Entangled photon pair generation using guided wave SPDC

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Outline

• Modes in waveguides
• Second order nonlinear optical effects
• Spontaneous parametric down conversion
  – Generation of photon pairs
• Domain engineering for generation of
  – Polarization entangled photon pairs
  – Modal and path entangled photon pairs
• Bragg reflection waveguides (BRW) for
  – Increasing pump acceptance bandwidth and
  – Narrow signal bandwidth
• Conclusions
Optical waveguides

• Optical waveguides:
  • High index region surrounded by lower index regions
  • Wave guidance by total internal reflection

• Materials
  • Glass, Lithium niobate, GaAs, Silicon etc.

Planar waveguide

Channel waveguide

Optical fiber
Modes of propagation

Mode:
• Certain electric field patterns that propagate unchanged
• Solution to Maxwell’s equations satisfying appropriate boundary conditions

• Characteristics:
  • Definite transverse electric field pattern
  • Definite phase and group velocity
  • Discrete set of guided modes

Similar to
• Modes of oscillation of a string fixed at two ends
• Eigenstates of a potential well in quantum physics
Modal field

\[ E(x, y, z, t) = \frac{1}{2} \left[ A(z) \psi(x, y) e^{i(\omega t - \beta z)} + cc \right] \]

- \( A \): Amplitude of the mode
- \( \psi(x,y) \): Transverse modal field distribution
- \( \beta \): Propagation constant of the mode

- Discrete number of guided modes
- Single mode waveguide: only a single guided mode is possible
- Each guided mode characterized by a different modal field pattern and propagation constant
Modes of propagation in a waveguide
Modes of coupled waveguides

Modes have different velocities

Symmetric mode

Anti-symmetric mode

Modes have different oscillation frequencies
Directional coupler

Used to split or combine optical signals

Input $P_1$  

Coupling region

Output $P_3$  

Output $P_4$
Power coupling in a directional coupler

Y-branch with single mode guides

Input

3 dB splitter

Inputs
In phase
π out of phase

No output
Nonlinear polarization

For light waves with high intensity, the electric fields are high and then polarization is a nonlinear function of electric field

\[ \rho = \varepsilon_0 \chi E + 2 \varepsilon_0 d E^2 + \varepsilon_0 \chi^{(3)} E^3 \ldots \]

Not a sine wave
Second order nonlinearity

\[ \varphi_{NL} = 2\varepsilon_0 d E^2 \]

If we consider a plane electromagnetic wave incident in the medium

\[ E = A \cos(\omega t - kz) \]

Then

\[ \varphi_{NL} = 2\varepsilon_0 d A^2 \cos^2(\omega t - kz) \]

\[ = \varepsilon_0 d A^2 + \varepsilon_0 d A^2 \cos[2(\omega t - kz)] \]

- The \( 2\omega \) term responsible for the generation of second harmonic electromagnetic field
Sum and difference frequency generation

Input electric field:

\[ E = A_1 \cos(\omega_1 t - k_1 z) + A_2 \cos(\omega_2 t - k_2 z) \]

Nonlinear polarization gets generated at the following frequencies:

- SHG: Second harmonic generation
- SFG: Sum frequency generation
- DFG: Difference frequency generation
Phase matching

In general the velocity of nonlinear polarization is not equal to the velocity of the electromagnetic wave at the same frequency that it is trying to generate.

For efficient generation, these two velocities have to be equal

**PHASE MATCHING CONDITION**

\[ k_2 = 2k_1 \quad \text{or} \quad n(2\omega) = n(\omega) \]

- Due to dispersion this is normally not possible
- Use birefringence of the crystal
- Use periodic interaction to compensate for mismatch
Photon picture

SHG can be considered as a fusion of two photons at frequency $\omega$ to form one photon at frequency $2\omega$.

For efficient interaction, we need to conserve momentum.

Vector diagram

Phase matching condition

$k_2 = 2k_1$
Sub harmonic generation

Second harmonic generation

\[ \omega \rightarrow 2\omega \]

Is the following possible?

Blue \rightarrow \omega

Red \rightarrow 2\omega

Nonlinear crystal

CLASSICALLY PROHIBITED PROCESS
Parametric fluorescence

- Incident photon at one frequency spontaneously generates a pair of photons at lower frequencies

Ref: Martin et al., Opt Exp 17 (2009) 1033
Spontaneous parametric down conversion (SPDC)

- One photon at $\omega_p$ splits *spontaneously* into one photon at $\omega_s$ and another at $\omega_i$.
- Explanation for the process is quantum mechanical.
- For efficient down conversion:
  - Energy conservation: $\omega_p = \omega_s + \omega_i$
  - Momentum conservation: $k_p = k_s + k_i$
SPDC using Quasi Phase Matching

• Periodic variation in the nonlinear coefficient
  • Spatial frequency $K$ chosen to compensate for phase mismatch
    $$k_p = k_s + k_i + K$$

• Most widely used technique for SPDC
  • Can be applied to any pair of signal and idler $\lambda$s
  • Use highest nonlinear coefficient tensor element
Bulk vs. waveguide configuration

- Lightwaves get guided through the device
- Photons generated in well defined spatial modes
  - Ease of collection and further processing
  - Due to restricted modal structure, much higher probability of emission into distinct modes
  - Effective decoupling of spectral and spatial degrees of freedom
- Novel configurations and integrated geometry

\[
\frac{A(\text{waveguide})}{A(\text{bulk})} \approx 10^{-4}
\]

- \( t \approx 2 \, \mu m \)
Entangled photons via SPDC

- Entanglement in different degrees of freedom
  - Polarization
  - Mode or path
- Generation SPDC using $\chi^{(2)}$ in waveguides
- Many existing schemes for polarization entanglement
  - need extra experimental steps to entangle signal and idler photon pairs
- Direct generation of non degenerate entangled signal and idler photons interesting
Type II quasi phase matching in LiNbO\textsubscript{3} 

- Using different QPM periods one can downconvert an o-polarized pump to either
  - An o-polarized signal and an e-polarized idler or
  - An e-polarized signal and an o-polarized idler

- In both cases the polarization states of output are well defined

\[
\frac{1}{\Lambda_1} = \left( \frac{n_{po} - n_{so} - n_{ie}}{\lambda_p - \lambda_s - \lambda_i} \right)
\]

\[
\frac{1}{\Lambda_2} = \left( \frac{n_{po} - n_{se} - n_{io}}{\lambda_p - \lambda_s - \lambda_i} \right)
\]
Doubly periodic poling

- It is possible to satisfy both QPM conditions simultaneously
- Variation of nonlinear coefficient $d$ along propagation direction is doubly periodic
- The grating contains two spatial frequencies required to phase match both the processes simultaneously

Ref: Thyagarajan, Lugani, Ghosh, Martin, Ostrowsky, Alibart, Tanzilli, PHYSICAL REVIEW A 80, 052321 (2009)
Doubly periodic poling

• x-variation of nonlinear coefficient

\[ \bar{d} = d_{24} f_1(x) f_2(x) \]

By choosing appropriate values of \( K_0 \) and \( K_p \), it is possible to achieve phase matching for both o->o+e and o->e+o processes
Polarization state of output photons

• Classically we would say that the output is either of the following:
  – Signal is H polarized and idler is V polarized or
  – Signal is V polarized and idler is H polarized

• According to quantum mechanics
  – *Polarization state of individual signal and idler photons are undefined but are orthogonal to each other*
  – *The output signal and idler photons are entangled in polarization*
Fields at pump, signal and idler

- Pump is taken to be a classical wave and signal and idler are treated quantum mechanical.

\[
Pump(o) \quad \vec{E}_{po} = \frac{1}{2} e_{po}(r) E_{po} \left( e^{(ik_p x - \omega_p t)} + e^{- (ik_p x - \omega_p t)} \right) \hat{y}
\]

\[
Signal(o) \quad \hat{E}_{so} = i \int d\omega_s e_{so}(\vec{r}) \sqrt{\frac{\hbar \omega_s}{2\varepsilon_{so} L_{int}}} \left( \hat{a}_{so} e^{ik_{so} x} - \hat{a}_{so}^+ e^{-ik_{so} x} \right) \hat{y}
\]

\[
Signal(e) \quad \hat{E}_{se} = i \int d\omega_s e_{se}(\vec{r}) \sqrt{\frac{\hbar \omega_s}{2\varepsilon_{se} L_{int}}} \left( \hat{a}_{se} e^{ik_{se} x} - \hat{a}_{se}^+ e^{-ik_{se} x} \right) \hat{z}
\]

\[
Idler(o) \quad \hat{E}_{io} = i \int d\omega_i e_{io}(\vec{r}) \sqrt{\frac{\hbar \omega_i}{2\varepsilon_{io} L_{int}}} \left( \hat{a}_{io} e^{ik_{io} x} - \hat{a}_{io}^+ e^{-ik_{io} x} \right) \hat{y}
\]

\[
Idler(e) \quad \hat{E}_{ie} = i \int d\omega_i e_{ie}(\vec{r}) \sqrt{\frac{\hbar \omega_i}{2\varepsilon_{ie} L_{int}}} \left( \hat{a}_{ie} e^{ik_{ie} x} - \hat{a}_{ie}^+ e^{-ik_{ie} x} \right) \hat{z}
\]
Interaction Hamiltonian

\[
\hat{H}_{\text{int}} = \int d\omega_s \left[ C_{oe}^{(1)} \left( \hat{a}_{so} \hat{a}_{ie} e^{-i\omega_p t} + \hat{a}_{so} \hat{a}_{ie} e^{i\omega_p t} \right) + C_{eo}^{(1)} \left( \hat{a}_{se} \hat{a}_{io} e^{-i\omega_p t} + \hat{a}_{se} \hat{a}_{io} e^{i\omega_p t} \right) \right]
\]

\[
C_{oe}^{(1)} = \left( \frac{4d_{24} E_{p0} \hbar \sqrt{\omega_s \omega_i} I_{oe}}{\pi^2 n_{so} n_{ie}} \right) \exp \left( -\frac{i \Delta k_{oe} L_{\text{int}}}{2} \right) \sin c \left( \frac{\Delta k_{oe} L_{\text{int}}}{2} \right)
\]

\[
C_{eo}^{(1)} = \left( \frac{4d_{24} E_{p0} \hbar \sqrt{\omega_s \omega_i} I_{eo}}{\pi^2 n_{se} n_{io}} \right) \exp \left( -\frac{i \Delta k_{eo} L_{\text{int}}}{2} \right) \sin c \left( \frac{\Delta k_{eo} L_{\text{int}}}{2} \right)
\]

\[
\Delta k_{oe} = \frac{2\pi}{\Lambda_1} - 2\pi \left( \frac{n_{po}}{\lambda_p} - \frac{n_{so}}{\lambda_s} - \frac{n_{ie}}{\lambda_i} \right), \quad \Delta k_{eo} = \frac{2\pi}{\Lambda_1} - 2\pi \left( \frac{n_{po}}{\lambda_p} - \frac{n_{se}}{\lambda_s} - \frac{n_{io}}{\lambda_i} \right)
\]
Output state

Entangled in polarization

$$|\psi\rangle = i \int d\omega_s \left( C_{oe} |s_o, i_e\rangle + C_{eo} |s_e, i_o\rangle \right)$$

- The relative magnitudes of the C coefficients would determine if the output is maximally entangled or not.
- We have defined the degree of entanglement as

$$\gamma = \frac{\min(C_{oe}, C_{eo})}{\max(C_{oe}, C_{eo})} \quad 0 < \gamma < 1$$
Ti:LiNbO$_3$ waveguide

- **Refractive index profile:**
  \[
  n^2(y, z) = n_b^2 + 2n_b \Delta n e^{-y^2/w^2} e^{-z^2/h^2}; \quad z < 0
  \]
  \[
  = n_c^2; \quad z > 0
  \]

- **Use standard variational analysis to calculate**
  - Effective indices at pump, signal and idler wavelengths
  - Mode field profiles of the interacting modes
Transverse mode distributions

Good modal overlap for both the processes
Spectrum of the two processes

\[ \text{sinc}^2(\Delta kL/2) \]

Signal wavelength (nm)
Entanglement vs. width

![Graph showing the relationship between entanglement measure and width with depth on the y-axis.](image)
Experimental verification

Interlaced domain structures

Ref: Thomas, Herrmann and Sohler, ECIO, Cambridge (2010), ThC4
Modal and path entangled photons

- Modal and path entangled photons can find applications in quantum information processing, lithography etc.

- Waveguide device supporting two spatial modes
  - Incident pump generates signal and idler photons entangled in modal degree of freedom
  - Modal entanglement can be converted to path entanglement using waveguide device
Waveguide device geometry

Mode Entangled photons

- Generating non-degenerate, co-polarised mode entangled photons.
- Satisfying QPM conditions of two different SPDC processes, simultaneously, leading to mode entangled pairs of photons.
- All waves have e-polarization and use $d_{33}$ coefficient

a. pump (0) -> signal(0) + idler(0)

$$K_1 = \frac{2\pi}{\Lambda_1} = 2\pi \left( \frac{n_{p0}}{\lambda_p} - \frac{n_{s0}}{\lambda_s} - \frac{n_{i0}}{\lambda_i} \right)$$

b. pump(0) -> signal(1) + idler(1)

$$K_2 = \frac{2\pi}{\Lambda_2} = 2\pi \left( \frac{n_{p0}}{\lambda_p} - \frac{n_{s1}}{\lambda_s} - \frac{n_{i1}}{\lambda_i} \right)$$
• Incident pump photon down converts
  – Either into symmetric signal and symmetric idler modes
  – Or into antisymmetric signal and antisymmetric idler modes
• Both processes almost equally efficient
Quantum mechanical analysis

- Pump is assumed to be classical and signal and idler treated quantum mechanically.
- The interaction Hamiltonian in the interaction picture under energy conservation and RWA is

\[
\hat{H}_{\text{int}} = \int d\omega_s \left[ C_0 (\hat{a}^+_s \hat{a}^+_i + \hat{a}_s \hat{a}_i) + C_1 (\hat{a}^+_s \hat{a}^+_i + \hat{a}_s \hat{a}_i) \right]
\]

\[
C_0 = \left( \frac{4d_{33}E_p t}{\pi^2 n_s n_{i0}} \right) e^{-i\Delta k_0 L/2} \text{sinc} \left( \frac{\Delta k_0 L}{2} \right)
\]

\[
C_1 = \left( \frac{4d_{33}E_p t}{\pi^2 n_{s1} n_{i1}} \right) e^{-i\Delta k_1 L/2} \text{sinc} \left( \frac{\Delta k_1 L}{2} \right)
\]

\[
I_0 = \iint e_p(y, z) e_{s0}(y, z) e_{i0}(y, z) dydz
\]

\[
I_1 = \iint e_p(y, z) e_{s1}(y, z) e_{i1}(y, z) dydz
\]
Output state

• Output state at the end of Region III:

\[ |\psi\rangle = i \int d\omega_s (C_0 |s_0, i_0\rangle + C_1 |s_1, i_1\rangle) \]

• Output represents a mode entangled state
  • 0: symmetric mode
  • 1: antisymmetric mode
Results

• We have carried out simulations for the planar domain engineered LN waveguide.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Δne</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>.0033</td>
</tr>
<tr>
<td>1452</td>
<td>.0026</td>
</tr>
<tr>
<td>1550</td>
<td>.0025</td>
</tr>
</tbody>
</table>

• The propagation constants vary with core separation
Modal patterns

- Titanium indiffused waveguides in lithium niobate
- The modes have very good modal overlap
- This leads to maximal modal entanglement

\[
pump(0) \rightarrow signal(0) + idler(0) \quad \text{and} \quad pump(0) \rightarrow signal(1) + idler(1)
\]
Bandwidth of the two processes

Bandwidths for both the processes are almost identical (16 nm)
• Fundamental symmetric modes exit from the upper waveguide having higher propagation constant
• First order antisymmetric modes exit from the lower waveguide with smaller propagation constant
• Output state is path entangled

\[ |\Psi\rangle = i \int d\omega_s \left( C_0 |s_u, i_u\rangle + C_1 |s_l, i_l\rangle \right) \]
SPDC with increased pump bandwidth

- SPDC source with
  - Signal photon at the telecom wavelength of 1550 nm

- Essential requirements:
  - High efficiency generation
  - Increased pump acceptance bandwidth so that femtosecond pump could be used
  - Narrow signal bandwidth so that signal photons can be used as flying qubits
Idea

- Efficiency of the down conversion varies as
  \[ \text{sinc}^2\left(\frac{\Delta \beta L}{2}\right) \]
  \( L \): Length of interaction
  \( \Delta \beta = \beta_p - \beta_s - \beta_i - K = 0 \)

- Now
  \[ \frac{d\beta_s}{d\lambda_p} = \frac{d}{d\lambda_p}(\beta_p - \beta_i) \]

- If waveguide design is such that
  \((\beta_p - \beta_i)\) exhibits a minimum at a specific \( \lambda_p \), then
  - As \( \lambda_p \) changes, \( \lambda_s \) will remain fixed
  - This will lead to
    - large pump acceptance bandwidth and
    - narrow signal bandwidth
Bragg reflection waveguides

- Variation of the propagation constants at different wavelengths depends on
  - Material dispersion
  - Waveguide dispersion
- We need to counter the strong material dispersion at pump wavelength by a strong waveguide dispersion at the idler wavelength
- Bragg reflection waveguides (BRW) can provide such a possibility

Ref: Thyagarajan, Das, Alibart, de Micheli, Ostrowsky, Tanzilli, Optics Express, 16 (2008) 3577.
Total internal reflection and Bragg reflection

- Total internal reflection
  - Reflection total
  - $n_c > n_{cl}$

- Bragg reflection
  - Reflection partial
  - No restrictions on $n_c$
TIR and Bragg modes

TIR modes: \( n_1 < n_{\text{eff}} < n_c \)
Bragg modes: \( n_{\text{eff}} < n_2 \)

- Dispersion of TIR and Bragg modes can be very different
- Large design space for dispersion of Bragg modes
Bragg modes exhibit very strong dispersion.

\[ n_{c}: \text{GaN} \]
\[ n_{s}: \text{AlGaN} \]

\[ n_{c}: \text{GaN} \]
\[ n_{1}: \text{Al}_{x}\text{Ga}_{1-x}\text{N} \]
\[ n_{2}: \text{Al}_{y}\text{Ga}_{1-y}\text{N} \]

\[ n_{\text{eff}} \]

Wavelength (\(\mu\text{m})\)
SPDC with BRW

- Pump (~0.8 μm): TIR mode
- Signal (~1.55 μm): TIR mode
- Idler (~1.653 μm): BRW mode
• Appearance of a minimum implies that as $\lambda_p$ changes
  – Phase matching condition will continue to be satisfied
Wavelength variation of signal and idler

- Modal dispersion designed so that as $\lambda_p$ changes, $\lambda_s$ remains the same
Bandwidth

- Pump acceptance bandwidth increased 30 times compared to conventional geometry
- Very small signal bandwidth; useful in quantum communication systems
Separable states

• The idea proposed also leads to separable signal-idler pair states
  – Useful for various applications like heralding identical single photon states etc.

\[
\frac{d}{d\lambda_p}(\beta_p - \beta_i) = 0
\]

• implies

\[
\frac{d\beta_p}{d\omega_p} = \frac{d\beta_i}{d\omega_i}
\]

• This condition leads to the generation of separable state
Conclusions

• Using waveguide geometries
  – Provides with additional degrees of freedom
  – Output photons in well defined discrete spatial modes
  – Higher efficiency

• Domain engineering
  – Direct generation of polarization, mode or path entangled photon pair

• BRWs have very interesting dispersion behaviour
  – Designs to achieve large pump acceptance bandwidth and small signal bandwidth or separable states

• Integrated quantum optical circuits should play a very important role in the future
Acknowledgement

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