Manifestation of Pointer state correlations in complex weak values

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Sequential Strong Measurement

preselected state $= |\psi_s\rangle\, |\phi(q)\rangle$ $\rightarrow$ $g\hat{A} \otimes \hat{p}$ $\nearrow$ $|a+\rangle\, |\phi(q+1)\rangle$ / $\searrow$ $|a-\rangle\, |\phi(q-1)\rangle$ $\rightarrow$ projective measurement with $H = \hat{B} \otimes \hat{p}$ would destroy information about $g\hat{A} \otimes \hat{p}$ that can be obtained from the pointer state

$\hat{A}$ is the dichotomic operator with eigenvectors $|a+\rangle$ and $|a-\rangle$ with eigenvalues 1 and -1 respectively.
Weak measurement is a way to retain some information about the first interaction by minimally disturbing the system and pointer state.

Basics of Standard Weak Measurement

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$$\text{State afer weak coupling} = e^{-i\int dt g(t)\hat{A}\otimes\hat{p}} |\psi\rangle_{pre} \text{ (expanding the exponential up to 1st order)}$$

$$\approx \int (1 - i\alpha\hat{A} \otimes \hat{p}) |\psi_i\rangle \otimes e^{-\frac{q^2}{4\sigma^2}} |q\rangle \, dq \left(\int g(t)dt = \alpha\right) \quad (1)$$

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post-selected pointer state$= \langle\psi_f| (1 - i\alpha\hat{A} \otimes \hat{p}) |\psi_i\rangle \otimes e^{-\frac{q^2}{4\sigma^2}} |q\rangle \, dq$

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$$\text{Final pointer state} = \langle \psi_f | \, \psi_i \rangle \int e^{-\frac{(q+\alpha (A)_w)^2}{\sigma^2}} \, |q\rangle \, dq \tag{2}$$

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$$\langle\hat{q}\rangle_f = \langle\hat{q}\rangle_{in} + 2\lambda(Im(A)_w)var(q) \qquad (4)$$

$$\langle\hat{p}\rangle_f = \langle\hat{p}\rangle_{in} - \lambda\mathrm{Re}(A)_w + m\lambda(\mathrm{Im}(A)_w)\frac{\partial var(q)}{\partial t} \qquad (5)$$

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$var(q) = \langle\hat{q}^2\rangle_{in} - \langle\hat{q}\rangle_{in}^2$, $\int g(t)dt = \lambda$, and m is the mass of the system in question[Phys. Rev. A 76, 044103(2007)]

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$$\lambda\sigma <<< \frac{1}{(A)_w} \tag{6}$$

sequential weak measurement [Phys. Rev. A 76, 062105(2007)]

preselected state — — — — $e^{i\hat{A}_1 \otimes \hat{q}_1}$ — — — $e^{i\hat{A}_2 \otimes \hat{q}_2}$ — — — postselected state

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The final pointer shift containing the effect of the weak interaction would follow Eqs. (4) and (5) for uncorrelated pointer state.

modification of pointer shift due to nonzero correlations

• Eqs. (4) and (5) holds true when weak interaction and projective measurement involve same pointer degree of freedom. In most general case, weak measurement involves two pointer degrees of freedom. One involved in the coupling corresponds to interaction and other for the coupling used for postselection.

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• For two mode pointer state Eqs. (4) and (5) holds true when

$$\phi(q_1, q_2) = \phi(q_1)\phi(q_2) \tag{7}$$

The probability distribution is uncorrelated in its respective degrees of freedom. i.e. Covariance matrix

$$\Sigma_{ij} = \langle q_i q_j \rangle - \langle q_i \rangle \langle q_j \rangle \tag{8}$$

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• We explored the consequence of having correlated pointer state (Σ is not diagonal) for weak measurement. [S. Kanjilal, G. Muralidhara, D. Home Phys. Rev. A 94, 052110]

Effect of correlated Pointer distribution in standard WM

It suffices to consider two mode preselected pointer state wave function $\phi_{in} = \phi_i(q_1, q_2) \neq \phi(q_1)_{in}\phi(q_2)_{in}$ We take $corr(q_1, q_2) \neq 0$ where $corr(A, B) = \langle AB\rangle - \langle A\rangle\langle B\rangle$. von Neumann coupling for weak interaction is $\hat{A} \otimes \hat{q}_1$. Subsequent postselection is done through projective measurement using another von Neumann coupling involving another system variable and pointer degree of freedom $\hat{q}_2$.

$$\langle\hat{q}_1\rangle_f = \langle\hat{q}_1\rangle_{in} + 2\lambda Im(A)_w var(q_1) \tag{9}$$

$$\langle\hat{q}_2\rangle_f = \langle\hat{q}_2\rangle_{in} + 2\lambda Im(A)_w corr(q_1, q_2) \tag{10}$$

$$\langle\hat{p}_1\rangle_f = \langle\hat{p}_1\rangle_{in} - \lambda Re(A)_w + m\lambda Im(A)_w \frac{\partial var(q_1)}{\partial t} \tag{11}$$

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Effect of correlated Pointer distribution in sequential WM

For pointer observable $\hat{M} = \hat{q}_{1,2,3}$

$$\langle \hat{q}_1 \rangle_f = \langle \hat{q}_1 \rangle_{in} + 2\lambda_1 Im(A_1)_w var(q_1) + 2\lambda_2 Im(A_2)_w corr(q_1, q_2) \quad (13)$$

$$\langle \hat{q}_2 \rangle_f = \langle \hat{q}_2 \rangle_{in} + 2\lambda_2 Im(A_2)_w var(q_2) + 2\lambda_1 Im(A_1)_w corr(q_1, q_2) \quad (14)$$

$$\langle \hat{q}_3 \rangle_f = \langle \hat{q}_3 \rangle_{in} + 2\lambda_1 Im(A_1)_w corr(q_1, q_3) + 2\lambda_2 Im(A_2)_w corr(q_2, q_3) \quad (15)$$

$corr(q_i, q_{j=i}) = \langle q_i q_{j\neq i} \rangle_{in} - \langle q_i \rangle_{in} \langle q_{j\neq i} \rangle_{in} \neq 0$.

Effect of correlated Pointer distribution in sequential WM

Similarly, for $\hat{M} = \hat{p}_{1,2,3}$, we can obtain

$$\langle \hat{p}_{1,2} \rangle_f = \langle \hat{p}_{1,2} \rangle_{in} - \lambda_{1,2} Re(A_{1,2})_w + m\lambda_{1,2} Im(A_{1,2})_w \frac{\partial var(q_{1,2})}{\partial t} + 2\lambda_{2,1} Im(A_{2,1})_w corr(p_1, q_2) \quad (16)$$

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$corr(q_i, p_{j\neq i}) = \langle \hat{q}_i \hat{q}_j \rangle_{in} - \langle \hat{q}_i \rangle_{in} \langle \hat{q}_j \rangle_{in} \neq 0$ The shift of the expectation value of this pointer degree of freedom contains contributions from the imaginary parts of both the weak values of the system observables $\hat{A}_1$ and $\hat{A}_2$

application of weak measurement with correlated pointer state: LG mode as pointer state

• Shikano et. al. [Phys. Rev. A 86, 053805(2012)] showed for two dimensional Laguerre Gauss modes with non-vanishing OAM l (which endows nonzero correaltion between x and y) as preselected pointer state and the weak interaction $H = g\delta(t - t_0)(\hat{A} \otimes \hat{p_x} + \hat{B} \otimes \hat{p_y})$

$$\langle\hat{x}\rangle_f - \langle\hat{x}\rangle_{in} = g[Re(A)_w + lIm(B)_w] \tag{18}$$

$$\langle\hat{y}\rangle_f - \langle\hat{y}\rangle_{in} = g[Re(B)_w - lIm(A)_w] \tag{19}$$

It can then be seen from Eqs. (18) and (19) that the shift of the each pointer position degree of freedom has contribution from both the relevant weak values. One can show that

$$corr(p_x, y) = corr(p_y, x) = \frac{l}{2} \tag{20}$$

and rest of the correlations are zero for LG mode.

application of weak measurement with correlated pointer state: joint weak value and higher orders of weak value

• Joint weak value $(AB)_w = \frac{\langle\psi_f|AB|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$ and higher orders of weak value $(A^2)_w = \frac{\langle\psi_f|A^2|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$ are important to experimentally demonstrate Hardy's paradox and modeling nonlinearities of optical mediums.

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• Steinberg et. al.[Phys. Rev. Lett. 92, 130402(2004).] showed that joint weak value $(AB)_w$ can be experimentally obtain by considering Hamiltonian, $H = g_1 A p_x + g_2 B p_y$ for the interaction and taking up to second order of the corresponding evolution operator and obtained the joint weak value $(AB)_w$ in the joint pointer expectation value $< XY >$.

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• Puentes et. al. improved obtained imaginary part of $(A^2)_w$ and $(B^2)_w$ along with real and imaginary part of the $< AB >_w$ for non-zero value of OAM l using LG pointer state.

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• One can give a condition on postselected pointer observable for which any correlated pointer state can be used to obtain real and imaginary part of the $< AB >_w$.


Thank
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