

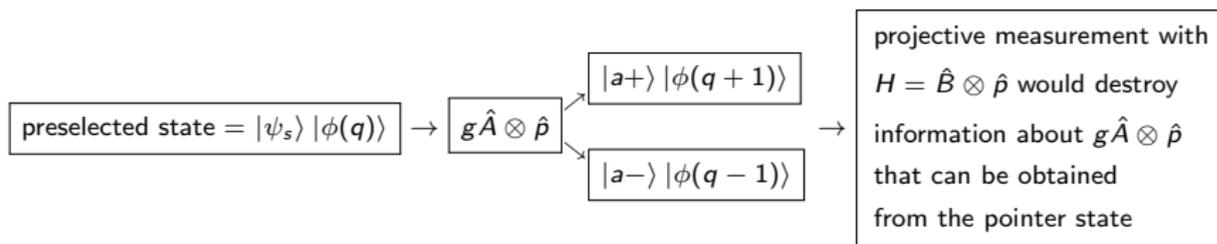
# Manifestation of Pointer state correlations in complex weak values

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# Sequential Strong Measurement



$\hat{A}$  is the dichotomic operator with eigenvectors  $|a+\rangle$  and  $|a-\rangle$  with eigenvalues 1 and  $-1$  respectively.

Weak measurement is a way to retain some information about the first interaction by minimally disturbing the system and pointer state.

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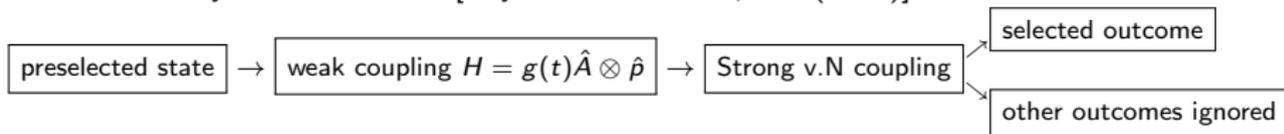
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$$\begin{aligned} \text{State after weak coupling} &= e^{-i \int dt g(t) \hat{A} \otimes \hat{p}} |\psi\rangle_{\text{pre}} \text{ (expanding the exponential up to 1st order)} \\ &\approx \int (1 - i\alpha \hat{A} \otimes \hat{p}) |\psi_i\rangle \otimes e^{-\frac{q^2}{4\sigma^2}} |q\rangle dq \left( \int g(t) dt = \alpha \right) \end{aligned} \quad (1)$$

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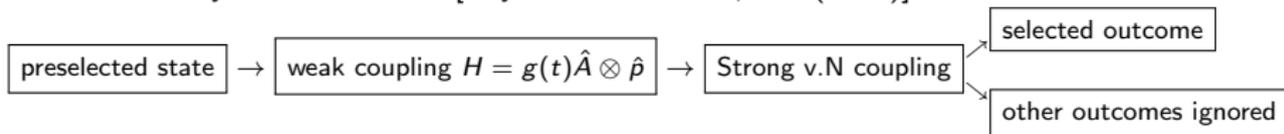
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$$\text{post-selected pointer state} = \langle \psi_f | (1 - i\alpha \hat{A} \otimes \hat{p}) |\psi_i\rangle \otimes e^{-\frac{q^2}{4\sigma^2}} |q\rangle dq$$

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$$\langle \hat{p} \rangle_f = \langle \hat{p} \rangle_{in} - \lambda\text{Re}(A)_w + m\lambda(\text{Im}(A)_w)\frac{\partial \text{var}(q)}{\partial t} \quad (5)$$

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$$\lambda\sigma \ll \ll \frac{1}{(A)_w} \quad (6)$$

## sequential weak measurement [Phys. Rev. A 76, 062105(2007)]



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The final pointer shift containing the effect of the weak interaction would follow Eqs. (4) and (5) for uncorrelated pointer state.

## modification of pointer shift due to nonzero correlations

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$$\phi(q_1, q_2) = \phi(q_1)\phi(q_2) \quad (7)$$

The probability distribution is uncorrelated in its respective degrees of freedom. i.e. Covariance matrix

$$\Sigma_{ij} = \langle q_i q_j \rangle - \langle q_i \rangle \langle q_j \rangle \quad (8)$$

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- We explored the consequence of having correlated pointer state ( $\Sigma$  is not diagonal) for weak measurement. [S. Kanjilal, G. Muralidhara, D. Home Phys. Rev. A 94, 052110]

## Effect of correlated Pointer distribution in standard WM

It suffices to consider two mode preselected pointer state wave function

$\phi_{in} = \phi_i(q_1, q_2) \neq \phi(q_1)_{in}\phi(q_2)_{in}$  We take  $corr(q_1, q_2) \neq 0$  where  $corr(A, B) = \langle AB \rangle - \langle A \rangle \langle B \rangle$ . von Neumann coupling for weak interaction is  $\hat{A} \otimes \hat{q}_1$ . Subsequent postselection is done through projective measurement using another von Neumann coupling involving another system variable and pointer degree of freedom  $\hat{q}_2$ .

$$\langle \hat{q}_1 \rangle_f = \langle \hat{q}_1 \rangle_{in} + 2\lambda Im(A)_w var(q_1) \quad (9)$$

$$\langle \hat{q}_2 \rangle_f = \langle \hat{q}_2 \rangle_{in} + 2\lambda Im(A)_w corr(q_1, q_2) \quad (10)$$

$$\langle \hat{p}_1 \rangle_f = \langle \hat{p}_1 \rangle_{in} - \lambda Re(A)_w + m\lambda Im(A)_w \frac{\partial var(q_1)}{\partial t} \quad (11)$$

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## Effect of correlated Pointer distribution in sequential WM

For pointer observable  $\hat{M} = \hat{q}_{1,2,3}$

$$\begin{aligned} \langle \hat{q}_1 \rangle_f &= \langle \hat{q}_1 \rangle_{in} + 2\lambda_1 \text{Im}(A_1)_w \text{var}(q_1) \\ &\quad + 2\lambda_2 \text{Im}(A_2)_w \text{corr}(q_1, q_2) \end{aligned} \quad (13)$$

$$\begin{aligned} \langle \hat{q}_2 \rangle_f &= \langle \hat{q}_2 \rangle_{in} + 2\lambda_2 \text{Im}(A_2)_w \text{var}(q_2) \\ &\quad + 2\lambda_1 \text{Im}(A_1)_w \text{corr}(q_1, q_2) \end{aligned} \quad (14)$$

$$\begin{aligned} \langle \hat{q}_3 \rangle_f &= \langle \hat{q}_3 \rangle_{in} + 2\lambda_1 \text{Im}(A_1)_w \text{corr}(q_1, q_3) \\ &\quad + 2\lambda_2 \text{Im}(A_2)_w \text{corr}(q_2, q_3) \end{aligned} \quad (15)$$

$$\text{corr}(q_i, q_{j=i}) = \langle q_i q_{j \neq i} \rangle_{in} - \langle q_i \rangle_{in} \langle q_{j \neq i} \rangle_{in} \neq 0 .$$

## Effect of correlated Pointer distribution in sequential WM

Similarly, for  $\hat{M} = \hat{p}_{1,2,3}$ , we can obtain

$$\langle \hat{p}_{1,2} \rangle_f = \langle \hat{p}_{1,2} \rangle_{in} - \lambda_{1,2} \text{Re}(A_{1,2})_w + m\lambda_{1,2} \text{Im}(A_{1,2})_w \frac{\partial \text{var}(q_{1,2})}{\partial t} + 2\lambda_{2,1} \text{Im}(A_{2,1})_w \text{corr}(p_1, q_2) \quad (16)$$

$$\langle \hat{p}_3 \rangle_f = \langle \hat{p}_3 \rangle_{in} + 2\lambda_1 \text{Im}(A_1)_w \text{corr}(q_1, p_3) + 2\lambda_2 \text{Im}(A_2)_w \text{corr}(q_2, p_3) \quad (17)$$

$\text{corr}(q_i, p_{j \neq i}) = \langle \hat{q}_i \hat{q}_j \rangle_{in} - \langle \hat{q}_i \rangle_{in} \langle \hat{q}_j \rangle_{in} \neq 0$  The shift of the expectation value of this pointer degree of freedom contains contributions from the imaginary parts of both the weak values of the system observables  $\hat{A}_1$  and  $\hat{A}_2$

## application of weak measurement with correlated pointer state: LG mode as pointer state

- Shikano et. al. [Phys. Rev. A 86, 053805(2012)] showed for two dimensional Laguerre Gauss modes with non-vanishing OAM  $l$  (which endows nonzero correlation between  $x$  and  $y$ ) as preselected pointer state and the weak interaction  $H = g\delta(t - t_0)(\hat{A} \otimes \hat{p}_x + \hat{B} \otimes \hat{p}_y)$

$$\langle \hat{x} \rangle_f - \langle \hat{x} \rangle_{in} = g[\text{Re}(A)_w + \text{Im}(B)_w] \quad (18)$$

$$\langle \hat{y} \rangle_f - \langle \hat{y} \rangle_{in} = g[\text{Re}(B)_w - \text{Im}(A)_w] \quad (19)$$

It can then be seen from Eqs. (18) and (19) that the shift of the each pointer position degree of freedom has contribution from both the relevant weak values. One can show that

$$\text{corr}(p_x, y) = \text{corr}(p_y, x) = \frac{l}{2} \quad (20)$$

and rest of the correlations are zero for LG mode.

## application of weak measurement with correlated pointer state: joint weak value and higher orders of weak value

- Joint weak value  $(AB)_w = \frac{\langle \psi_f | AB | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$  and higher orders of weak value  $(A^2)_w = \frac{\langle \psi_f | A^2 | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$  are important to experimentally demonstrate Hardy's paradox and modeling nonlinearities of optical mediums.

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- Steinberg et. al. [Phys. Rev. Lett. 92, 130402(2004).] showed that joint weak value  $(AB)_w$  can be experimentally obtain by considering Hamiltonian,  $H = g_1 A p_x + g_2 B p_y$  for the interaction and taking up to second order of the corresponding evolution operator and obtained the joint weak value  $(AB)_w$  in the joint pointer expectation value  $\langle XY \rangle$ .

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- One can give a condition on postselected pointer observable for which any correlated pointer state can be used to obtain real and imaginary part of the  $\langle AB \rangle_w$ .

