Probing hierarchy of temporal correlation requires either generalised measurement or nonunitary evolution

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- Probing hierarchy of temporal correlation requires either generalised measurement or nonunitary evolution.
  S. Mal, A S Majumdar, D. Home, arXiv: 1510.00625 (2015).

EPR paradox

- In their seminal 1935 paper Einstein, Podolsky and Rosen (EPR) presented an argument demonstrating the incompatibility between local realism and quantum mechanics.
- Schrödinger seems to have been the first to name the situation a paradox as he could not believe with EPR that QM is incomplete but neither found any flaw in their argument.
- Necessary condition of completeness:every element of the physical reality must have a counterpart in the physical theory. Sufficient condition of reality: If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

EPR's dilemma

- EPR's dilemma: i) the quantum-mechanical description of reality given by the wave function is not complete or ii) when the operators corresponding to physical quantities do not commute the quantities cannot have simultaneous reality.
- $|\Psi\rangle = \sum_n c_n|\psi_n\rangle \otimes |u_n\rangle = \sum_s c'_s|\phi_s\rangle \otimes |v_s\rangle$.
- Necessary condition of locality: No real change can take place in the second system in consequence of anything that may be done to the first system.

EPR-Schrödinger steering

- EPR conclude that as it is possible to assign two different wave functions to the same reality, QM is incomplete.
- Schrödinger did not agree with EPR's conclusion. In disentangling measurement at experimenter's mercy one is able to affect the other system.
- It is discomfortable that the theory allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of her having no access to it.

EPR-Schödinger steering

- He reject the idea of LHV for explaining steering and due to Bell theorem we now know he is correct.
- Unlike EPR being discomfortable with that he attempt to resolve the paradox by arguing that there being no sufficient experimental evidence of steering in nature.
- The essence of the above arguments involved perfect correlations and therefore could not be directly tested in nonideal situation in laboratory without additional assumptions.
- Criteria for experimental demonstration of EPR-Schödinger steering is first derived by Reid in 1989 for continuous variable scenario and then for discrete variable system by E G Cavalcanti et al.

Reid's criteria

- Reids extension of EPRs sufficient condition of reality: If, without in any way disturbing a system, we can predict with some specified uncertainty the value of a physical quantity, then there exists a stochastic element of physical reality which determines this physical quantity with at most that specific uncertainty.
- Since Alice can, by measuring either position x^A or momentum p^B, infer with some uncertainty $\Delta_{inf} x_B$ or $\Delta_{inf} p_B$, the outcomes of the corresponding experiments performed by Bob.
- Since by the locality condition of EPR there must be simultaneous stochastic elements of reality which determine x^B and p^B with at most those uncertainties.

Reid's criteria

- Now by Heisenbergs uncertainty principle, quantum mechanics imposes a limit to the precision with which one can assign values to observables corresponding to noncommuting operators such as $\hat{x}$ and $\hat{p}$.
- Therefore, if quantum mechanics is complete and the locality condition holds, by use of the extended sufficient condition of reality and EPRs necessary condition for completeness, the limit with which one could determine the average inference variances above is
- $\Delta x_{inf} \Delta p_{inf} \geq 1$.
- $\Delta J_x^B \Delta J_y^B \geq \frac{1}{2} |\langle J_z^B|$.

Locality models

- In 2007 Wiseman et al. formalise the concept of steering by demonstrating that EPR steering constitute a different class of nonlocality intermediate between the class of non-separability and Bell-nonlocality.
- A distinction was made between three locality models, the failure of each correspond to three strictly distinct form of nonlocality.
- Suppose $\lambda \in \Lambda$ any variable associated with events in the union of the past light cones of a, A, b, B which are relevant to the experimental situation.

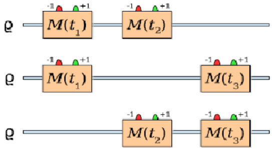
Condition of steering

- Wiseman and co-workers considered EPR-steering to occur iff it is not the case that there exists a decomposition of Bob's reduced state, $\rho_B = \sum_\lambda p(\lambda)\rho(\lambda)$, s.t. for all measurements and outcomes of Alice there exists a stochastic map $p(a|A,\lambda)$ for which $\tilde{\rho}_a^A = \sum_\lambda p(\lambda)p(a|A,\lambda)\rho_\lambda$.
- One certainly does not want to consider it as an example of steering when the ensembles prepared by Alice are just different coarse grainings of some underlying ensemble of states.

Locality models

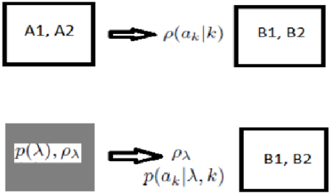
- A phenomenon has LHV model iff
$P(a_i, b_j|A_i, B_j) = \sum_\lambda p(\lambda)p(a_i|A_i, \lambda)p(b_j|B_j, \rho_\lambda)$
- A phenomenon has quantum separable iff
$P(a_i, b_j|A_i, B_j) = \sum_\lambda p(\lambda)p_Q(a_i|A_i, \lambda)p_Q(b_j|B_j, \rho_\lambda)$
- Strictly intermediate between LHV and separable models is the LHS model for Bob
$P(a_i, b_j|A_i, B_j) = \sum_\lambda p(\lambda)p(a_i|A_i, \lambda)p_Q(b_j|B_j, \rho_\lambda)$
- It was shown that for pure states, entangled states, steerable states, and Bell-nonlocal states are all equivalent classes.
- For Werner class of states $\eta_{ent} < \eta_{ster} < \eta_{Bell}$.

Temporal correlation



- Kochen-Specker no-go theorem: No non-contextual model for quantum theory (1967). Probed through violation of noncontextuality inequalities for dimension atleast three.
- No macro-realist/noninvassive-realist model for quantum theory (1985). Probed through violation of Leggett-Garg inequality for dimension atleast two.

Temporal steering



- Alice measures A_1 or A_2 according to the request from Bob, sends the post-measurement state to him through some quantum channel and announces her outcome publicly.
- Classical mimicry of the above case. Alice picks ρ_λ with probability p_λ, announces outcome a_k with probability $p(a_k|\lambda, k)$ and sends the particle to Bob.

Different temporal correlations

- In the *non-invasive realist model* (NIRM), a hidden variable model (HVM) pertinent to the LG scenario, the joint probabilities can be written as

$$P(A_i = a_i, B_j = b_j) = \sum_{\lambda} p(\lambda) p(a_i|A_i, \lambda) p(b_j|B_j, \lambda) \tag{1}$$

  This NIRM leads to an LGI. Quantum violation of this inequality has been linked with information processing tasks
- Entanglement in Time and Temporal Communication Complexity, S. Taylor, S. Cheung, C. Brukner, V. Vedral, AIP Conference Proceedings 734, 281 (2004).
- Temporal correlations and device-independent randomness, S Mal, M Banik, S K Choudhury, Quantum Inf Process 15, 2993(2016).

Different temporal correlation

- There exists a hidden state model (HSM) for Bob when Alice is not capable of steering, and joint probabilities can be written as
$$P(A_i = a_i, B_j = b_j) = \sum_{\lambda} p(\lambda) p(a_i|A_i, \lambda) p^Q(b_j|B_j, \rho_\lambda) \quad (2)$$
Violation of any inequality derived from this is a demonstration of temporal steering.
- When this is valid for all measurements performed by Alice and Bob then there is HSE for Bob
$$\tilde{\rho}(a_k|k) = \sum_{\lambda} p(\lambda) p(a_k|\lambda, k) \rho_\lambda \quad (3)$$
- Temporal steering inequality, Yueh-Nan Chen et al., Phys. Rev. A 89, 032112 (2014)

Equivalence under unitary evolution and projective measurements

- **Theorem 2**: *For arbitrary initial state under unitary evolution and projective measurements HSM and HVM for the temporal correlations are equivalent*.
- This is proved by showing HSM $\Rightarrow$ HVM and conversely HVM $\Rightarrow$ HSM .

Equivalence under unitary evolution and projective measurements

- **Lemma 1**: Consider a group G with unitary representation $U(g)$ on the Hilbert space of the system. Suppose, $\forall A \in \mathcal{M}_A$ (which Alice can measure), $\forall a$ and $\forall g \in G$, if we have $U_1^\dagger(g) A U_1(g) \in \mathcal{M}_A$ and $\tilde{\rho}_a^{U_1^\dagger(g) A U_1(g)} = U_2(g) \tilde{\rho}_a^A U_2^\dagger(g)$, then there exists a G-covariant optimal ensemble: $\{\rho_\lambda^\star, p_\lambda^\star\} = \{U_2(g) \rho_\lambda^\star U_2^\dagger(g), p_\lambda^\star\}$. $U_1(g)$ and $U_2(g)$ are unitary operations applied by Alice and Bob respectively.

Equivalence under unitary evolution and projective measurements

- For any initial state under unitary evolution and projective measurements lemma holds. This is because firstly $U_1^\dagger(g) A U_1(g) \in \mathcal{M}_A$.
- For the other condition suppose, Alice measures $U_1^\dagger(g) A U_1(g)$, then the unnormalised state becomes $\tilde{\rho}_a^{U_1^\dagger(g)AU_1(g)} = P_a^{U_1^\dagger(g)AU_1(g)} \rho P_a^{U_1^\dagger(g)AU_1(g)} \propto P_a^{U_1^\dagger(g)AU_1(g)}$.
- Again, $U_2(g)\tilde{\rho}_a^A U_2^\dagger(g) = U_2(g) P_a^A \rho P_a^A U_2^\dagger(g) \propto U_2(g) P_a^A U_2^\dagger(g)$.
- Now, two pure states $P_a^{U_1^\dagger(g)AU_1(g)}$ and $U_2(g)P_a^A U_2^\dagger(g)$ are always connected by some unitary, rather they are identical when $U_2(g) = U_1^\dagger(g)$. Hence, conditions of the lemma are satisfied.

Equivalence under unitary evolution and projective measurements

- let us take the set of all pure states, $\{|\lambda\rangle \in C^d || \langle\lambda|\lambda\rangle| = 1\}$, for constructing HVM. The set of pure states together with the probability measure taken as the Haar measure over the unitary groups defines an unique optimal covariant ensemble.
- HSM $\Rightarrow$ HVM. This follows trivially by simply denoting $p(b_j|B_j,\lambda) = p^Q(b_j|B_j,\rho_\lambda)$.
- $P(A_i = a_i, B_j = b_j) = \sum_\lambda p(\lambda)p(a_i|A_i,\lambda)p(b_j|B_j,\lambda)$.
- $P(A_i = a_i, B_j = b_j) = \sum_\lambda p(\lambda)p(a_i|A_i,\lambda)p^Q(b_j|B_j,\rho_\lambda)$.

Equivalence under unitary evolution and projective measurements

- Now we show for projective measurements and unitary evolution that the converse of the above implication, *i.e.,* HVM $\Rightarrow$ HSM is also true.
- This is to say that Alice can simulate $\tilde{\rho}_a^A$ using the HSE, $\{p_\lambda, \rho_\lambda\}$, with the same p_λ and $p(a|A, \lambda)$ appearing in the HVM.

Equivalence under unitary evolution and projective measurements

- According to steering protocol, Bob asks Alice to measure A, and after measuring she announces the outcome a. Then Bob gets an unnormalised state $\tilde{\rho}_a^A = p(a|A)\rho_a^A$.
- Now, for unitary evolution and projective measurements as the lemma is satisfied for arbitrary initial states, there exists an optimal ensemble, and without loss of generality let it consist of pure states with the Haar measure $\{\rho_\lambda^\star, p_\lambda^\star\}$, s.t. $\tilde{\rho}_a^A = \sum p_\lambda^\star \rho_\lambda^\star p^\star(a|A,\lambda)$.

Equivalence under unitary evolution and projective measurements

- Now, for unitary evolution and projective measurements as the lemma is satisfied for arbitrary initial states, there exists an optimal ensemble, and without loss of generality let it consist of pure states with the Haar measure $\{\rho_\lambda^\star, p_\lambda^\star\}$, s.t. $\tilde{\rho}_a^A = \sum p_\lambda^\star \rho_\lambda^\star p^\star(a|A,\lambda)$.
- From the existence of the HVM, we have $p(a|A) = \sum_\lambda p_\lambda p(a|A,\lambda)$, and since the optimal ensemble exists, we must have $\sum_\lambda p_\lambda p(a|A,\lambda) = \sum_\lambda p_\lambda^\star p^\star(a|A,\lambda)$.
- As ρ_λ and $\rho_\lambda^\star$ are pure states, they are unitarily related and the invariance of the Haar measure over all spherical rotations implies Alice can construct another HSE, $\{p_\lambda, \rho_\lambda\}$, with $p_\lambda = p_\lambda^\star$.

Equivalence under unitary evolution and projective measurements

- Consequently, we have $p(a|A,\lambda) = p^{\star}(a|A,\lambda)$. Thus from the knowledge of HVM Alice can simulate $\tilde{\rho}_a^A$. Hence, the theorem.
- The above theorem states that under unitary evolution and projective measurements the existence of HSM implies the existence of HVM, and *vice-versa*, from which it logically follows that violation of LGI implies violation of TSI, and *vice-versa*.

Probing Hierarchy

- Necessity and sufficiency of nonjoint measurability to demonstrate temporal steering has been demonstrated. Here we find that there exist nonjoint measurable observables that can demonstrate steering without leading to LGI violation.
- A quadratic steering inequality for measurements in $N = 2$ or 3 mutually unbiased basis is given by

$$S_N = \sum_{i=1}^{N} E[\langle B_i \rangle_{A_i}^2] \leq 1. \tag{4}$$

- where, $E[\langle B_i \rangle_{A_i}^2] = \sum_{a_i=\pm 1} p(A_i = a_i)\langle B_i \rangle_{A_i=a_i}^2$, with $p(A_i = a_i)$ being the probability of getting a_i at t_A, and $\langle B_i \rangle_{A_i=a_i}^2$ is the expectation value of B_i at t_B on the state measured by Alice at t_A.

Probing hierarchy

- Let us consider three dichotomic POVMs acting on the two dimensional Hilbert space as $M^k(a_k) = \frac{1}{2}(\mathbb{I} + \eta a_k \sigma_k)$. This is an example of an unsharp measurement with sharpness parameter η.
- The system evolves under the Hamiltonian $U = e^{-i\sigma_x \omega t/2}$ when A_1, A_2 are measured and $V = e^{-i\sigma_y \omega t/2}$ when A_3 is measured.
- Going to the Heisenberg picture, Bob's observables are given by $B_{1(2)} = U^\dagger \sigma_z(\sigma_y) U$, and $B_3 = V^\dagger \sigma_x V$.
- With the above choices we get $S_3 = 3\eta^2 \cos^2\theta$. It is now straightforward to see that is violated for $\eta > \frac{1}{\sqrt{3}} (= 0.57735)$.

Probing hierarchy

- Now consider a large class of LGI of the form)

$$K_n = C_{21} + C_{32} + ... + C_{n(n-1)} - C_{n1}. \tag{5}$$

- K_n is the n-term LG sum derived from outcome statistics of measurements of an observable, Q at times $t_1, t_2 ... t_n$.
- Under the assumptions of macrorealism this quantity is bounded by $-n \leq K_n \leq n-2;\ \ n \geq 3$, for odd n, and by $-(n-2) \leq K_n \leq n-2;\ \ n \geq 4$, for even n.
- It has been shown that for $\eta \leq \sqrt{(n-2)/(n \cos \frac{\pi}{n})}$, no violation can be found.

Probing hierarchy

- As we want to compare with the three measurement steering scenario, the relevant LGIs are K_5 and K_6.
- Both of them can be mapped to a situation where Alice and Bob measure three different observables on the same system at time t_A and t_B sequentially, with no time evolution of the state between the measurements of Alice and Bob.
- $K_5 = C_{21} + C_{32} + C_{43} + C_{54} - C_{51} \leq 3$.
- in order to reproduce two point correlations yielding maximal violation in the mapped situation, Alice's choice of measurements are the three observables $A_1 = Q(\pi/5), A_2 = Q(3\pi/5), A_3 = Q(\pi)$, and Bob's choices are $B_1 = Q(2\pi/5), B_2 = Q(4\pi/5), B_3 = Q(\pi/5)$, where $Q(\theta) = (\sigma_z \cos\theta + \sigma_x \sin\theta)$.

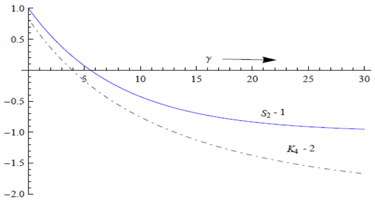
Probing hierarchy

- The correlation $C_{21} = \langle A_1 B_1 \rangle$ means Alice first measures A_1 and then Bob measures B_1 sequentially on the same system. Similarly for
  $C_{32} = \langle A_2 B_1 \rangle, C_{43} = \langle A_2 B_2 \rangle, C_{54} = \langle A_3 B_2 \rangle, C_{51} = \langle A_3 B_3 \rangle.$
- for dichotomic observables, C_{ij}s are independent of the order of the measurements as, $C_{ij} = \frac{1}{2} tr[\rho\{A_i, B_j\}]$.
- With these choices K_5 becomes 4.04 and for $\eta \leq 0.861186$, no violation of the LGI is possible in this case.
- Temporal steering is possible for $\eta > 0.57735$ as $S_3 > 1$. Hence, in the range $0.861186 > \eta > 0.57735$ steering can be shown but no LGI violation can be demonstrated.

SHARP MEASUREMENT WITH NONUNITARY EVOLUTION

- We now show that with sharp measurement under noisy evolution, temporal steering is possible even when the violation of LGI is washed out by noise. Consider a qubit under Hamiltonian, $H = -\frac{\omega}{2}\sigma_z$, sent through an amplitude damping channel in Lindblad form is described by
- $\frac{d\rho}{dt} = -\frac{i}{h}[H, \rho] + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho + \rho\sigma_+\sigma_-)$.
- $K_4 = 3\exp^{-\gamma\delta t}\cos(\omega\delta t) - \exp^{-3\gamma\delta t}\cos(3\omega\delta t) \leq 2$.
- $S_2 = 2\exp^{-2\gamma\delta t}\cos^2(\omega\delta t) \leq 1$.

SHARP MEASUREMENT WITH NONUNITARY EVOLUTION



- we plot the functions $K_4 - 2$ and $S_2 - 1$ versus the damping parameter γ. It is clear from the figure that after the damping parameter γ exceeds a certain value, the violation of LGI disappears, but temporal steering persists upto a greater value of γ.

Thank You