Closing the Debates on Quantum Locality and Reality: EPR Theorem, Bell's Theorem, and Quantum Information from the Brown-Twiss Vantage

C. S. Unnikrishnan
Fundamental Interactions Laboratory
Tata Institute of Fundamental Research,
Mumbai 400005
tifr.res.in/~filab

Resolution of nonlocality puzzle in EPR paradox, C. S. Unnikrishnan, Current Science, **79**, 195, (2000).

Quantum correlations from local amplitudes and the resolution of the EPR nonlocality puzzle, C. S. Unnikrishnan, Optics and Spectroscopy, **91**, 358, (2001).

Is the quantum mechanical description of physical reality complete ? Resolution of the EPR puzzle, C. S. Unnikrishnan, Found. Phys. Lett., **15**, 1-25 (2002).

Proof of absence of spooky action at a distance in quantum correlations, C. S. Unnikrishnan, (Proc. Winter Institute in Foundations of Quantum Mechanics and Quantum Optics) Pramana **59**, 295 (2002)

Conservation laws, Correlations functions and Bell's inequalities, C. S. Unnikrishnan, Europhysics Letters **69**, 489-495 (2005).

The incompatibility between local hidden variable theories and the fundamental conservation laws, C. S. Unnikrishnan, Pramana – Jl. Phys. **65**, 359 (2005)

End of Several Quantum Mysteries, C. S. Unnikrishnan, arXiv:1102.1187

Main thread of earlier work:

1) Einstein Locality is strictly valid in nature, even in those phenomena involving several particles that share a common past of interactions.

2) All correlations observed by measurement with spatially separated (or any) detectors can be understood as due to local correlations set up at the source of multiple particles (point of interaction) and encoded as a <u>fixed relative phase on the different particles</u>. (2000-2002)

3) While classical correlations obey Einstein locality by encoding on dynamical variables (energy, momentum etc.) quantum correlations are encoded on <u>phases induced by the dynamical variables, at source</u>. What is encoded is simply a conservation law, in both cases. (2005-06)

4) Quantum correlation functions are direct consequence of a fundamental conservation law or a constraint applicable on the average over the ensemble of systems – therefore, a theory of correlations with a different correlation function (LHVT, super-correlations etc.) are incompatible with conservation laws and are physically invalid theories.

Plan:

Demonstrate explicitly how quantum correlations result due to prior encoding of a conservation law at source, obeying strict Einstein locality.

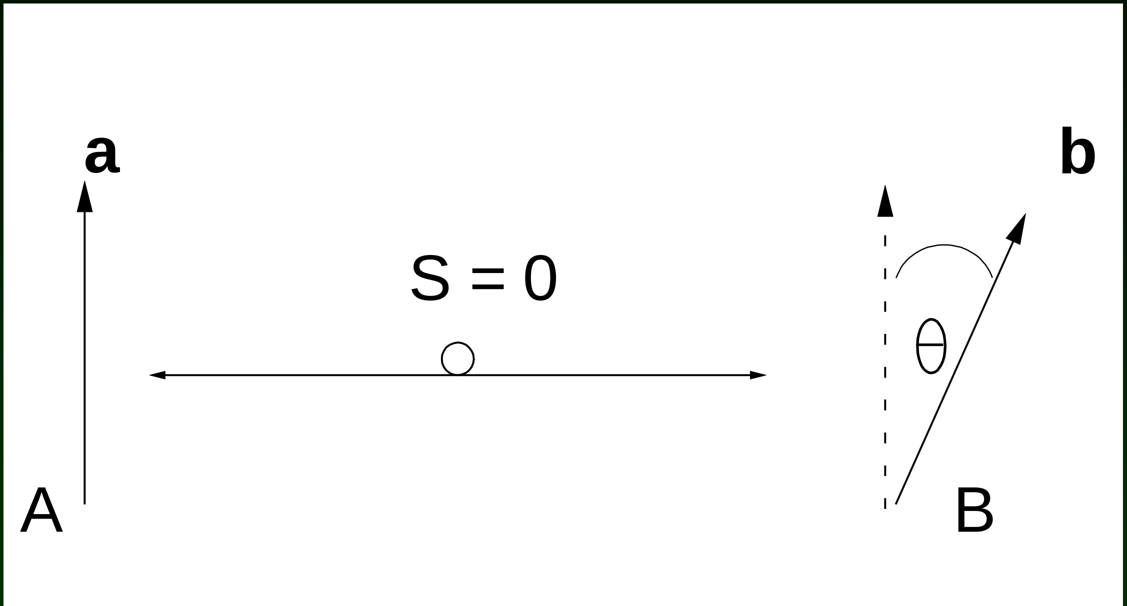
Start from a situation of ‘multi-detector correlations’ in classical optics - the Hanbury Brown-Twiss interferometer - to see the origin of ‘fringes’ and coherence. Strict Einstein locality is valid in this case , being classical wave phenomena.

Show how exactly the same considerations give rise to ‘correlation fringes’ when the system is ‘quantum mechanical’ .

Explain how individual measurements give random results and a spatially separated multi-point measurement returns a perfect correlation without violating Einstein locality.

The case of two 'spin-half' particles:

$$\Psi_S = \frac{1}{\sqrt{2}}\left(|+1\rangle_1 |-1\rangle_2 - |-1\rangle_1 |+1\rangle_2\right)$$

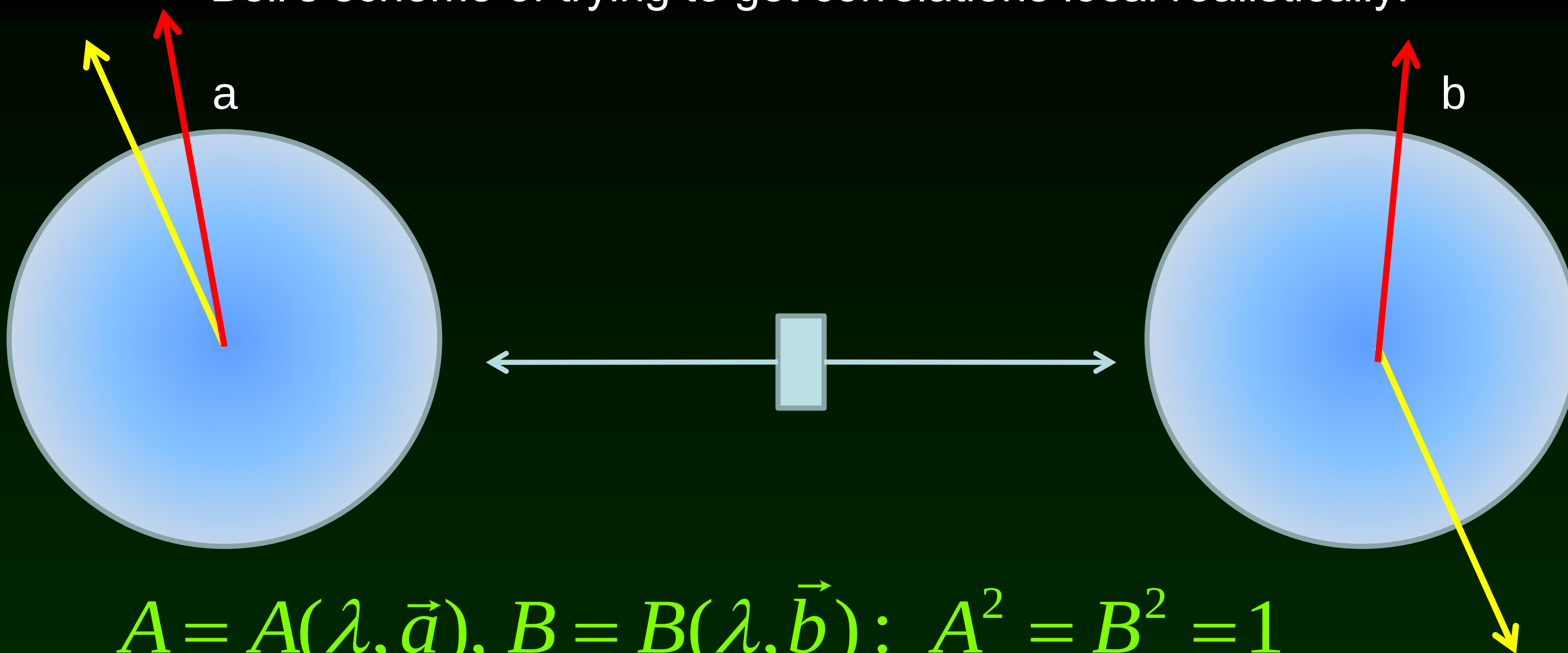


$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i \quad : A_i, B_i = \pm 1$$

Important input

Quantum Mechanics: $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} = -\cos\theta$

Bell's scheme of trying to get correlations local realistically:



$$A = A(\lambda, \vec{a}),\ B = B(\lambda, \vec{b}):\ A^2 = B^2 = 1$$

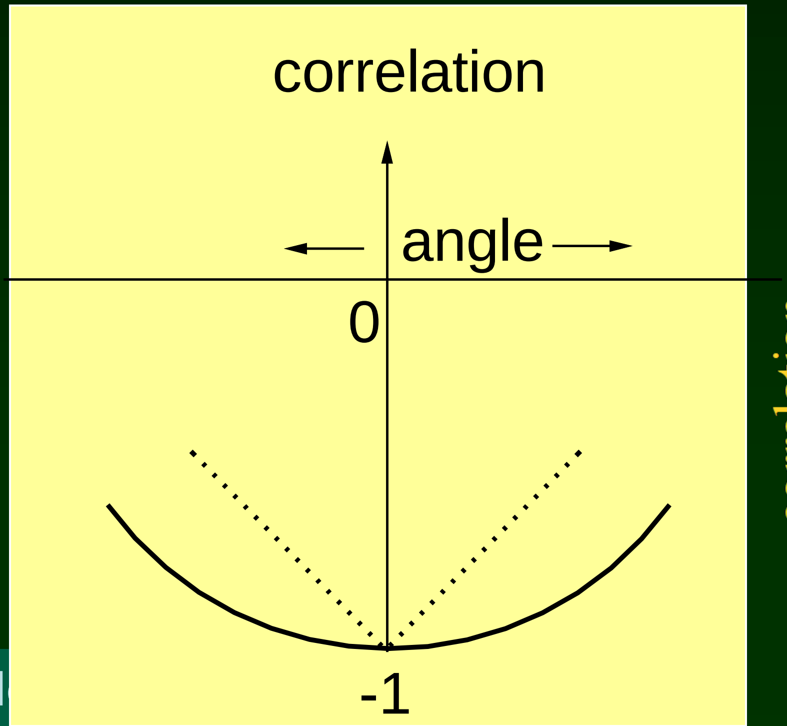
$$\text{Outcome: Sign}(\vec{\lambda} \cdot \vec{a}) \text{ and Sign}(\vec{\lambda} \cdot \vec{b})$$

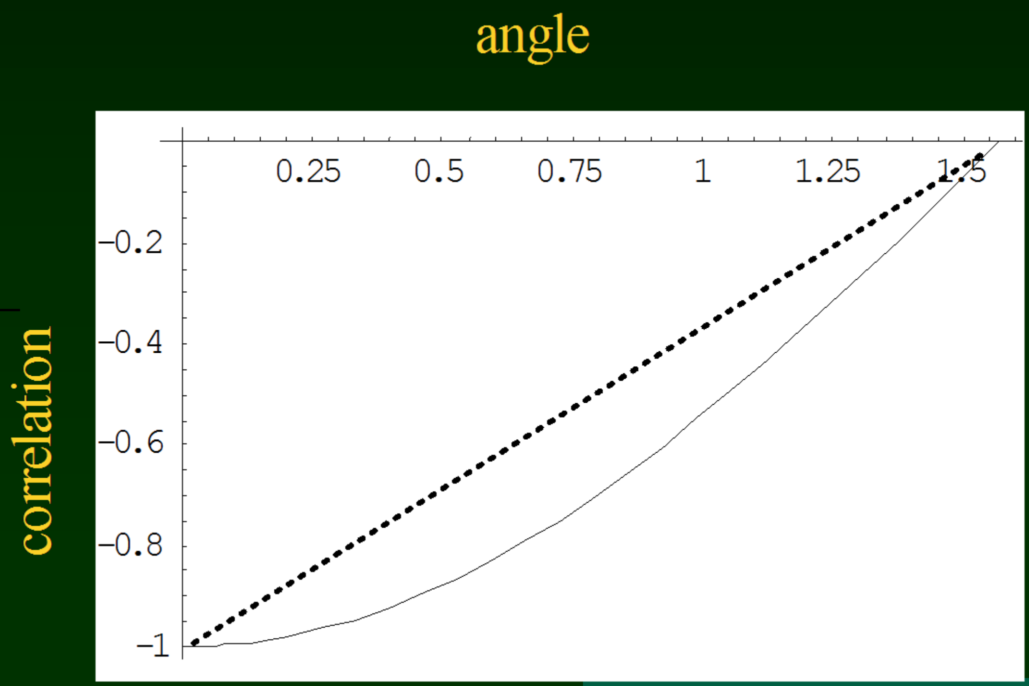
This prescription will reproduce P(a,b) for some angles, and the perfect correlation at zero relative angle. But, this does not reproduce the QM correlation.

$$P(\vec{a},\vec{b})_{QM} = \langle \Psi_S | \sigma_1 \cdot \vec{a} \otimes \sigma_2 \cdot \vec{b} | \Psi_s \rangle = -\vec{a}\cdot\vec{b}$$

$$P(\vec{a},\vec{b})_{Bell} = \int A(\vec{a},h) B(\vec{b},h) \rho(h) dh$$

The essence of Bell's theorem is that these two correlation functions have distinctly different dependences on the angle between the settings of the apparatus (difference of about 30% at specific angles).





Correlations measured in experiments do exceed the bound specified for LHVTs and they agree well with QM predictions.

So, what does the experimental confirmation of the violation of Bell's inequality imply as valid theoretical statements that are <u>logically rigorous</u>?

1) Quantum mechanics is validated as a good theory of correlations…

2) OR…a classical hidden variable theory in which statistically distributed valued of the HV determine measurement outcomes is validated as a good theory of correlations <u>to replace</u> QM provided there is violation of Einstein locality.

The common mistake is to mix the two and claim that experiments prove nonlocality or that Experiments prove QM is nonlocal !

EPR argument as described by Einstein?

Excerpts from Einstein's letter to Popper

"Should we regard the wave-function whose time dependent changes are, according to Schrödinger equation, deterministic, as a *complete* description of physical reality,…?

The answer at which we arrive is the wave-function should not be regarded as a complete description of the physical state of the system.
We consider a composite system, consisting of the partial systems A and B which interact for a short time only.

We assume that we know the wave-function of the composite system *before* the interaction – a collision of two free particles, for example – has taken place. Then Schrodinger equation will give us the wave-function for the composite system *after* the interaction.

Assume that now (after the interaction) an optimal measurement is carried out upon the partial system A, which may be done in various ways, however depending on the variables which one wants to measure precisely – for example, the momentum or the position co-ordinate. Quantum mechanics will then give us the wave-function for the partial system B, and it will give us *various wave-functions that differ, according to the kind of measurement which we have chosen to carry out upon A.*

Now it is unreasonable to assume that the physical state of B may depend upon some measurement carried out upon a system A which by now is separated from B (so that it no longer interacts with B); and this means that the two different wave-functions belong to one and the same physical state of B. Since a *complete* description of a physical state must necessarily be an *unambiguous* description (apart from superficialities such as units, choice of the co-ordinates etc.) it is therefore not possible to regard the wave-function as the *complete* description of the state of the system."

Anything beyond this in the EPR Phys. Rev. paper is superfluous and irrelevant as far as Einstein's point is concerned. No statement of violation of uncertainty relation. The validity of QM and superposition is assumed for the proof.

In particular there is no reference or wish regarding a possible completion of QM using some classical statistical hidden variables.

In condensed form, the argument is just that locality implies no instant change in a physical state possible after the particle is spatially separated whereas QM implies instant changes in the description of the physical state. Therefore it is not a complete unambiguous description.

$$P_B(\vec{a},\vec{b}) = \int \rho(h)dh\ A(\vec{a},h)B(\vec{b},h), \quad \int \rho(h)dh = 1$$

Since $A(\vec{a}) = -B(\vec{a})$ and $P_B(\vec{a},\vec{a}) = -1,$ Bell wrote

$$P_B(\vec{a},\vec{b}) = -\int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)$$

Simultaneous definite values for quantum mechanically non-commuting observables

Clearly not part of a program to complete QM by adding additional features to QM.

A physically correct program of completing QM should never have simultaneous values for ‘conjugate’ observables before measurement – that is not consistent with even basic wave-particle duality.

CSU, Proc. SPIE Photonics 2007

Quantum correlations and Classical Conservation Laws

Assumption: Fundamental conservation laws related to space-time symmetries are valid on the average over the quantum ensemble and measurements are made with finite number of discrete outcomes. (conservation check is not possible event-wise)

Result: Unique two-particle and multi-particle correlation functions can be derived from the assumption of validity of conservation laws alone. Interestingly, they are identical to the ones derived using formal quantum mechanics with appropriate operators and states.

CSU, Europhys. Lett, 2005, Pramana-J.Phys (2006)

1) Correlation functions of quantum mechanics are direct consequence of the CLASSICAL conservation laws arising in space-time symmetries (fundamental conservation laws), applied to ensembles.

2) Any theory that has a correlation function different from the ones in QM is incompatible with the fundamental conservation laws and space-time symmetries, and therefore it is unphysical. Local hidden variable theories fall in this class. Bell's inequalities can be obeyed (in the general case) only by violating a fundamental conservation law, making them redundant in physics.

1) No less, no more

2) Closing loopholes will improve agreement with QM!
(better tally with conservation principle)

CSU, Europhys. Lett, 2006, Pramana-J.Phys (2006)

1) No experiment to date proves violation of Einstein locality

2) Quantum correlations functions are direct consequence of conservation laws applicable at source , just as in the case of classical correlations.

Now I proceed to demonstrate that the observed correlations of microscopic physical systems (like the spin-1/2 singlet in QM) are realized in nature preserving strictly Einstein locality.

My approach to address the issues (2000-2004):

1) Notice that conservation constraints and wave-particle duality hold the key.

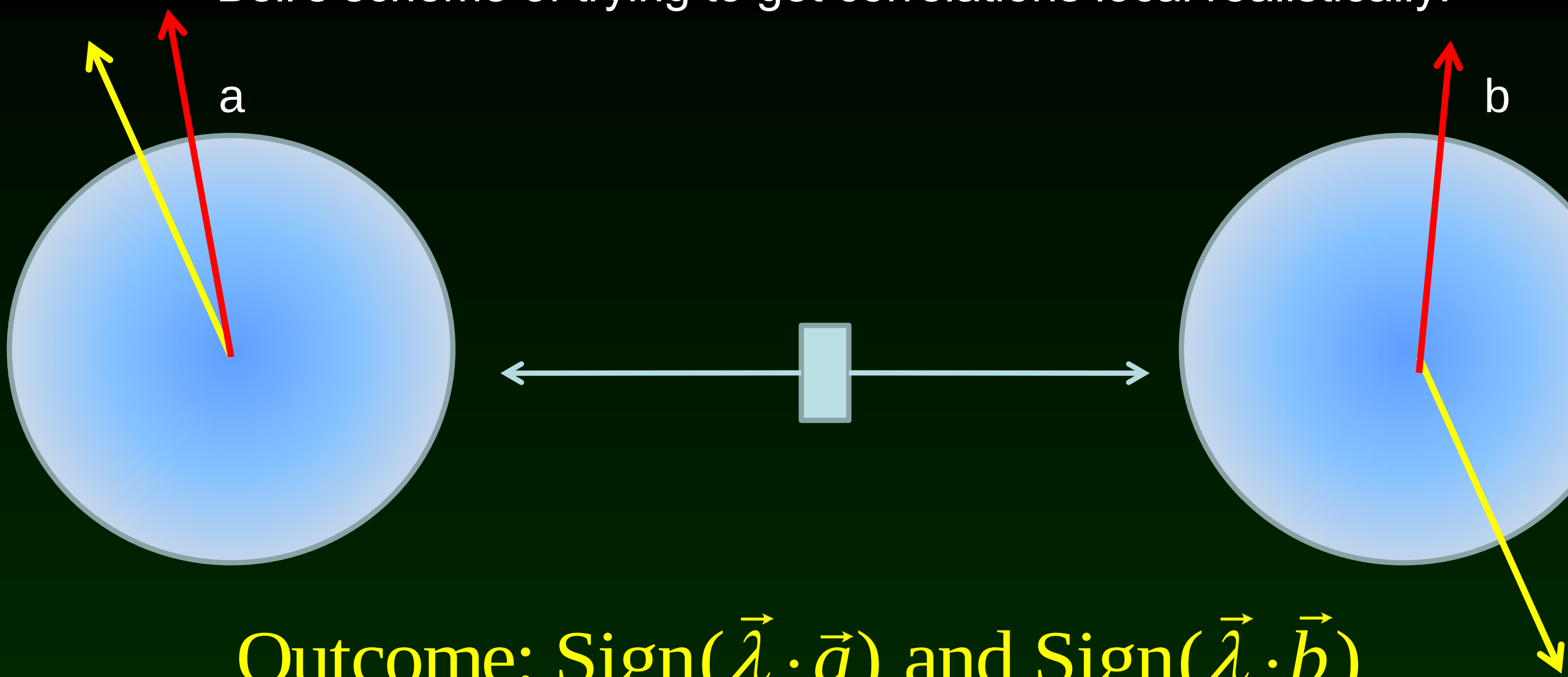
2) Notice that the conservation constraint directly reflects as a phase constraint for multi-particle systems

$$\text{Conservation law: } p_1 + p_2 = 0 \rightarrow \exp\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)$$

The assertion was that a local phase constraint (relative phase being fixed, while individual phases are random) at the source or interaction point determines the correlations, and that Einstein locality is preserved.

Unnikrishnan, Current Science (2000), Found. Phys. Lett **15**, 1-25 (2002), Ann. Fondation L. de Broglie (2002)…

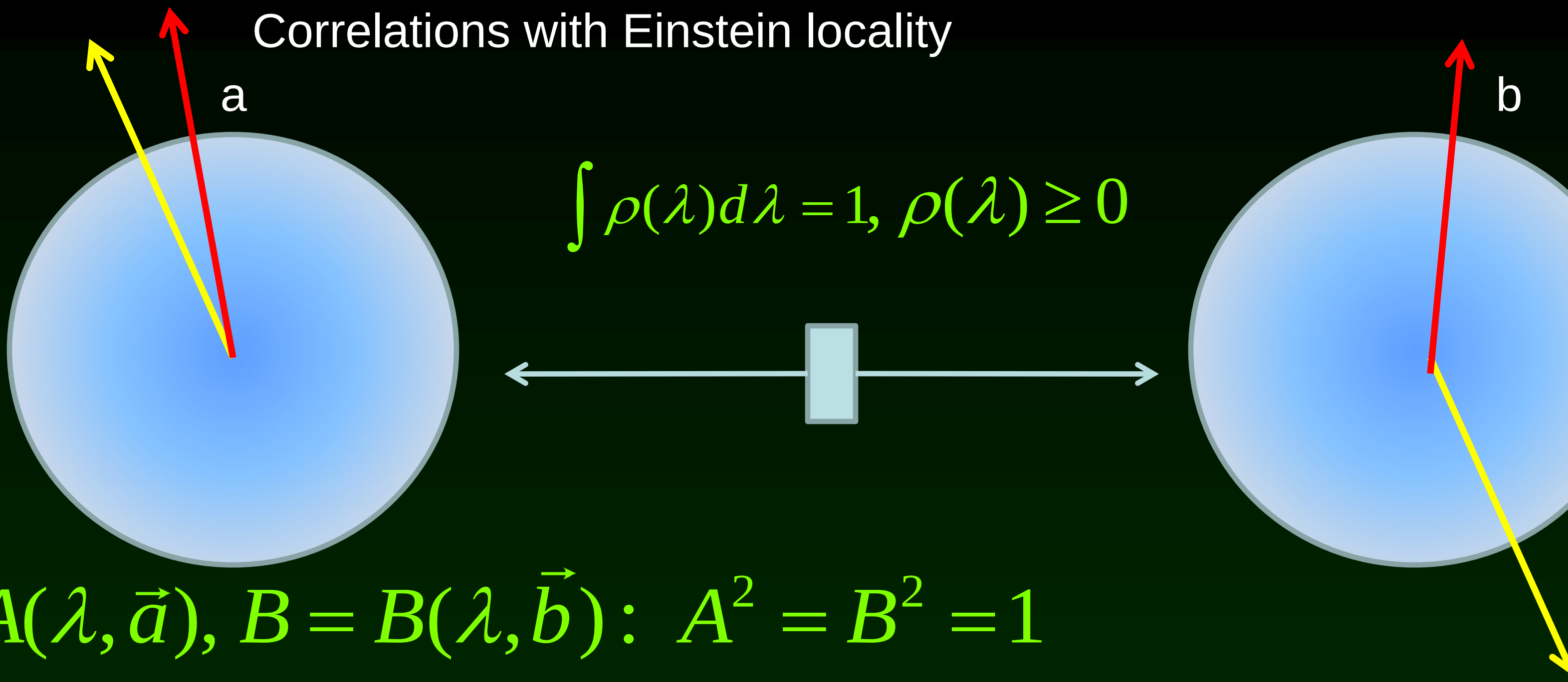
Bell's scheme of trying to get correlations local realistically:



Outcome: $\text{Sign}(\vec{\lambda}\cdot\vec{a})$ and $\text{Sign}(\vec{\lambda}\cdot\vec{b})$

Contrast with conservation law: $\mathrm{s}_1 + s_2 = 0 \rightarrow \exp\frac{i}{\hbar}(s_1\theta_1 + s_2\theta_2)$

Correlations with Einstein locality



$$\int \rho(\lambda)\, d\lambda = 1,\ \rho(\lambda) \geq 0$$

$$A = A(\lambda, \vec{a}),\ B = B(\lambda, \vec{b}):\ \ A^2 = B^2 = 1$$

Outcomes: A=$(\vec{\lambda}\cdot\vec{a})$ and B=$(-\vec{\lambda}\cdot\vec{b})$

$$(\vec{\lambda}\cdot\vec{a})^2 = (\vec{\lambda}\cdot\vec{b})^2 = 1$$

Correlation: $-\left\langle (\vec{\lambda}\cdot\vec{a})\ (\vec{\lambda}\cdot\vec{b}) \right\rangle_{\lambda}$

End of Several Quantum Mysteries, C. S. Unnikrishnan, arXiv:1102.1187

Correlation: $\left\langle (\vec{\lambda}\cdot\vec{a})(\vec{\lambda}\cdot\vec{b}) \right\rangle_\lambda$

$$\left(\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3\right)\left(\lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3\right)$$

$$= \lambda_1^2 a_1 b_1 + \lambda_2^2 a_2 b_2 + \lambda_1^2 a_3 b_3 +$$

$$\lambda_1\lambda_2 a_1 b_2 + \lambda_1\lambda_3 a_1 b_3 + \lambda_2\lambda_1 a_2 b_1 + \lambda_2\lambda_3 a_2 b_3 + \lambda_3\lambda_1 a_3 b_1 + \lambda_3\lambda_2 a_3 b_2$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 +$$

$$\lambda_1\lambda_2(a_1 b_2 - a_2 b_1) + \lambda_1\lambda_3\left(a_1 b_3 - a_3 b_1\right) + \lambda_2\lambda_3\left(a_2 b_3 - a_3 b_2\right)$$

$$= \vec{a}\cdot\vec{b} + i\vec{\lambda}\cdot(\vec{a}\times\vec{b}) \text{ with } \boxed{\lambda_i^2 = 1,\ \lambda_i\lambda_j = -\lambda_j\lambda_i \text{ and } \lambda_i\lambda_j = i\lambda_k}$$

$$-\left\langle \vec{a}\cdot\vec{b} + i\vec{\lambda}\cdot(\vec{a}\times\vec{b}) \right\rangle_\lambda = -\vec{a}\cdot\vec{b}$$

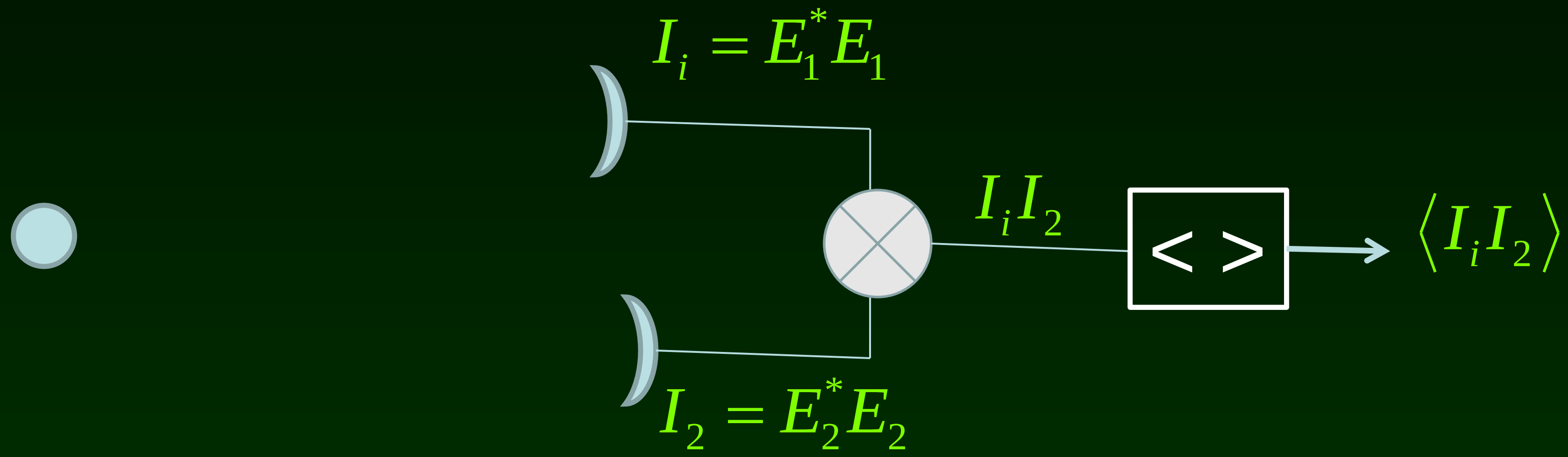
How is it possible to get correlations that vary as cos(theta) between spatially separated measurements, which by themselves are totally random between different realizations, with strict Einstein locality?

↓

No phase information retained in local individual measurements and no stable phase in the physical system, and yet, there is a coherent correlation.

Hanbury Brown – Twiss Interferometry

Second Order or Intensity-Intensity Correlator $g^{(2)}(x1, x2)$



$$g^{(2)}(x1, x2) = \frac{\langle I_1(x1) I_2(x2) \rangle}{\langle I_1(x1) \rangle \langle I_2(x2) \rangle} \geq 1$$

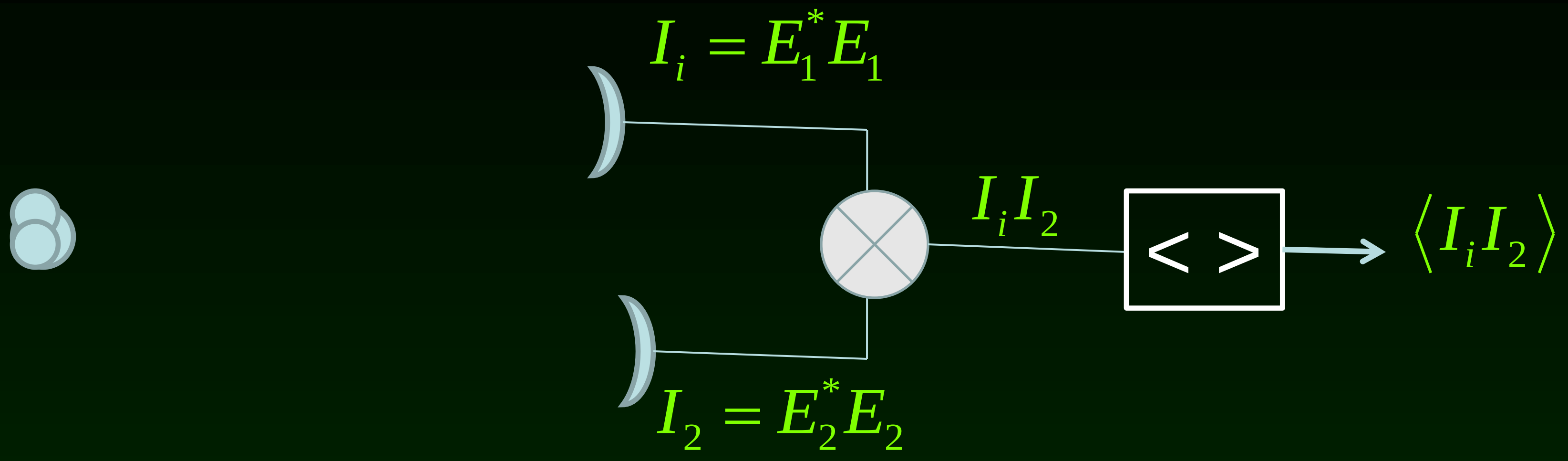
Random photocurrents in individual detectors, but perfect correlations possible in the averaged product. Indeed, this is a two-photon correlation when the detectors are single photon sensitive.

The physics content of HBT correlations:

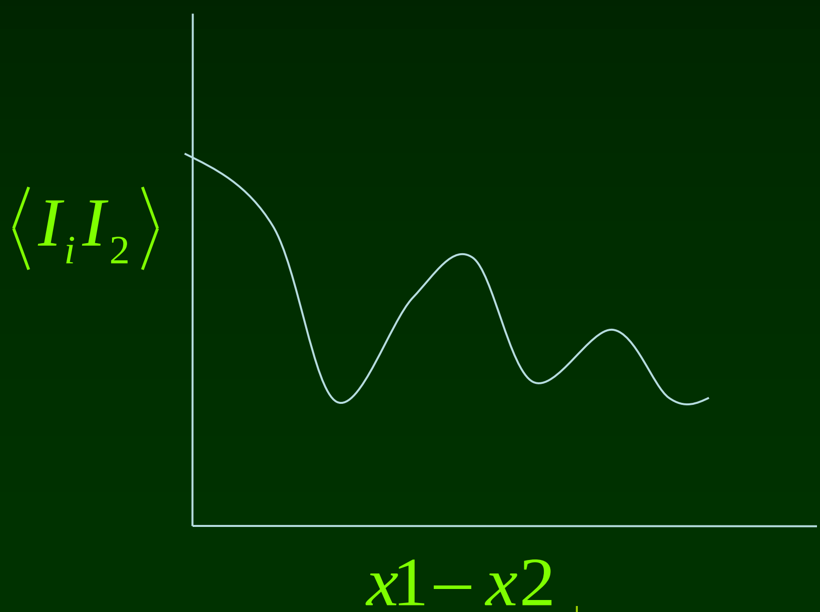
1) Spectral content of the source is directly related to Intensity fluctuations

2) If light from a spatially coherent region of the source is sampled by two squaring detectors (Intensity) they will detect the same fluctuations sampled at different times determined by the separation of the detectors. In particular they detect the same fluctuations with zero time delay. So, perfect correlation while individual intensity fluctuations are random.

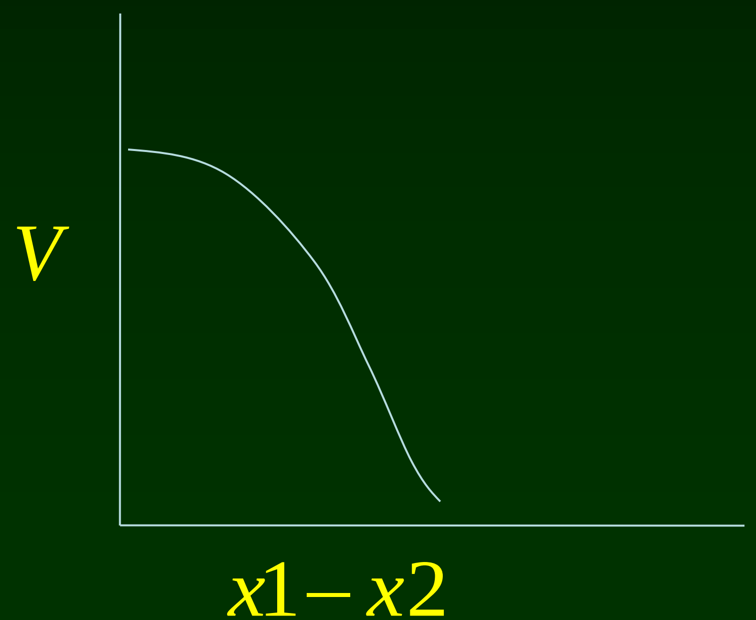
3) The first order interference visibility (square) is related to the Intensity-Intensity correlation. There are interference fringes in I-I correlations.



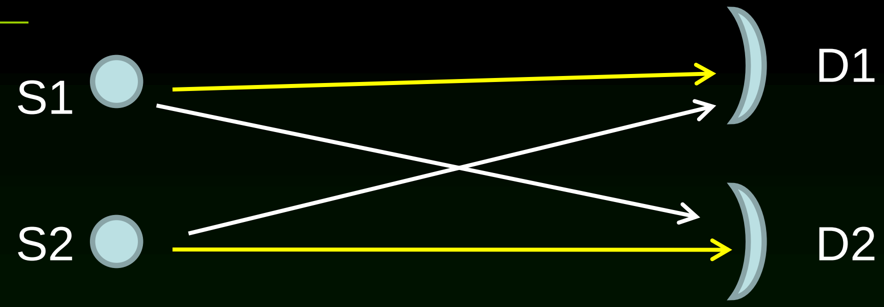
$I_i = E_1^* E_1$
$I_i I_2$
$< >$
$\langle I_i I_2 \rangle$
$I_2 = E_2^* E_2$



$\langle I_i I_2 \rangle$
$x1 - x2$



V
$x1 - x2$



$$E_{D1} = ae^{ikr_{11}+i\phi_1} + be^{ikr_{21}+i\phi_2}$$

$$E_{D2} = ae^{ikr_{12}+i\phi_1} + be^{ikr_{22}+i\phi_2}$$

$$I_{D1} = |a|^2 + |b|^2 + a^* b e^{ik(\Delta r + \Delta\phi)} + ab^* e^{-ik(\Delta r + \Delta\phi)}$$

$$\langle I_{D1} \rangle = \langle I_{D2} \rangle = |a|^2 + |b|^2$$

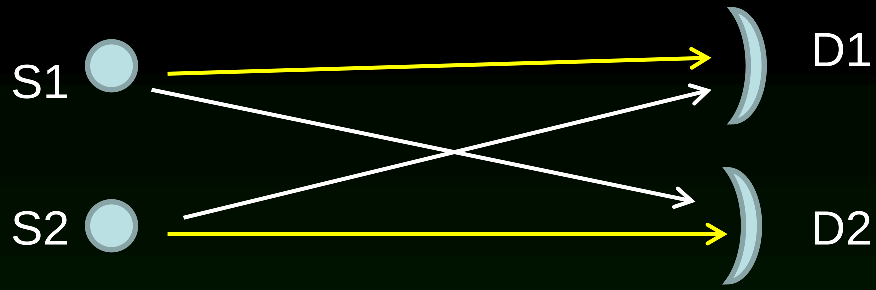
$$\langle I_{D1} I_{D2} \rangle = |a|^4 + |b|^4 + 2|a|^2 |b|^2 (1 + \cos k(\Delta r_1 + \Delta r_2))$$

$$\langle I_{D1}\rangle = \langle I_{D2}\rangle = |a|^2 + |b|^2$$

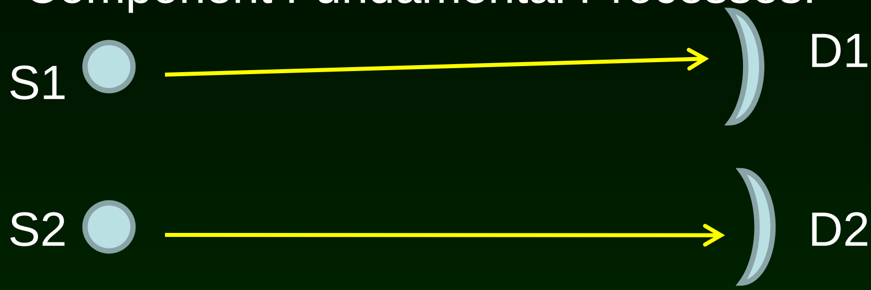
$$\langle I_{D1} I_{D2}\rangle = |a|^4 + |b|^4 + 2|a|^2 |b|^2 (1 + \cos k(\Delta r_1 + \Delta r_2))$$

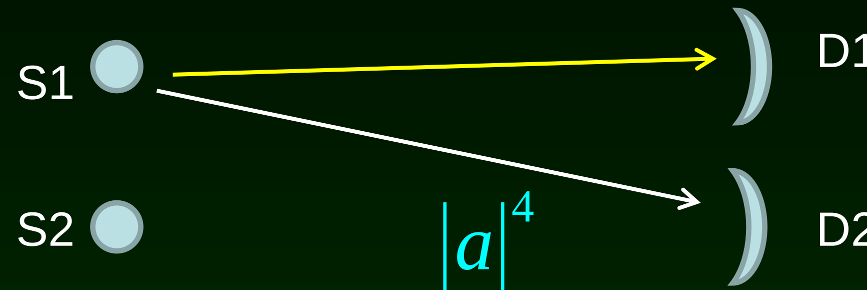
$$C(s) = \frac{\langle I_{D1} I_{D2}\rangle}{\langle I_{D1}\rangle\langle I_{D2}\rangle} = 1 + 2\frac{\langle |a|^2\rangle\langle |b|^2\rangle}{\left(\langle |a|^2\rangle + \langle |b|^2\rangle\right)^2} \cos k(\Delta r_X - \Delta r_{II})$$

Maximum Visibility =50%

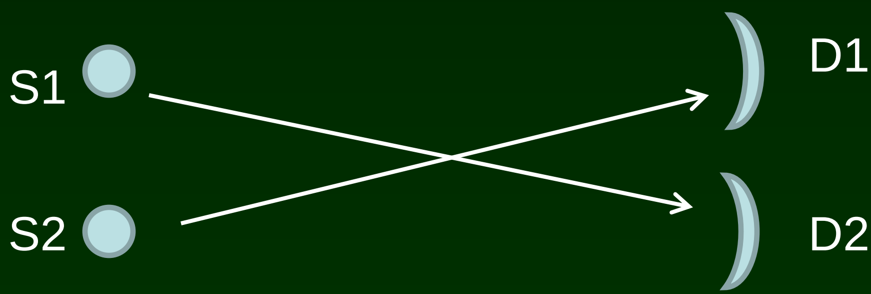


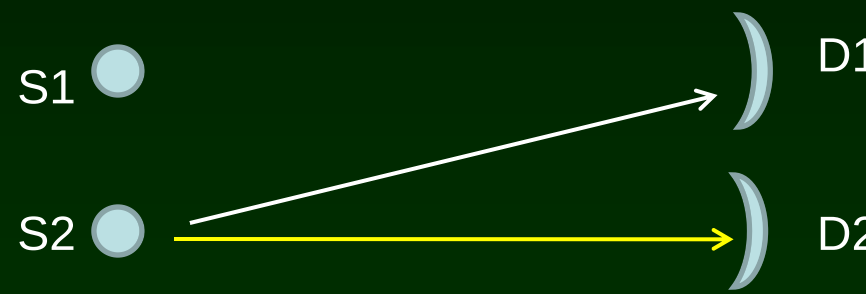
Component Fundamental Processes:





$$|a|^4$$



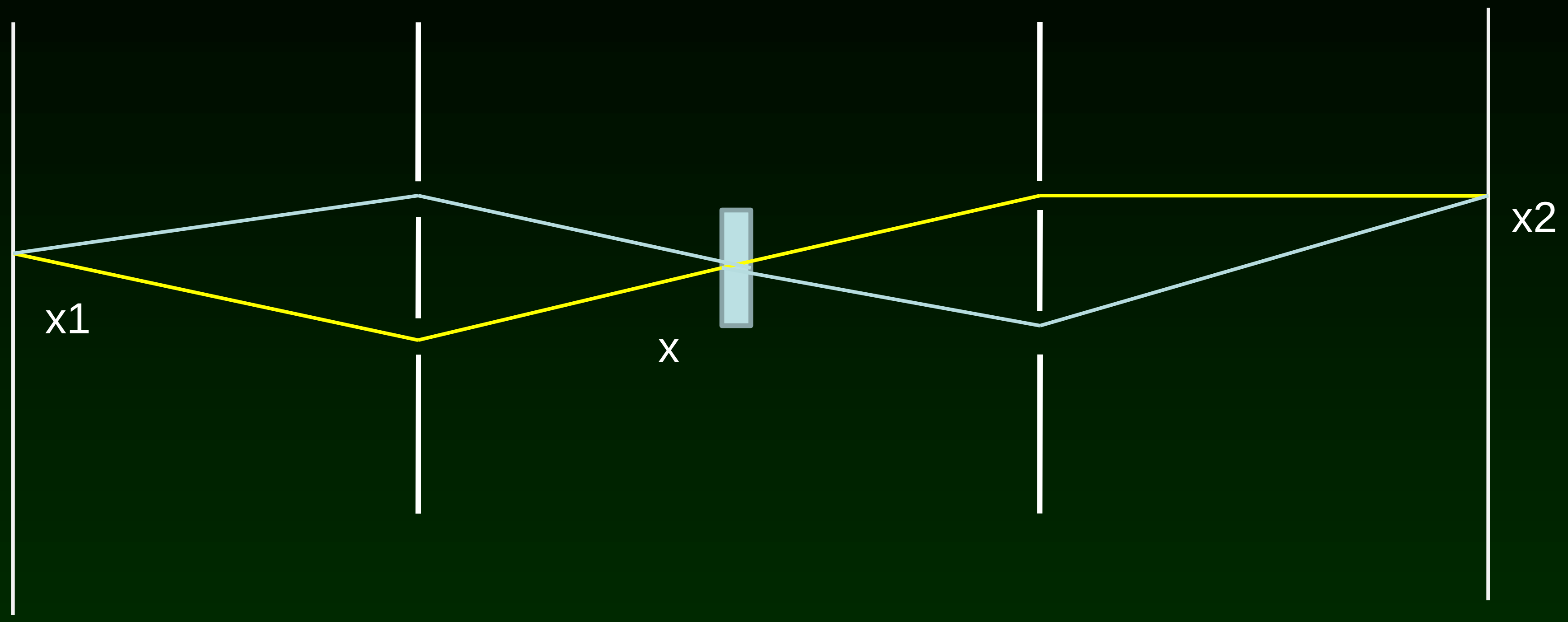


$$|b|^4$$

$$C(s) = 1 + \cos k(\Delta r_X - \Delta r_{II})$$

Clearly, local restriction on source that remove these two will give 100% visibility in Int-Int correlations!

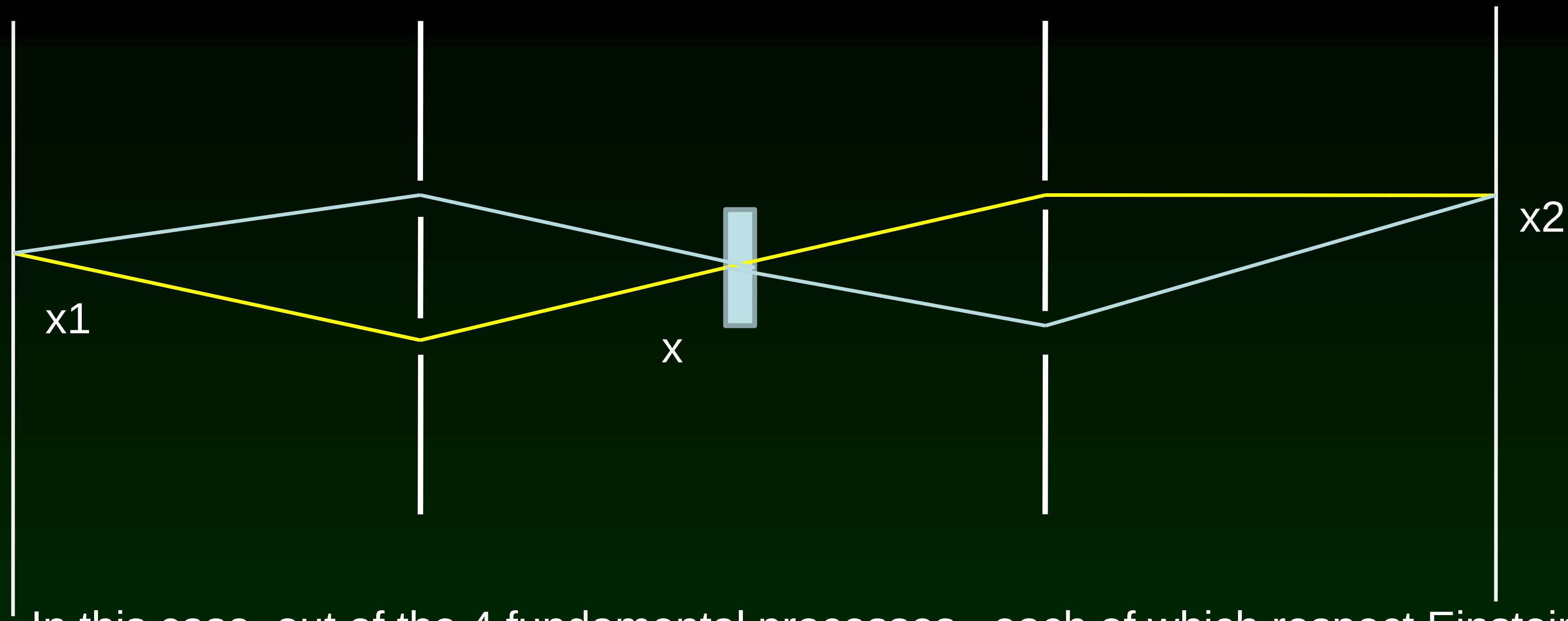
Now I apply these ideas to quantum correlations:



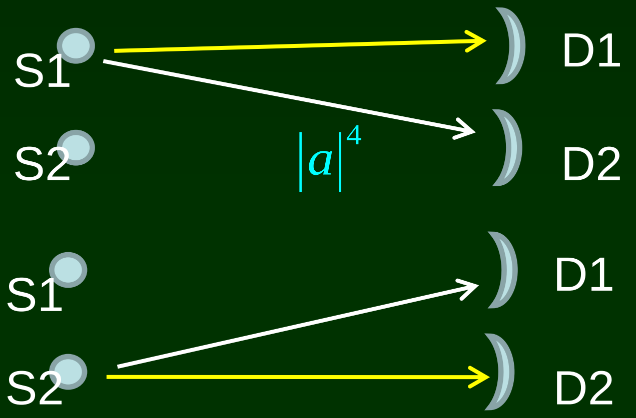
1) No single photon fringes on S1 and S2, when x1 and x2 are scanned.

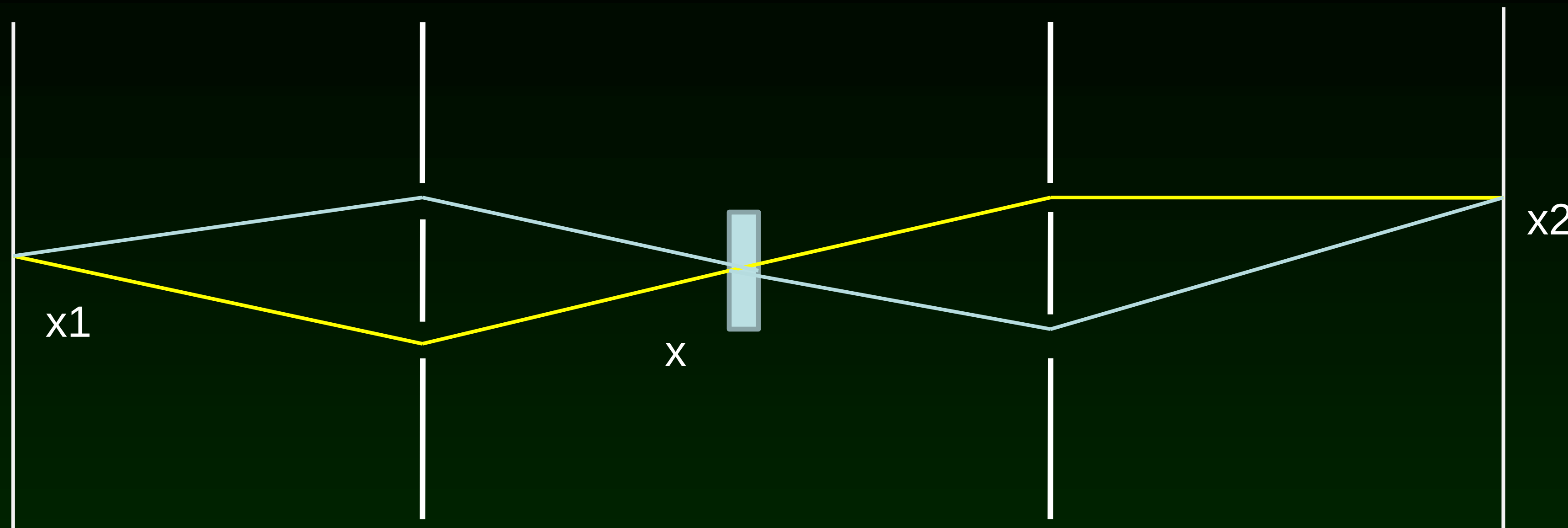
2) Two-photon Correlations (fringes) as (x1-x2) is varied, with 100% visibility

3) If source is made very small, the situation reverses.



In this case, out of the 4 fundamental processes , each of which respect Einstein locality, only two can operate BECAUSE the conservation law imposes a correlation right at source (oppositely directed momenta). Therefore, correlations exceed classical bound, and there is 100% visibility (maximal violation).





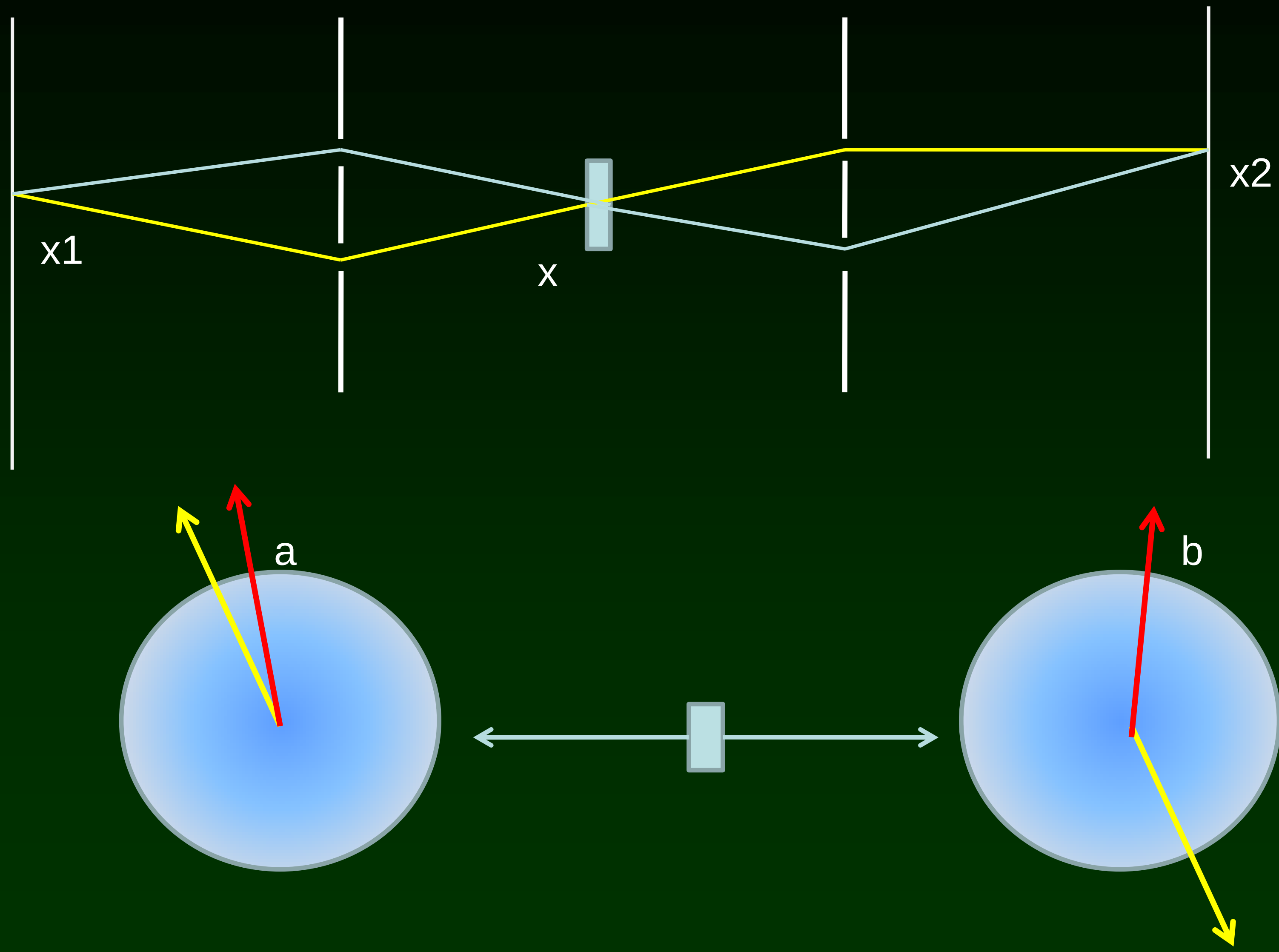
$$A_1 = \cos k\theta(x1 + x) \qquad A_1 = \cos k\theta(x2 + x)$$

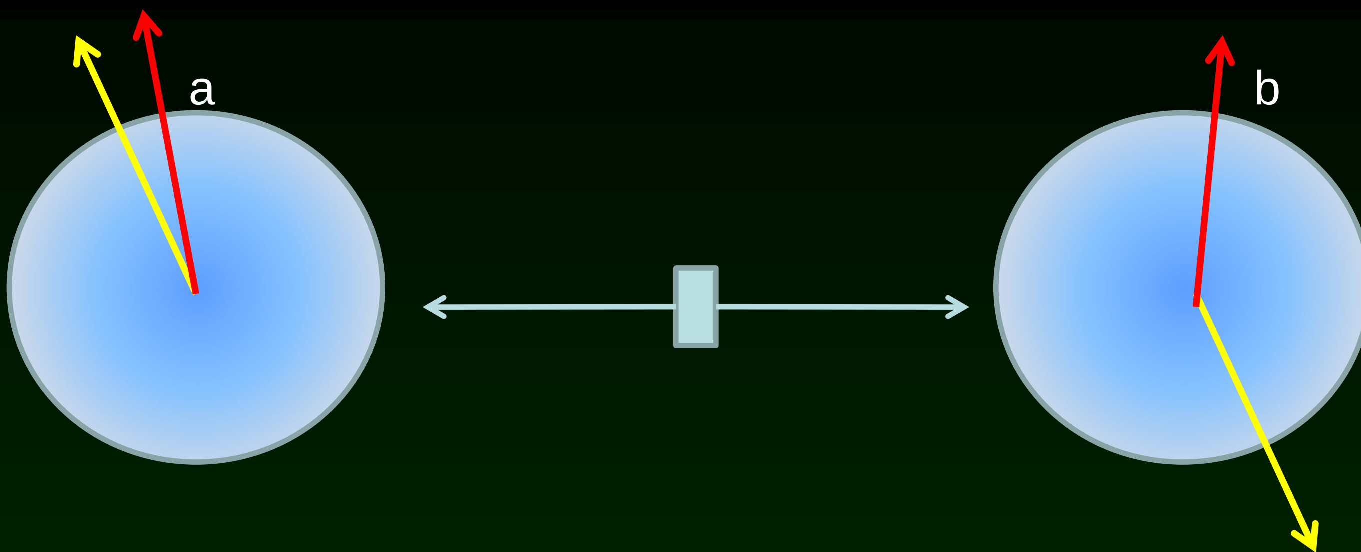
$$\langle A \rangle_C = \langle A_1 A_2 \rangle = \int_{source} \cos k\theta(x1 + x)\cos k\theta(x2 + x)dx$$

$$= \tfrac{1}{2}\cos k\theta(x1 - x2)$$

$$P(x1 - x2) \propto \cos^2(k\theta(x1 - x2))$$

Now this local causal analysis can be applied to spin-singlet and similar cases





$$A_1 = \cos\left(\theta_a - \phi_r\right) \qquad A_1 = \cos\left(\theta_b - \phi_r\right)$$

$$\left\langle A \right\rangle_C = \left\langle A_1 A_2 \right\rangle = \int_{\phi_r} \cos\left(\theta_a - \phi_r\right) \cos\cos\left(\theta_a - \phi_r\right) d\phi_r$$

$$\propto \cos\left(\theta_a - \theta_b\right)$$

$$C(\Delta\theta) = \cos^2\left(\Delta\theta\right) - \sin^2\left(\Delta\theta\right)$$

Summary of Results Discussed:

1) Brown Twiss correlations in classical optics respects Einstein locality.

2) Fundamental restrictions on the source due to conservation requirements can increase the HBT two-particle correlations beyond the classical bound, up to 100% visibility. Einstein locality continues to be valid.

3) Exactly same thing happens in two particle correlations of entangled systems. The correlation is the result of the product of two local amplitudes with random phases, with all phases contributing simultaneously .

4) This results in correlation that depends on the cosine of the difference in the settings of the measurement apparatus, all obeying Einstein locality. The entire phase information is at the source and no nonlocal effects are required.

We have demonstrated how quantum correlations arise from conservation constraint encoded a priori in relative phases at source. The 'hidden variable' was hiding in the theory itself – the correlated random phases reflecting the nature of the source. Einstein locality is strictly valid in quantum correlations.

The trio, EPR, argued that the outcome of a measurement on any physical system is determined prior to and independent of the measurement (realism) and that the outcome cannot depend on any actions in space-like separated regions (Einstein's locality). They used the perfect correlations of entangled states (thus often called EPR states) to define elements of reality, a notion which according to them was missing in quantum theory.

Elements of reality are deterministic predictions for a measurement result, which can be established without actually performing the measurement, and without physically disturbing the (sub-)system to which they pertain. As elements of reality in the studied case were argued to exist necessarily even for pairs of non-commuting observables, they claimed they are contradicting the Heisenberg uncertainty relation.