On Monogamy of Measurement Induced Non-locality

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Main Objective of the talk

- To discuss different measures of correlation, entanglement and beyond entanglement scenario.
- To discuss shareability of different measures of correlation, in particular, the monogamous nature of measurement induced nonlocality.
- To begin with, we want to explain some of the background materials.

Non-locality: Is it meaningful?

- The term non-locality is one of the most debatable word in the last century.
- If we break the whole period in some parts in the sense that due to non-local (!) behavior someone may consider quantum mechanics is not a complete theory or we may think or try to understand the word in some operational way through the results from quantum mechanics.
- For the first part we usually look for hidden variable approaches and for the second part we find some fascinating results almost counterintuitive in nature.

Main factor

- If we restrict ourselves with the results from quantum mechanics only, we find the behavior of quantum systems is not fully understood whenever there are more number of parties involved.
- In other words, there exist a peculiar type correlation between the parties involved which is not explainable by classical scenario.

Entanglement

- This is possibly the most wonderful invention of quantum mechanics.
- Initially everyone thinks the correlation which is responsible for non-local behavior of quantum systems is nothing but the entanglement.
- However, findings in different quantum systems show there are other candidates also. E.g.,
- The local-indistinguishibility of a complete set of orthonormal product states in 3x3 system.

- Therefore people are searching for new measures of correlation which would help us to understand the non-local behavior of multipartite quantum systems.
- There are other issues to study beyond entanglement also.
- As entanglement is used as a resource in many information processing tasks, therefore, the characterization and quantification problems are the some fundamental issues generated in the last two decades.
- However, there are lot of difficulties. Firstly, we consider entanglement in bipartite systems.

Bipartite Entanglement

- As far as bipartite entanglement is concerned we have at least some knowledge how to deal with entangled states.
- For pure bipartite states there exists a unique measure of entanglement calculated by Von-Neumann entropy of reduced density matrices.

- However for mixed entangled states there is no unique measure of entanglement. One has to look on different ways to quantify entanglement
- Some of the measures of entanglement are distillable entanglement, entanglement cost, entanglement of formation, relative entropy of entanglement, logarithmic negativity, squashed entanglement, etc.

Difficulty

- In most of the cases it is really hard to calculate exactly the measures of entanglement. Only for some few classes of states, actual values are available.
- A similar problem is that it is hard to find whether a mixed bipartite state is entangled or not.

Multipartite Entanglement

- But the situation in multipartite case is more complex than that of bipartite case. E.g., how could we define a measure of entanglement for multipartite states at least for pure states are concerned. It is also very difficult to define maximally entangled states in multipartite systems.

- Consider a mixed entangled state in a multipartite system with the property that it has maximal entanglement w.r.t. any bipartite cut (i.e., reduced density matrices corresponding to the cut is proportional to the identity operator), then we observe that for n-qubit ($n \geq 3$) system, there does not exist any maximally entangled states for $n=4$ and $n \geq 8$.

Therefore one has to think how to define maximally entangled states for such situations. Gour and others have defined maximally entangled states in 4-qubit system considering some operational interpretation.

A possible way: the average bipartite entanglement w.r.t. all possible bipartite cuts the state is maximal.

Depending upon different entanglement measures, such as, tangle, Tasllis and Renyi α-entropies one could find different states which are maximally entangled w.r.t. the entangled measures considered.

Another attempt to quantify entanglement of a multipartite state, through the distance measures. E.g., geometric measure.

Correlation measures beyond entanglement:

Consider two newly introduced measures of correlation:

1. Quantum Discord
2. Measurement Induced Non-locality

Quantum Discord

- Consider the following state:

$$\rho = \frac{1}{4}\left[|+\rangle\langle+|\otimes|0\rangle\langle 0| + |-\rangle\langle-|\otimes|1\rangle\langle 1| + |0\rangle\langle 0|\otimes|-\rangle\langle-| + |1\rangle\langle 1|\otimes|+\rangle\langle+|\right]$$

- The above state is separable. However, it has non-zero quantum discord which is defined by difference of measuring mutual information in two different ways, viz., D(A,B)= I(A:B)-J(A:B) where,
- I(A:B)= S(A)-S(A|B) and
- J(A:B)=S(A)-min Σ pj S(A|j)
- {Πj} j

The above quantity is a measure of non-classical correlation. It has zero value if and only if there exists a von Neumann-measurement $\Pi_k = |\Psi_k\rangle\langle\Psi_k|$ such that the bipartite state

$$\rho = \sum_k \Pi_k \otimes I \, \rho \, \Pi_k \otimes I$$

States of the above kind are known as classical-quantum state.

Some Comments

- One could interpret Discord in terms of consumption of entanglement in an extended quantum state merging protocol thus enabling it to be a measure of genuine quantum correlation.
- Physically, discord quantifies the loss of information due to the measurement.
- This correlation measure is invariant under LU but may change under other local operation. It is asymmetric w.r.t the parties.

- The set of Classical-Quantum states is non convex.
- Due to the optimization problem, it is in general very hard to find analytic expression for discord. Exact analytical result is available only for a few classes of states.
- It was found that Quantum discord is always non-negative and it reduces to Von Neumann Entropy of the reduced density matrix for pure bipartite states.

- Recently, different measures of quantum discord and their extensions to multipartite systems have been proposed.E.g.,
- Geometric discord:

$D(\rho)= \min ||\rho-\chi||$ where the minimum is taken over all zero discord state χ.

- Exact analytical formula for geometric discord is also available for only a few class of states. A tight lower bound is found recently.
- One could also define discord in terms of relative entropy: $D(\rho) = \min S(\rho || \chi)$
- The correlation measure discord actually generates the possibility of research beyond entanglement.

Criteria for measures of correlation

- To formalize the new paradigm beyond entanglement one could set some properties for a measure of correlation.
- Modi et al., provided a set of conditions for a measure of correlation.
- Necessary conditions:
- Product states have no correlation
- Invariance under local unitarity
- Non=negativeness
- Classical states do not have quantum correlation(!)

Other conditions

- Reasonable conditions:
- Continuity under small perturbations
- Other type of strong and weak continuities
- Questionable/debatable conditions:
- For pure bipartite states total, classical and quantum correlations could be defined by nthe marginals
- Additivity total= classsical+quantum
- Classical and/or quantum are nonincreasing under LOCC
- Symmetry under interchange of subsystems

Measurement Induced Nonlocality

- Consider the state, $\rho = \frac{1}{2}\left[|00\rangle\langle 00| + |11\rangle\langle 11|\right]$
- The state has non-zero value of a new measure of correlation is the Measurement Induced Non-Locality(MIN) .
- It is defined as,

$$N(\rho) = \max \|\rho - \Pi(\rho)\|$$

where the maximum is taken over all Von-Neumann measurements that preserves density matrix of the first party.

- Physically, MIN quantifies the global effect caused by locally invariant measurement.
- MIN vanishes for product state and remains positive for entangled states. For pure bipartite state MIN reduces to linear entropy like geometric discord.
- It has explicit formula for $2\otimes m$ system, $m\otimes n$ system(if reduced density matrix of first party is non-degenerate) system.

- MIN is invariant under local unitary.
- The set of states with zero MIN is a proper subset of the set of states with zero Discord. Thus, it signifies the existence of non-locality without Discord. The set of all zero MIN states is also non-convex.

Shareability

- Unlike classical case there are differences between shareability of states with shareability of correlations.
- E.g., if a pure state is shared between three parties A,B,C in such a way that two parties(say, A,B) shared a maximally entangled state then there must not be any entanglement between A with C and B with C. It is therefore useful for secret sharing.
- Again, if A,B are correlated in such a way that they violet CHSH inequality, then neither A nor B could be correlated with C.

Shareability contd..

- Thus the question of shareability of different quantum correlations is now an important issue of study.

- There are lot of results obtained for the last few years in this field, in particular, the study of monogamous behavior of different correlation measures.

- Entanglement in pure states of multipartite systems has monogamous behavior.

Shareability contd..

- However, it is not the case for other measures of correlations, in general. In some cases, rather polygamous.

- Actually recent results on discord show sometimes even in pure state case discord is not monogamous.

- We will now discuss on monogamous behavior of measurement induced non-locality.

Monogamy of MIN

- Firstly, we write down the formula for MIN of any state in 2xn dimensional system:
- Suppose ρ_{AB} has the form,

$$\rho_{AB} = \frac{1}{\sqrt{2n}} \frac{I}{\sqrt{2}} \otimes \frac{I}{\sqrt{n}} + \sum_{i=1}^{3} x_i X_i^A \otimes \frac{I}{\sqrt{n}} +$$

$$\frac{I}{\sqrt{2}} \otimes \sum_{i=1}^{n^2-1} y_i Y_i^B + \sum_{i=1}^{3} \sum_{j=1}^{n^2-1} t_{ij} X_i^A Y_i^B$$

- Where X_i^A and Y_i^B are orthonormal operator bases for system A and B respectively, then

Monogamy of MIN contd..

- MIN is given by,

$$N(\rho_{AB}) = \begin{cases} trTT^t - \frac{1}{\|x\|^2} x^t TT^t x & if \quad x \neq 0 \\ trTT^t - \lambda_3 & if \quad x = 0 \end{cases}$$

- Where the matrix $T = (t_{ij})$ with λ_3 is the minimum eigenvalue of TT^t
- Now consider the three qubit state,

$$|\psi_{ABC}\rangle = \lambda_0|000\rangle + \lambda_1 e^{i\theta}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle$$

Monogamy of MIN contd..

- Then we find for $\|x\| = 0$,

$$N\left(\rho_{AB}\right)+N\left(\rho_{AC}\right)\geq 0.5=N\left(\rho_{A|BC}\right)$$

- So, monogamy is violated for most of the states. W states satisfies with equality.
- For $\|x \neq 0\|$,
- $$N\left(\rho_{AB}\right)+N\left(\rho_{AC}\right)\leq 0.5=N\left(\rho_{A|BC}\right)$$

- In this case momogamy is satisfied. GHZ belongs to this class.

Four qubit system

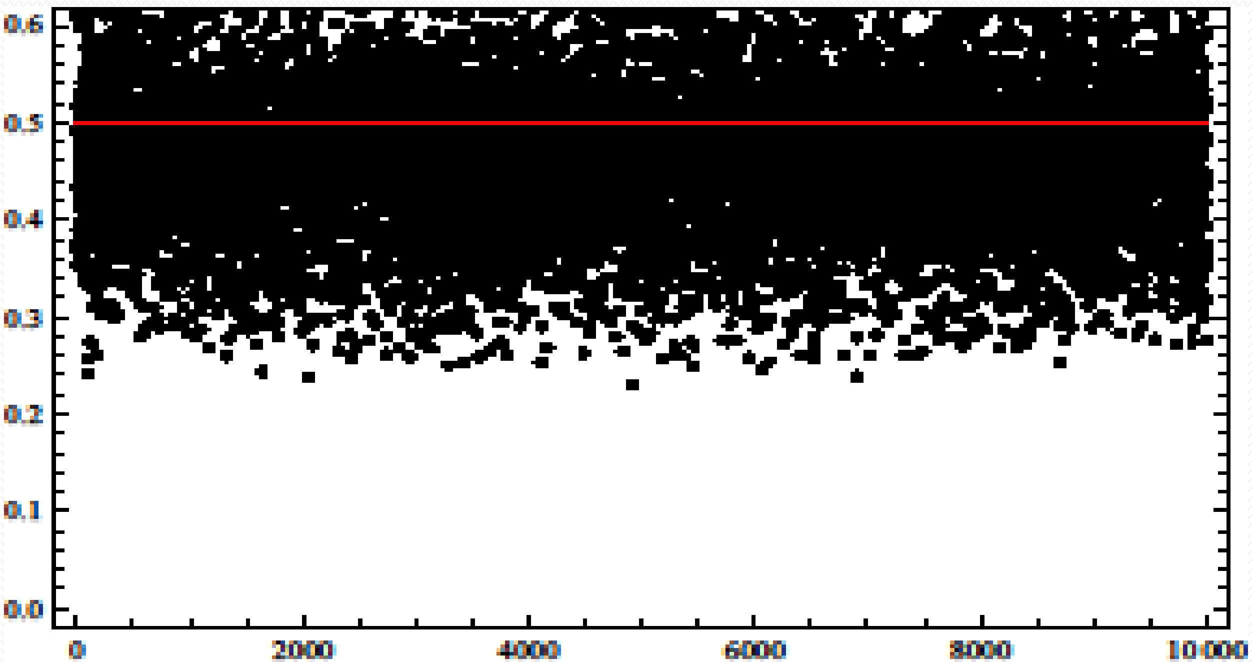
- Firstly, consider the 4-qubit generic class,

$$|\Phi\rangle = \beta_1|B_1B_1\rangle + \beta_2|B_2B_2\rangle + \beta_3|B_3B_3\rangle + \beta_4|B_4B_4\rangle,$$

- where Bi are Bell states.
- We have calculated analytically the values of $N(\rho_{AX})$ for all X=B,C,D and also $N(\rho_{A|BCD}) = 0.5$
- We find monogamy is satisfied for large no. of states in this class.

Contd..

- Numerical simulation of 10^6 states shows 66% of them satisfies monogamy relation.



contd..

- There are two important subclasses of the generic class, one is M and other is τ_{min} where for M, the sum of the squares of the coefficients is equal to zero and for τ_{min} all coefficients are real. Four qubit GHZ belongs to τ_{min} .
- We find MIN is monogamous in the subclass M, but is not monogamous in the subclass τ_{min} . So for GHZ monogamy does not hold.
- For W states monogamy holds with equality.

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