Experimentally freezing quantum discord in a dissipative environment using dynamical decoupling

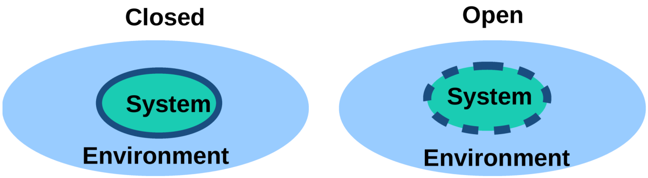
Harpreet Singh Kavita Dorai Arvind

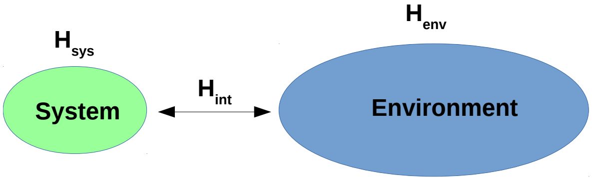
Department of Physical Sciences
Indian Institute of Science Education Mohali

Young Quantum-2017

Outline

Introduction





- The interaction of a quantum system with its environment causes the rapid destruction of crucial quantum properties, such as quantum superpositions and of quantum correlations in composite systems.

Quantum correlations for two-qubit BD state

- Consider a two-qubit system with Hilbert space $\mathcal{H}$ and computational base $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.
- A generic two-qubit state

$$\rho_{AB} = \frac{1}{4}\left(I + \vec{u}\vec{\sigma}^A \otimes I + I \otimes \vec{v}\vec{\sigma}^B + \sum_{j,k=1}^{3} w_{jk}\sigma_j^A \otimes \sigma_k^B\right)$$

- We are interested in system undergoing phase damping, initially in the state

$$\rho_{AB} = \frac{1}{4}\left(I_{AB} + \sum_{i=1}^{3} c_i\sigma_i^A \otimes \sigma_i^B\right)$$

With $c_1 = 1$ and $c_2 = -c_3$.

- Dynamics of phase damping are goverened by the master equation:

$$\dot{\rho}_{AB}(t) = -\frac{i}{\hbar}[\ H_{AB}, \rho]\ + \sum_{i,\alpha}(L_{i,z}\rho L_{i,z}^{\dagger} - \frac{1}{2}\{L_{i,z}^{\dagger}L_{i,z}, \rho\}) \qquad (1)$$

where the Lindblad operator $L_{i,z} = \sqrt{\gamma_i}\sigma_z^{(i)}$ acts on the ith qubit and describes decoherence and $\sigma_z^{(i)}$ denotes the Pauli matrix of the ith qubit. The constant γ_i is approximately equal to the inverse of decoherence time.

- Solving above equation

$c_1(t) = c_1(0)exp(-2\gamma t)$, $c_2(t) = c_2(0)exp(-2\gamma t)$, and $c_3(t) = c_3(0) = c_3$.

- Total mutual information of system is given by

$$\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

where $\rho_{A(B)}$ is the reduced density matrix of subsystem and $S(\rho) = -Tr\{\rho log_2 \rho\}$ is the Von Neumann entropy.
For BD type of states the mutual information is:

$$\mathcal{I}(\rho_{AB}) = 2 + \sum_{l=0}^{3} \lambda_l log_2 \lambda_l$$

where λ_l are the eigenvalues of ρ_{AB} given by
$\lambda_0 = [1 + c_1 - c_2 + c_3]/4$, $\lambda_1 = [1 - c_1 + c_2 + c_3]/4$,
$\lambda_2 = [1 + c_1 - c_2 - c_3]/4$, and $\lambda_3 = [1 - c_1 + c_2 - c_3]/4$

- Classical correlations[1] $\mathcal{C}(\rho_{AB})$ are given by:

$$\mathcal{C}(\rho_{AB}) = \max_{\{\Pi_k\}}[S(\rho_A) - S(\rho_{AB}|\{\Pi_k\})], \tag{2}$$

  where the maximum is taken over the set of projective measurements $\{\Pi_k\}$ and $S(\rho_{AB}|\{\Pi_k\}) = \sum_k p_k S(\rho_k)$ is the conditional entropy of A, given the knowledge of the state of B, with $\rho_k = Tr_B(\Pi_k \rho_{AB} \Pi_k)/p_k$ and $p_k = Tr_{AB}(\rho_{AB}\Pi_k)$.

- For BD state

$$\mathcal{C}(\rho_{AB}) = \sum_{j=1}^{2} \frac{1+(-1)^j\chi}{2} \log_2[1+(-1)^j\chi]$$

  where $\chi = \max\{|c_1|, |c_2|, |c_3|\}$.

[1] S. Luo, Phys. Rev. A 77, 022301 (2008).

$$
\begin{aligned}
\mathcal{C}(\rho(t)) &= \sum_{j=1}^{2} \frac{1+(-1)^j \chi(t)}{2} \log_2[1+(-1)^j \chi(t)] \\
\mathcal{I}(\rho(t)) &= \sum_{j=1}^{2} \frac{1+(-1)^j c_1(t)}{2} \log_2[1+(-1)^j c_1(t)] \\
&\quad + \sum_{j=1}^{2} \frac{1+(-1)^j c_3}{2} \log_2[1+(-1)^j c_3] \\
\mathcal{D}(\rho) &\equiv \mathcal{I}(\rho) - \mathcal{C}(\rho) \qquad (3)
\end{aligned}
$$

where $\chi(t) = \max\{|c_1(t)|, |c_2(t)|, |c_3(t)|\}$.

- For a time $t < \bar{t} = \frac{1}{2\gamma} ln(\frac{c_1(0)}{c_3(0)})$,

$$\mathcal{D}(\rho) = \sum_{j=1}^{2} \frac{1+(-1)^j c_3}{2} \log_2[1+(-1)^j c_3]$$

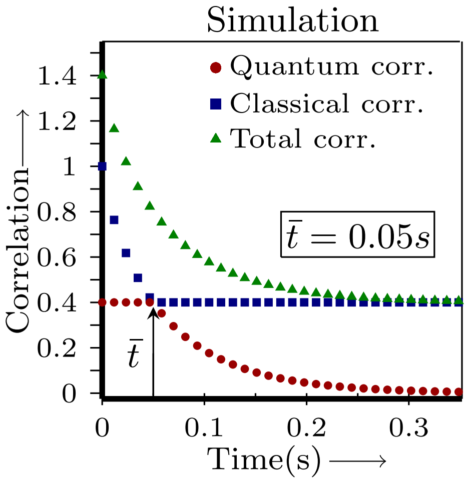
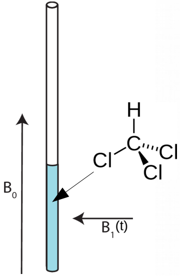
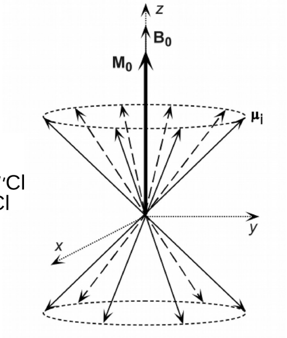
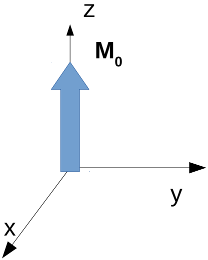


Figure: Time evolution of total correlations (green triangles), classical correlations (blue squares) and quantum discord (red circles) of the BD state. With intial $c_1 = 1$, $c_2 = 0.7$, $c_3 = -0.7$, and $2\gamma = 7.5$.

Nuclear magnetic interaction



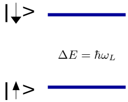




NMR Qubits

- A molecule contains spin $I = \frac{1}{2}$ nuclei placed in B_0 represents a single qubit, with internal Hamiltonian:

$$H_0 = -\mu.B = -\mu_z B_0 = -\gamma_n \hbar B_0 I_z = -\hbar \omega_L I_z \tag{4}$$



- A molecule containing N-coupled spin $I = \frac{1}{2}$ nuclei placed in B_0 represents a N-qubit, with internal Hamiltonian

$$H_0 = \sum_{i=1}^{N} -\omega_i I_z^i + 2\pi \sum_{i<j}^{N} J_{ij} I_z^i . Iz^j \tag{5}$$

where J_{ij} is the scalar coupling and ω_i is the Larmor frequency of i^{th} spin.

NMR States

- At temperature T, NMR qubits are at thermal equilibrium state

$$\rho_{eq} = \frac{e^{-H_0/k_B T}}{Tr[e^{-H_0/K_B T}]} \tag{6}$$

- For high temperature, $E_i << K_B T$

$$\rho_{eq} \approx \frac{I - H_0/k_B T}{Tr[I - H_0/k_B T]} \approx \frac{I - H_0/k_B T}{Tr[I]} \approx \frac{I}{2^N} - \frac{H_0}{2^N k_B T} \tag{7}$$

$$\Delta\rho \approx -\sum_{i=0}^{2^N-1} \epsilon I_z^i \tag{8}$$

where $\epsilon = \frac{\hbar\omega_i}{2^N k_B T} \approx 10^{-5}$

Pseudo-Pure State

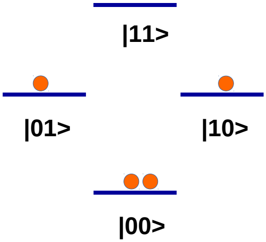


Figure: At thermal equilibrium for $\rho = 10^5 I + \Delta\rho$, for two-qubit Homo-nuclear system $\Delta\rho = I_{z1} + I_{z2}$

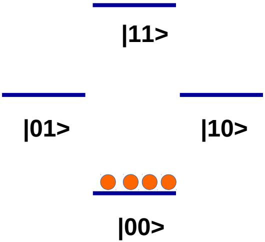


Figure: Pseudo-Pure $\Delta\rho = I_{z1} + I_{z2} + 2I_{z1}.I_{z2}$

NMR System Details

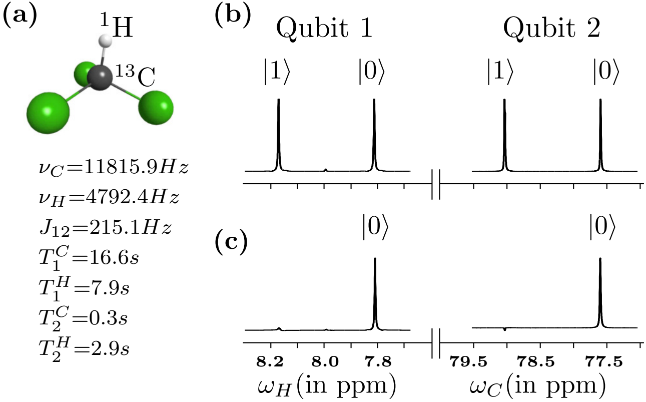


Figure: (a) Chloroform-^{13}C molecular structure with ^{1}H and ^{13}C labeling the first and second qubits, respectively. Tabulated system parameters are: chemical shifts ν_i, the scalar coupling interaction strength J_{12} (in Hz) and relaxation times T_1 and T_2 (in seconds). NMR spectrum of (b) the thermal equilibrium state after a $\pi/2$ readout pulse and (c) the pseudopure $|00\rangle$ state.

- We aim to prepare an initial BD state with the parameters $c_1(0) = 1$, $c_2(0) = 0.7, c_3(0) = -0.7$.

$$\rho_{\textbf{BD}}^{\textbf{th}} = \begin{pmatrix} 0.07 & 0.00 & 0.00 & 0.07 \\ 0.00 & 0.43 & 0.43 & 0.00 \\ 0.00 & 0.43 & 0.43 & 0.00 \\ 0.07 & 0.00 & 0.00 & 0.07 \end{pmatrix}$$

- The two-qubit system was initialized into the pseudopure state $|00\rangle$ using the spatial averaging technique, with the density operator given by

$$\rho_{00} = \frac{1-\epsilon}{4} I + \epsilon |00\rangle\langle 00| \tag{9}$$

with a thermal polarization $\epsilon \approx 10^{-5}$ and I being a 4×4 identity operator.

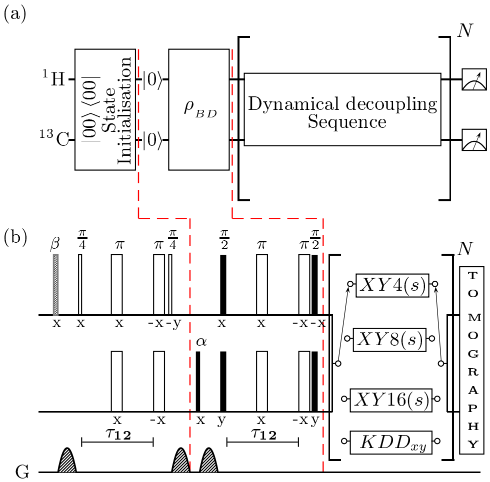


Figure: (a) Quantum circuit for the initial pseudopure state preparation, followed by the block for BD state preparation. The next block depicts the DD scheme used to preserve quantum discord. The entire DD sequence is repeated N times before measurement. (b) NMR pulse sequence corresponding to the quantum circuit. The rf pulse flip angles are set to $\alpha = 46^\circ$ and $\beta = 60^\circ$, while all other pulses are labeled with their respective angles and phases.

- The experimentally achieved $\rho_{\rm BD}^{\rm E}$ (reconstructed using the maximum likelihood method [2]) had parameters $c_1(0) = 1.0$, $c_2(0) = 0.68$ and $c_3(0) = -0.68$ and a computed fidelity of 0.99.

$$\rho_{\textbf{BD}}^{\textbf{exp}} = \begin{pmatrix} 0.080 & 0.003+0.000i & 0.003+0.000i & 0.080+0.000i \\ 0.003-0.000i & 0.42 & 0.420+0.001i & 0.003+0.000i \\ 0.003-0.000i & 0.420-0.001i & 0.420 & 0.003+0.000i \\ 0.080-0.000i & 0.003-0.000i & 0.003-0.000i & 0.080 \end{pmatrix}$$

- All experimental density matrices were reconstructed using a reduced tomographic protocol, with the set of operations given by $\{II, IX, IY, XX\}$ being sufficient to determine all 15 variables for the two-qubit system.

- $$F = \left(Tr \left(\sqrt{\sqrt{\rho_{\rm theory}}\rho_{\rm expt}\sqrt{\rho_{\rm theory}}} \right) \right)^2 \qquad (10)$$

where $\rho_{\rm theory}$ and $\rho_{\rm expt}$ denote the theoretical and experimental density matrices respectively.

[2] H. Singh, Arvind, and K. Dorai, Phys. Lett. A, 380, 3051 (2016)

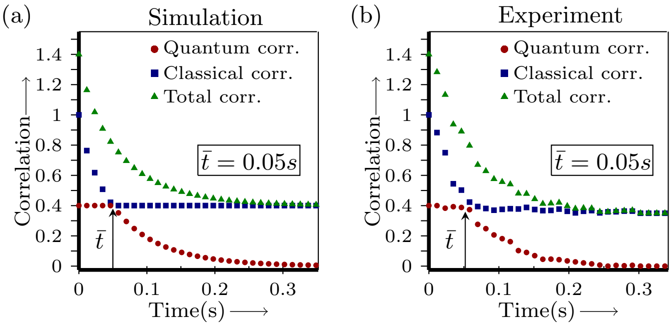


Figure: Time evolution of total correlations (green triangles), classical correlations (blue squares) and quantum discord (red circles) of the BD state.(a)Simulated with $\gamma_{zq} \approx 6.9/s$ $\gamma_{dq} \to 9.8/s$. (b) experimental results.

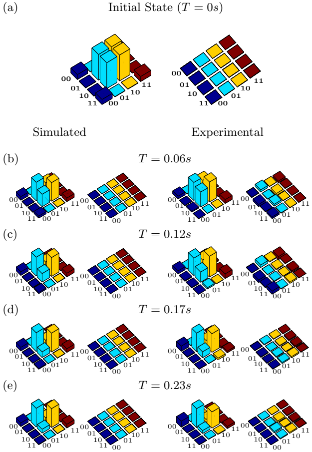
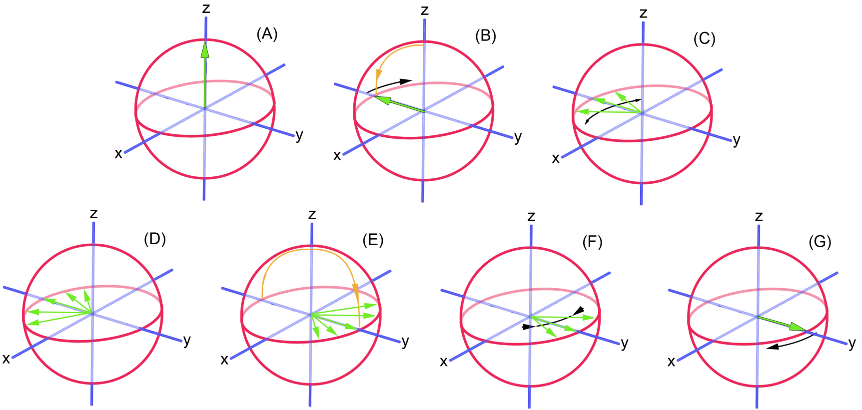


Figure: Real (left) and imaginary (right) parts of the experimental tomographs of the (a) Bell Diagonal (BD) state, with a computed fidelity of 0.99. (b)-(e) depict the state at $T = 0.06, 0.12, 0.17, 0.23$s, with the tomographs on the left and the right representing the simulated and experimental state, respectively. The rows and columns are labeled in the computational basis ordered from $|00\rangle$ to $|11\rangle$.

Spin echo



Dynamical Decoupling strategies

- When the state is known.
- When the subspace is known.
- Interested in protection coherence only.

(a)

(b)

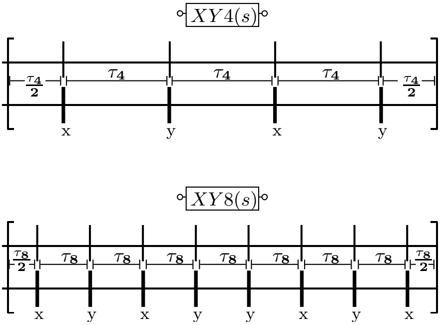


Figure: NMR pulse sequence corresponding to DD schemes (a) XY4(s), and (b) XY8(s). All the pulses are of flip angle π and are labeled with their respective phases. The pulses act on both qubits simultaneously.

- We implemented the symmetrized XY4 DD scheme for $\tau_4 = 0.58ms$ and an experimental time for one run of 2.43 ms.
- We implemented the XY8 DD scheme [a] (providing a better compensation for pulse errors), for $\tau_8 = 0.29ms$ and an experimental time for one run of 2.52 ms.

[a] Phys.Rev. A, 87, 042309 (2013)

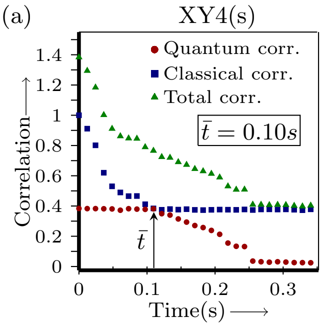


Figure: Time evolution of total correlations (green triangles), classical correlations (blue squares) and quantum discord (red circles) of the BD state. With dynamical decoupling sequence (a)XY4(s) (b) XY8(s)

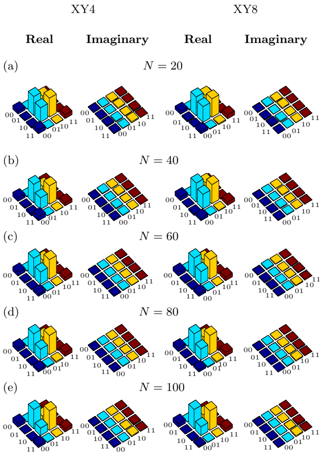


Figure: Real (left) and imaginary (right) parts of the experimental tomographs of the (a)-(e) depict the BD state at $N = 20, 40, 60, 80, 100$, with the tomographs on the left and the right representing the BD state after applying the XY4 and XY8 scheme, respectively. The rows and columns are labeled in the computational basis ordered from $|00\rangle$ to $|11\rangle$.

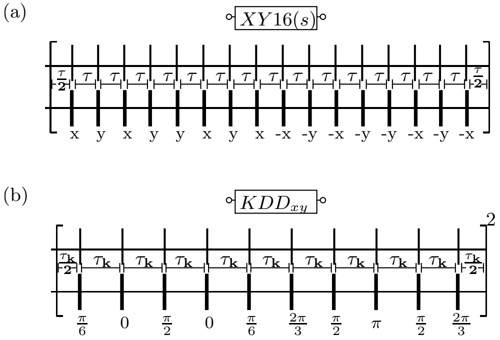


Figure: NMR pulse sequence corresponding to DD schemes (a) XY16(s), and (b) KDD_{xy}. All the pulses are of flip angle π and are labeled with their respective phases. The pulses act on both qubits simultaneously.

- We implemented the symmetrized XY16 DD scheme for $\tau = 0.145ms$ and an experimental time for one run of $2.75ms$.
- We implemented the KDD_{xy} DD scheme [a], $\tau_k = 0.116$ ms and an experimental time for one run of 2.86 ms.

[a] Phys. Rev.Lett., 106, 240501 (2011).

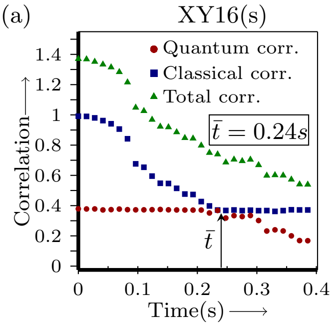


Figure: Time evolution of total correlations (green triangles), classical correlations (blue squares) and quantum discord (red circles) of the BD state.With dynamical decoupling sequence (a)XY16(s) (b) KDD_{xy}

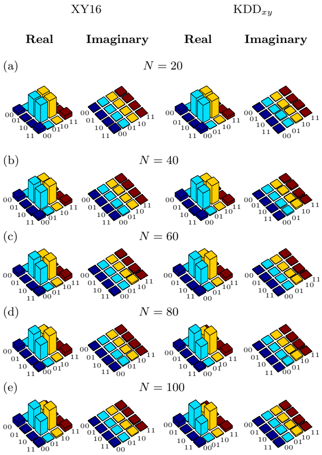


Figure: Real (left) and imaginary (right) parts of the experimental tomographs in (a)-(e) depict the Bell Diagonal (BD) state at $N = 20, 40, 60, 80, 100$, with the tomographs on the left and the right representing the BD state after applying the XY16 and KDD$_{xy}$ preserving DD schemes, respectively. The rows and columns are labeled in the computational basis ordered from $|00\rangle$ to $|11\rangle$.

Conclusion

- Our results show that quantum discord, which remains unaffected under certain decoherence regimes, can be preserved for very long times using dynamical decoupling schemes.
- Our experiments have important implications in situations where persistent quantum correlations have to be maintained in order to carry out quantum information processing tasks.

Thanks