Noise against noise

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What is quantum: Superposition!

Isn't it so?

Adapted from: A. Pathak, Elements of Quantum Computation and Quantum Communication, CRC Press, Boca Raton, USA (2013).

No.

Unlike a classical stochastic theory, in quantum mechanics, there is a *special basis* (the basis set the quantum state belongs to) in which it may be measured deterministically.

Mermaid remains mermaid in the special basis.

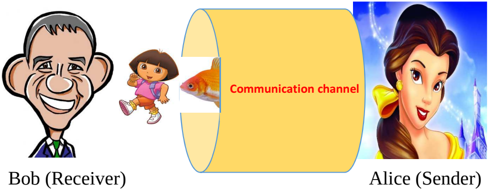
Entanglement is superposition in tensor product space violating separability condition. These are the facts (also, no-cloning, uncertainty principle . . .) exploited to enhance performance in quantum communication and computation.

A. Pathak, Elements of Quantum Computation and Quantum Communication. CRC Press, Boca Raton, USA (2013)

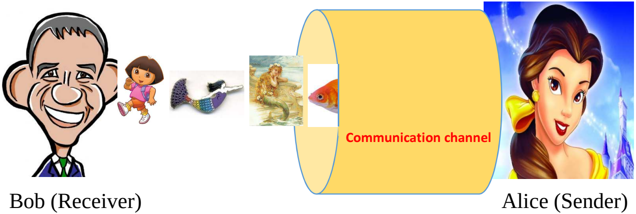
M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information. Cambridge University Press, New Delhi (2008)

Examples of quantum enhanced protocols

Classical communication



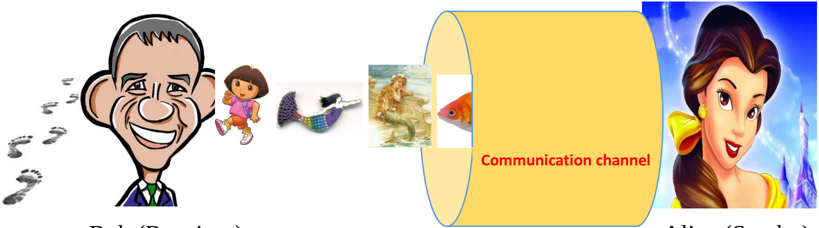
Quantum communication



Examples of quantum enhanced protocols: QKD

Eavesdropping by an intruder (Eve) will leave *detectable traces*.

This is famous BB84 quantum key distribution (QKD) protocol.



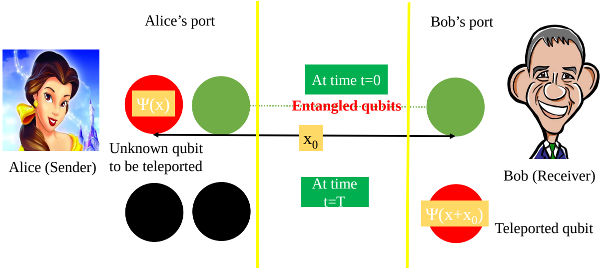
Bob (Receiver)   Alice (Sender)

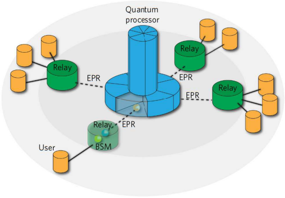
Therefore, quantum computation endangers classical cryptography but quantum cryptography is our solace (*provides unconditional security*).

Creates the problem on the one hand and solves it on the other.

C. H. Bennett, G. Brassard, In Proceedings of the IEEE International Conference on Computers, Systems, and Signal Processing, Bangalore, India 175 (1984).

Examples of quantum enhanced protocols: Teleportation



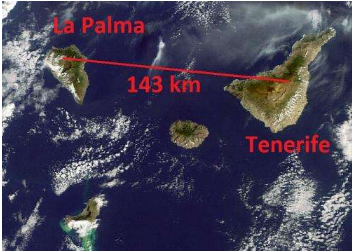


Transfer of an unknown quantum state using prior shared entanglement, where the information do not exist between Alice and Bob.

Proposals for using teleportation in quantum networks (fiber-based communication). (Various open-air and fiber-based quantum networks already exist [see Wikipedia]).

C. H. Bennett, et al., Phys. Rev. Lett. 70, 1895 (1993).
Q. C. Sun, et al., Nature Photonics (2016).

Major breakthroughs in quantum communication



Teleportation and QKD over 143 kms
(open-air communication)
(source: http://phys.org/news/2012-09-km-physicists-quantum-teleportation-distance.html)

Secure earth to satellite communication
(source: https://www.theguardian.com/world/2016/aug/16/china-launches-quantum-satellite-for-hack-proof-communications)

X.-S. Ma, et al., Nature 489, 269 (2012)
E. Gibney, Nature 535, 478 (2016)

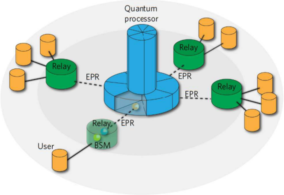
What else . . . : Our contributions

Teleportation QINP 16, 76 (2017) & Controlled teleportation QINP 14, 2599 (2015) & QINP 14, 4601 (2015)

Hierarchical quantum communication arxiv:1605.07399 (2016)

Direct secure quantum communication arxiv:1608.06071 (2016) & Asymmetric quantum dialogue QINP 16, 49 (2017)

Quantum voting IJQI 15, 1750007 (2017) & Decoy qubits QINP 15, 1703 (2016) & QINP 15, 4681 (2016)



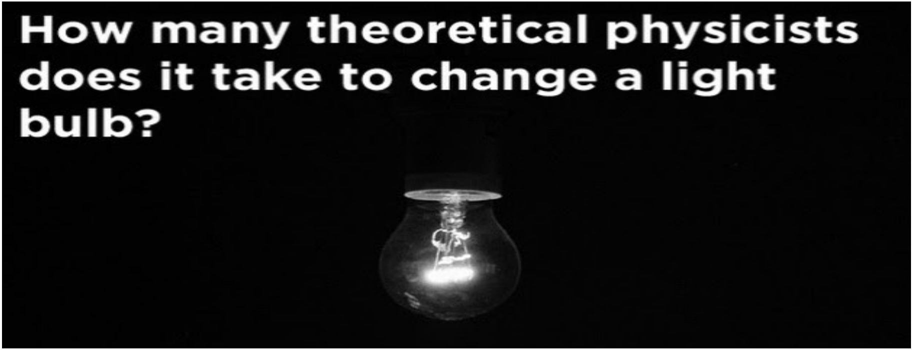
Quantum key distribution arxiv:1609.07473v1 (2016) & Quantum conference arxiv:1702.00389v1 (2017) & Quantum e-commerce

Controlled direct secure quantum communication arxiv:1608.06071 (2016)

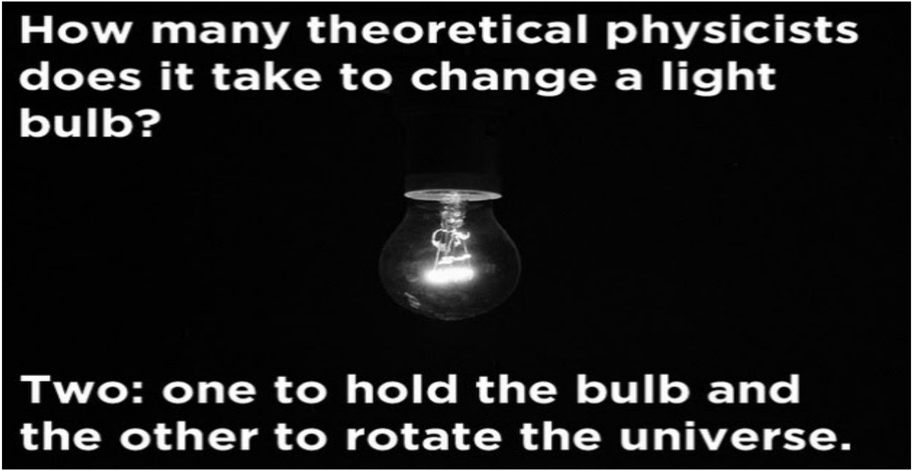
Quantum sealed bid auction arxiv:1612.08844v1 (2016)

Quantum private comparison arxiv:1608.00101v1 (2016)

Let's talk some realistic scenario



Let’s talk some realistic scenario



Your and our contribution

Is life that simple?

No, the trouble is our contribution.

Do you have a role to play?

Numerous quantum computations are running round the clock on IBM Quantum Experience delivered via IBM Cloud.

Yes! you have.

You contribute in **decoherence** caused due to environment.

Adapted from: A. Pathak, Elements of Quantum Computation and Quantum Communication (CRC Press, Boca Raton, USA (2013)).

Your contribution....

But it may cost someone his nose.

Adapted from: A. Pathak, Elements of Quantum Computation and Quantum Communication (CRC Press, Boca Raton, USA (2013)).

Mathematically handling your contribution: Open quantum system formalism

- Quantum non-demolition (QND) Evolution

To obtain a tomogram of a spin-$\frac{1}{2}$ atomic coherent state under QND evolution, we can write the density matrix in terms of Wigner-Dicke state as

$$\rho^{(j)} \equiv \rho^{(j)}(t) = \sum_{m,m'=-j}^{j} \rho^{(j)}_{m,m'} |j,m\rangle\langle j,m'|.$$

The different elements of this density matrix $\rho^{(j)}_{m,m'} = \langle m|\rho^{(j)}|m'\rangle$ can be obtained, with $m,\ n \rightarrow m,\ m'$, for $j = \frac{1}{2}$, $m,\ m' = \pm\frac{1}{2}$. Subsequently, the density matrix is obtained as

$$\rho^{(1/2)} = \begin{bmatrix} \sin^2\left(\frac{\alpha}{2}\right) & \frac{1}{2}e^{-i\omega t}e^{-(\hbar\omega)^2\gamma(t)}\sin\alpha e^{-i\beta} \\ \frac{1}{2}e^{i\omega t}e^{-(\hbar\omega)^2\gamma(t)}\sin\alpha e^{i\beta} & \cos^2\left(\frac{\alpha}{2}\right) \end{bmatrix}.$$

We can easily check that the trace of the density matrix is one, i.e., $\sum_{m=-1/2}^{1/2} \rho^{(1/2)}_{m,m} = 1$.

***K. Thapliyal**, S. Banerjee, A. Pathak, Annals of Phys. 366, 148 (2016).*

Open quantum system formalism...

- Squeezed generalized amplitude damping (SGAD) channel

The same initial state evolved under SGAD channel can be written as

$$\rho^{s}(t)=\left[\begin{array}{cc}\langle\frac{1}{2}|\rho^{s}(t)|\frac{1}{2}\rangle & \langle\frac{1}{2}|\rho^{s}(t)|-\frac{1}{2}\rangle\\ \langle-\frac{1}{2}|\rho^{s}(t)|\frac{1}{2}\rangle & \langle-\frac{1}{2}|\rho^{s}(t)|-\frac{1}{2}\rangle\end{array}\right],$$

where the various terms are

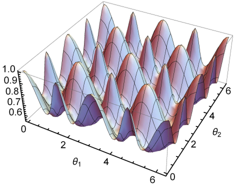
$$\begin{aligned}\langle\tfrac{1}{2}|\rho^{s}(t)|\tfrac{1}{2}\rangle &= \sin^{2}\left(\tfrac{\alpha}{2}\right)e^{-\gamma^{\beta}t}+\frac{\gamma_{-}}{\gamma^{\beta}}\left(1-e^{-\gamma^{\beta}t}\right),\\ \langle\tfrac{1}{2}|\rho^{s}(t)|-\tfrac{1}{2}\rangle &= \tfrac{1}{2}\sin\alpha\left[\left\{\cosh(\alpha't)-\frac{i\omega}{\alpha'}\sinh(\alpha't)\right\}\right.\\ &\times\left. e^{-i\beta}-\frac{\gamma_{0}M}{\alpha'}\sinh(\alpha't)e^{i\beta}\right]e^{-\frac{\gamma^{\beta}t}{2}},\\ \langle-\tfrac{1}{2}|\rho^{s}(t)|\tfrac{1}{2}\rangle &= \tfrac{1}{2}\sin\alpha\left[\left\{\cosh(\alpha't)+\frac{i\omega}{\alpha'}\sinh(\alpha't)\right\}\right.\\ &\times\left. e^{i\beta}-\frac{\gamma_{0}M^{*}}{\alpha'}\sinh(\alpha't)e^{-i\beta}\right]e^{-\frac{\gamma^{\beta}t}{2}},\\ \langle-\tfrac{1}{2}|\rho^{s}(t)|-\tfrac{1}{2}\rangle &= \cos^{2}\left(\tfrac{\alpha}{2}\right)e^{-\gamma^{\beta}t}+\frac{\gamma_{+}}{\gamma^{\beta}}\left(1-e^{-\gamma^{\beta}t}\right),\end{aligned}$$

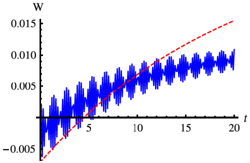
and the density matrix can be seen to be normalized, $\sum_{m=-1/2}^{1/2}\langle m|\rho^{s}(t)|m\rangle=1.$

***K. Thapliyal**, S. Banerjee, A. Pathak, Annals of Phys. 366, 148 (2016).*

Quantitative analysis of your contribution

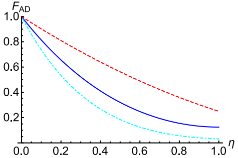
Fidelity of state in teleportation

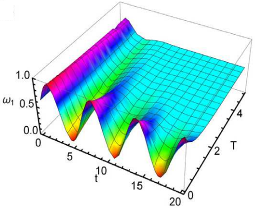




Wigner function for atomic/optical systems

Fidelity in secure quantum communication





Tomograms for atomic/optical systems

***K. Thapliyal**, S. Banerjee, A. Pathak, S. Omkar, and V. Ravishankar, Ann. Phys. 362, 261 (2015).*
***K. Thapliyal**, S. Banerjee, A. Pathak, Ann. Phys. 366, 148 (2016).*
*A. Banerjee, C. Shukla, **K. Thapliyal**, A. Pathak, and P. K. Panigrahi, QINP 16, 49 (2017).*
***K. Thapliyal** and A. Pathak, QINP 14, 2599 (2015).*

Controlled quantum dialogue

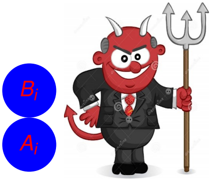
Controller

Bob

Alice

K. Thapliyal *and A. Pathak, Quantum Inf. Process. 14, 2599-2616 (2015).*
K. Thapliyal*, A. Pathak, and S. Banerjee, arxiv:1608.06071 (2016).*

Controlled quantum dialogue



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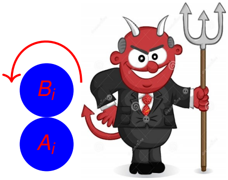

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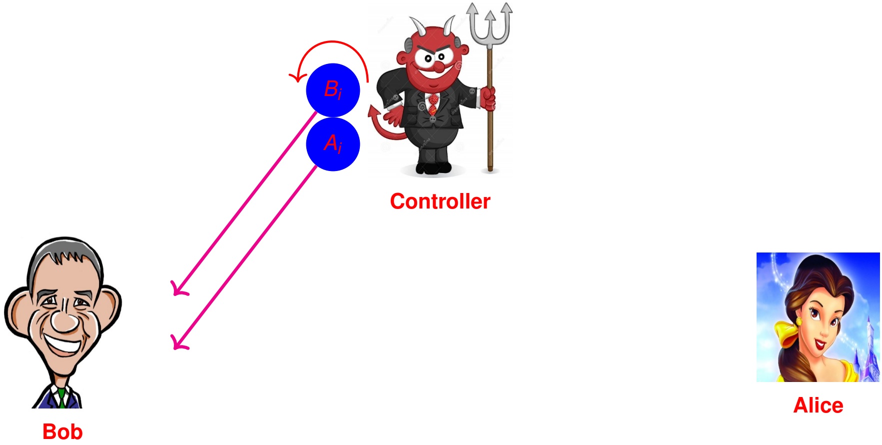

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Controlled quantum dialogue

Controller

B_i

A_i

Bob

Alice

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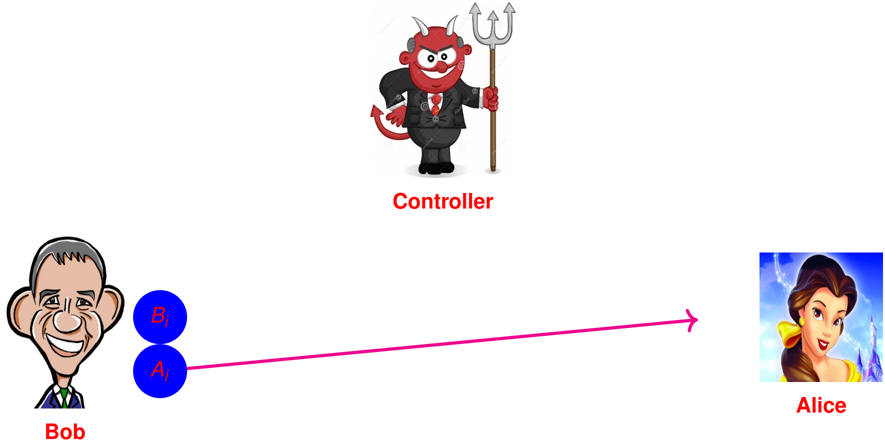

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Controlled quantum dialogue

Controller

B_i

Bob

A_i

Alice

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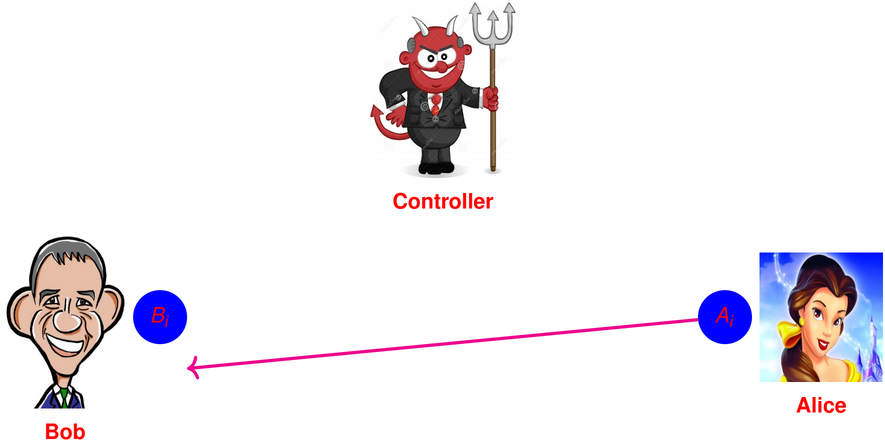

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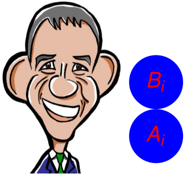

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Alice

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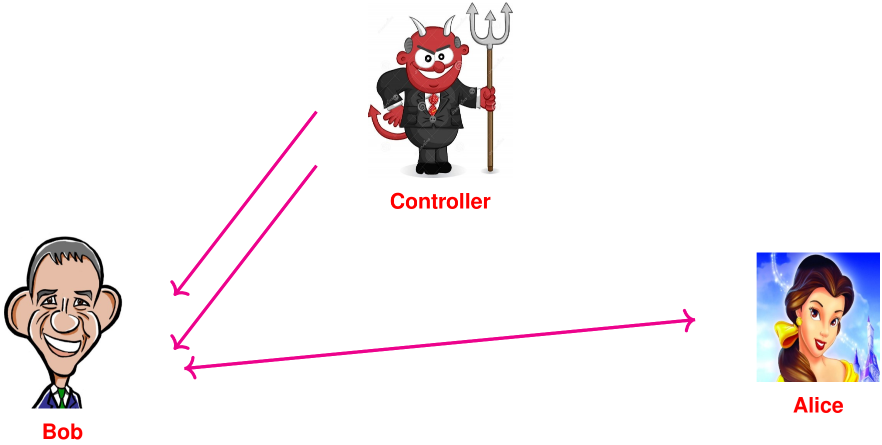

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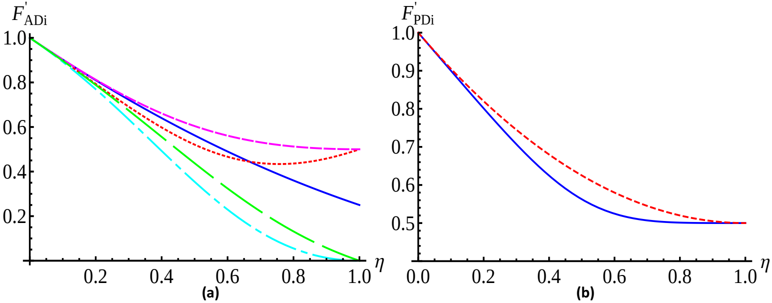
Controlled quantum dialogue: Markovian environment



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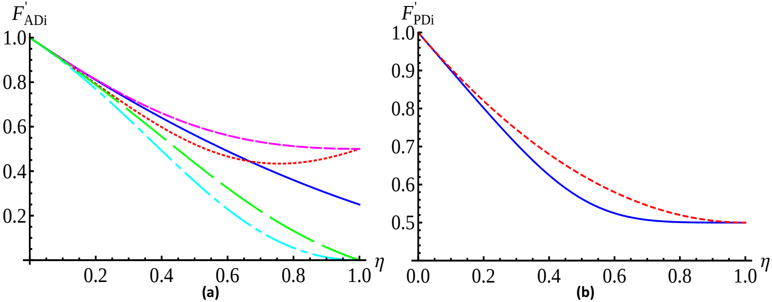
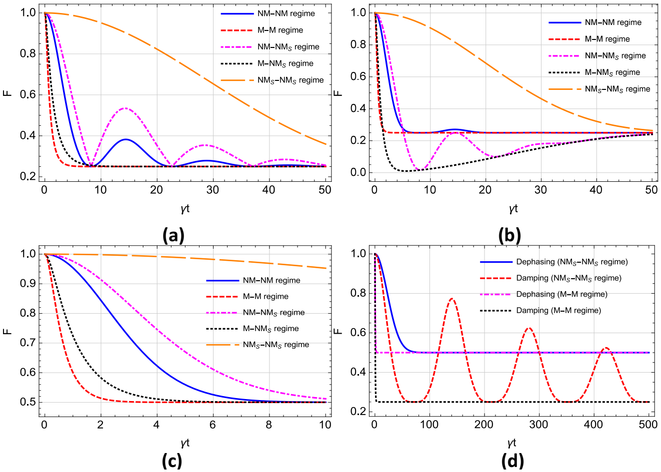
Controlled quantum dialogue: Markovian environment



From both the plots we can conclude that fidelity ($F = \langle\psi|\rho'|\psi\rangle$) falls with increasing decoherence rate.

K. Thapliyal *and A. Pathak, Quantum Inf. Process. 14, 2599-2616 (2015).*

Controlled quantum dialogue: non-Markovian environment



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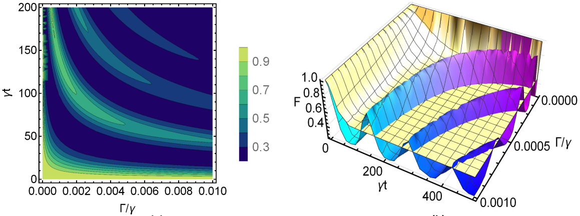
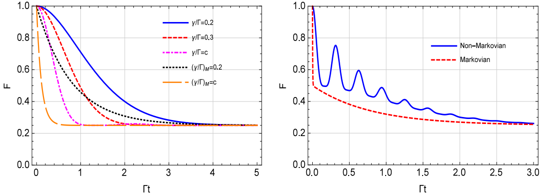
Controlled quantum dialogue: non-Markovian environment



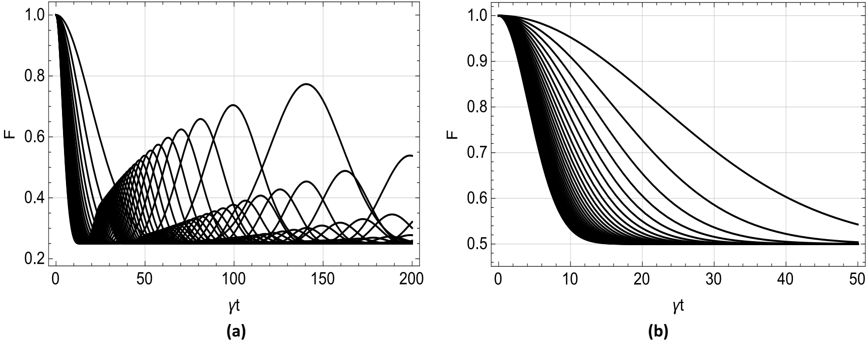
(a) **(b)**



(a) **(b)**

***K. Thapliyal**, A. Pathak, and S. Banerjee, arxiv:1608.06071 (2016).*

Controlled quantum dialogue: non-Markovian environment



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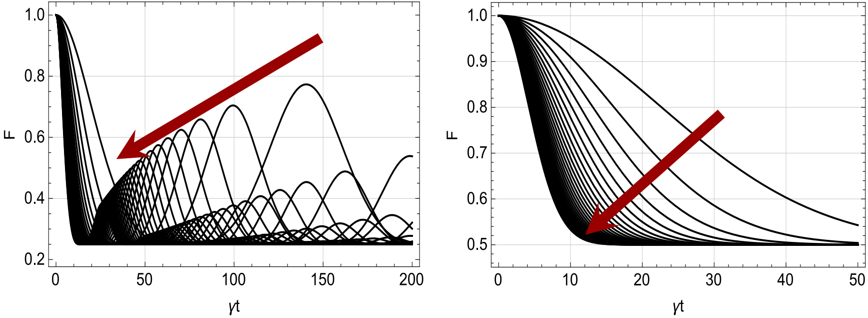
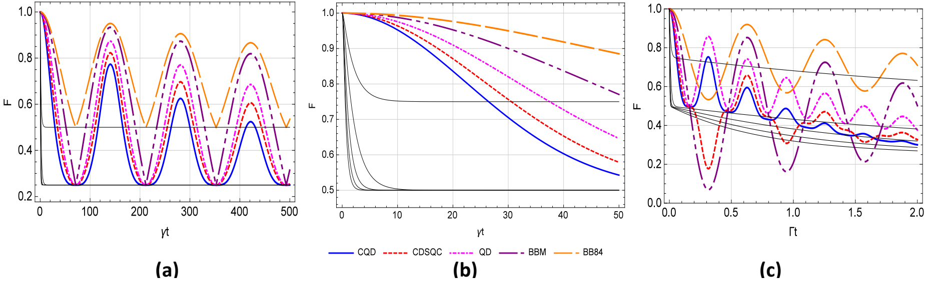
Controlled quantum dialogue: non-Markovian environment



(a) (b)

__K. Thapliyal__, A. Pathak, and S. Banerjee, arxiv:1608.06071 (2016).

Quantum cryptography over non-Markovian channels

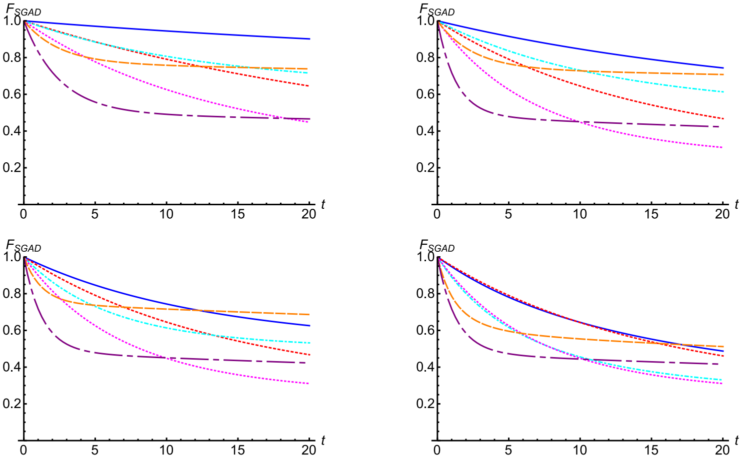


K. Thapliyal, *A. Pathak, and S. Banerjee, arxiv:1608.06071 (2016).*

Quantum cryptography over dissipative channels

QKD

QKA

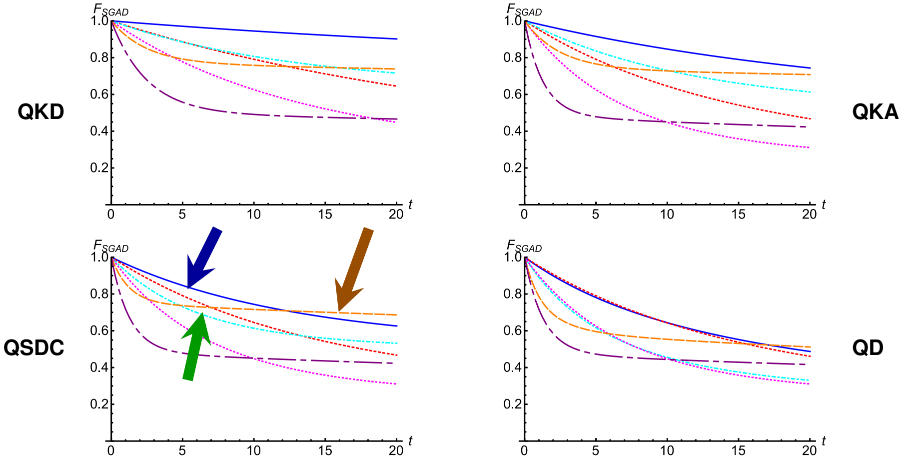


QSDC

QD

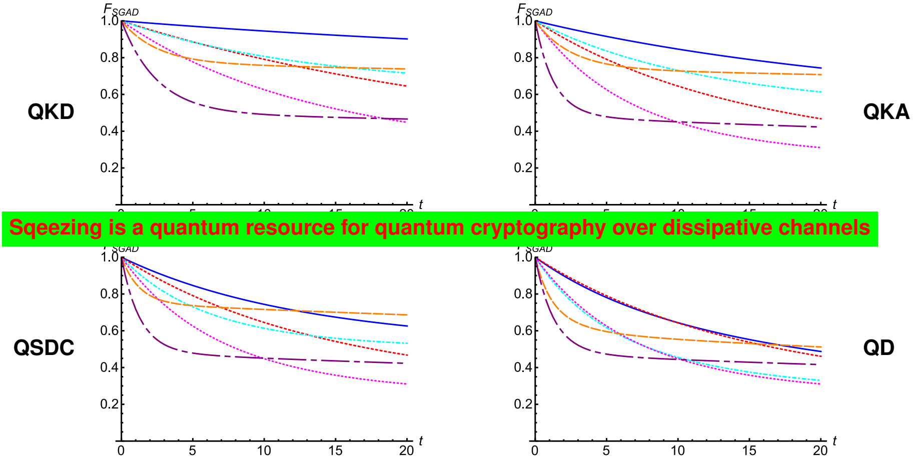
V. Sharma, **K. Thapliyal**, A. Pathak, and S. Banerjee, Quantum Inf. Process. 15, 4681 (2016).

Quantum cryptography over dissipative channels



V. Sharma, **K. Thapliyal**, A. Pathak, and S. Banerjee, Quantum Inf. Process. 15, 4681 (2016).

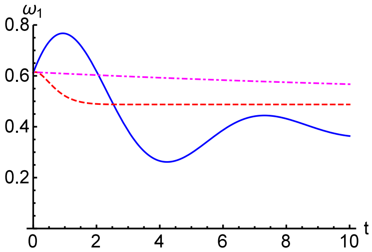
Quantum cryptography over dissipative channels

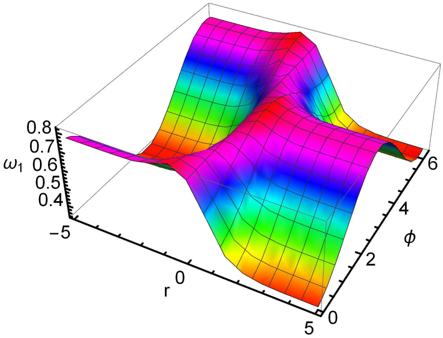


Sqeezing is a quantum resource for quantum cryptography over dissipative channels

*V. Sharma, **K. Thapliyal**, A. Pathak, and S. Banerjee, Quantum Inf. Process. 15, 4681 (2016).*

Squeezing works for atomic systems, too





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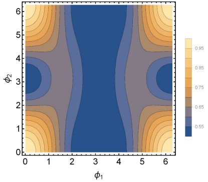
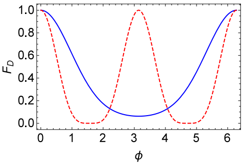
***K. Thapliyal**, S. Banerjee, A. Pathak, S. Omkar, V. Ravishankar, Annals of Phys. 362, 261 (2015).*

Let's count on optical-fiber-based quantum communication

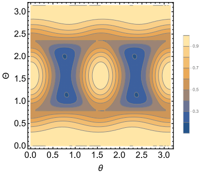
Performance of quantum dialogue over collective dephasing channel

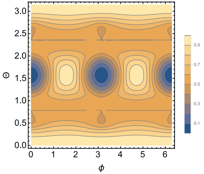




Decoy qubits commonly used in secure communication over collective dephasing channel

HJRSP under the effect of collective rotation noise





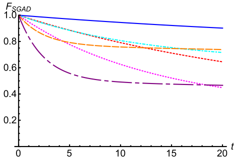
Effect of collective dephasing noise on HJRSP

*V. Sharma, **K. Thapliyal**, A. Pathak, S. Banerjee, Quantum Inf. Process. 15, 4681 (2016).*
*R. D. Sharma, **K. Thapliyal**, A. Pathak, A. K. Pan, A. De, Quantum Inf. Process. 15, 1703 (2016).*
*C. Shukla, **K. Thapliyal**, A. Pathak, arxiv:1605.07399 (2016).*

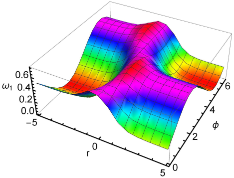
Conclusion: How to suppress your contribution?

Squeezing in the reservoir (environment)

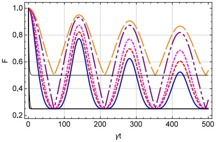
Squeezing enhancing performance in QKD



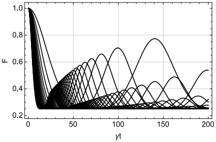
Effect of squeezing via tomogram



Strong coupling with the environment



Non-Markovianity performing the same task



Effect of coupling on quantum cryptography

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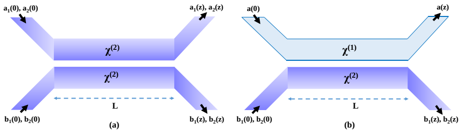
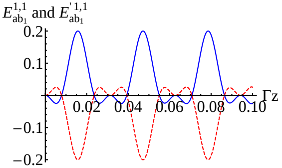
***K. Thapliyal**, S. Banerjee, A. Pathak, Annals of Phys. 366, 148 (2016).*

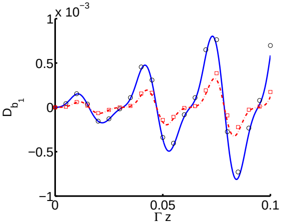
Generation of nonclassical states: Optical systems

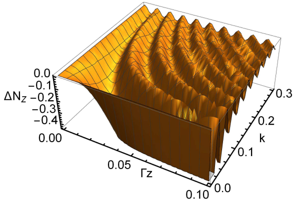




Entanglement in nonlinear optical couplers

Antibunching in nonlinear optical couplers





Quantum Zeno and anti-Zeno effect in nonlinear optical couplers

***K. Thapliyal**, A. Pathak, B. Sen, and J. Peřina, Phys. Rev. A 90, 013808 (2014).*
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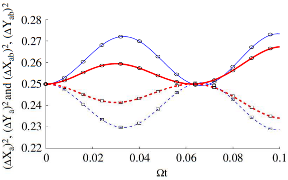
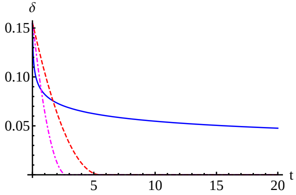
Generation and characterization of nonclassical states: Atomic systems

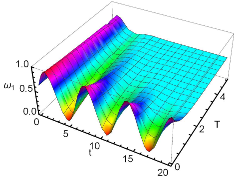
Squeezing and intermodal squeezing in an atom-molecule BEC

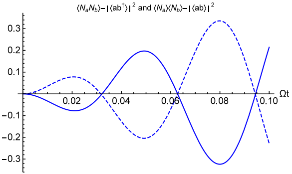




Quantification (using Wigner volume) and degradation of nonclassicality in a single qubit spin state

The effect of temperature on tomogram of a single qubit spin state





Atomic and molecular BEC modes found to be always entangled

*S. K. Giri, **K. Thapliyal**, B. Sen, A. Pathak, Physica A 466, 140 (2017).*
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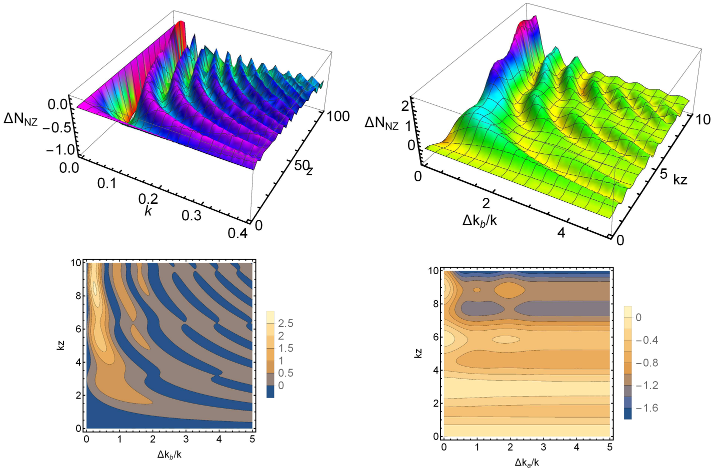
(Non-)Linear quantum Zeno and anti-Zeno effects: Counter-factual computation and communication



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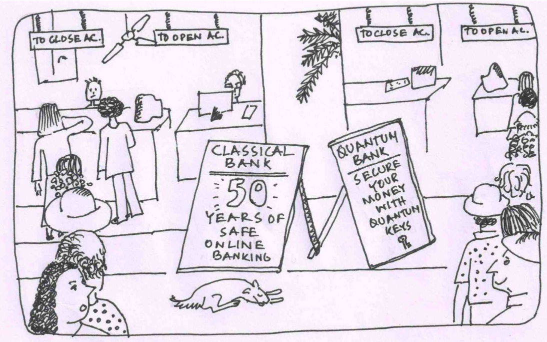
Our future: Quantum world



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1. M. Sisodia, V. Verma, **K. Thapliyal**, and A. Pathak, Teleportation of a qubit using entangled non-orthogonal states: A comparative study, Quantum Inf. Process. 16, 76 (2017).
2. A. Banerjee, C. Shukla, **K. Thapliyal**, A. Pathak, and P. K. Panigrahi, Asymmetric quantum dialogue in noisy environment, Quantum Inf. Process. 16, 49 (2017).
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5. V. Sharma, **K. Thapliyal**, A. Pathak, and S. Banerjee, A comparative study of protocols for secure quantum communication under noisy environment: single-qubit-based protocols versus entangled-state-based protocols, Quantum Inf. Process. 15, 4681–4710 (2016).
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9. **K. Thapliyal**, A. Verma and A. Pathak, A general method for selecting quantum channel for bidirectional controlled state teleportation and other schemes of controlled quantum communication, Quantum Inf. Process. 14, 4601–4614 (2015).

विद्या तत्व ज्योतिसम:

Some of the relevant publications

10. **K. Thapliyal**, S. Banerjee, A. Pathak, S. Omkar, V. Ravishankar, Quasiprobability distributions in open quantum systems: spin-qubit systems, Annals of Phys. 362, 261–286 (2015).
11. **K. Thapliyal** and A. Pathak, Applications of quantum cryptographic switch: Various tasks related to controlled quantum communication can be performed using Bell states and permutation of particles, Quantum Inf. Process. 14, 2599–2616 (2015).
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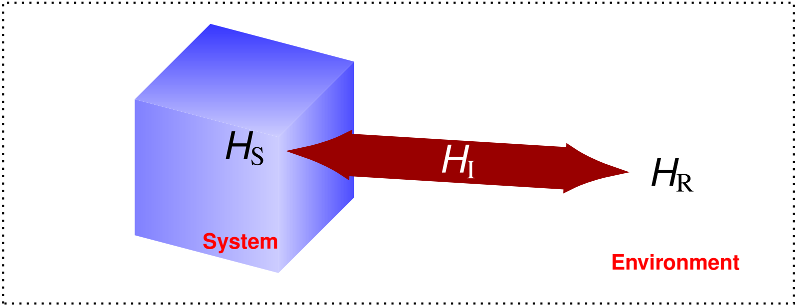
Dr. S. Banerjee*, Indian Institute of Technology Jodhpur, Jodhpur, India;*
Dr. Biswajit Sen*, Vidyasagar Teachers' Training College, Midnapore, India;*
Prof. V. Ravishankar*, Indian Institute of Technology Delhi, Hauz Khas, New Delhi, India;*
Prof. P. K. Panigrahi*, Indian Institute of Science Education and Research Kolkata, Mohanpur, India.*

Appendices

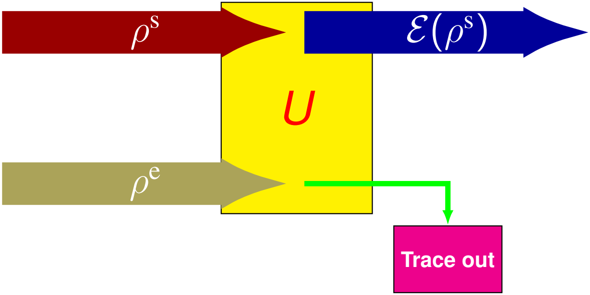
Open quantum system



Hamiltonian: $H = H_S + H_R + H_I$, where H_s is the system Hamiltonian, H_I is the system-reservoir interaction Hamiltonian, and the reservoir Hamiltonian H_R is given by $H_R = \sum_j \frac{p_j^2}{2m_j} + \frac{1}{2}m_j\omega_j^2 x_j^2$.

Quantum non-demolition systems: $[H_S, H_I] = 0$.

Open quantum system



Evolution of the system-bath combination is unitary and is given by Liouville-von Neumann equation

$$\dot{\rho}(t) = -i[H, \rho(t)],$$

where $\rho = \rho^S \otimes \rho^e$ is the quantum state in combined Hilbert space $H^S \otimes H^e$.

Markovian and non-Markovian channels

- Tracing over environment degrees of freedom, one can obtain, for a quantum state ρ^S in N-dimensional Hilbert space. $\dot{\rho}^S = \mathcal{L}[\rho^S]$.
- The construction of most general form of generator $\mathcal{L}$ leads to the Lindblad equation.
- Following assumptions are involved in writing Lindblad form of master equation.
  Born approximation: Weak coupling between system (S) and reservoir (R).
  Markov approximation: Memoryless (when the time scale associated with the reservoir correlations is much smaller than the time scale over which the state varies appreciably, which is easily justified for weak S–R coupling and high temperature).
  Rotating wave approximation: Fast system dynamics compared to relaxation time are averaged over.
- In operator-sum (or Kraus representation), a superoperator $\mathcal{E}$ acting on a system due to interaction with ambient environment is given by $\rho \to \mathcal{E}(\rho) = \Sigma_k \langle e_k \left| U(\rho \otimes |f_0\rangle\langle f_0|) U^\dagger \right| e_k\rangle = \Sigma_j E_j \rho E_j^\dagger$, where U is the unitary operator for free evolution of system, reservoir, and interaction between them. Here, $|f_0\rangle$ is the environment's initial state, and $|e_k\rangle$ is a basis of environment.
- This gives $E_j = \langle e_k |U| f_0\rangle$ as the Kraus operators satisfying completeness condition $E_j^\dagger E_j = \mathbb{I}$.

Non-Markovian channels

- Typically, this is due to the fact that the relevant environmental correlation times are not small compared to the system's relaxation or decoherence time, rendering the standard Markov approximation impossible.
- The violation of this separation of time scales can occur, for example, in the cases of strong system-environment couplings, structured or finite reservoirs, low temperatures, or large initial system environment correlations.
- The Kraus operators for the **damping noise under non-Markovian effects** are given by

$$K_0 = |0\rangle\langle 0| + \sqrt{p}|1\rangle\langle 1|, \qquad K_1 = \sqrt{1-p}|0\rangle\langle 1|,$$

where $p \equiv p(t) = \exp(-\Gamma t)\left\{\cos\left(\frac{dt}{2}\right) + \frac{\Gamma}{d}\sin\left(\frac{dt}{2}\right)\right\}^2$ with $d = \sqrt{2\gamma\Gamma - \Gamma^2}$. Here, Γ is the line width which depends on the reservoir correlation time $\tau_r \approx \Gamma^{-1}$; and γ is the coupling strength related to qubit relaxation time $\tau_s \approx \gamma^{-1}$. In the domain of large reservoir correlation time in comparison to qubit relaxation time, memory effects come into play. The memory effects are characteristic of non-Markovian nature of dissipation.
- Similarly, the Kraus operators for **purely dephasing non-Markovian noise** are

$$K_0 = |0\rangle\langle 0| + p|1\rangle\langle 1|, \qquad K_1 = \sqrt{1-p^2}|1\rangle\langle 1|,$$

where $p \equiv p(t) = \exp\left[-\frac{\gamma}{2}\left\{t + \frac{1}{\Gamma}\left(\exp(-\Gamma t) - 1\right)\right\}\right]$.
- Finally, a **non-Markovian depolarizing channel** can be described by the Kraus operators $K_i = \sqrt{\mathcal{P}_i}\sigma_i$, where $\sigma_0 \equiv I$ and σ_is are the three Pauli matrices. The $\mathcal{P}_i$s should remain positive to ensure the complete positivity for all values of $\frac{\gamma_j}{\Gamma_j}$.