Complementarity between tripartite quantum correlation and bipartite Bell inequality violation in three qubit states

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Plan of the talk

- Motivation of the study
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- Bipartite Bell inequality violation in three qubit states
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P. Pandya, AM, I. Chakrabarti, PRA 94, 052126, 2016
Motivations

• Nonlocality \rightarrow key resource for key distribution, quantum randomness generation etc.
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- Linking quantum correlations and nonlocality in multiparty systems $\rightarrow$ important yet challenging problem.
  Horodecki et al., PLA, 222, 21 (1996)
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- Linking quantum correlations and nonlocality in multiparty systems $\rightarrow$ important yet challenging problem.
  Horodecki et al., PLA, 222, 21 (1996)

- Statistics of two or one body $\rightarrow$ crucial for $\rightarrow$ many body nonlocality and entanglement criterion.
  J. Tura et al., Science 344, 1256 (2014)
For an arbitrary two-qubit state, $\rho$, maximum Bell-CHSH $S_{\rho}$ value

$$S_{\rho} = 2\sqrt{M(\rho)}$$
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$M(\rho) = m_1 + m_2$, with $m_1$ and $m_2$ being the two largest eigenvalues of $T_\rho^T T_\rho$

$$(T_\rho)_{ij} = \text{Tr}(\sigma_i \otimes \sigma_j \rho)$$
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$$M(\rho) > 1$$

The amount of Bell-CHSH inequality violation is $B(\rho_{AB}) = \max\{0, M(\rho_{AB}) - 1\}$
Monogamy of Bell Inequality Violation

For a three qubit quantum state, if the quantum state shared by any two subparts of the three party system leads to the Bell inequality violation, then it precludes its violation for the states which the two subparts share with the third party of the tripartite system.
Monogamy of Bell Inequality Violation

For a three qubit quantum state, if the quantum state shared by any two subparts of the three party system leads to the Bell inequality violation, then it precludes its violation for the states which the two subparts share with the third party of the tripartite system.

Figure: Monogamy of Bell inequality violation.
Monogamy of Bell Inequality Violation

For a three qubit quantum state, if the quantum state shared by any two subparts of the three party system leads to the Bell inequality violation, then it precludes its violation for the states which the two subparts share with the third party of the tripartite system.

\[ B(\psi) = \max\{B(\rho_{AB}), B(\rho_{BC}), B(\rho_{AC})\} \]

*Figure:* Monogamy of Bell inequality violation.
Genuinely entangled three qubit pure states  

\( |\psi\rangle_{GHZ} = \sqrt{K} \left( c_\delta |000\rangle + s_\delta e^{i\phi} |\varphi_A\rangle |\varphi_B\rangle |\varphi_C\rangle \right) \)

\( |\psi\rangle_{W} = \sqrt{W} \left( \sqrt{d} |000\rangle + \sqrt{a} |001\rangle + \sqrt{b} |010\rangle + \sqrt{c} |100\rangle \right) \)

\( W = (a + b + c + d) \) is the normalizing constant.
Genuinely entangled three qubit pure states

\[ |\psi\rangle_{GHZ} = \sqrt{K} \left( c_\delta |000\rangle + s_\delta e^{i\phi} |\varphi_A\rangle |\varphi_B\rangle |\varphi_C\rangle \right) \]

\( c_\delta \) and \( s_\delta \) stand for \( \cos \delta \) and \( \sin \delta \) respectively

\[ K = \left( 1 + c_\alpha c_\beta c_\gamma c_\phi s^2_\delta \right)^{-1} \] is the normalizing constant

\[ |\varphi_A\rangle = c_\alpha |0\rangle + s_\alpha |1\rangle , |\varphi_B\rangle = c_\beta |0\rangle + s_\beta |1\rangle , |\varphi_C\rangle = c_\delta |0\rangle + s_\delta |1\rangle \]

\( \alpha, \beta, \gamma \in (0, \pi/2] \), \( \delta \in (0, \pi/4] \) and \( \phi \in [0, 2\pi) \)
Genuinely entangled three qubit pure states

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\[ \alpha, \beta, \gamma \in (0, \pi/2], \delta \in (0, \pi/4] \text{ and } \phi \in [0, 2\pi) \]

\[ |\psi\rangle_{GHZ^R} = \sqrt{K} \left( c_\delta |000\rangle + s_\delta |\varphi_A\rangle |\varphi_B\rangle |\varphi_C\rangle \right) \]
Genuinely entangled three qubit pure states

\[ |\psi\rangle_{\text{GHZ}} = \sqrt{K} \left( c_\delta |000\rangle + s_\delta e^{i\phi} |\varphi_A\rangle |\varphi_B\rangle |\varphi_C\rangle \right) \]

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\( \alpha, \beta, \gamma \in (0, \pi/2], \delta \in (0, \pi/4] \) and \( \phi \in [0, 2\pi) \)

\[ |\psi\rangle_{\text{GHZR}} = \sqrt{K} \left( c_\delta |000\rangle + s_\delta |\varphi_A\rangle |\varphi_B\rangle |\varphi_C\rangle \right) \]

\[ |\psi\rangle_W = \frac{1}{\sqrt{\mathcal{W}}} \left( \sqrt{d} |000\rangle + \sqrt{a} |001\rangle + \sqrt{b} |010\rangle + \sqrt{c} |100\rangle \right) \]

\( \mathcal{W} = (a + b + c + d) \) is the normalizing constant

\[ a, b, c > 0 \text{ and } d = 1 - (a + b + c) \geq 0. \]
Tangle vs Bell inequality violation

\[
\tau(\psi_{ABC}) = C_{A:BC}^2 - C_{AB}^2 - C_{AC}^2.
\]

\[
C(\rho_{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},
\]

\(\lambda_i\) are the square roots of the eigenvalues of \(\rho_{AB}\tilde{\rho}_{AB}\) in decreasing order,

\[
\tilde{\rho}_{AB} = (\sigma_y \otimes \sigma_y)\rho_{AB}^*(\sigma_y \otimes \sigma_y); \rho_{AB}^* \text{ in computational basis.}
\]

CKW, PRA 61, 052306 (2000)
Tangle vs Bell inequality violation

**Theorem:** If the tangle of a three qubit pure state $|\psi\rangle_{\text{GHZ}}$ (or $|\psi\rangle_{\text{W}}$) is equal to the tangle of another three qubit pure state $|\psi\rangle_m$, i.e., $\tau(\psi_{\text{GHZ}}/\psi_{\text{W}}) = \tau(\psi_m)$, then the bipartite Bell inequality violations necessarily follow,

$$B(\psi_m) \geq B(\psi_{\text{GHZ}}/\psi_{\text{W}}).$$

$$|\psi\rangle_m = \frac{|000\rangle + |111\rangle + m( |010\rangle + |101\rangle )}{\sqrt{2+2m^2}}, \text{ where } m \in [0, 1]$$

The maximally Bell inequality violating state (MBV)
Tangle vs Bell inequality violation

**Theorem:** If the tangle of a three qubit pure state $|\psi\rangle_{\text{GHZ}}$ ($/|\psi\rangle_W$) is equal to the tangle of another three qubit pure state $|\psi\rangle_m$, i.e., $\tau(\psi_{\text{GHZ}}/\psi_W) = \tau(\psi_m)$, then the bipartite Bell inequality violations necessarily follow,

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The maximally Bell inequality violating state (MBV)

$$B(\psi) + \tau(\psi) \leq 1$$
Complementary relation: Tangle

Figure: Complementary relation between tangle and bipartite Bell inequality violation.
Conjecture proved! :-)

Anisotropic Invariance and the Distribution of Quantum Correlations

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We report the discovery of two new invariants for three-qubit states which, similarly to the three-tangle, are invariant under local unitary transformations and permutations of the parties. These quantities have a direct interpretation in terms of the anisotropy of pairwise spin correlations. Applications include a universal ordering of pairwise quantum correlation measures for pure three-qubit states; trade-off relations for anisotropy, three-tangle and Bell nonlocality; strong monogamy relations for Bell inequalities, Einstein-Podolsky-Rosen steering inequalities, geometric discord and fidelity of remote state preparation (including results for arbitrary three-party states); and a statistical and reference-frame-independent form of quantum secret sharing.

DOI: 10.1103/PhysRevLett.118.010401
Extension to mixed states

\[ \tau(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i \tau(\psi_i) \]
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\( \tau(\rho) \) by definition convex
Extension to mixed states

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\tau(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i \tau(\psi_i)
\]

\(\tau(\rho)\) by definition convex

\[
S(\rho_{AB}) = |\text{Tr}[(O_{AB}^m \otimes I_C)\rho_{ABC}]|
= \left| \sum_i p_i \left( \text{Tr}[(O_{AB}^m \otimes I_C) |\psi_{ABC}^i\rangle\langle\psi_{ABC}^i|] \right) \right|
\]
Extension to mixed states

\[ \tau(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i \tau(|\psi_i\rangle) \]

\(\tau(\rho)\) by definition convex

\[
S(\rho_{AB}) = |\text{Tr}[(O_{AB}^m \otimes I_C) \rho_{ABC}]| \\
= \left| \sum_i p_i \left( \text{Tr}[(O_{AB}^m \otimes I_C) |\psi_{ABC}^i\rangle\langle\psi_{ABC}^i|] \right) \right| \\
\]

\[
S(\rho_{AB}) \leq \sum_i p_i |\text{Tr}[(O_{AB}^m \otimes I_C) |\psi_{ABC}^i\rangle\langle\psi_{ABC}^i|]| \\
= \sum_i p_i |\text{Tr}[O_{AB}^m \rho_{AB}^i]| \\
\]
Extension to mixed states

\[ S(\rho_{AB}) \leq \sum_i p_i \left| \text{Tr}[O^{i,m}_{AB} \rho_{AB}] \right| , \]

\[ S \left( \sum_i p_i \rho_{AB}^i \right) \leq \sum_i p_i S(\rho_{AB}^i) \leq \sum_i p_i B(\rho_{AB}^i) \]
Extension to mixed states

\[ S(\rho_{AB}) \leq \sum_i p_i \left| \text{Tr}[O_{AB}^{i,m} \rho_{AB}^i] \right|, \]

\[ S \left( \sum_i p_i \rho_{AB}^i \right) \leq \sum_i p_i S(\rho_{AB}^i), \]

\[ \sqrt{M \left( \sum_i p_i \rho_{AB}^i \right)} \leq \sum_i p_i \left( \sqrt{M(\rho_{AB}^i)} \right). \]
Extension to mixed states

\[ S(\rho_{AB}) \leq \sum_i p_i \left| \text{Tr}[O_{AB}^{i,m} \rho_{AB}^i] \right| , \]
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\[ \sqrt{M \left( \sum_i p_i \rho_{AB}^i \right)} \leq \sum_i p_i \left( \sqrt{M(\rho_{AB}^i)} \right) \]
\[ B(\rho_{AB}) \leq \sum_i p_i B(\rho_{AB}^i) \]
Extension to mixed states

\[ S(\rho_{AB}) \leq \sum_i p_i \left| \text{Tr}[O_{AB}^i \rho_{AB}^i] \right| , \]

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\[ \sqrt{M \left( \sum_i p_i \rho_{AB}^i \right)} \leq \sum_i p_i \left( \sqrt{M(\rho_{AB}^i)} \right) \]

\[ \mathcal{B}(\rho_{AB}) \leq \sum_i p_i \mathcal{B}(\rho_{AB}^i) \]

Similarly, for \( \rho_{AC} \) and \( \rho_{BC} \)…
Extension to mixed states

\[ B(\rho_{AB}) \leq \sum p_i B(\rho_{AB}^i) \leq \sum p_i [\max\{B(\rho_{AB}^i), B(\rho_{BC}^i), B(\rho_{AC}^i)\}] \leq \sum p_i [B(|\psi_{ABC}^i\rangle)] \]
Extension to mixed states

\[
\mathcal{B}(\rho_{AB}) \leq \sum_i p_i \mathcal{B}(\rho_{iAB}) \\
\leq \sum_i p_i \left[ \max\{\mathcal{B}(\rho_{iAB}), \mathcal{B}(\rho_{iBC}), \mathcal{B}(\rho_{iAC})\} \right] \\
\leq \sum_i p_i \left[ \mathcal{B}(|\psi_{ABC}^i\rangle) \right]
\]

Therefore,

\[
\mathcal{B}(\rho_{ABC}) \leq \sum_i p_i \left[ \mathcal{B}(|\psi_{ABC}^i\rangle) \right]
\]
Extension to mixed states

\[ B(\rho_{AB}) \leq \sum_i p_i B(\rho_{iAB}) \]
\[ \leq \sum_i p_i [\max\{B(\rho_{iAB}), B(\rho_{iBC}), B(\rho_{iAC})\}] \]
\[ \leq \sum_i p_i [B(|\psi_{iABC}\rangle)] \]

Therefore,

\[ B(\rho_{ABC}) \leq \sum_i p_i [B(|\psi_{iABC}\rangle)] \]

\[ B(\rho) + \tau(\rho) \leq 1 \]
Genuine Tripartite Correlation vs Nonlocality

GGM vs Bell inequality violation

Generalized Geometric Measure of $|\psi_N\rangle$

\[
G(|\psi_N\rangle) = 1 - \Lambda_{\text{max}}^2(|\psi_N\rangle)
\]

\[
\Lambda_{\text{max}}(|\psi_N\rangle) = \max |\langle \chi |\psi_N\rangle|, |\chi\rangle : \rightarrow \text{not genuinely multiparty entangled.}
\]
Genuine Tripartite Correlation vs Nonlocality

**GGM vs Bell inequality violation**

Generalized Geometric Measure of $|\psi_N\rangle$

$$G(|\psi_N\rangle) = 1 - \Lambda_{\text{max}}^2(|\psi_N\rangle)$$

$$\Lambda_{\text{max}}(|\psi_N\rangle) = \max |\langle \chi |\psi_N\rangle|, |\chi\rangle : \rightarrow \text{not genuinely multiparty entangled.}$$

$$G(|\psi_n\rangle) = 1 - \max\{\lambda_{I:L}^2 |I \cup L = \{A_1, \ldots, A_N\}, I \cap L = \emptyset\}$$

$$\lambda_{I:L} : \rightarrow \text{maximal Schmidt coefficient in the bipartite split } I : L.$$
Genuine Tripartite Correlation vs Nonlocality

Lemma: If for a three qubit pure state $|\psi\rangle_{GHZ}^R (\text{or} |\psi\rangle_W)$, the GGM is obtained from, say the $A:BC$ bipartite split, then the only reduced bipartite system of $|\psi\rangle_{GHZ}^R (\text{or} |\psi\rangle_W)$ that can violate the Bell inequality is $\rho_{BC}$.
Lemma: If for a three qubit pure state $|\psi\rangle_{GHZ^R}$ ($|\psi\rangle_{W}$), the GGM is obtained from, say the $A:BC$ bipartite split, then the only reduced bipartite system of $|\psi\rangle_{GHZ^R}$ ($|\psi\rangle_{W}$) that can violate the Bell inequality is $\rho_{BC}$.

See also K. Sharma et al. PRA 93, 062344 (2016)
Genuine Tripartite Correlation vs Nonlocality

**GGM vs Bell inequality violation**

**Theorem:** If the GGM of a three qubit pure state $|\psi\rangle_{\text{GHZ}}$ (or $|\psi\rangle_{W}$) is equal to the GGM of another three qubit pure state $|\psi\rangle_m$, i.e., $G(\psi_{\text{GHZ}}/\psi_{W}) = G(\psi_m)$, then the bipartite Bell inequality violations necessarily follow,

$$B(\psi_m) \geq B(\psi_{\text{GHZ}}/\psi_{W}) .$$

$$|\psi\rangle_m = \frac{|000\rangle + |111\rangle + m(|010\rangle + |101\rangle)}{\sqrt{2+2m^2}} , \text{ where } m \in [0, 1]$$

The maximally Bell inequality violating state (MBV)
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The maximally Bell inequality violating state (MBV)

$$B(\psi) + 4G(\psi)(1 - G(\psi)) \leq 1$$
Complementary Relation: GGM

Figure: Complementary relation between GGM and bipartite Bell inequality violation.
Quantum Discord

The difference of total correlation and classical correlation

Total Correlation is quantified by the mutual information

\[ I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \]
\[ = S(\rho_{AB} || \rho_A \otimes \rho_B) \]

\[ S(\rho) = -\text{tr}(\rho \log \rho) \]
\[ S(\rho || \sigma) = -S(\rho) - \text{tr}(\rho \log \sigma) \]

The classical correlation is given in terms of the measured conditional entropy

\[ J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B}) \]
DMS vs Bell inequality violation

\[ J(\rho_{AB}) = \max_{\{P_i\}} I(\rho'_{AB}), \]
\[ \rho'_{AB} = \sum_i (\mathbb{I}_A \otimes P_i) \rho_{AB} (\mathbb{I}_A \otimes P_i) \]

Quantum Discord: \[ \rightarrow D(\rho_{AB}) = I(\rho_{AB}) - J(\rho_{AB}). \]
DMS vs Bell inequality violation

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Quantum Discord: \[ \rightarrow D(\rho_{AB}) = I(\rho_{AB}) - J(\rho_{AB}). \]

Discord Monogamy Score
Genuine Tripartite Correlation vs Nonlocality

DMS vs Bell inequality violation

\[ J(\rho_{AB}) = \max_{\{P_i\}} I(\rho'_{AB}), \]
\[ \rho'_{AB} = \sum_i (\mathbb{I}_A \otimes P_i) \rho_{AB} (\mathbb{I}_A \otimes P_i) \]

Quantum Discord: \( D(\rho_{AB}) = I(\rho_{AB}) - J(\rho_{AB}) \).

Discord Monogamy Score

\[ \delta_D(|\psi\rangle_{ABC}) = D(|\psi\rangle_{A|BC}) - D(\rho_{AB}) - D(\rho_{AC}) \]

Complementary Relation: DMS

Figure: Complementary relation between discord monogamy score and bipartite Bell inequality violation.
Conclusion

- We have seen that there exists a complementary relation between genuine tripartite quantum correlations and bipartite Bell inequality violation in three qubit states.
Conclusion

- We have seen that there exists a complementary relation between genuine tripartite quantum correlations and bipartite Bell inequality violation in three qubit states.

- The MBV states $|\psi\rangle_m$, exhibit maximum Bell inequality violation for a fixed amount of genuine tripartite correlation.
thanks!