Coupled systems as quantum thermodynamic machines

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March 1, 2017

Young Quantum 2017, HRI Allahabad

Talk layout

- Introduction
- Simple models of quantum heat engine
- Coupled systems as heat engines
- Refrigerator model
- Comparison of performances

Motivation

Quantum thermodynamics

To extend thermodynamics in quantum domain.

- Superposition and entanglement
- Quantum processes such as quantum adiabatic process
- A few particle system in a thermal environment

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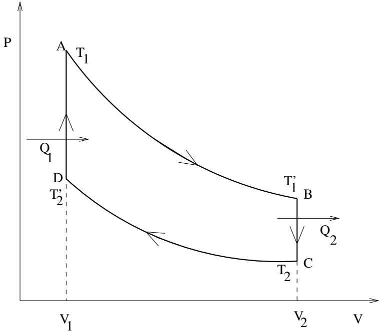
Models

Quantum thermodynamic machines

Otto cycle

- Nikolaus Otto was the first person to build a working model of four-stroke engine in 1861.
- Four-stroke gasoline engines used nowadays are generally called Otto engines.

Classical Otto cycle



Work and Efficiency

$$Q_1 = C_v(T_1 - T_2'), \qquad Q_2 = C_v(T_2 - T_1')$$

$$W = Q_1 + Q2$$

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_1'}{T_1} = 1 - \left(\frac{V_1}{V_2}\right)^{(\gamma-1)}.$$

MASER as a heat engine

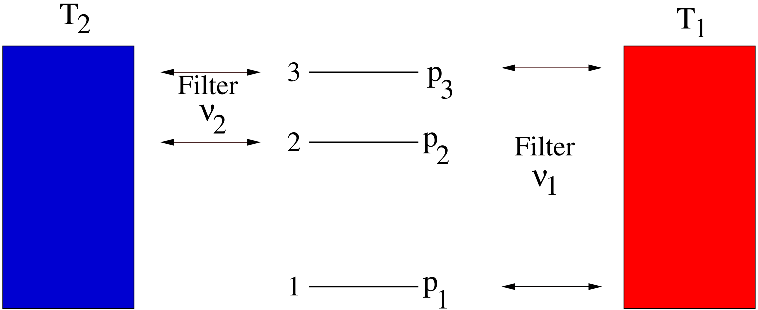


Figure : Three level MASER as a heat engine [1]

$\eta_M = 1 - \frac{\nu_2}{\nu_1} \leq 1 - \frac{T_2}{T_1}$

[1] H. E. D. Scovil and E. O. Schulz-DuBois, Phys. Rev. Lett. **2**, 262 (1959).

Mean energy

$$U = \mathrm{Tr}(H\rho),$$

where H is the Hamiltonian and ρ is the density matrix.

First law of thermodynamics

$$\begin{aligned} dU &= \mathrm{Tr}(H\,d\rho) + \mathrm{Tr}(\rho\,dH) \\ dQ &= \mathrm{Tr}(H\,d\rho) \\ dW &= \mathrm{Tr}(\rho\,dH) \end{aligned}$$

Adiabatic process

Adiabatic process in thermodynamics

No heat is exchanged between the system and its surroundings, $dQ = 0$.

Quantum adiabatic process

Suppose the system is initially in the nth eigenstate of the initial Hamiltonian (H_{ini}), then the Hamiltonian is changed gradually from H_{ini} to H_{fin}. According to Quantum adiabatic theorem, the system will remain in the nth eigenstate of the instantaneous Hamiltonian.

A model of quantum heat engine

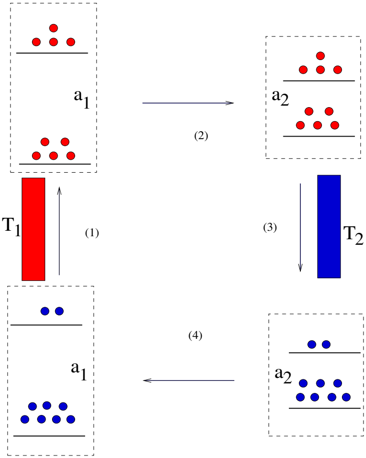


Figure : Two level system as heat engine.

Heat, work and efficiency

$$Q_h = \frac{a_1}{2}\left(\tanh\left[\frac{\beta_2 a_2}{2}\right] - \tanh\left[\frac{\beta_1 a_1}{2}\right]\right) > 0$$

$$Q_c = \frac{a_2}{2}\left(\tanh\left[\frac{\beta_1 a_1}{2}\right] - \tanh\left[\frac{\beta_2 a_2}{2}\right]\right) < 0$$

$$W = Q_h + Q_c > 0 \qquad \eta = W/Q_h$$

Condition for engine

$\beta_2 a_2 \geq \beta_1 a_1$ and $a_1 > a_2$

$$\eta = 1 - \frac{a_2}{a_1} \leq 1 - \frac{T_2}{T_1}$$

T. D. Kieu, Phys. Rev. Lett. 93, 140403 (2004).

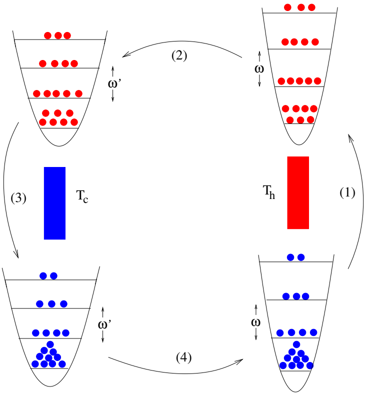


Figure : Harmonic oscillator as a model of heat engine.

Work and efficiency

$$Q_h = \frac{\omega}{2}\left(\coth\left[\frac{\beta_h\omega}{2}\right] - \coth\left[\frac{\beta_c\omega'}{2}\right]\right) > 0$$

$$Q_c = \frac{\omega'}{2}\left(\coth\left[\frac{\beta_c\omega'}{2}\right] - \coth\left[\frac{\beta_h\omega}{2}\right]\right) < 0$$

$$W = Q_1 + Q_2 > 0$$

Condition for engine

$\beta_c\omega' \geq \beta_h\omega$ and $\omega > \omega'$

$$\eta = 1 - \frac{\omega'}{\omega} \leq 1 - \frac{T_c}{T_h}$$

Comparison

Oscillator model

$$W^{\mathrm{os}} = \frac{(\omega - \omega')}{2}\left(\coth\left[\frac{\beta_h \omega}{2}\right] - \coth\left[\frac{\beta_c \omega'}{2}\right]\right)$$

$$\eta^{\mathrm{os}} = \frac{W^{\mathrm{os}}}{Q_h^{\mathrm{os}}} = 1 - \frac{\omega'}{\omega}$$

Spin model

$$W^{\mathrm{sp}} = \frac{(\omega - \omega')}{2}\left(\tanh\left[\frac{\beta_c \omega'}{2}\right] - \tanh\left[\frac{\beta_h \omega}{2}\right]\right)$$

$$\eta^{\mathrm{sp}} = \frac{W^{\mathrm{sp}}}{Q_h^{\mathrm{sp}}} = 1 - \frac{\omega'}{\omega}$$

$$\eta^{\mathrm{os}} = \eta^{\mathrm{sp}} \qquad W^{\mathrm{os}} > W^{\mathrm{sp}}$$

Coupled systems as thermodynamic machines[2]

[2] G. Thomas, M. Banik, and S. Ghosh, arXiv:1607.00994 [quant-ph] (2016).

Coupled systems

$$H^{\mathrm{os}} = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{m\Omega^2}{2}x_1^2 + \frac{m\Omega^2}{2}x_2^2 + 2\left(\frac{m\Omega}{2}\lambda_x x_1 x_2 + \frac{1}{2m\Omega}\lambda_p p_1 p_2\right)$$

$$\begin{aligned} H^{\mathrm{os}} &= \left(c_1^\dagger c_1 + c_2^\dagger c_2 + 1\right)\Omega + \frac{(\lambda_x + \lambda_p)}{2}(c_1^\dagger c_2 + c_1 c_2^\dagger) \\ &\quad + \frac{(\lambda_x - \lambda_p)}{2}(c_1 c_2 + c_1^\dagger c_2^\dagger) \end{aligned}$$

Heisenberg XY model

$$\begin{aligned} H^{\mathrm{sp}} &= (S_1^+ S_1^- + S_2^+ S_2^- + 1)\Omega + \frac{(J_x + J_y)}{2}(S_1^+ S_2^- + S_1^- S_2^+) \\ &\quad + \frac{(J_x - J_y)}{2}(S_1^+ S_2^+ + S_1^- S_2^-) \end{aligned}$$

Co-ordinate Transformation

$$x_A = \frac{x_1 + x_2}{\sqrt{2}}, \qquad x_B = \frac{x_1 - x_2}{\sqrt{2}};$$
$$p_A = \frac{p_1 + p_2}{\sqrt{2}}, \qquad p_B = \frac{p_1 - p_2}{\sqrt{2}}.$$

$$H^{\rm os} = \frac{p_A^2}{2M_A} + \frac{M_A \Omega_A^2}{2} x_A^2 + \frac{p_B^2}{2M_B} + \frac{M_B \Omega_B^2}{2} x_B^2$$

$$H^{\rm os} = \left(c_A^\dagger c_A + \frac{1}{2} \right) \Omega_A + \left(c_B^\dagger c_B + \frac{1}{2} \right) \Omega_B$$

$$\Omega_{A/B} = \sqrt{(\Omega \pm \lambda_p)(\Omega \pm \lambda_x)}.$$

Heat, work and efficiency

$$
\begin{aligned}
Q &= \frac{\omega_A}{2}\left(\coth\left[\frac{\beta_h \omega_A}{2}\right] - \coth\left[\frac{\beta_c \omega'_A}{2}\right]\right) \\
&\quad + \frac{\omega_B}{2}\left(\coth\left[\frac{\beta_h \omega_B}{2}\right] - \coth\left[\frac{\beta_c \omega'_B}{2}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
W &= \frac{(\omega_A - \omega'_A)}{2}\left(\coth\left[\frac{\beta_h \omega_A}{2}\right] - \coth\left[\frac{\beta_c \omega'_A}{2}\right]\right) \\
&\quad + \frac{(\omega_B - \omega'_B)}{2}\left(\coth\left[\frac{\beta_h \omega_B}{2}\right] - \coth\left[\frac{\beta_c \omega'_B}{2}\right]\right)
\end{aligned}
$$

$$
\eta = \frac{W_A + W_B}{Q_A + Q_B}; \qquad \eta_A = \frac{W_A}{Q_A}; \qquad \eta_B = \frac{W_B}{Q_B}
$$

Heat, work and efficiency

$$Q = \frac{\omega_A}{2}\left(\coth\left[\frac{\beta_h\omega_A}{2}\right] - \coth\left[\frac{\beta_c\omega'_A}{2}\right]\right) + \frac{\omega_B}{2}\left(\coth\left[\frac{\beta_h\omega_B}{2}\right] - \coth\left[\frac{\beta_c\omega'_B}{2}\right]\right)$$

$$W = \frac{(\omega_A - \omega'_A)}{2}\left(\coth\left[\frac{\beta_h\omega_A}{2}\right] - \coth\left[\frac{\beta_c\omega'_A}{2}\right]\right) + \frac{(\omega_B - \omega'_B)}{2}\left(\coth\left[\frac{\beta_h\omega_B}{2}\right] - \coth\left[\frac{\beta_c\omega'_B}{2}\right]\right)$$

$$\eta = \frac{W_A + W_B}{Q_A + Q_B}; \qquad \eta_A = \frac{W_A}{Q_A}; \qquad \eta_B = \frac{W_B}{Q_B}$$

$$\min\{\eta_A, \eta_B\} \leq \eta \leq \max\{\eta_A, \eta_B\}$$

Bounds of the efficiency

$$\min\{\eta_A, \eta_B\} \leq \eta \leq \max\{\eta_A, \eta_B\}$$

When the Hamiltonian of the coupled system (at all stages of the cycle) can be decoupled (as two independent modes) in some suitably chosen co-ordinate system, then the efficiency of the coupled system is bounded (both from above and below) by the efficiencies of the independent modes, provided both the modes work as engines.

XX-model

Now we consider the following case $\lambda_x = \lambda_p = \lambda_J$ and $J_x = J_y = \lambda_J$.

$$H^{\mathrm{os}} = (\Omega + \lambda_J)(c_A^\dagger c_A + \frac{1}{2}) + (\Omega - \lambda_J)(c_B^\dagger c_B + \frac{1}{2})$$

$$H^{\mathrm{sp}} = (\Omega + \lambda_J)(S_A^+ S_A^- + \frac{1}{2}) + (\Omega - \lambda_J)(S_B^+ S_B^- + \frac{1}{2})$$

Consider the cycle $\Omega : \omega \to \omega' \to \omega$

$$\eta_A = \frac{(\omega - \omega')}{(\omega + \lambda_J)} \quad \text{and} \quad \eta_B = \frac{(\omega - \omega')}{(\omega - \lambda_J)}$$

Efficiency

$$\eta^{\mathrm{os}} = 1 - \frac{\omega'}{\omega} \quad + \quad \frac{\gamma\left(T_c \mathrm{csch}^2\left[\frac{\omega}{2T_h}\right] - T_h \mathrm{csch}^2\left[\frac{\omega'}{2T_c}\right]\right)\lambda_J^2}{2\left(\coth\left[\frac{\omega}{2T_h}\right] - \coth\left[\frac{\omega'}{2T_c}\right]\right)} \quad + \quad O[\lambda_J^4]$$

$$\eta^{\mathrm{sp}} = 1 - \frac{\omega'}{\omega} \quad + \quad \frac{\gamma\left(T_h \mathrm{sech}^2\left[\frac{\omega'}{2T_c}\right] - T_c \mathrm{sech}^2\left[\frac{\omega}{2T_h}\right]\right)\lambda_J^2}{2\left(\tanh\left[\frac{\omega}{2T_h}\right] - \tanh\left[\frac{\omega'}{2T_c}\right]\right)} \quad + \quad O[\lambda_J^4]$$

$$\eta^{\mathrm{os}} - \eta^{\mathrm{sp}} = \gamma\left(T_c \mathrm{csch}\left[\frac{\omega}{T_h}\right] + T_h \mathrm{csch}\left[\frac{\omega'}{T_c}\right]\right)\lambda_J^2 > 0$$

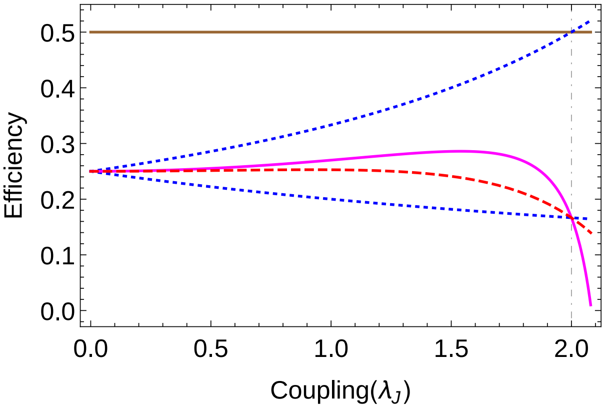


Figure : The two dotted curves show the upper bound (η_B) and lower bound (η_A) . The continuous curve represents the efficiency of the coupled oscillator. Efficiency of the coupled spin system is denoted by the dashed curve. When the upper bound reaches Carnot value, $\eta_B = 1 - T_c/T_h$ for $\lambda_J = \lambda_c$ (represented by vertical dashed-dotted line), then we get $\eta^{\mathrm{os}} = \eta^{\mathrm{sp}} = \eta_A$. Here we take $T_h = 2$, $T_c = 1$, $\omega = 4$ and $\omega' = 3$.

$$W = W_A + W_B = \frac{(\omega_A - \omega'_A)}{2}\left(\coth\left[\frac{\beta_h\omega_A}{2}\right] - \coth\left[\frac{\beta_c\omega'_A}{2}\right]\right) + \frac{(\omega_B - \omega'_B)}{2}\left(\coth\left[\frac{\beta_h\omega_B}{2}\right] - \coth\left[\frac{\beta_c\omega'_B}{2}\right]\right)$$

where $\omega_A = \omega_A(\omega, \lambda_x, \lambda_p)$, $\omega_B = \omega_B(\omega, \lambda_x, \lambda_p)$, $\omega'_A = \omega'_A(\omega', \lambda_x, \lambda_p)$, and $\omega'_B = \omega'_B(\omega', \lambda_x, \lambda_p)$

Suppose W_A is maximum, when $\omega_A = \omega_A*$ and $\omega'_A = \omega'_A*$. If the subsystem A provides subsystem work, then the work obtained from the subsystem B may not be optimal. Therefore we have

$$W^{\max}_{\lambda\neq 0} \leq W^{\max}_{\lambda=0}$$

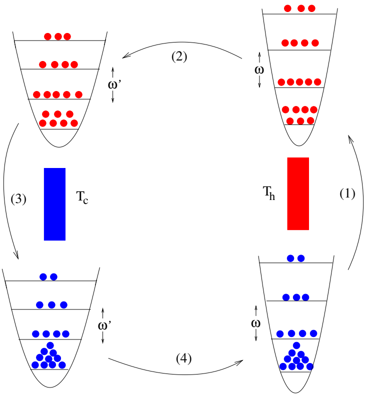


Figure : Harmonic oscillator as a model of refrigerator.

Work and COP

$$Q_h = \frac{\omega}{2}\left(\coth\left[\frac{\beta_h\omega}{2}\right] - \coth\left[\frac{\beta_c\omega'}{2}\right]\right) < 0$$

$$Q_c = \frac{\omega'}{2}\left(\coth\left[\frac{\beta_c\omega'}{2}\right] - \coth\left[\frac{\beta_h\omega}{2}\right]\right) > 0$$

$$W = Q_1 + Q_2 < 0$$

Condition for refrigerator

$\beta_c\omega' \leq \beta_h\omega$ and $\omega > \omega'$

$$\zeta = \frac{Q_c}{|W|} = \frac{\omega'}{\omega - \omega'} \leq \frac{T_c}{T_h - T_c}$$

Coupled system as refrigerator

$$\min\{\zeta_A, \zeta_B\} \leq \zeta \leq \max\{\zeta_A, \zeta_B\}$$

The global coefficient of performance (COP) is bounded (both from above and below) by the COPs of the independent modes.

Coupled system as refrigerator

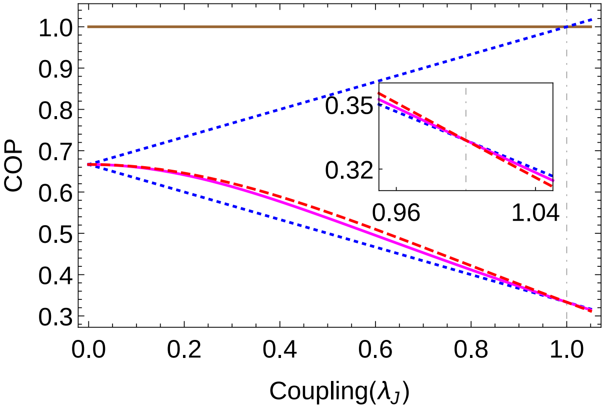


Figure : The upper bound (ζ_A) and the lower bound (ζ_B) are shown with the dotted curves. The continuous curve represents the COP of the coupled oscillator and COP of the coupled spin system is denoted by the dashed curve. The horizontal line represents the Carnot value for the refrigerator. When the independent subsystems work in refrigerator mode, the global COP of the coupled system is bounded by ζ_A and ζ_B. The plot also shows that the global COP of the coupled spins is higher than that of the coupled oscillators for small values of λ_J. When the upper bound achieves Carnot value, $\zeta_A = T_c/(T_h - T_c)$ for $\lambda_J = \lambda_c'$ (vertical dashed-dotted line), then we get $\zeta^{\rm os} = \zeta^{\rm sp} = \zeta_B$. Inset shows the enlarged region near $\lambda_J = \lambda_c'$. Here we take $T_h = 2$, $T_c = 1$, $\omega = 5$ and $\omega' = 2$.

XY model

Now we consider the following case $\lambda_x = -\lambda_p = \lambda_J$ and $J_x = -J_y = \lambda_J$.

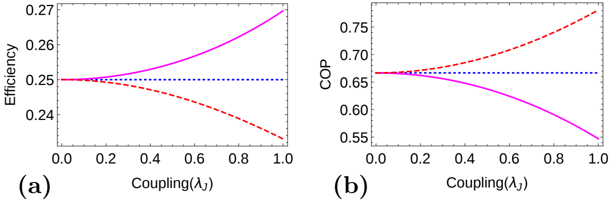


Figure : (a) The continuous and the dashed curves represent the efficiencies of coupled oscillators and coupled spins respectively. The efficiency of the uncoupled oscillator (or spin) is shown by the horizontal dotted line. The parameter values are $T_h = 2$, $T_c = 1$, $\omega = 4$ and $\omega' = 3$. (b) The COPs of coupled oscillators and spins are shown by continuous and dashed curves respectively. Horizontal dotted line represents COP of uncoupled oscillator (or spin). Here we used $T_h = 2$, $T_c = 1$, $\omega = 5$ and $\omega' = 2$.

Thank you all !!