New Bell inequalities for three-qubit pure states

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Introduction : Nonlocality and Entanglement

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[2] N. Gisin, Phys. Lett. A, 154 (1991) 201.

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- Unlike pure bipartite case, the relationship between entanglement and nonlocality is not simple even for pure multipartite states. Using Hardy's argument it was shown that all pure entangled states violate a single Bell inequality [4].

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[2] N. Gisin, Phys. Lett. A, 154 (1991) 201.

[3] J. F. Clauser *et al.*, Phys. Rev. Lett., 23 (1969) 880.

[4] Sixia Yu *et al.*, Phys. Rev. Lett., 109 (2012) 120402.

Tripartite Entanglement and Nonlocality

- A state $|\psi\rangle$ is a pure separable or product state if it can be written in the form $|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle$, a pure biseparable state if it can be written as $|\psi_1\rangle \otimes |\psi_{23}\rangle$ or in other permutations and is genuinely entangled if it cannot be written in a product form.

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- Idea of non-separability according to Bell locality comes from the inability of construction of a LHV model for observed correlations. If the joint probability can be written as, $P(a_1 a_2 a_3|A_1 A_2 A_3) = \int d\lambda \rho(\lambda) P_\lambda(a_1|A_1) P_\lambda(a_2|A_2) P_\lambda(a_3|A_3)$, then the model is called the well known LHV model. The intermediate case is the hybrid local-nonlocal model, first considered by Svetlichny[5]. And the last situation is genuine tripartite nonlocality, where three particles are allowed to share arbitrary correlations.

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[5] G. Svetlichny, Phys. Rev. D, 35 (1987) 3066.

Multipartite Bell inequalities

- Violation of MABK inequalities[6] gives sufficient criteria to distinguish separable states from entangled ones. But it is not a necessary condition as $|\psi\rangle = \cos\alpha\,|0...0\rangle + \sin\alpha\,|1...1\rangle$ would not violate MABK inequalities[7] for $\sin 2\alpha \leq 1/\sqrt{2^{N-1}}$.

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- A class of states (generalized GHZ states within a specified parameter range for odd number of qubits)[8] do not violate any correlation Bell inequalities, WWŻB inequalities.[9]

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[7] V. Scarani and N. Gisin, J. Phys. A, 34 (2001) 6043.

[8] M. Żukowski *et al.*, Phys. Rev. Lett., 88 (2002) 210402.

[9] R. F. Werner and M. M. Wolf, Phys. Rev. A., 64 (2001) 032112; M. Żukowski and Č. Brukner, Phys. Rev. Lett., 88 (2002) 210401.

Distinguishing Entangled states

- In general, it is very difficult to discriminate between biseparable and genuinely entangled states. MABK inequalities give sufficient condition to distinguish them[10].

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- They can not discriminate between the correlations due to tripartite genuine entanglement, local-nonlocal hybrid model and genuine tripartite nonlocality[11] .

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- Svetlichny's inequality[12] holds for three-particle local-nonlocal hybrid model and its violation guarantees the presence of tripartite genuine nonlocality. But, some tripartite genuinely entangled states do not violate Svetlichnys inequality[13].

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- Svetlichny's inequality[12] holds for three-particle local-nonlocal hybrid model and its violation guarantees the presence of tripartite genuine nonlocality. But, some tripartite genuinely entangled states do not violate Svetlichnys inequality[13].
- By a strictly weaker definition of genuine tripartite nonlocality[14], it was conjectured that genuine tripartite entanglement and genuinely tripartite nonlocality are same.

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[14] J. D. Bancal *et al.*, Phys. Rev. A, 88 (2013) 014102.

New inequalities : Context

- We introduce a new set of twelve Bell inequalities, such that if one or more violations from the set is obtained for a three qubit state then the state is entangled. Each inequalities within the set is violated by all generalized GHZ states, though they don't always violate MABK or WWZB inequalities.

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- Our set inequalities can always distinguish between separable, biseparable and genuinely entangled three qubit pure states from the pattern of their violations.

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- Our set inequalities can always distinguish between separable, biseparable and genuinely entangled three qubit pure states from the pattern of their violations.
- Numerically we have got evidence that any three qubit pure state will violate atleast one Bell inequality from the set. This may extend the Gisin's theorem without invoking Hardy's argument.

List of Bell inequalities

$$A_2B_1(C_1+C_2)+B_2(C_1-C_2) \leq 2, \quad (1)$$
$$B_1(C_1+C_2)+A_1B_2(C_1-C_2) \leq 2, \quad (2)$$
$$A_2B_1(C_1+C_2)+A_1(C_1-C_2) \leq 2, \quad (3)$$
$$A_2(C_1+C_2)+A_1B_2(C_1-C_2) \leq 2, \quad (4)$$
$$(B_1+B_2)C_2+A_2(B_1-B_2)C_1 \leq 2, \quad (5)$$
$$A_1(B_1+B_2)C_2+(B_1-B_2)C_1 \leq 2, \quad (6)$$
$$A_1(B_1+B_2)+A_2(B_1-B_2)C_1 \leq 2, \quad (7)$$
$$A_1(B_1+B_2)C_2+A_2(B_1-B_2) \leq 2, \quad (8)$$
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Motivation behind the inequalities

- To motivate these inequalities, our starting point is CHSH inequality: $A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 \leq 2$. From Tsirelson's bound [15], maximum value this operator can achieve for quantum states is $2\sqrt{2}$. This value is achieved for the maximally entangled states - Bell states.

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- For a suitable measurements choice : $A_1 = \sigma_x$, $A_2 = \sigma_z$, $B_1 = 1/\sqrt{2}(\sigma_x + \sigma_z)$ and $B_2 = 1/\sqrt{2}(\sigma_x - \sigma_z)$, the CHSH operator takes the form $\sqrt{2}(\sigma_x \otimes \sigma_x + \sigma_z \otimes \sigma_z)$, with $|\phi^+\rangle$ as its eigenstate with eigenvalue $2\sqrt{2}$[16].

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- Now, we want to construct an operator for three-qubit pure states such that, the GHZ state of three qubits will be the eigenstate of this operator with highest eigenvalue.

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- Now, we want to construct an operator for three-qubit pure states such that, the GHZ state of three qubits will be the eigenstate of this operator with highest eigenvalue.
- The GHZ state, $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, is the eigenstate of the operator $\sqrt{2}(\sigma_x \otimes \sigma_x \otimes \sigma_x + \sigma_z \otimes \sigma_z \otimes I)$ with eigenvalue $2\sqrt{2}$.

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Motivation behind the inequalities (contd.)

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- Also from previous discussion, it is clear that to obtain violations for all pure entangled states, correlation Bell inequalities are not enough. So, it seems that non-correlation Bell inequalities may work.
- A general state need not have any symmetry. So, we have to consider a set of Bell inequalities instead of one, such that one measurement is done on either Alice or Bob or Charlie. The one measurement by one of the parties is necessary.

Violation by generalized GHZ states

We will now show that for states in generalized GHZ class we can always obtain violations of these inequalities. Let's consider,

$$|GGHZ\rangle = \alpha\,|000\rangle + \beta\,|111\rangle \tag{13}$$

Then we choose the inequality to be,

$$A_1(B_1 + B_2) + A_2(B_1 - B_2)C_1 \leq 2 \tag{14}$$

Now, we choose the operators such that,

$$A_1 = \sigma_z, \;\; A_2 = \sigma_x \tag{15}$$

$$B_1 = \cos\theta\sigma_x + \sin\theta\sigma_z, \;\; B_2 = -\cos\theta\sigma_x + \sin\theta\sigma_z \tag{16}$$

$$C_1 = \sigma_x \tag{17}$$

Violation by generalized GHZ states (contd.)

We will now calculate the expectation value of the mentioned Bell operator for the state for these measurement settings, i.e

$$\langle GGHZ|\, A_1(B_1 + B_2) + A_2(B_1 - B_2)C_1 \,|GGHZ\rangle \tag{18}$$

Which comes out to be,

$$2[2\alpha\beta\cos\theta + (\alpha^2 + \beta^2)\sin\theta] = 2[2\alpha\beta\cos\theta + \sin\theta] \tag{19}$$

Now,

$$a\sin\phi + b\cos\phi \leq \sqrt{a^2 + b^2} \tag{20}$$

Hence, we get,

$$\langle GGHZ|\, A_1(B_1 + B_2) + A_2(B_1 - B_2)C_1 \,|GGHZ\rangle \leq 2\sqrt{1 + 4\alpha^2\beta^2} \tag{21}$$

Which is always greater than 2 for nonzero α, β and gives maximum value $2\sqrt{2}$ for the conventional GHZ state.

Quantifying Entanglement?

- Quantification of entanglement in multipartite scenario is a messy business. Unlike pure bipartite system, there is no unique measure of entanglement for multipartite states.[17] [18] We will use the average of Von Neumann entropy over each bipartition as a suitable measure of multipartite entanglement.

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- From the plot it will be clear that this entanglement measure and the maximum amount of Bell violation for generalized GHZ states are monotonically related to each other. We can say that the more is the entanglement of a state the more nonlocal it is.

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Plots

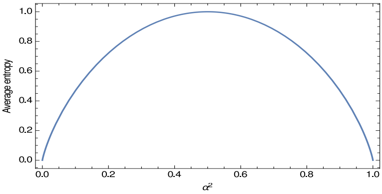

Figure: Average Von Neumann entropy over three bipartitions vs α^2 plot.

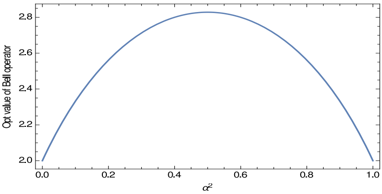

Figure: Maximum expectation value of the Bell operator for a generalized GHZ state vs α^2 plot.

Any separable state obeys all the inequalities

All separable pure three-qubit states can be written, after applying some convenient local unitary transformation as as $|0\rangle\,|0\rangle\,|0\rangle$. We use one of the operators, say $A_1 B_2(C_1 + C_2) + B_1(C_1 - C_2)$. Now,

$$A_1 = \sin\theta_1 \cos\phi_1 \sigma_x + \sin\theta_1 \sin\phi_1 \sigma_y + \cos\theta_1 \sigma_z \tag{22}$$

and similarly for other observables A_2, B_1, B_2, C_1, C_2, for which the parameters are θ_2, ϕ_2; θ_3, ϕ_3; θ_4, ϕ_4; θ_5, ϕ_5; θ_6, ϕ_6 respectively. Putting these measurement settings, we get the expectation value to be,

$$\cos\theta_2(\cos\theta_5 - \cos\theta_6) + \cos\theta_1 \cos\theta_3(\cos\theta_5 + \cos\theta_6). \tag{23}$$

We can write this as : $X_1(Y_1 + Y_2) + X_2(Y_1 - Y_2)$, where $X_1 = \cos\theta_1 \cos\theta_3$, $X_2 = \cos\theta_2$, $Y_1 = \cos\theta_5$, $Y_2 = \cos\theta_6$, and $X_1, X_2, Y_1, Y_2 \leq 1$. So, clearly the maximum possible value of this operator is 2.

Discriminating different types of entanglement

We can rewrite any biseparable state by local unitary transformations equivalent form of $|0\rangle (\alpha |0\rangle |0\rangle + \beta |1\rangle |1\rangle)$. Inequalities, which can explore the entanglement between the second and the third qubit will be violated. For example, $A_1 B_2 (C_1 + C_2) + B_1 (C_1 - C_2) \leq 2$ will be violated, as CHSH type operator for second and third qubits is embedded in this operator. So, the amount of violation will be exactly same as in the case of two-qubit entangled state and the CHSH operator. There are other three inequalities within this set, which will also be violated.

$$
\begin{aligned}
B_1(C_1 + C_2) + A_1 B_2 (C_1 - C_2) &\leq 2, & (24)\\
A_1(B_1 + B_2) C_2 + (B_1 - B_2) C_1 &\leq 2, & (25)\\
(B_1 + B_2) C_2 + A_2 (B_1 - B_2) C_1 &\leq 2. & (26)
\end{aligned}
$$

No other states (except biseparable pure states) will have same kind of violations, i.e exactly four violations with the same maximal amount.

Proposition for genuine three qubit entangled pure states

- Any genuinely entangled three-qubit pure state can be written in a canonical form[19] with six parameters,

$$|\psi\rangle = \lambda_0 |0\rangle |0\rangle |0\rangle + \lambda_1 e^{i\phi} |1\rangle |0\rangle |0\rangle + \lambda_2 |1\rangle |0\rangle |1\rangle + \lambda_3 |1\rangle |1\rangle |0\rangle + \lambda_4 |1\rangle |1\rangle |1\rangle , \quad (27)$$

where $\lambda_i \geq 0$, $\sum_i {\lambda_i}^2 = 1$, $\lambda_0 \neq 0$, $\lambda_2 + \lambda_4 \neq 0$, $\lambda_3 + \lambda_4 \neq 0$ and $\phi \in [0, \pi]$.

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  where $\lambda_i \geq 0$, $\sum_i {\lambda_i}^2 = 1$, $\lambda_0 \neq 0$, $\lambda_2 + \lambda_4 \neq 0$, $\lambda_3 + \lambda_4 \neq 0$ and $\phi \in [0, \pi]$.

- We have randomly generated 35,000 states and tested our set of Bell inequalities. The expectation value of a Bell operator is optimized by considering all possible measurement settings for all observables. Starting from the inequality (1) from the set, we continued with other inequalities one after one until all the generated states violate one inequality from the set.

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Some plots

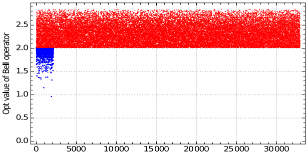

Figure: Optimum value of the Bell operator (1). Out of 35000 states, 2099 states do not violate this inequality. States which violate the inequality are shown by red points and those do not are shown by blue points.

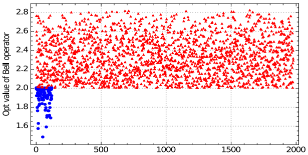

Figure: Optimum value of the Bell operator (2). Out of 2099 states, 120 states do not violate this inequality. States which violate the inequality are shown by red points and those do not are shown by blue points.

Multiparty generalization

- This extension for multi-qubit scenario is straight-forward. One will have to distinguish between two cases – odd number of qubits and even number of qubits. For even n, there will be a set of n inequalities; while for odd n, the it is $2n(n-1)$.

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- When n is odd. n-qubit GHZ states is the eigenstate of $\sqrt{2}(\sigma_x \otimes \sigma_x \otimes \sigma_x \otimes \cdots \otimes \sigma_x^{nth} + \sigma_z \otimes \sigma_z \otimes \cdots \otimes \sigma_z^{(n-1)th} \otimes I)$ with the highest eigenvalue $2\sqrt{2}$.

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- The first two Bell inequalities (1) and (2) can be easily generalized for n-qubit pure states as,

$$A_1 A_2 A_3 A_4 A_5 ..(A_n + A'_n) + A'_2 A'_3 A'_4 A'_5 ..(A_n - A'_n) \leq 2, \quad (28)$$

$$A_2 A_3 A_4 A_5 ..(A_n + A'_n) + A_1 A'_2 A'_3 A'_4 A'_5 ..(A_n - A'_n) \leq 2. \quad (29)$$

Multiparty generalization (contd.)

- One can prove like before that any of these $2n(n-1)$ inequalities are violated maximally by all generalized GHZ states with odd n.

Multiparty generalization (contd.)

- One can prove like before that any of these $2n(n-1)$ inequalities are violated maximally by all generalized GHZ states with odd n.
- GHZ state of n qubits (n is even) is the eigenstate of $\sqrt{2}(\sigma_x \otimes \sigma_x \otimes \sigma_x \otimes \cdots \otimes \sigma_x^{nth} + \sigma_z \otimes \sigma_z \otimes \cdots \otimes \sigma_z^{(n-1)th} \otimes \sigma_z)$ with highest eigenvalue $2\sqrt{2}$. This suggests that correlation Bell inequalities are required in this case.

Multiparty generalization (contd.)

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&\qquad (A_1 - A_1')A_2'A_3'A_4'A_5'..A_n' \leq 2. \qquad (30)
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- The fact that generalized GHZ states with even number of qubits violate a correlation Bell inequality within the set of all correlation Bell inequalities was known[20].

[20] M. Żukowski *et al.*, Phys. Rev. Lett., 88 (2002) 210402.

Conclusions

- We have presented a new set of twelve non-correlation Bell inequalities such that all three qubit generalized GHZ states violate them with maximal violation $2\sqrt{2}$. In our inequalities one party makes only one measurement in place of two.

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- From numerical evidence we concluded that every three qubit pure state would violate atleast one inequality within the set.
- Lastly, we have generalized the set of inequalities for n qubit pure states, for both even and odd n. Non-correlation Bell inequalities are required for odd n, whereas correlation Bell inequalities are sufficient for even n.

THANK YOU