczyk (SDP 2016)

- We now would like to turn our abstract mathematical result into a form which could be tested in a laboratory. Let us consider the source of independent pairs of photons entangled in their polarizations i.e. many copies of singlet states: $|\Psi\rangle = |\psi_-\rangle^{\otimes k}$. We assume a single pair fidelity $F < 1$ and let us take into account the efficiency of detectors $\eta$ at Alice side, and let us assume the relaxed MUB condition $C \leq \sqrt{d^{\epsilon-1}}$. In this case the local dimension of the Hilbert space is $d = 2^k$ and we take the number of setting growing slower that dimension, $N = d^{1-\sigma}$ , $0 \leq \sigma < 1$. This leads to the ratio $V_Q^\eta = \frac{\left(2^{1-\sigma}\eta F\right)^k}{1+(2^{k-1}-1)2^{\frac{\epsilon-1}{2}k}}$ which leads to an unbounded violation if only $\epsilon + 2\sigma < 1 + 2\log_2(\eta F)$ . It is remarkable that for any fidelity and efficiency satisfying $\eta F > \frac{1}{\sqrt{2}}$ exist $\epsilon$ such that the unbounded in fact exponential in the number k of the entangled pairs, violation is possible.

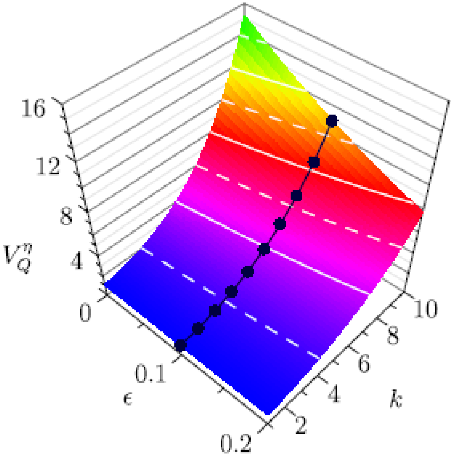



Figure: Here we illustrate the dependence of the the parameter $V_Q$ illustrating violation of the steering inequality on the number k of the qubit pairs involved and the relaxed MUB parameter $\epsilon$. The fidelity has been choosen $F = 0.98$ and the detector efficiency $\eta = 0.95$

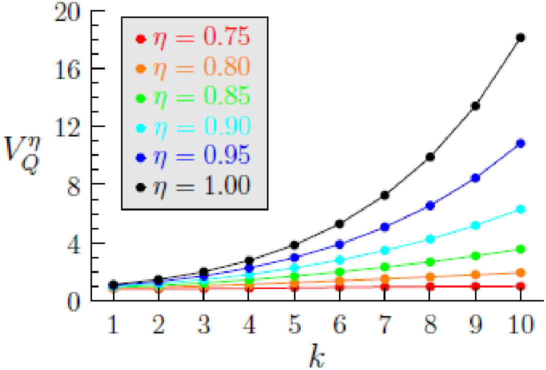



Figure: Quantum violation $V_Q^\eta$ of steering inequality computed for relaxed MUB with $\epsilon = 0.2$, $F = 0.98$ and different values of efficiency $\eta$.

- Now we will employ a quantum-optical scheme based on a parametric-down-conversion source, generating polarization entangled squeezed vacuum states. Their quantum correlations posses the same rotational invariance as a usual two-photon polarization singlet and can be seen as two copies of approximate original EPR correlations. Using the same key feature and implementing the relaxed MUBs by simple polarization rotations, we will show that entangled squeezed vacuum states lead to unbounded violation of our steering inequality.
- Entangled squeezed vacuum is a superposition of $2d$-photon polarization Bell-singlet states $|\psi_d\rangle = \frac{1}{\sqrt{d+1}}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)^d |0\rangle$ with a probability amplitude $\lambda_d$, $|\Psi\rangle = \sum_{d=0}^{\infty} \lambda_d |\psi_d\rangle$, where $a^\dagger$ $(b^\dagger)$ is creation operator for a spatial mode $a$ $(b)$ and $H$ $(V)$ denotes horizontal (vertical) polarization. Perfect correlations present in each multi-particle polarization singlet are manifested by equal photon numbers in orthogonal polarizations in the spatial modes:

$$|\psi_d\rangle = \frac{1}{\sqrt{d+1}} \sum_{m=0}^{d} (-1)^m |m_H, (d-m)_V\rangle_a |(d-m)_H, m_V\rangle_b . \tag{25}$$

*Proposition. Given a set of N Bob's measurement bases* $\{|\phi_x^m\rangle\} := \{|\phi^m(\theta_x)\rangle\}$ *with* $m = 0, \ldots, d$ *and* $x = 1, \ldots, N$*, defined by some set of angles* $0 \leq \theta_x < \frac{\pi}{2}$*, the maximal overlap* $C = \max_{x,y,a,b} |\langle \phi_x^a | \phi_y^b \rangle|$ *equals the maximal overlap between* $\{|\phi^m(0)\rangle\}$ *and* $\{|\phi^m(\theta)\rangle\}$ *with* $\theta = \min_{x,y} \left|\theta_x - \theta_y\right|$*:*

$$C(\theta, d) = \max_{m,n} \left| \langle \phi^n(0) | \phi^m(\theta) \rangle \right| = \\ = \sqrt{\binom{d}{q_{\theta,d}}} (\cos\theta)^d (\tan\theta)^{q_{\theta,d}} \tag{26}$$

*where* $q_{\theta,d} := \lfloor d \sin^2\theta - \cos^2\theta \rfloor + 1$ *and* $\lfloor \ldots \rfloor$ *denotes the floor function.* $C(\theta, d)$ *goes to zero as fast as* $1/\sqrt[4]{d}$*.*

- Including experimental imperfections in their simplest form, we assume equal efficiency $\eta$ for all the detectors (two at Bob's and two at Alice's). For the multi-particle Bell-singlet states (25) this modifies the quantum value of the steering functional to $\eta^d S_\mathrm{Q}$ and condition (8) to $V_\mathrm{Q}^\eta \geq \dfrac{\eta^d N(d)}{1+(N(d)-1)C(\theta,d)}$.

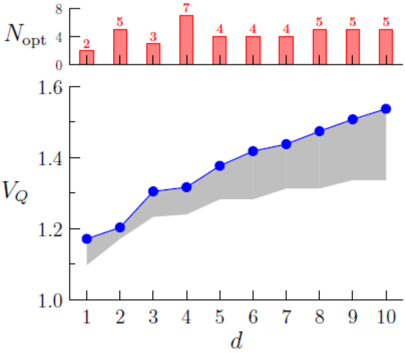



Figure: Quantum violation $V_Q^\eta$ of steering inequality computed for relaxed MUB with $\epsilon = 0.2$, $F = 0.98$ and different values of efficiency $\eta$.

# Outline

- We have provided the sufficient condition for unbounded violation of steering inequalities
- This is the first quantum steering inequality which is formulated in an error-tolerant way
- Violation of this inequality with multi-particle quantum correlations seems feasible:
- applied to multiple copies of a singlet state may enable violation of order of $O(\sqrt{d})$.
- multi-particle bipartite steering based on polarization entangled squeezed vacuum allows violation of order of $O(\sqrt[4]{d})$
- what is necessary condition?

Thanks for Your attention