Quantum mechanical violation of macrorealism for large spin and its robustness against coarse-grained measurements

Debarshi Das

Bose Institute, Kolkata

Plan of the talk

1. Some remarks on Macrorealism (MR) - Realism + Noninvasive Measurability (NIM).

2. Proposal of various necessary conditions for Macrorealism: *Leggett-Garg inequality* (LGI), *Wigner's form of the Leggett-Garg inequality* (WLGI), and the *No-Signalling in Time* (NSIT) condition.

3. Results of recent studies concerning the quantum mechanical (QM) violation of LGI, WLGI & NSIT in the limit of *large spin* for coarse-grained measurement.

Some Fundamental Quantum Questions

1. What is the relationship between the weird behaviour of the microscopic world described by quantum physics and the everyday macroscopic world of human experience?

2. Under what conditions do classical laws emerge out of quantum mechanics (QM)?

3. To what extent it is possible to test QM in the macrolimit?

Some Significant Experimental Developments regarding the Probing of QM in the Macro-Domain

- Quantum Teleportation achieved over a distance of 143 km.
- Quantum Interference of C_{60} molecules (size $\sim$ 1nm) with mass $=$ 720 amu
- Quantum Interference of bigger biomolecules of size $\sim$ 2nm and mass $\sim 2 \times 10^3$ amu
- Quantum Interference of organic molecules of size $\sim$ 6 nm with 430 atoms and masses up to $\sim 7 \times 10^3$ amu
- Expt. tests of Macroscopic Quantum Coherence for SQUID (Superconducting Quantum Interference Device) systems involving superposition of micro-nano amperes current involving $\sim 10^{15}$ electrons flowing along clockwise and anticlockwise directions.

The term 'macroscopic' here is taken to denote a system with high dimensionality, or a low-dimensional system with large mass, or involving large value of any other relevant parameter.

The notion of Macrorealism (MR)

- *Macrorealism (MR)* entails the conjunction of the notions of *Realism* and *Noninvasive Measurability (NIM)*.
- *Realism*: At *any* instant, irrespective of measurement, a system is in a *definite* one of the available states and *all* its observable properties have *definite values*.

  ⇒Most natural point of view based on everyday experience. (A tossed coin lying on the floor is either heads or tails, even if we don't know which)
- *Noninvasive Measurability (NIM)*: It is possible, at least in principle, to *determine which* of the states the system is in, *without affecting* the state itself or the system's subsequent evolution.

  ⇒Natural point of view in the classical conception. (One can measure the temperature of a glass of water without perturbing its thermal properties)

Various necessary conditions proposed for testing macrorealism (MR)

1. *Leggett-Garg Inequality (LGI)*: [A. J. Leggett and A. Garg, *Phys. Rev. Lett.* **54**, 857 (1985)]

   Derived as a testable algebraic consequence of the deterministic form of Macrorealism.

2. *Wigner's form of LGI (WLGI)*: [D. Saha, S. Mal, P. K. Panigrahi, D. Home, *Phys. Rev. A* **91**, 032117 (2015)]

   Derived as a testable algebraic consequence of the probabilistic form of Macrorealism.

3. *No-Signalling in Time (NSIT)*: [J. Kofler and C. Brukner, *Phys. Rev. A* **87**, 052115 (2013)]

   This condition is formulated as a statistical version of NIM to be satisfied by any macrorealist theory. Violation of NSIT implies violation of NIM at an individual macrorealist level.

Significance of studying the QM violation of MR

- QM violation of a necessary condition of MR can be used as a tool for revealing *nonclassicality* in a context that is usually thought to be entailing classical behaviour.

  The work discussed in this talk will illustrate this feature and compare the efficacy of different necessary conditions of MR in this context.

- QM violation of a necessary condition of MR can be invoked for ruling out a class of realist models, besides displaying nonclassicality in a classical-like context.

  Experimental setup for demonstrating this significance necessarily requires an unambiguous implementation of *Negative Result Measurement*.

Leggett-Garg Inequality (LGI)

- We consider temporal evolution for a two state system where the available states are, say, 1 and 2.
- Let Q(t) be on observable quantity such that, whenever measured, it is found to take a value+1 (-1) depending on whether the system is in 1 (2).
- Next, consider a collection of sets of experimental runs, each set of runs startging from the identical initial state such that on the first set of runs Q is measured at times t_1 and t_2, on the second at t_2 and t_3 and on the third at t_1 and t_3 ($t_1 < t_2 < t_3$)
- One can then use the following deterministic consequence of the assumptions of realism and NIM. For any set of runs corresponding to the *same initial state* at, say, $t = 0$, any individual $Q(t_i)$ has the same definite value, irrespective of the pair in which it occurs, i.e., the value of $Q(t_i)$ in any pair *does not* depend on whether any prior or subsequent measurement has been made on the system.
- Let us divide the whole ensemble of runs into three subensembles, S1, S2, and S3 and consider measurement of Q on each subensemble of runs at the times (t_1, t_2) for S1, (t_2, t_3) for S2 and (t_1, t_3) for S3 corresponding to the same initial state at t =0. Then, since for any collection of runs chosen respectively from S_1, S_2 and S_3

$$Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3) = +1 \text{ or } -3$$

one obtains for the respective subensemble averages

$$\langle Q(t_1)Q(t_2)\rangle_{S_1} + \langle Q(t_2)Q(t_3)\rangle_{S_2} - \langle Q(t_1)Q(t_3)\rangle_{S_3} \leq 1$$

Leggett-Garg Inequality (LGI)

Next, applying the principle of induction (if one measures a quantity on a subset chosen at random from a given set, the result one gets should be typical of the set as a whole), replacing the respective subensemble averages by the whole ensemble average

$$\langle ....\rangle_{S_1} = \langle ....\rangle, \langle ....\rangle_{S_2} = \langle ....\rangle, \langle ....\rangle_{S_3} = \langle ....\rangle$$

one obtains

$$C \equiv C_{12} + C_{23} - C_{13} \leq 1 \tag{1}$$

where the temporal correlation

$$C_{ij} = \langle Q(t_i)Q(t_j)\rangle$$

LHS of the inequality (1) is an experimentally measurable quantity. This is the Leggett-Garg inquality imposing realist constraint on the temporal correlations pertaining to any two level system.

Wigner's form of Leggett-Garg inequality (WLGI)

Here again consider temporal evolution of a two state system where the available states are, say, 1 & 2 and consider measurement of Q at t_1, t_2 and t_3 ($t_1 < t_2 < t_3$).

Here the notion of realism implies the existence of overall joint probabilities $\rho(Q_1, Q_2, Q_3)$ pertaining to different combinations of definite values of observables or outcomes for the relevant measurements, while the assumption of NIM implies that the probabilities of such outcomes would be unaffected by measurements. Hence, by appropriate marginalization, the observable probabilities can be obtained.

For example, the observable joint probability $P(Q_2+, Q_3-)$ of obtaining the outcomes +1 and -1 for the sequential measurements of Q at the instants t_2 and t_3, respectively, can be written as

$$P(Q_2+, Q_3-) = \sum_{Q_1=\pm 1} \rho(Q_1, +, -) = \rho(+, +, -) + \rho(-, +, -)$$

Writing similar expressions for the other measurable marginal joint probabilities $P(Q_1-, Q_3-)$ and $P(Q_1+, Q_2+)$, we get

$$P(Q_1+, Q_2+) + P(Q_1-, Q_3-) - P(Q_2+, Q_3-) = \rho(+, +, +) + \rho(-, -, -)$$

Then invoking non-negativity of the joint probabilities occuring on the RHS of the above Equation, the following form of WLGI is obtained in terms of three pairs of two-time joint probabilities.

$$P(Q_2+, Q_3-) - P(Q_1+, Q_2+) - P(Q_1-, Q_3-) \leq 0$$

Similarly, other forms of WLGI involving any number of pairs of two-time joint probabilities can be derived by using various combinations of the observable joint probabilities.

No-Signalling in Time (NSIT)

Statement: The measurement outcome statistics for any observable at any instant is *independent* of whether any prior measurement has been performed.

Let us consider a system whose time evolution occurs between two possible states. Probability of obtaining the outcome $+1$ for the measurement of a dichotomic observable Q at an instant, say, t_2 *without* any earlier measurement being performed, is denoted by $P(Q_2 = +1)$.

NSIT requires that $P(Q_2 = +1)$ should remain *unchanged* even when an earlier measurement is made at t_1

$$P(Q_2 = +1) = P(Q_1 = +1, Q_2 = +1) + P(Q_1 = -1, Q_2 = +1)$$

QM violation of MR for large spin and its robustness against coarse-grained measurements

Collaboration with Shiladity Mal (SNBNCBS, Kolkata) and Dipankar Home (Bose Institute, Kolkata), Phys. Rev. A **94**, 062117 (2016)

Emergence of classicality from QM

- Sharp measurement $\Rightarrow$ One can distinguish each and every Eigenvalue of an Hermitian operator corresponding to an observable under consideration individually.

  Coarse-grained measurement $\Rightarrow$ One *cannot* distinguish each and every Eigenvalue of an Hermitian operator corresponding to an observable under consideration individually.

- For *unrestricted measurement accuracy* involving projections onto individual levels (so that the consecutive eigenvalues can be resolved), QM violation of MR persists for arbitrary large spin. [C. Budroni and C. Emary, Phys. Rev. Lett. **113**, 050401 (2014)]

  Similar result also holds for the QM violation of Local Realism (LR) for the entangled systems. [D. Home and A. S. Majumdar, Phys. Rev. A **52**, 4959 (1995); A. Cabello, Phys. Rev. A **65**, 062105 (2002)]

- On the other hand, if for a certain time evolution, instead of resolving consecutive eigenvalues of a spin-component observable (sharp measurement), one can experimentally resolve only those eigenvalues which are well separated (one form of coarse-grained measurement), the system dynamics mimics the rotation of a classical spin coherent state and the assumption of NIM becomes valid.
  $\Rightarrow$ Classicality emerges out of QM under the restriction of coarse-grained measurement modelled in a specific way. [J. Kofler and C. Brukner, *Phys. Rev. Lett.* **99**, 180403 (2007); J. Kofler and C. Brukner, *Phys. Rev. Lett.* **101**, 090403 (2008)]

QM violation of MR for large spin

Motivation

No study yet probing emergence of classicality for higher dimensional quantum systems by modelling coarse-graining of measurements in a very general way for varying ability of resolving eigenvalues and by including fuzziness of measurement of each eigenlevel.

The key result obtained

Our study reveals that by employing QM violation of MR as a tool classicality does *not* emerge in large limit of spin, whatever be the *unsharpness* and degree of *coarse-graining of the measurements*. For this purpose, employing the different necessary conditions of MR (LGI, WLGI and NSIT), their relative efficacy in demonstrating non-classicality is compared – NSIT is found to be most effective in this specific context.

Specifying the Hamiltonian, initial condition and measurement times

- Consider a QM spin j system in a uniform magnetic filed of magnitude B_0 along the x direction. The relevant Hamiltonian is ($\hbar = 1$)

$$H = \Omega J_x$$

  where $\Omega \rightarrow$ angular precession frequency ($\propto B_0$), $J_x \rightarrow$ x component of spin angular momentum.

- We initialize the system so that at t=0, the system is in the state $|-j;j\rangle$; where $|m;j\rangle$ denotes the eigen state of J_z operator with eigenvalue m.
- Consider measurements of Q at times t_1, t_2 & t_3 ($t_1 < t_2 < t_3$) & set the measurement times as $\Omega t_1 = \pi$ and $\Omega(t_2 - t_1) = \Omega(t_3 - t_2) = \frac{\pi}{2}$
- Now, consider the following form of 3-term LGI:

$$K_{LGI} = C_{12} + C_{23} - C_{13} \leq 1$$

  The following form of 3-term WLGI:

$$K_{WLGI} = P(Q_2+, Q_3+) - P(Q_1-, Q_2+) - P(Q_1+, Q_3+) \leq 0$$

  and the following form of NSIT:

$$K_{NSIT} = P(Q_3 = -1) - [P(Q_2 = +1, Q_3 = -1) + P(Q_2 = -1, Q_3 = -1)] = 0$$

Modelling coarse grained measurement in an arbitrary spin system

Considering measurements of spin-z component (J_z) observable in a spin-j system, the outcomes of J_z measurements are denoted by m, m takes the values $-j$, $-j+1$, $-j+2$, ..., $+j$. For modelling coarse grained measurement through appropriate dichotomization, different number of measurement outcomes are clubbed together into two groups, the grouping scheme being characterized by a particular value of x.

Let Q be such an observable that
$Q = -1$ for $m = -j, ..., -j + x$ (No. of outcomes in this group $= x + 1$),
$Q = +1$ for $m = -j + x + 1, ..., +j$ (No. of outcomes in this group $= 2j - x$), where $0 \leq x \leq$ integer part (j) and x being integer.

• The asymmetry in the number of measurement outcomes clubbed together into two groups characterizes the biasness of coarse graining of the measurement outcomes.

• The asymmetry decreases and, hence, the biasness of coarse graining of the measurement outcomes decreases with an increasing value of x.

• $x =$ integer part (j), signifying equal number of outcomes in the two groups denotes the grouping scheme corresponding to the most unbiased coarse graining of the measurement outcomes.

Modelling coarse grained measurement in an arbitrary spin system

• In this modelling, clubbing of the measurement outcomes into two groups makes the measurement coarse grained. However, the boundary between the two groups of outcomes is assumed to be precise which, in general, is *not* true in the realization of the macrolimit.

• Employing, in conjunction, *unsharp measurement* corresponding to each eigenvalue makes this boundary also imprecise.

• Thus, by simultaneously clubbing the different measurement outcomes together and by invoking unsharp measurement one can capture in a more *general way* what is entailed by the coarse graining of the measurement outcomes.

Modelling fuzziness of a measurement in an arbitrary spin system

Consider measurements of spin-z component (J_z) observable in a spin-j system. In order to capture the effect of fuzziness or imprecision involved in a measurement, in the formalism of unsharp measurement, a parameter (λ) known as the sharpness parameter is introduced to characterize non-idealness of a measurement by defining what is referred to as the effect operator given by

$$F_m = \lambda P_m + (1-\lambda)\frac{\mathbb{I}}{d}$$

where λ is the sharpness parameter ($0 < \lambda \leq 1$);

P_m is the projector $|m;j\rangle\langle m;j|$, where $|m;j\rangle$ is the eigenvector of J_z operator with eigenvalue m;

$\mathbb{I}$ is the identity operator;

d is the dimension of the system (for spin j system, $d = 2j+1$).

For an unsharp measurement pertaining to an initial state ρ_0, the probability of an outcome, say, m is given by $tr(\rho_0 F_m)$ for which the post-measurement state is given by $(\sqrt{F_m}\rho_0\sqrt{F_m})/tr(\rho_0 F_m)$.

Summary of the key Results: Considering projective measurement ($\lambda = 1$)

• For projective measurement, for any j (including for arbitrarily large value), QM violation of any necessary condition of MR decreases with increasing values of x, i.e., with decreasing biasness of coarse graining of the measurement outcomes. On the other hand, for a fixed and finite value of x, QM violation of any necessary condition of MR increases with increasing values of j, i.e., with increasing spin of the system.

j	**Magnitude of the QM Violation of**					
	LGI for		**WLGI for**		**NSIT for**	
	$x = 10$	$x = 20$	$x = 10$	$x = 20$	$x = 10$	$x = 20$
40	1.52	1.32	0.76	0.66	0.76	0.66
60	1.61	1.46	0.81	0.73	0.81	0.72
80	1.67	1.53	0.83	0.77	0.83	0.76
100	1.70	1.58	0.85	0.79	0.85	0.79

Summary of the key Results: Considering projective measurement ($\lambda = 1$)

• For projective measurement, for any j (including for arbitrarily large value), QM violation of LGI does not occur for *unbiased* coarse graining of the measurement outcomes. On the other hand, for this type of coarse graining of measurement outcomes, QM violations of WLGI and NSIT persist for any j.

j	For unbiased coarse grained measurement (x = integer part(j)), the magnitude of the QM violation of		
	LGI	**WLGI**	**NSIT**
10	No violation	0.20	0.48
20	No violation	0.21	0.49
30	No violation	0.22	0.50

Summary of the key Results: Considering unsharp measurement ($0 < \lambda \leq 1$)

• For a fixed value of spin and fixed biasness of coarse graining of the measurement outcomes (fixed j and fixed x), QM violations of the different necessary conditions of MR decrease with increasing fuzziness of the measurement.

• For fixed and finite values of j and x, the range of λ for which the QM violation of MR persists is the maximum for NSIT, followed by WLGI and then LGI.

For any j, the ranges of λ for which QM violations of LGI and WLGI persist become smaller for increasing values of x.

On the other hand, for any fixed and finite value of x, the ranges of λ for which QM violations of LGI and WLGI persist become larger for increasing values of j.

j	The range of λ for which the QM violation of					
	LGI persists for			WLGI persists for		
	$x = 5$	$x = 7$	$x = 9$	$x = 5$	$x = 7$	$x = 9$
10	$(0.64, 1]$	$(0.75, 1]$	$(0.92, 1]$	$(0.53, 1]$	$(0.61, 1]$	$(0.72, 1]$
20	$(0.49, 1]$	$(0.55, 1]$	$(0.59, 1]$	$(0.40, 1]$	$(0.44, 1]$	$(0.48, 1]$
30	$(0.42, 1]$	$(0.47, 1]$	$(0.51, 1]$	$(0.33, 1]$	$(0.37, 1]$	$(0.40, 1]$
40	$(0.38, 1]$	$(0.42, 1]$	$(0.45, 1]$	$(0.29, 1]$	$(0.33, 1]$	$(0.36, 1]$

• The range of λ for which the QM violation of NSIT persists is always $(0, 1]$ for any value of j and x.

Summary of the key Results: Considering unsharp measurement ($0 < \lambda \leq 1$)

- The results show that for the most *unbiased* coarse graining of the measurement outcomes, i.e., when the number of outcomes in the two groups becomes almost equal (equal for half integer spin system and difference of outcomes is 1 for integer spin system), then the QM violation of NSIT persists for *any* degree of fuzziness of the measurement; the QM violation of WLGI persists up to a *certain* degree of fuzziness of the measurement, while the QM violation of LGI *does not* occur in this case even for sharp measurement.

j	**Range of λ for which the QM violation of WLGI persists for unbiased coarse grained measurement ($x =$ integer part(j))**	**Range of λ for which the QM violation of NSIT persists for unbiased coarse grained measurement ($x =$ integer part(j))**
10	$(0.78, 1]$	$(0, 1]$
20	$(0.76, 1]$	$(0, 1]$
30	$(0.75, 1]$	$(0, 1]$

Outlook

These results signify that, in the limit of arbitrarily large spin system, for the most *unbiased* coarse graining of the measurement outcomes, i.e., when the number of outcomes in the two groups becomes almost equal, even then classicality does *not* emerge out of QM for *any* degree of fuzziness of the measurement. This is best illustrated through the QM violation of NSIT, followed by that of WLGI.

In this work we have not considered the *coarse graining of measurement times* [H. Jeong et al. PRL 112, 010402 (2014)]. It remains an open question as to what extent the coarse graining of measurement times, in conjunction with coarse-graining of measurement outcomes, would affect the above mentioned results, taking into account LGI, WLGI as well as NSIT.

thank you!

Appendix 1: The idea of Negative Result Measurement (NRM)

- Consider a two-state system whose available states are, say, 1 and 2. Let Q(t) be an observable quantity such that, whenever measured, it is found to take a value +1(-1) depending on whether the system is in the state 1(2).
- The measuring apparatus be such that if $Q(t_1)$ is, say, $+1$, the probe is *triggered*, while if $Q(t_1) = -1$, it is *not*. One then uses the results of *those runs* for which $Q(t_1) = -1$, followed by the measurement of Q at $t_2 \rightarrow$ These results are then used for determining the joint probabilities $P_{-+}(t_1, t_2)$ and $P_{--}(t_1, t_2)$.
- One can then invert the measuring setup so that for a value of $Q(t_1) = -1$ the probe is *triggered, while for* $Q(t_1) = +1$, it is *not*. In this case, one then uses the results of *those runs* for which $Q(t_1) = +1$, followed by the measurement of Q at $t_2 \rightarrow$ These results are then used for determining $P_{+-}(t_1, t_2)$ and $P_{++}(t_1, t_2)$.

Appendix 1: The idea of Negative Result Measurement (NRM)

- Thus, in this way, one can evaluate the temporal correlations like

$$C_{12} = P_{++}(t_1, t_2) - P_{+-}(t_1, t_2) - P_{+}(t_1, t_2) + P_{--}(t_1, t_2)$$

- For interpreting the test of any macrorealist condition involving temporal correlations based on NRM, if one assumes the validity of the condition of Realism, then it becomes difficult to allow for the possibility of a state being affected by a measurement process such as the NRM proceedure, in which the measurement information is obtained even if *no interaction occurs*. Thus, in the context of NRM or, interaction-free measurement, NIM seems a natural corollary of the notion of realism.
- Two experimental claims to date using NRM for implementing NIM:
  G. Knee et al., *Nature Communications* 3, 606 (2012) → Spin-bearing phosphorus impurities in silicon sample.
  C. Robens et al. *Physical Review X* 5, 011003 (2015) → Quantum Walks in a lattice having cesium atoms.
  Though criticisms persist about the effectiveness of implementing ideal NRM in the above mentioned experiments.

Appendix 2: Experiments showing QM violations of LGI

1. Superconducting qubit → Continuous Weak Measurements → Palacios-Laloy et al. [*Nat. Phys.* **6**, 442 (2010)]
2. Superconducting qubit → Weak/Semi-weak point Measurements → Groen et al. [*Phys. Rev. Lett.* **111**, 090506 (2013)]
3. Nitrogen-vacancy centre → Weak Measurements → George et al. [*Proc. Natl. Acad. Sci. USA* **110**, 3777 (2013)]
4. Nuclear magnetic resonance → Projective Measurements → Athalye et al. [*Phys. Rev. Lett.* **107**, 130402 (2011)], Souza et al. [*New J. Phys.* **13**, 053023 (2011)].
5. Nuclear magnetic resonance → Ideal Negative Measurements → Katiyar et al. [*Phys. Rev. A* **87**, 052102 (2013)].
6. Photons → Weak/Semi-weak point Measurements → Goggin et al. [*Proc. Natl. Acad. Sci. USA* **108**, 1256 (2011)], Dressel et al. [*Phys. Rev. Lett.* **106**, 040402 (2011)], Suzuki et al. [*New J. Phys.* **14**, 103022 (2012)].
7. Photons → Projective Measurements → Xu et al. [*Sci. Rep.* **1**, 101 (2011)].
8. Phosphorus impurities in Silicon → Ideal Negative Measurements → Knee et al. [*Nat. Commun.* **3**, 606 (2012)].

Appendix 3: Coarse-grained measurement

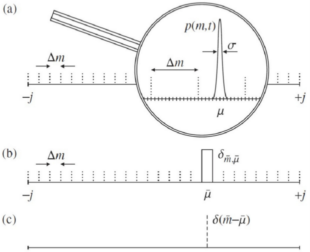

FIG. 1. An initial spin-j coherent state $|\vartheta_0, \varphi_0\rangle$ precesses into the coherent state $|\vartheta, \varphi\rangle$ at time t under a quantum time evolution. (a) The probability $p(m, t)$ for the outcome m in a measurement of the spin's z-component is given by a Gaussian distribution with width σ and mean μ, which can be seen under the magnifying glass of sharp measurements. (b) The measurement resolution Δm is finite and subdivides the $2j + 1$ possible outcomes into a smaller number of coarse-grained "slots." If the measurement accuracy is much poorer than the width σ, i.e., $\Delta m \gg \sqrt{j}$, the sharply peaked Gaussian cannot be distinguished anymore from the Kronecker delta $\delta_{\bar{m},\bar{\mu}}$ where $\bar{m}$ is numbering the slots and $\bar{\mu}$ is the slot in which the center μ of the Gaussian lies. (c) In the limit $j \to \infty$, the slots *seem* to become infinitely narrow and $\delta_{\bar{m},\bar{\mu}}$ becomes the delta function $\delta(\bar{m} - \bar{\mu})$.

Appendix 4: Tool for calculation

- Let $R = e^{-i\frac{\pi}{2}J_x}$, the time evolution operators are as follow:

$$U(t_1 - 0) = e^{-i\pi J_x} = R^2$$

$$U(t_2 - 0) = R^3$$

$$U(t_2 - t_1) = R$$

$$U(t_3 - t_2) = R$$

$$U(t_3 - t_1) = R^2$$

So here the time evolution operators have the form of rotation operators about x axis.

- We, therefore, take the help of *Wigner's D Matrix* to evaluate the matrix elements of the rotation operators in arbitrary spin system.

- It is a square matrix of dimension $2j + 1$.