Control of coherence in many-body quantum systems

Florian Mintert

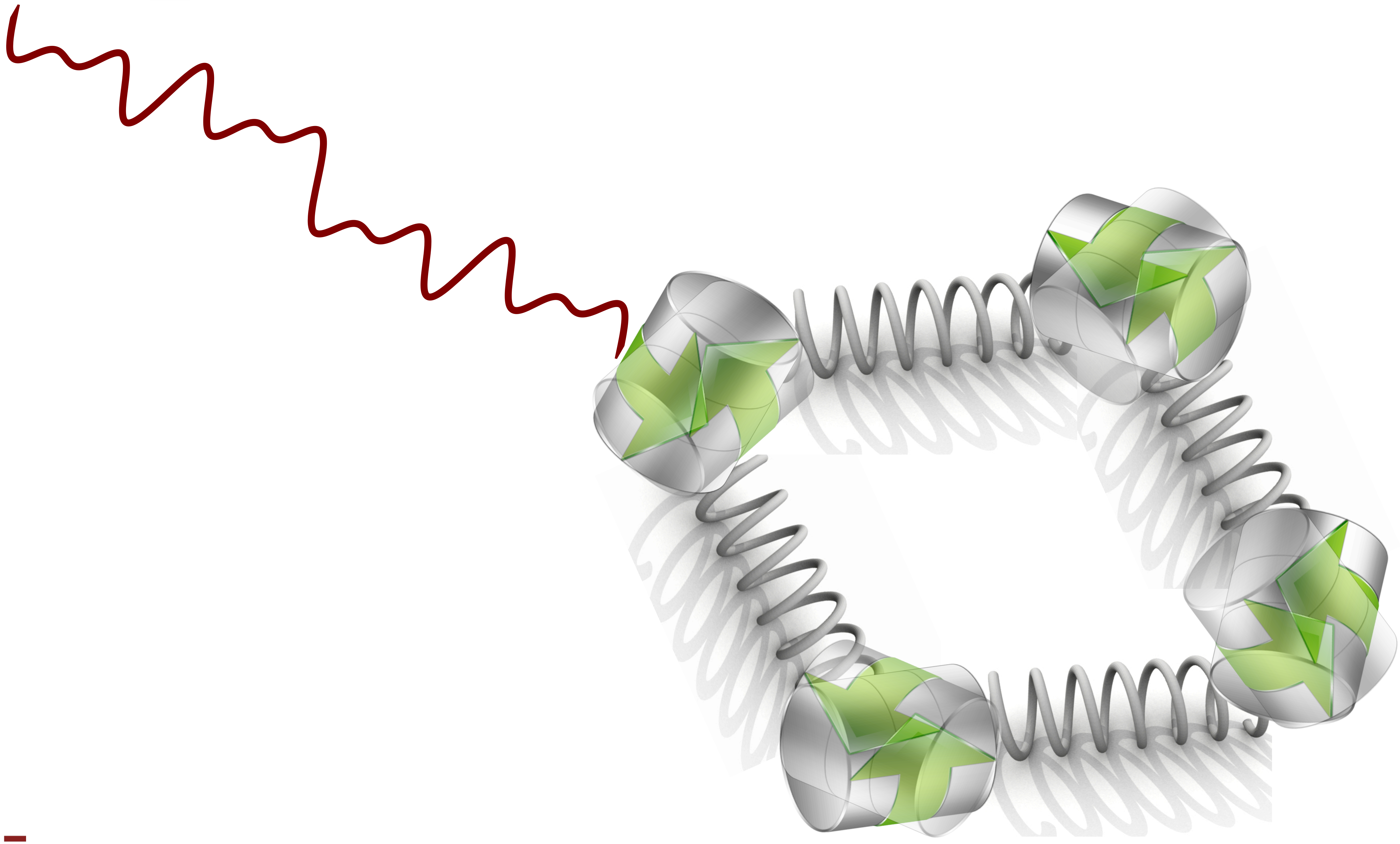
FRIAS
FREIBURG INSTITUTE FOR ADVANCED STUDIES

Coherent dynamics in many-body quantum systems

Felix Platzer

Fabian Bohnet-Waldraff

Control of entanglement



Entanglement as target

Typical : define target state and maximize fidelity

continuum of equally entangled states ... $|\Phi\rangle = \mathcal{U}_1 \otimes \mathcal{U}_2 \ldots \mathcal{U}_n |\Psi\rangle$

... with different dynamical properties !

$$E(\varrho) = \inf \sum_i p_i E(\Psi_i)$$

optimization ... impractical

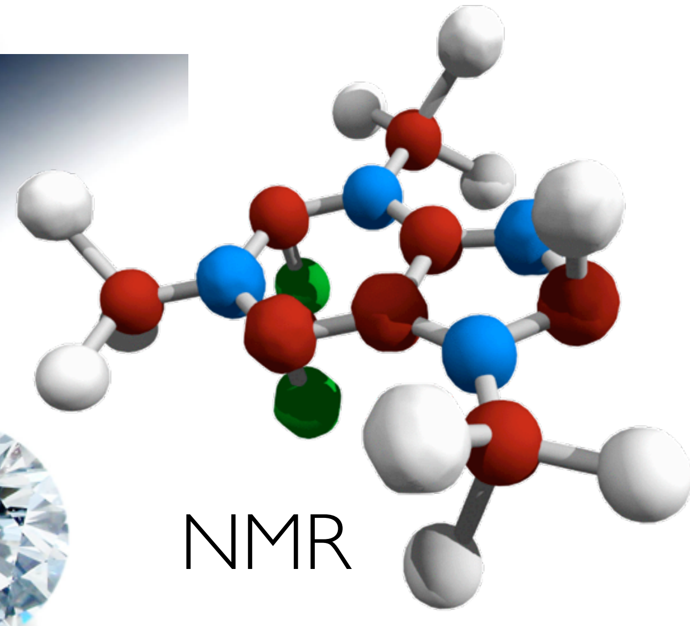
our approach : observable entanglement measures

$$\tau(\varrho) = \mathrm{Tr}\, \varrho \otimes \varrho\, A$$

Asher Peres and William Wootters PRL 1119, **66** (1991)
Todd Brun, Quant. Inf. Comp. 401, **4** (2004)
F.M., Marek Kuś and A. Buchleitner, PRL **95**, 260502 (2007)
F.M. and A. Buchleitner, PRL **98**, 140505 (2007)

advantages of entanglement measures and practibility

The playground



NMR

Many-body system

Intrinsic interaction

$$H = \sum_{i \neq j} \lambda_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

Control induces local unitary dynamics

$$H_c = \sum_{ij} h_{ij}(t) \sigma_j^{(i)}$$

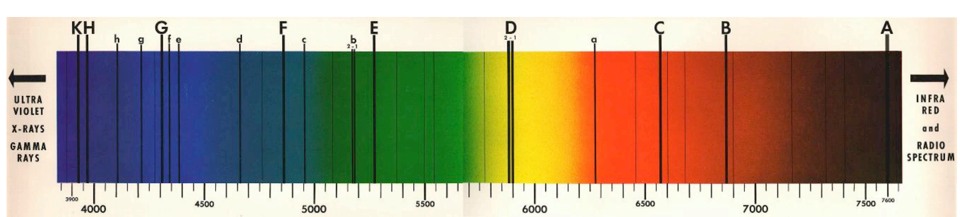
Control disorder and noise :

drive system towards robust strongly entangled states

Program

control of entanglement

GRAPE



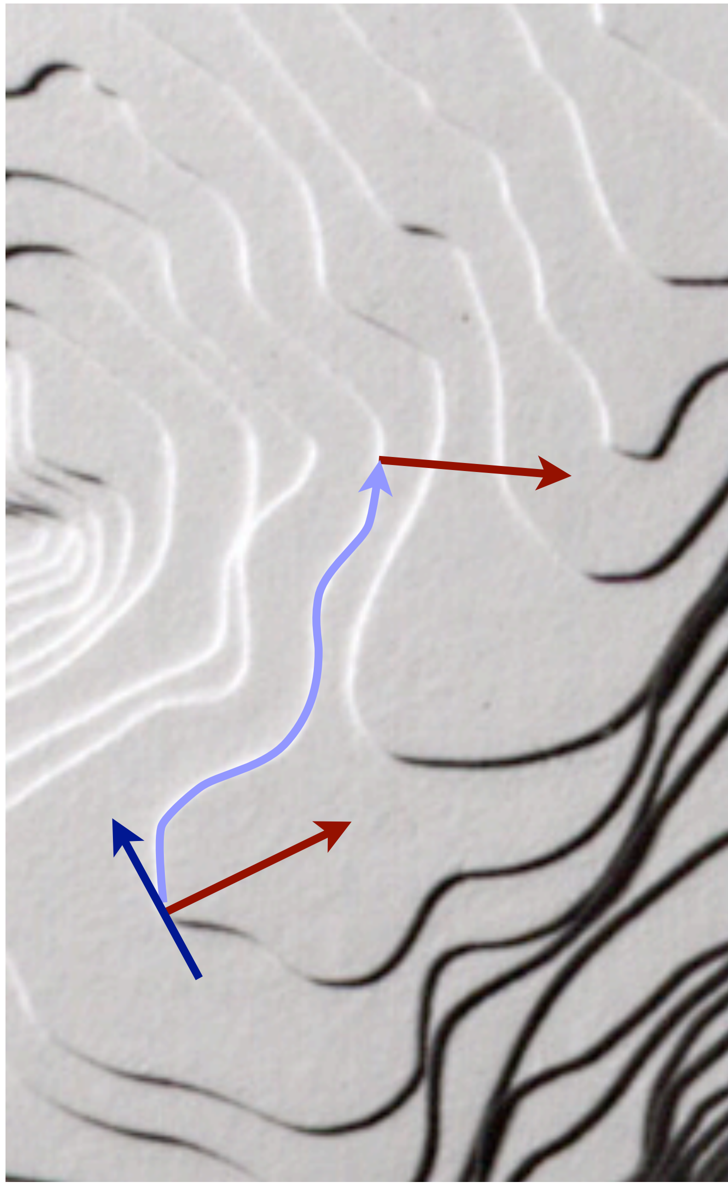
control in frequency space

Local control

entanglement is independent of local unitary dynamics

$$\frac{\partial \tau}{\partial t}$$ independent of control pulse

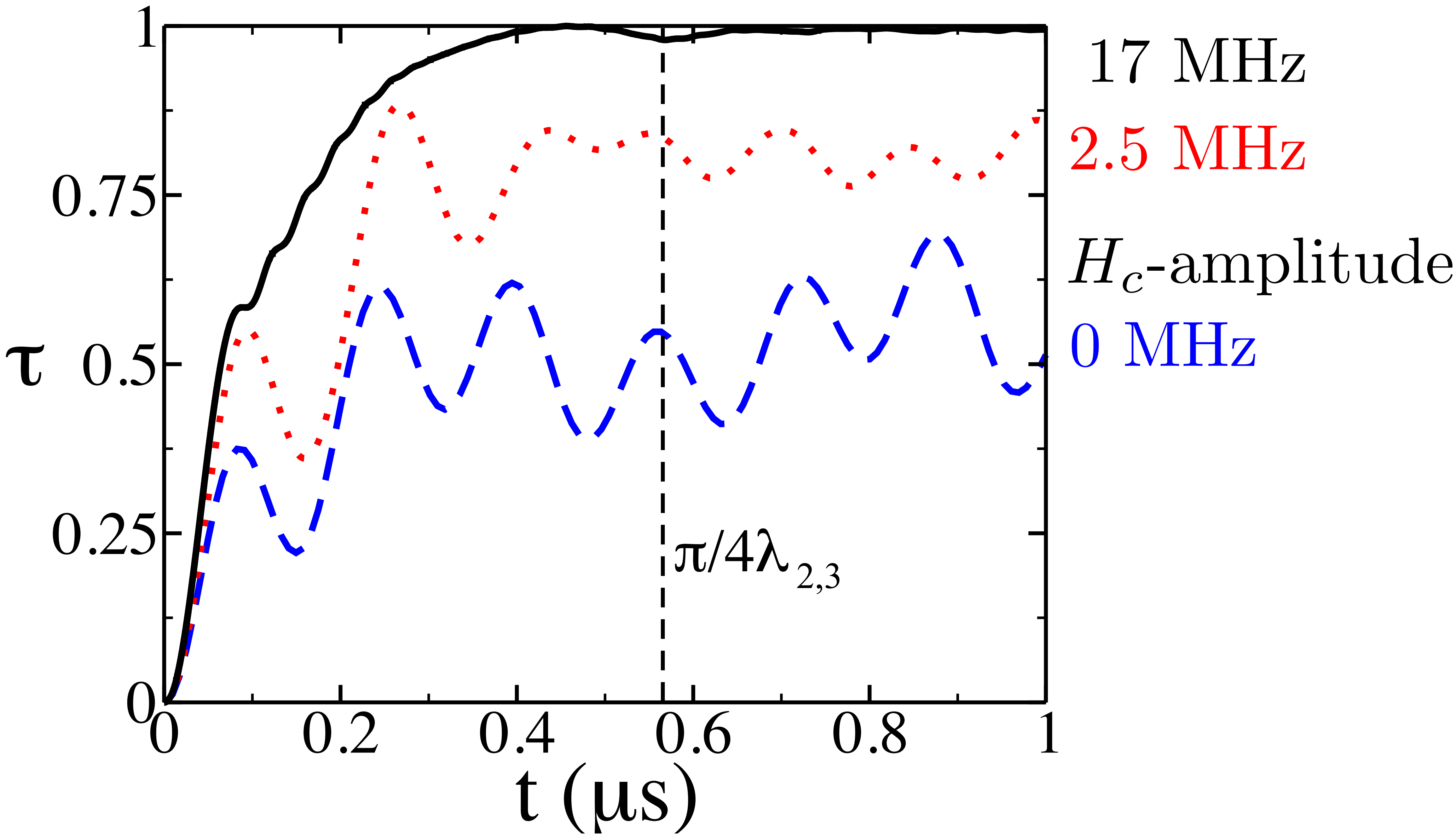
$$\frac{\partial^2 \tau}{\partial t^2}$$ interplay of interaction/decoherence and control



$$\boxed{\frac{\partial^2 \tau}{\partial t^2} = \vec{X}\vec{H}_c + \ddot{\tau}_0}$$

Control of 4 spins

$(\lambda_{1,2} = 9.8 \text{ MHz}, \lambda_{3,4} = 2.7 \text{ MHz}, \lambda_{2,3} = 1.3 \text{ MHz})$



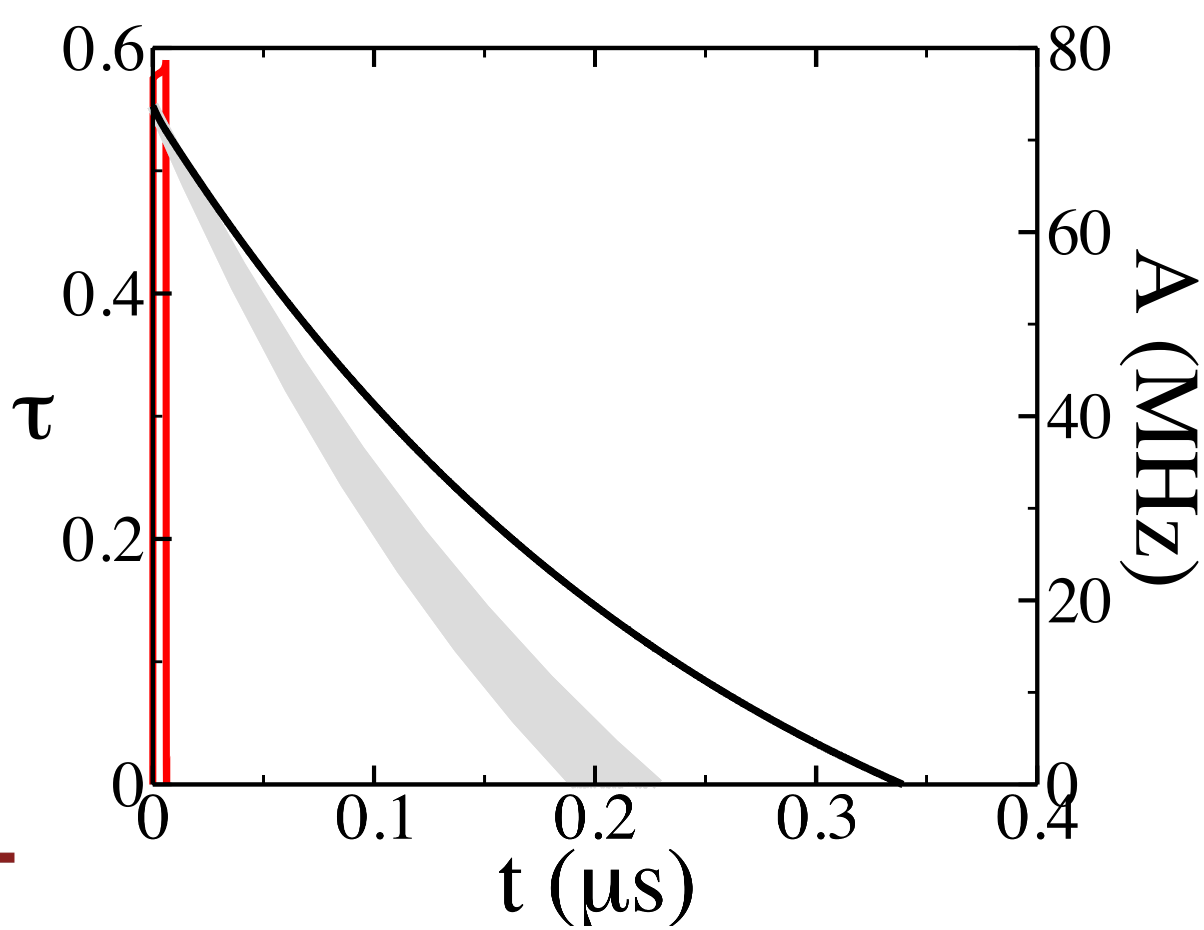
Felix Platzer, FM, A. Buchleitner, PRL **105**, 020501 (2010)

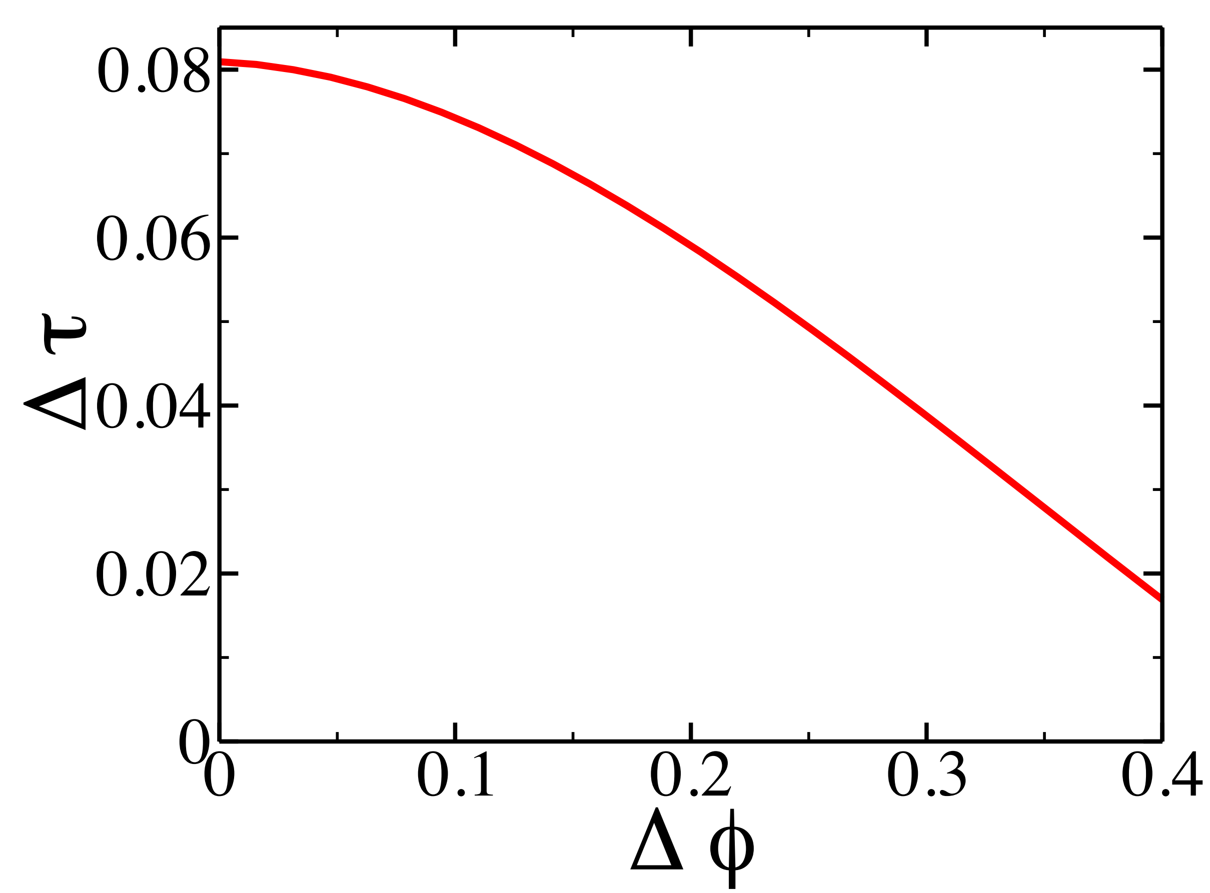
Robust states

no interaction: $$\dot{\varrho} = \frac{1}{i\hbar}[H_c, \varrho] + \mathcal{L}(\varrho)$$

maximize $\ddot{\tau}$ → results in maximial $\dot{\tau}$

→ maximially robust states





Program

control of entanglement

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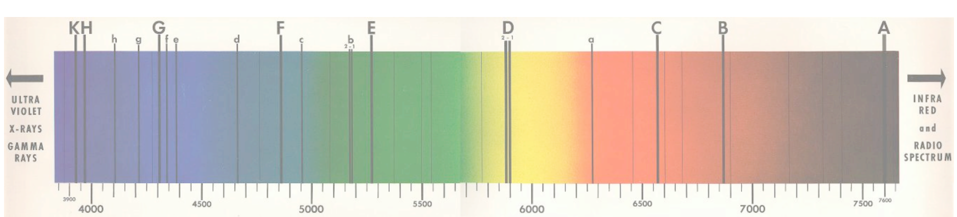


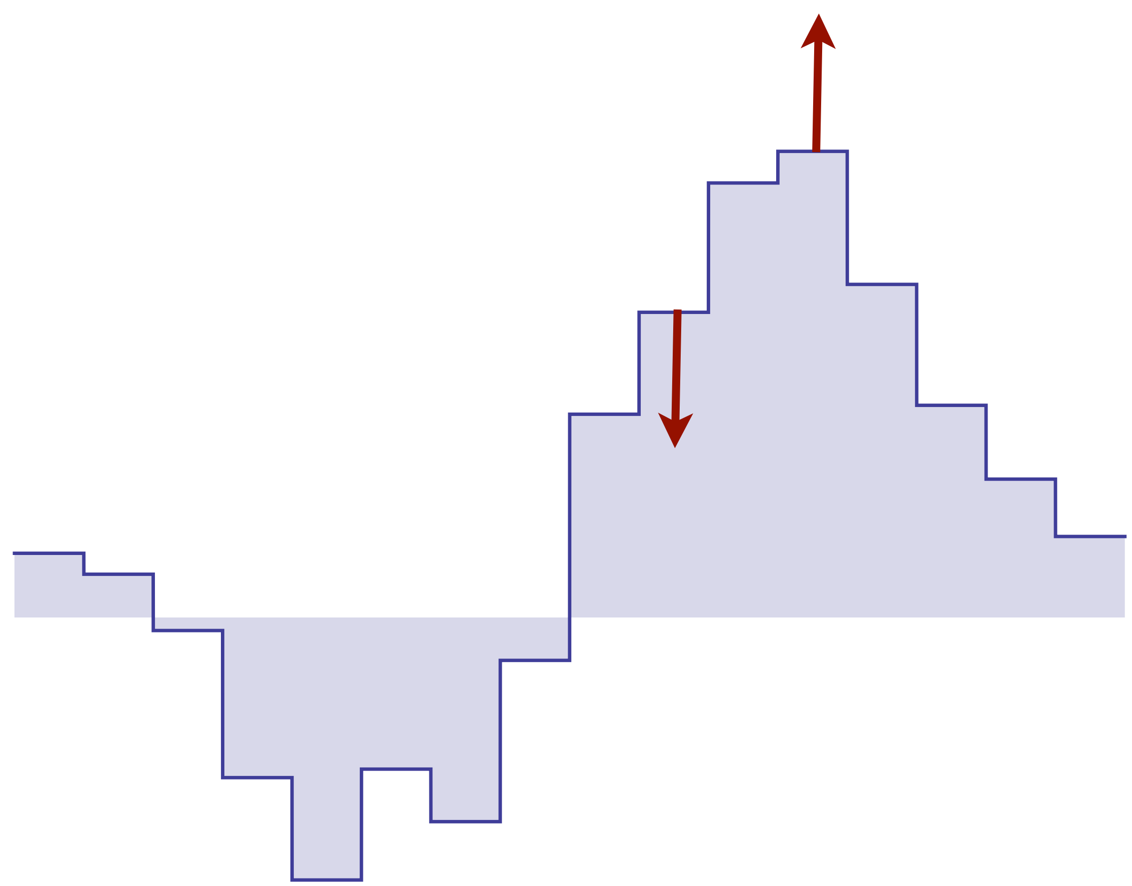
control in frequency space

GRAPE

GRADIENT ASCENT PULSE ENGINEERING

Navin Khaneja, T. Reiss, C. Kehlet, Thomas Schulte-Herbrüggen & Steffen Glaser

J. Magn. Reson. **172**, 296 (2005)



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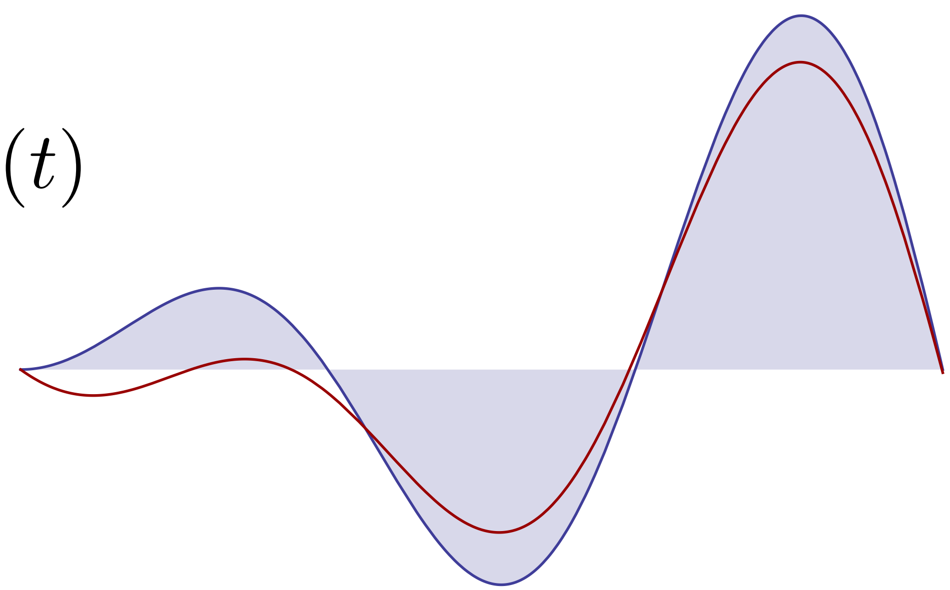
$$\mathcal{H}(t) = \mathcal{H}(a(t))$$

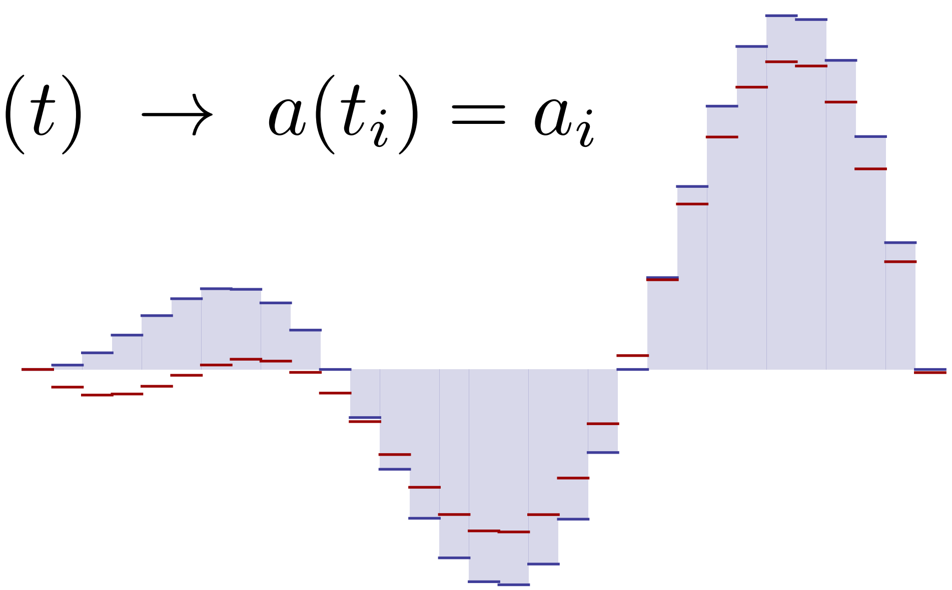
discretize pulse

$a(t)$



$\rightarrow$

$$a(t) \;\rightarrow\; a(t_i) = a_i$$



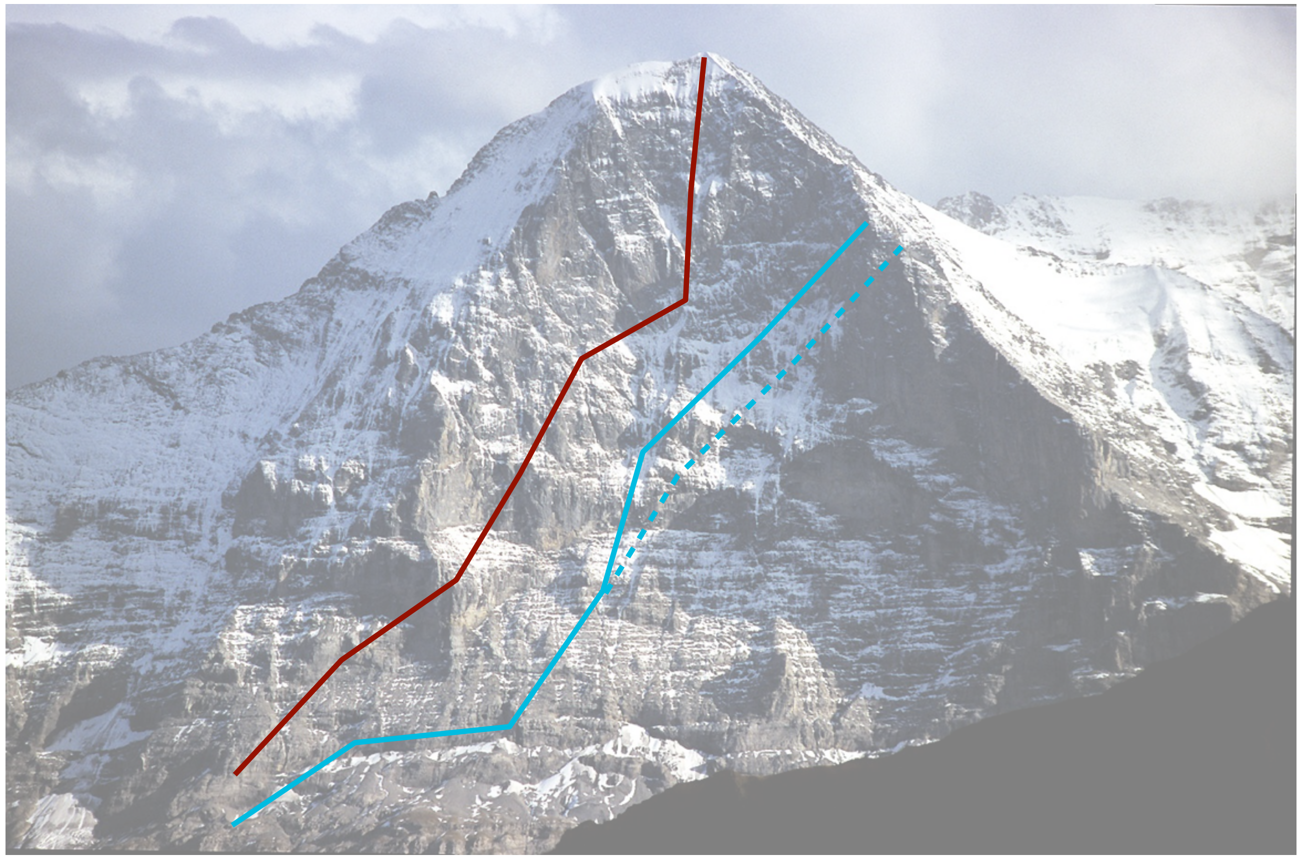
time evolution

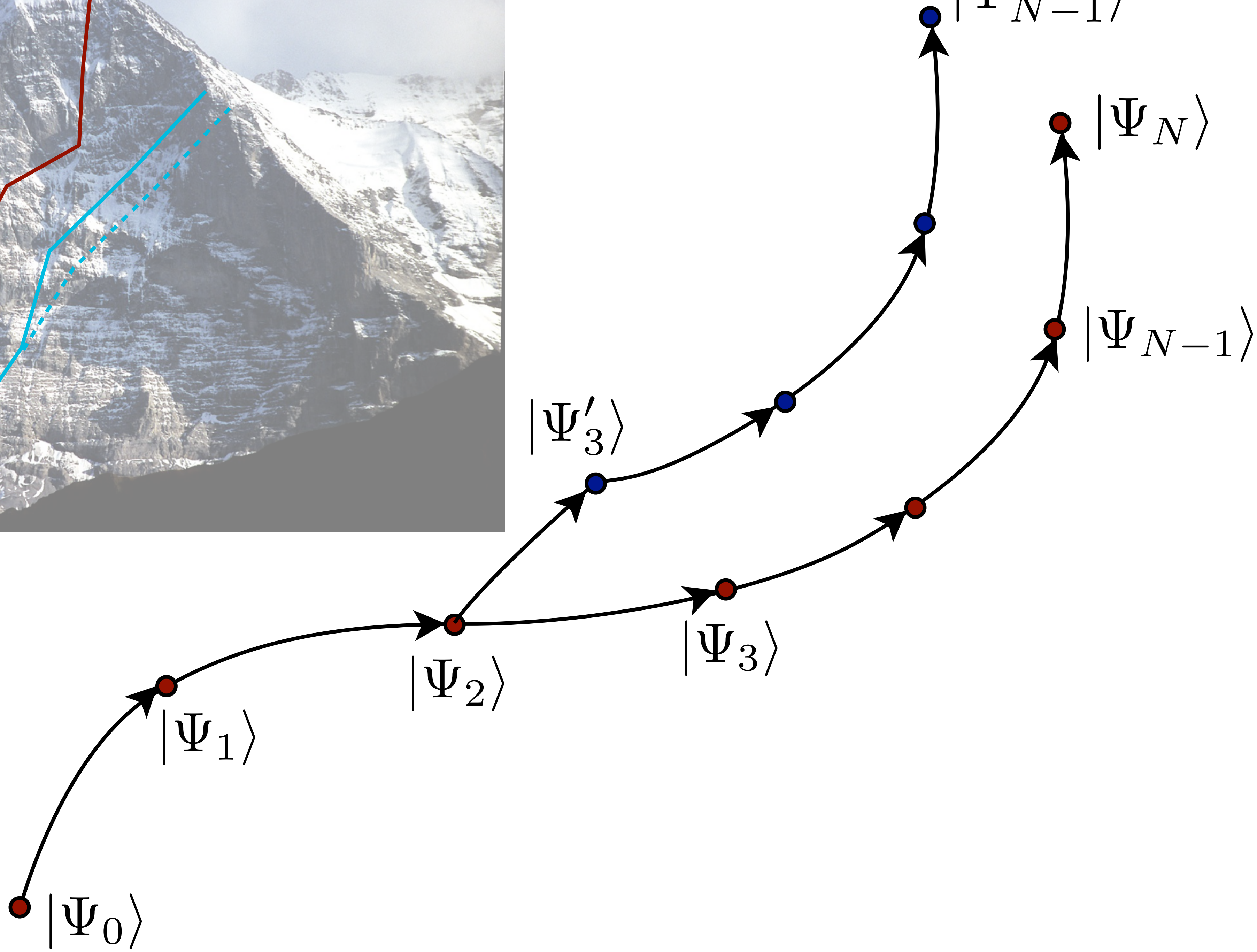
$$\mathcal{U} = \mathcal{T} e^{-i \int dt \;\; \mathcal{H}(a(t))} \qquad\qquad \mathcal{U} \;\rightarrow\; \mathcal{U}_N \mathcal{U}_{N-1} \ldots \mathcal{U}_1$$

derivatives

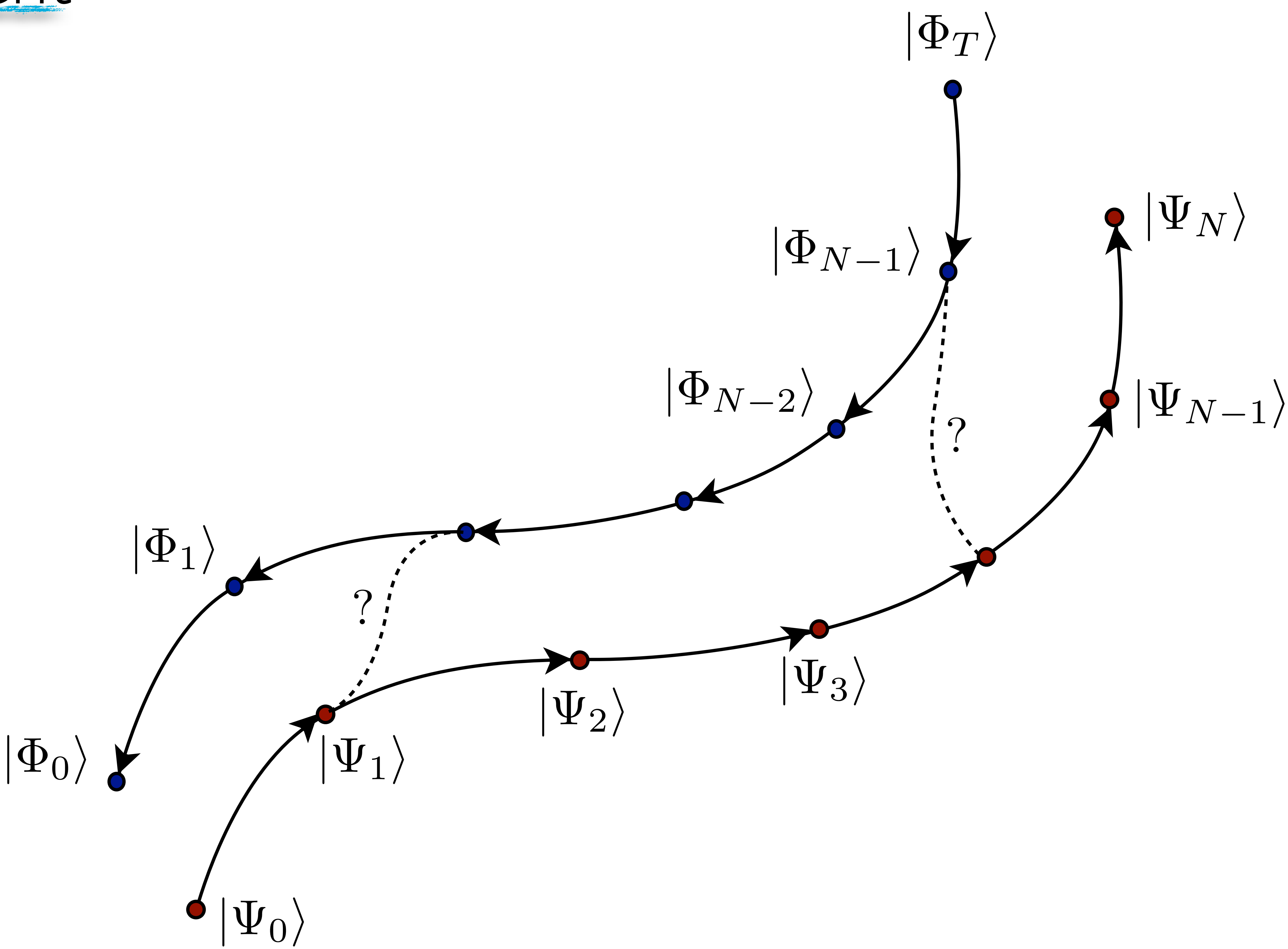
$$\frac{\delta \mathcal{U}}{\delta a} \qquad\qquad \frac{\partial \mathcal{U}}{\partial a_i}$$

Gradient




$|\Psi'_{N-1}\rangle$
$|\Psi_N\rangle$
$|\Psi_{N-1}\rangle$
$|\Psi'_3\rangle$
$|\Psi_3\rangle$
$|\Psi_2\rangle$
$|\Psi_1\rangle$
$|\Psi_0\rangle$

Gradient


$|\Phi_T\rangle$
$|\Phi_{N-1}\rangle$
$|\Psi_N\rangle$
$|\Phi_{N-2}\rangle$
$|\Psi_{N-1}\rangle$
?
$|\Phi_1\rangle$
?
$|\Psi_3\rangle$
$|\Psi_2\rangle$
$|\Phi_0\rangle$
$|\Psi_1\rangle$
$|\Psi_0\rangle$

Gradient

$$\frac{\partial}{\partial a_i}\langle\Phi_T|\mathcal{U}_N\ldots\mathcal{U}_{i+1}\mathcal{U}_i\mathcal{U}_{i-1}\ldots\mathcal{U}_1|\Psi\rangle = \underbrace{\langle\Phi_T|\mathcal{U}_N\ldots\mathcal{U}_{i+1}}_{\langle\Phi_i|}\frac{\partial\mathcal{U}_i}{\partial a_i}\underbrace{\mathcal{U}_{i-1}\ldots\mathcal{U}_1|\Psi\rangle}_{|\Psi_{i-1}\rangle}$$

$$|\Phi_i\rangle = \mathcal{U}_{i+1}^{-1}\ldots\mathcal{U}_N^{-1}|\Phi_T\rangle$$

$$\boxed{\mathcal{G}_i = \langle\Phi_i|\frac{\partial\mathcal{U}_i}{\partial a_i}|\Psi_{i-1}\rangle}$$

perturbation theory

Robustness

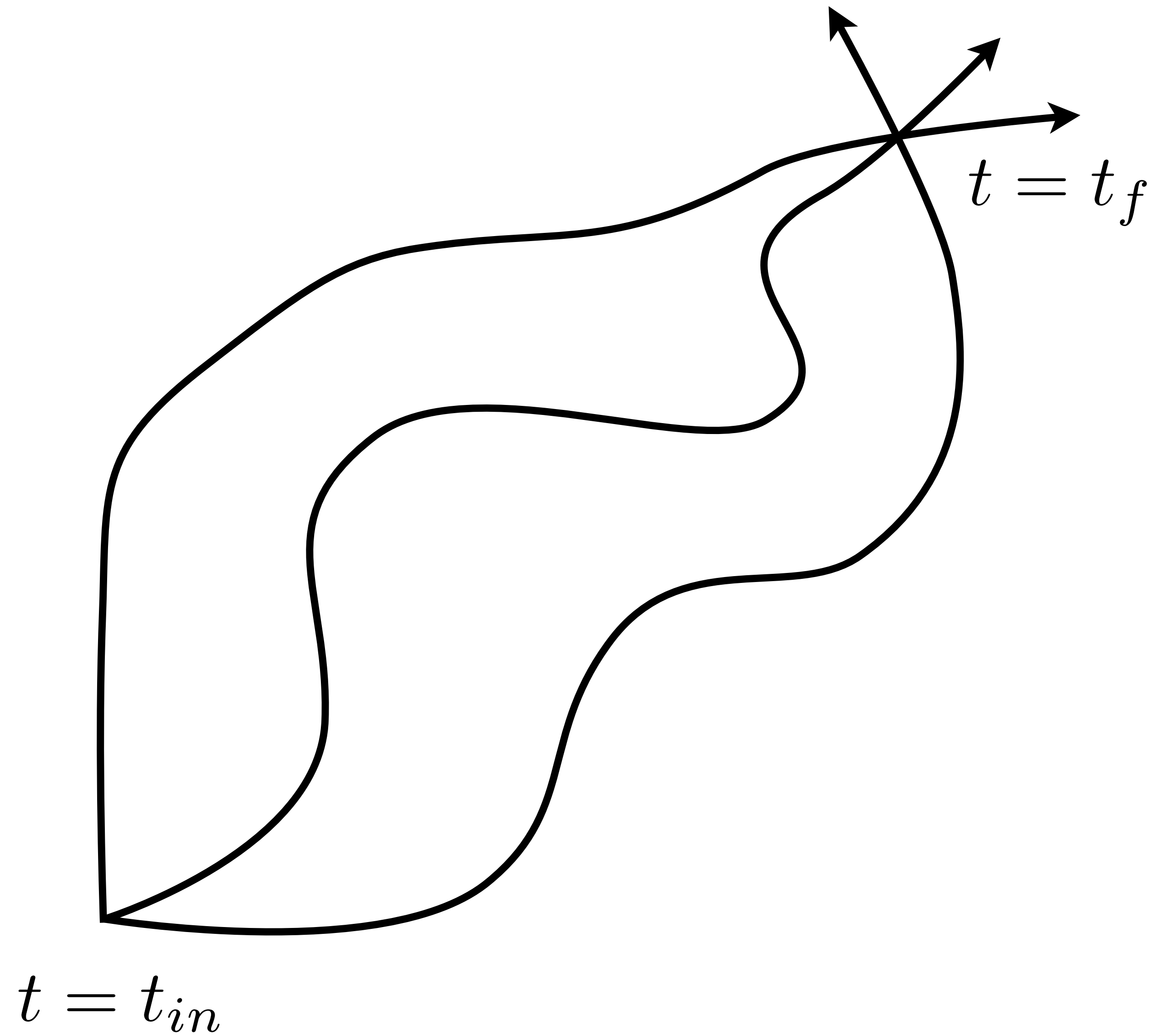
ensemble of Hamiltonians, one pulse

fluctuations in resonance frequency

$$H(\omega) = \frac{\omega}{2}\sigma_z$$

fluctuations in coupling constants

$$H(\vec{\lambda}) = \sum_{i \neq j} \lambda_{ij}\ \sigma_z^{(i)}\sigma_z^{(j)}$$



What target ?

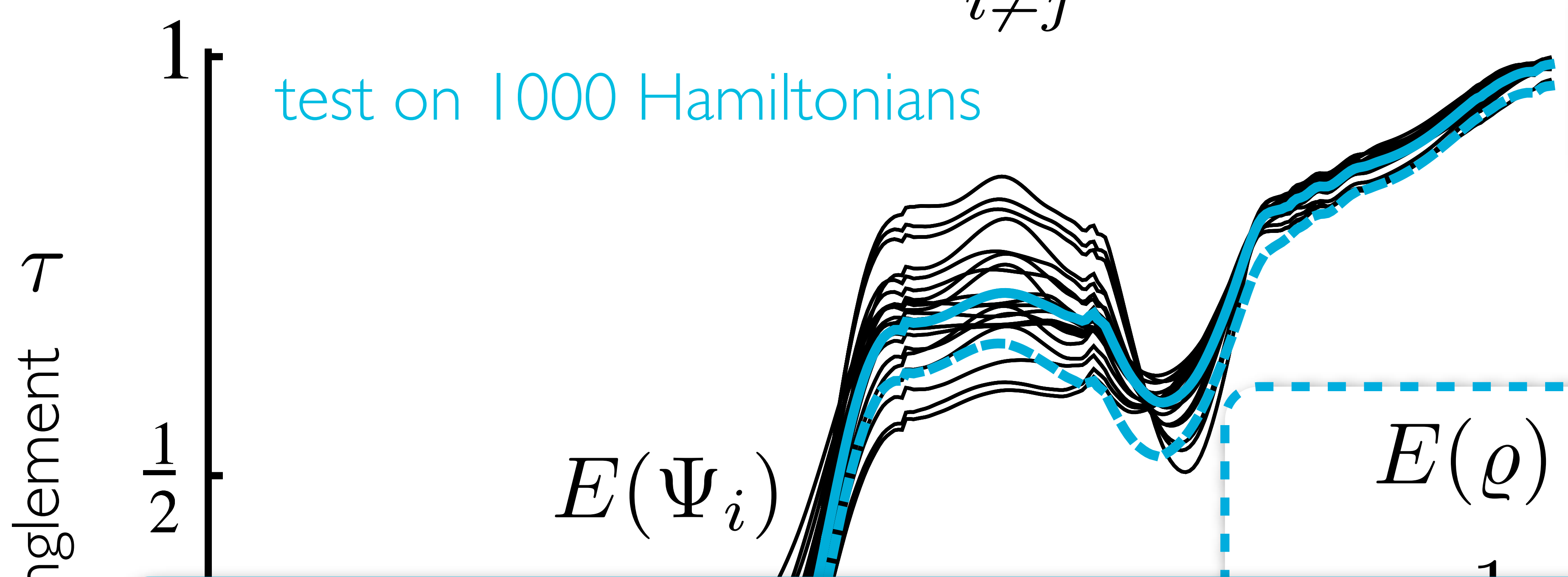
average entanglement or entanglement of average state

$$\langle E \rangle = \frac{1}{N}\sum_i E(\Psi_i) \qquad E(\varrho) \qquad \varrho = \frac{1}{N}\sum_i |\Psi_i\rangle\langle\Psi_i|$$

Robust Pulses

ensemble with 20 Hamiltonians $H_s = \sum_{i \neq j} \lambda_{ij} \; \sigma_z^{(i)} \sigma_z^{(j)}$ $\pm 35\%$ variation

$\langle E(\Psi_i) \rangle$



$E(\varrho)$

$\ldots i\rangle\langle\Psi_i|$

optimized

$\overline{E(\rho)}_{20} = 0.9919$

$E(\bar{\rho})_{20} = 0.9737$

averaged

$\overline{E(\rho)}_{1000} = 0.9908$

$E(\bar{\rho})_{1000} = 0.9625$

Program

control of entanglement

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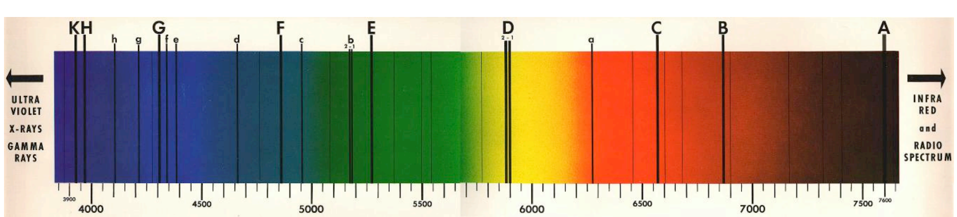
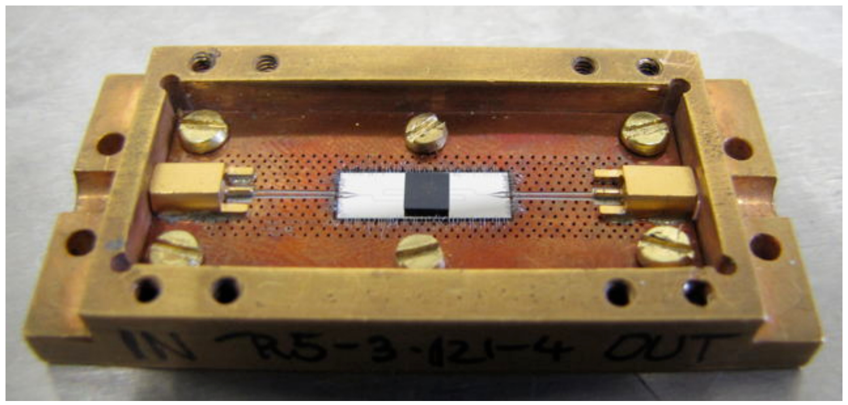
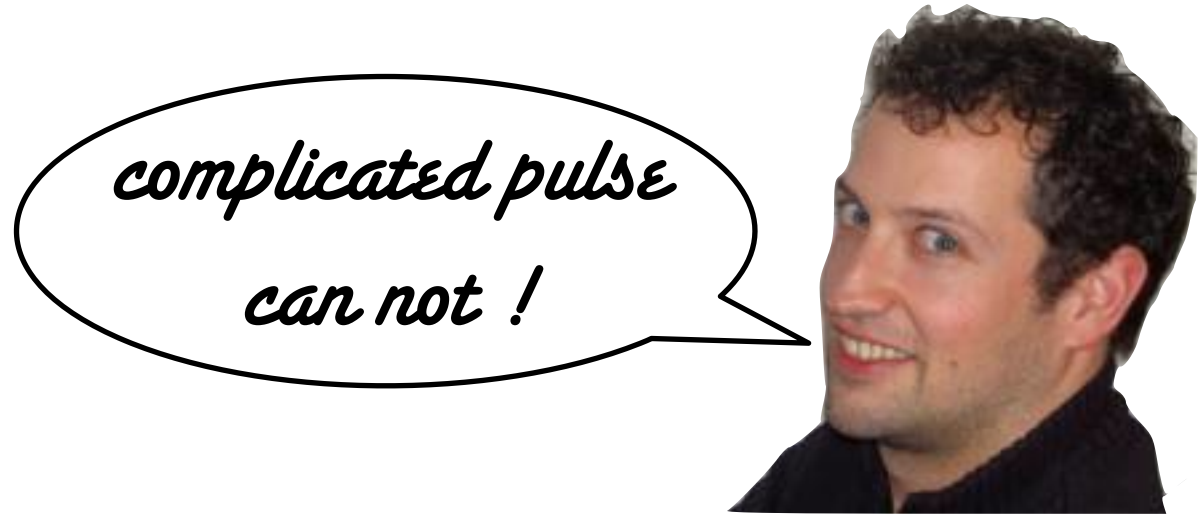
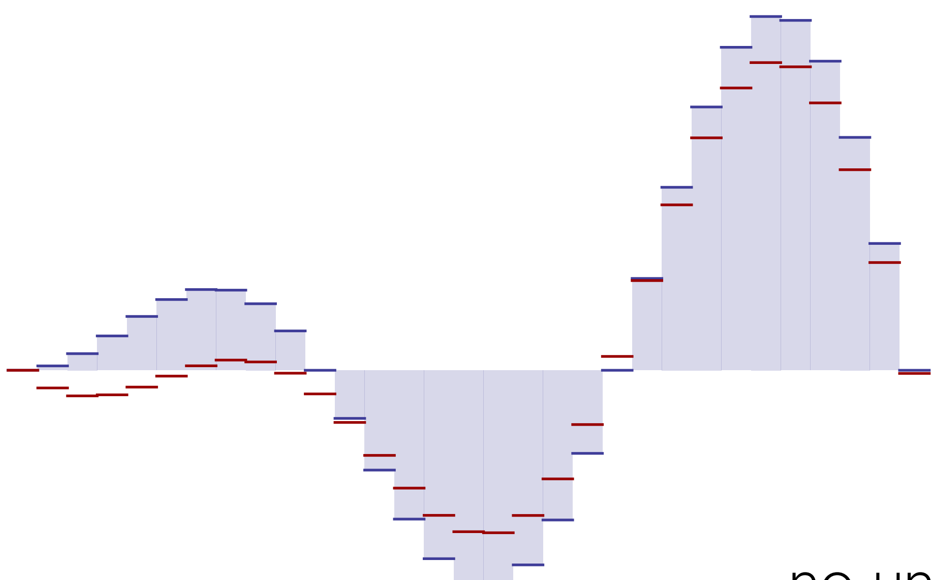


control in frequency space

Smooth control

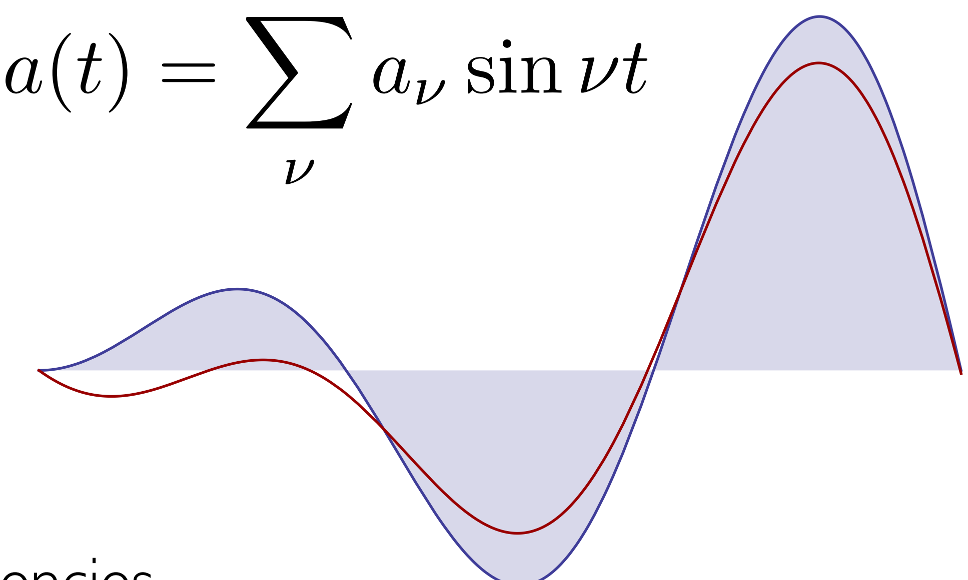






$$\rightarrow$$

$$a(t) = \sum_{\nu} a_\nu \sin \nu t$$



- no undesired frequencies
- few parameters to optimize
- boundary conditions
- also curvature available
- time averaged targets

Smooth control

time-periodic Hamiltonian

$$\mathcal{H}(t) = \mathcal{H}(t+T)$$

$\xrightarrow{\hspace{3cm}}$ Floquet theory

complete solution

$$|\Psi_i(t)\rangle = e^{i\varepsilon_i t}|\Phi_i(t)\rangle$$

eigenvalue problem

$$\mathcal{K} = \mathcal{H} - i\frac{\partial}{\partial t} \qquad \mathcal{K}|\Phi_i\rangle = \varepsilon_i|\Phi_i\rangle$$

gradient and curvature through perturbation of $\mathcal{K}$

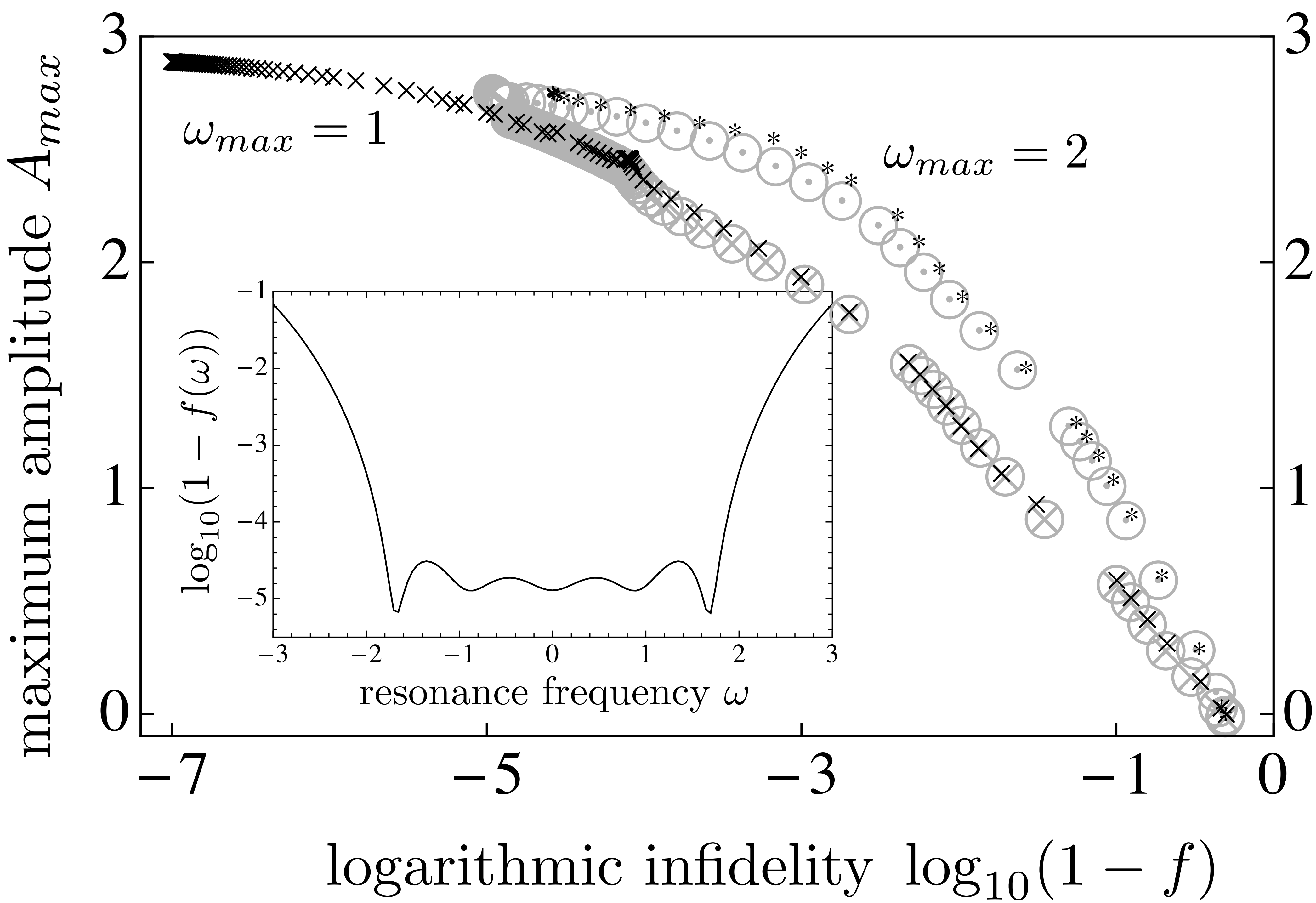
Ensemble of single spins

$$\mathcal{H}_\omega = \frac{\omega}{2}\sigma_z + A(t)\sigma_y \qquad A(t) = \sin t \; (\sum_{i=1}^{i_{max}} a_i \sin it)$$

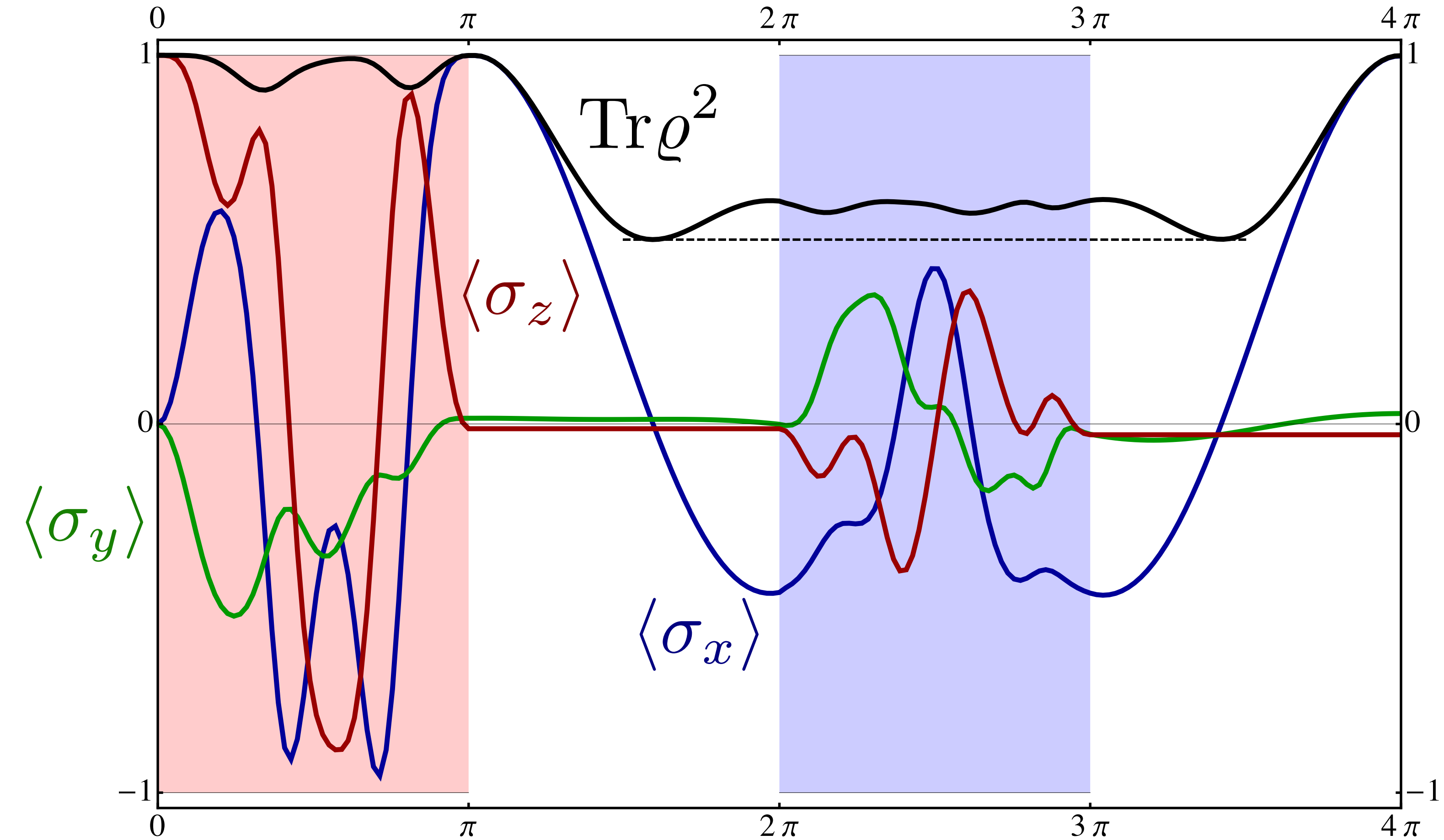
ensemble : $\omega_{max} \geq \omega \geq -\omega_{max}$

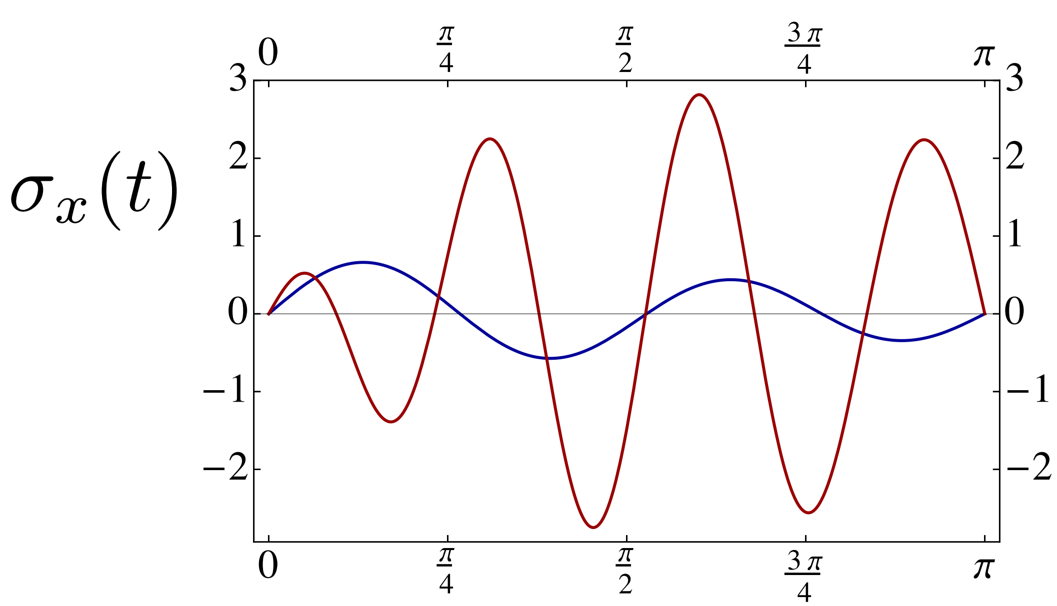
targeted operation : $|\Psi(0)\rangle = |0\rangle \;\rightarrow\; |\Psi(\pi)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

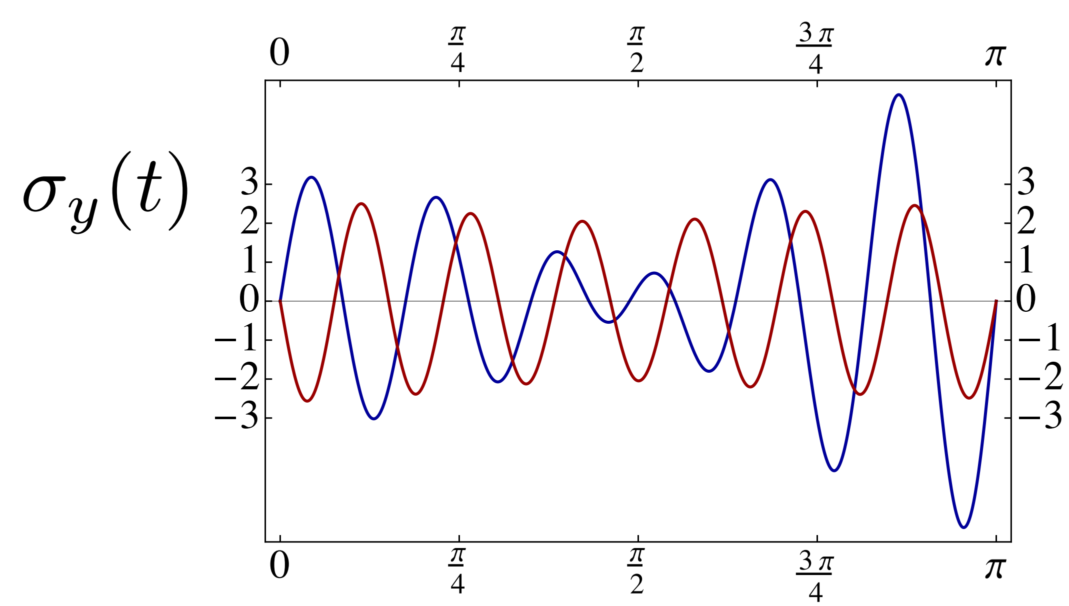
Approaching the target



Flip and rephasing







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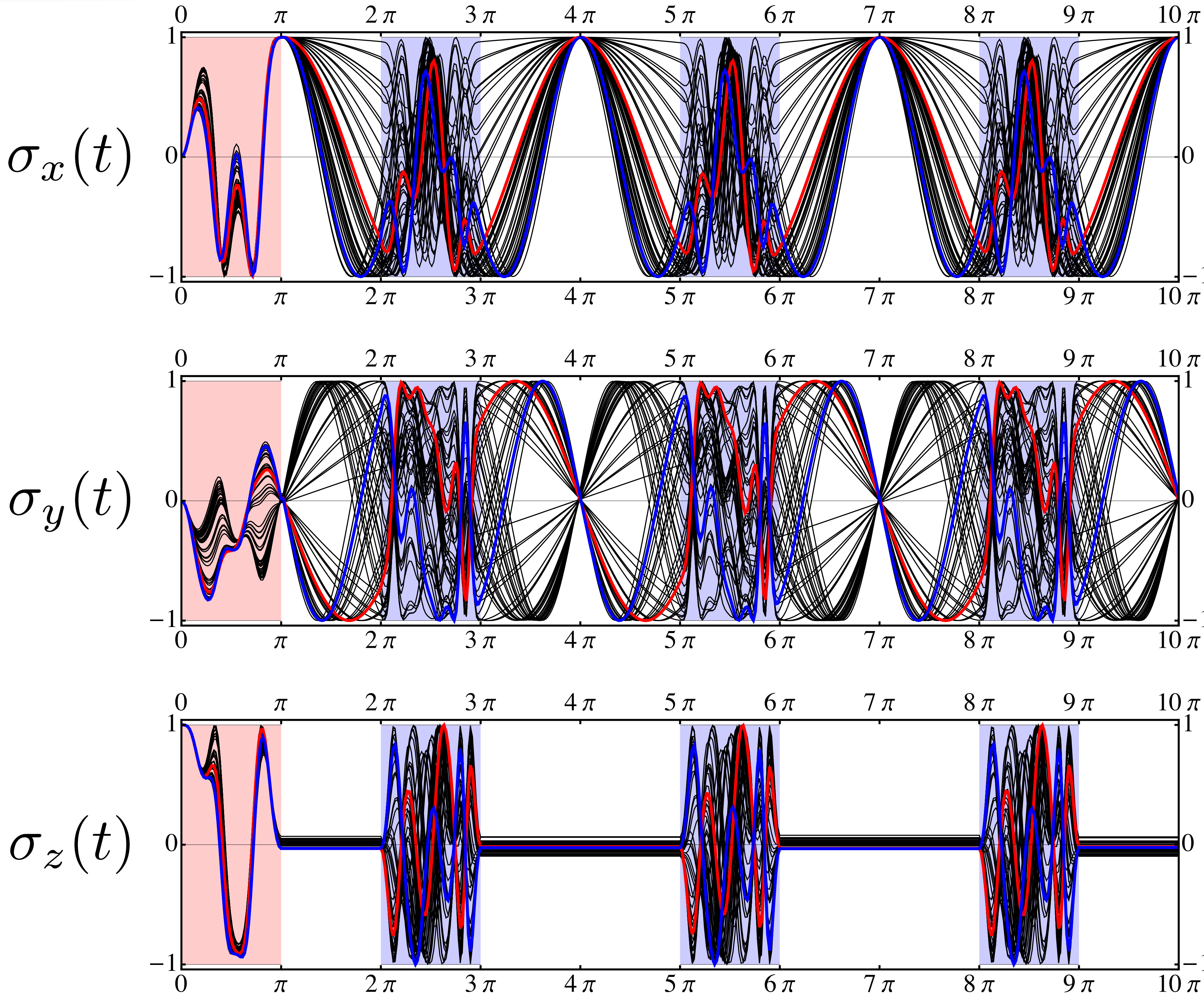

Conclusions

don’t exert unnecessarily stringent control

identify ideally suited states

significantly enhanced robustness through time-nonlocal control

smooth pulses through control in frequency space