# Detecting genuine tripartite entanglement in steering scenarios

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February 28, 2017

# Overview

1. Motivation
2. Genuine tripartite steering
   - Tripartite steering inequality proposed by me
3. Discussion

# Classification of Entanglement detection

## Entanglement certification of an unknown bipartite quantum system

- **Local tomography**: Alice (**A**) and Bob (**B**) have access to suitable trusted measurement devices to reconstruct given $\rho_{AB}$. By checking whether $\mathrm{Tr}\,\rho_{AB} W < 0$, where $W$ is an entanglement witness operator, **A** and **B** can aviod full tomographic knowledge.
- **Steering**: **A** performs a set of black-box (unknown) measurements, whereas **B** performs trusted measurements to know his conditional states $\sigma_{a|x}$. If $\{\sigma_{a|x}\}_{a,x}$ violates a steering inequality, then entanglement is detected.
- **Bell nonlocality**: Both **A** and **B** perform black-box measurements on the given $\rho_{AB}$. If $p(ab|xy)$ violates a Bell inequality, then entanglement is detected.

# Definition of steering

## Spooky action at a distance

- Local quantum measurements on one part of a bipartite system can be used to prepare genuinely different ensembles on the other part. This spooky action at a distance which was first noticed in the context of EPR paradox [a] was called steering [Schrodinger 1936].
- In 2007, Wiseman etal [b] gave a formal definition for the above idea by introducing the so called local hidden state (LHS) models.

[a] Einstein, Podolsky and Rosen: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47 (10), 777780 (1935)

[b] Wiseman, Jones and Doherty: Steering, Entanglement, Nonlocality, and the EPR Paradox, Phys. Rev. Lett. 98, 140402 (2007)

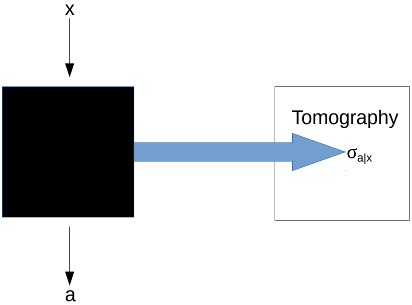


Figure: One-sided device-independent scenario.

$$\sigma_{a|x} = \mathrm{Tr}_A \left( M_{a|x} \otimes \mathbf{1} \rho_{\mathbf{AB}} \right) = p(a|x)\rho_{a|x} \quad \text{("assemblage")} \tag{1}$$

Suppose the given assemblage can be explained as follows with some hidden varibale $\lambda$:

$$\sigma_{a|x} = \sum_{\lambda} p(\lambda)p(a|\lambda, x)\rho_{\lambda} \quad \text{(LHS model).} \tag{2}$$

Then the one-sided device-independent scenario does not demonstrate steering.

## Operational definition of steering [Wiseman, Jones, Doherty (2007)]

- Suppose there exists a separable states that can reproduce the given assemblage for some measurements $M_{a|x}$. Then Alice does not demonstrates steerability to Bob.
- In other words, the correlations between Alice's measurement results and Bob's conditional states certify entanglement if the scenario demonstrates steering.

- Detecting entanglement through quantum steering requires fewer assumptions than the standard one and is a less experimentally demanding approach than device independent techniques.
- In Ref. [Q. Y. He and M. D. Reid, Phys. Rev. Lett. 111, 250403 (2013)], Genuine multipartite Einstein-Podolsky-Rosen steering has been proposed as a resource for one-sided device-independent quantum secret sharing.
- In Nat. Commun. 6, 7941 (2015), Cavalcanti etal have proposed an effiecient method to detect all kinds of multipartite entanglement in asymmetric quantum networks.

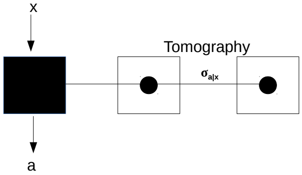


Figure: Tripartite one-sided device-independent scenario.

$$\sigma^{BC}_{a|x} = \mathrm{Tr}_A\left(M_{a|x} \otimes \mathbf{1} \rho_{\mathbf{ABC}}\right) = p(a|x)\rho^{BC}_{a|x}$$

## Definition of genuine steering [a]

[a] D. Cavalcanti etal (2015).

Suppose $\rho^{BC}_{a|x}$ could not be reproduced by a biseparable state which has the form,

$$\begin{aligned} \rho^{ABC}_{bisep} &= \sum_\lambda p^{A:BC}_\lambda \rho^A_\lambda \otimes \rho^{BC}_\lambda \\ &+ \sum_\mu p^{B:AC}_\mu \rho^B_\mu \otimes \rho^{AC}_\mu \\ &+ \sum_\nu p^{AB:C}_\nu \rho^{AB}_\nu \otimes \rho^C_\nu . \end{aligned}$$

Then the scenario demonstrates genuine steering.

- In the multipartite scenario, the observation of genuine nonlocality through the violation of a Bell-type inequality (e.g. the inequality derived by Svetlichny in 1987) implies genuine entanglement in a fully device-independent way. Consider the Svetlichny inequality,

$$\begin{aligned}&\langle A_0B_0C_1 + A_0B_1C_0 + A_1B_0C_0 - A_1B_1C_1\rangle \\ &+ \langle A_0B_1C_1 + A_1B_0C_1 + A_1B_1C_0 - A_0B_0C_0\rangle \leq 4. \qquad (3)\end{aligned}$$

  Here $\langle A_xB_yC_z\rangle = \sum_{abc} abc \cdot P(abc|xyz)$.
- The violation of the above inequality implies that the correlation cannot reproduced by the hybrid local-nonlocal model,

$$\begin{aligned}P(abc|xyz) = &\sum_\lambda p_\lambda P_\lambda(a|x)P_\lambda(bc|yz) \\ &+ \sum_\lambda q_\lambda P_\lambda(ab|xy)P_\lambda(c|z) \\ &+ \sum_\lambda r_\lambda P_\lambda(ac|xz)P_\lambda(b|y). \qquad (4)\end{aligned}$$

- An intuitive approach to Svetlichny inequality was presented to understand its violation by quantum correlations [Bancal etal 2011]:

$$\langle A_1 CHSH_{BC} + A_0 CHSH'_{BC} \rangle \leq 4, \tag{5}$$

where $CHSH_{BC} = B_0C_0 + B_0C_1 + B_1C_0 - B_1C_1$.

- Bancal etal found that considering the bipartition $A/BC$, $BC$ play the average game $\pm CHSH_{BC} \pm CHSH'_{BC}$. It can be checked that this average game satisfies the bound 4 for the correlations that have a hybrid local-nonlocal model.

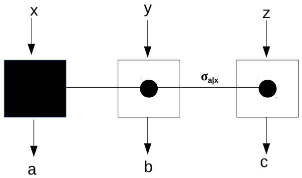


Figure: $p(abc|xyz) = \mathrm{Tr}\left(\Pi_{b|y} \otimes \Pi_{c|z}\sigma^{BC}_{a|x}\right)$.

## Theorem

*If $p(abc|xyz)$ violates the following steering inequality,*

$$\langle A_1 CHSH_{BC} + A_0 CHSH'_{BC}\rangle^{NLHS}_{?\times 2\times 2} \leq 2\sqrt{2}, \tag{6}$$

*then genuine tripartite steering from Alice to Bob and Charlie is demonstrated. Here $? \times 2 \times 2$ indicates that Alice and Bob have access to known qubit projective measurements that demonstrate Bell nonlocality of certain states, while Charlie's measurements are uncharacterized.*

If a correlation violates the above steering inequality, then it cannot be explained by the following nonlocal LHV-LHS (NLHS) model,

$$\begin{aligned} P(abc|xyz) = &\sum_\lambda p_\lambda P_\lambda(a|x) P(bc|yz, \rho^\lambda_{BC}) \\ &+ \sum_\lambda q_\lambda P(ab|xy, \rho^\lambda_{AB}) P_\lambda(c|z) \\ &+ \sum_\lambda r_\lambda P(ac|xz, \rho^\lambda_{AC}) P_\lambda(b|y), \end{aligned} \tag{7}$$

where $P(ab|xy, \rho^\lambda_{AB})$ and $P(ac|xz, \rho^\lambda_{AC})$ in general detect steerability of a $d \times 2$ system and $P(bc|yz, \rho^\lambda_{BC})$ is in general detects $2 \times 2$ nonseparability.

- Consider Svetlichny family defined as

$$P_{Sv}(abc|xyz) = \frac{2 + abc(-1)^{xy\oplus xz\oplus yz\oplus x\oplus y\oplus z\oplus 1}\sqrt{2}V}{16}, \tag{8}$$

which violates the Svetlichny inequality iff $V > 1/\sqrt{2}$.

- For $V \leq 1/\sqrt{2}$, the Svetlichny family is local. However, it violates the following steering inequality,

$$\langle A_1 CHSH_{BC} + A_0 CHSH'_{BC}\rangle \leq 2\sqrt{2}, \tag{9}$$

with $B_0 = (\sigma_x - \sigma_y)/\sqrt{2}$, $B_1 = (\sigma_x + \sigma_y)/\sqrt{2}$, $C_0 = \sigma_x$ and $C_1 = -\sigma_y$, for $V > 1/2$.

- In the above scenario, the noisy three-qubit GHZ state, $\rho = V|\Phi_{GHZ}\rangle\langle\Phi_{GHZ}| + (1 - V)\mathbf{1}/\mathbf{8}$, where $|\Phi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, gives rise to the Svetlichny family if Alice has performed the measurements $A_0 = \sigma_x$ and $A_1 = \sigma_y$. Note that the noisy GHZ state given above is genuinely entangled iff $V > 0.429$ (Guhne and Seevinck (2010)).

## Observation

*In the one-sided device-independent scenario where there is no assumption on which measurements Bob and Charlie perform, Svetlichny family does not detect Alice to Bob-Charlie steerability for* $V \leq 1/\sqrt{2}$.

## Proof.

Consider the $4 \times 2 \times 2$ classical-quantum state [Moroder etal, 2016],

$$\rho_{ABC} = \frac{1}{4} \sum_{i,j=0,1} |i,j\rangle\langle i,j| \otimes \omega_{ij}, \tag{10}$$

where $|i,j\rangle$ label measurements $M_{i|0} = |i\rangle\langle i| \otimes \mathbf{1}$ and $M_{j|1} = \mathbf{1} \otimes |\mathbf{j}\rangle\langle\mathbf{j}|$. For $V = 1/\sqrt{2}$, the Svetlichny family can arise from the above state with $\omega_{ij}$ are the four Bell states for the measurements $B_0 = \sigma_x$, $B_1 = \sigma_y$, $C_0 = \sigma_x$ and $C_1 = -\sigma_y$. □

- Recently, a tight connection between nonjoint measureability and steerability has been established [Quintino etal and Uola etal 2014].
- Consider the steering scenario where Bob and Charlie perform projective measurements along the directions $\hat{b}_0 = \hat{x}$, $\hat{b}_1 = \hat{y}$, $\hat{c}_0 = (\hat{x} - \hat{y})/\sqrt{2}$ and $\hat{c}_1 = (\hat{x} + \hat{y})/\sqrt{2}$. Suppose Alice has performed noisy projective measurements with visibility $\eta$,

$$M^{\eta}_{\pm|\hat{a}_x} = \eta \Pi_{\pm|\hat{a}_x} + (1-\eta)\frac{\mathbf{1}}{2}; \quad 0 \leq \eta \leq 1, \tag{11}$$

along the directions $\hat{a}_0 = \hat{x}$ and $\hat{a}_1 = -\hat{y}$ on the GHZ state. Then the statistics arising from the steering scenario are equivalent to the Svetlichny family with $V$ replaced by $\eta$.

- Note that in the above measurement scenario, Alice's measurements are nonjointly measureable iff $\eta > 1/\sqrt{2}$. However, the statistics violate the steering inequality for $\eta > 1/2$.

**Remark**

*Quantum correlations can also detect genuine tripartite steerability from Alice to Bob and Charlie even if Alice has performed jointly measureable measurements. This holds if the trusted parties Bob and Charlie are restricted to use two particular noncommuting measurements as we have seen above.*

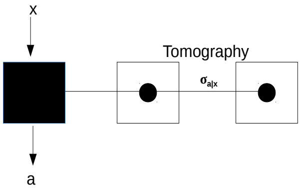


Figure: One-sided device-independent scenario.

In the one-sided device-independent scenario where Bob and Charlie can do local state tomography without any restriction on which measurements they perform, the assemblage satisfies the following constraint:

$$\sigma_{a|x}^{BC} = \Gamma_{a|x}^{A:BC} + \Gamma_{a|x}^{A:BC} + \Gamma_{a|x}^{A:BC}, \tag{12}$$

if there is no genuine steering from Alice to Bob and Charlie. That is, the assemblage can arise from a biseparable state,

$$\rho_{bisep}^{ABC} = \sum_\lambda p_\lambda^{A:BC} \rho_\lambda^A \otimes \rho_\lambda^{BC} + \sum_\mu p_\mu^{B:AC} \rho_\mu^B \otimes \rho_\mu^{AC} + \sum_\nu p_\nu^{AB:C} \rho_\nu^{AB} \otimes \rho_\nu^C,$$

for some suitable measurements $M_{a|x}$.

Under the above constraint, Cavalcanti etal [1] derived the following 3-setting steering inequality,

$$\begin{aligned} & 1+0.1547\,\langle Z_B Z_C\rangle-\frac{1}{3}(\langle A_3 Z_B\rangle+\langle A_3 Z_C\rangle+\langle A_1 X_B X_C\rangle-\langle A_1 Y_B Y_C\rangle \\ - & \langle A_2 X_B Y_C\rangle-\langle A_2 Y_B X_C\rangle)\geq 0. \qquad (13)\end{aligned}$$

For $A_i = X, Y, Z$, the noisy GHZ states $\rho = V|\Phi_{GHZ}\rangle\langle\Phi_{GHZ}| + (1-V)\mathbf{1/8}$ violate the above steering inequality for $V \gtrsim 0.54$. Whereas, my 2-setting steering inequality can be violated by these states even for any $V > 0.5$.

[1] D. Cavalcanti, P. Skrzypczyk, G. H. Aguilar, R. V. Nery, P. S. Ribeiro, and S. P. Walborn, Nat. Commun. 6, 7941 (2015).

# Summary

- In this talk, I presented a tripartite steering inequality to detect genuine tripartite entanglement in an asymmetric tripartite quantum network where one of the parties perform two black-box dichotomic measurements, while the other two parties perform two mutually unbiased qubit measurements.
- In terms of tolerance to noise, my steering inequality has advantage over the tripartite steering inequality derived by Cavalcanti etal.

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I acknowledge support through the project SR/S2/LOP-08/2013 of the DST, Govt. of India for my postdoctoral research associate position at S.N.Bose Centre Kolkata.

# Thank you for your attention!