Entangled photon pair generation using guided wave SPDC

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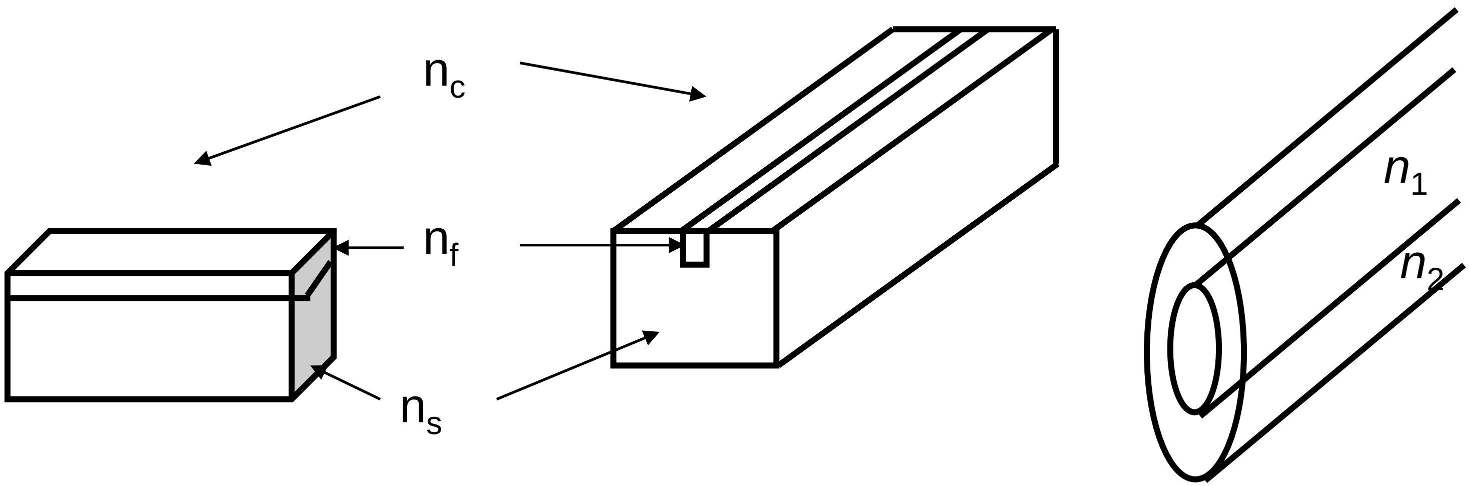
Outline

- Modes in waveguides
- Second order nonlinear optical effects
- Spontaneous parametric down conversion
  - Generation of photon pairs
- Domain engineering for generation of
  - Polarization entangled photon pairs
  - Modal and path entangled photon pairs
- Bragg reflection waveguides (BRW) for
  - Increasing pump acceptance bandwidth and
  - Narrow signal bandwidth
- Conclusions

Optical waveguides

- Optical waveguides:
  - High index region surrounded by lower index regions
  - Wave guidance by total internal reflection
- Materials
  - Glass, Lithium niobate, GaAs, Silicon etc.



Planar waveguide

Channel waveguide

Optical fiber

Modes of propagation

Mode:

- Certain electric field patterns that propagate unchanged
- Solution to Maxwell's equations satisfying appropriate boundary conditions
- Characteristics:
  - Definite transverse electric field pattern
  - Definite phase and group velocity
  - Discrete set of guided modes

Similar to

- Modes of oscillation of a string fixed at two ends
- Eigenstates of a potential well in quantum physics

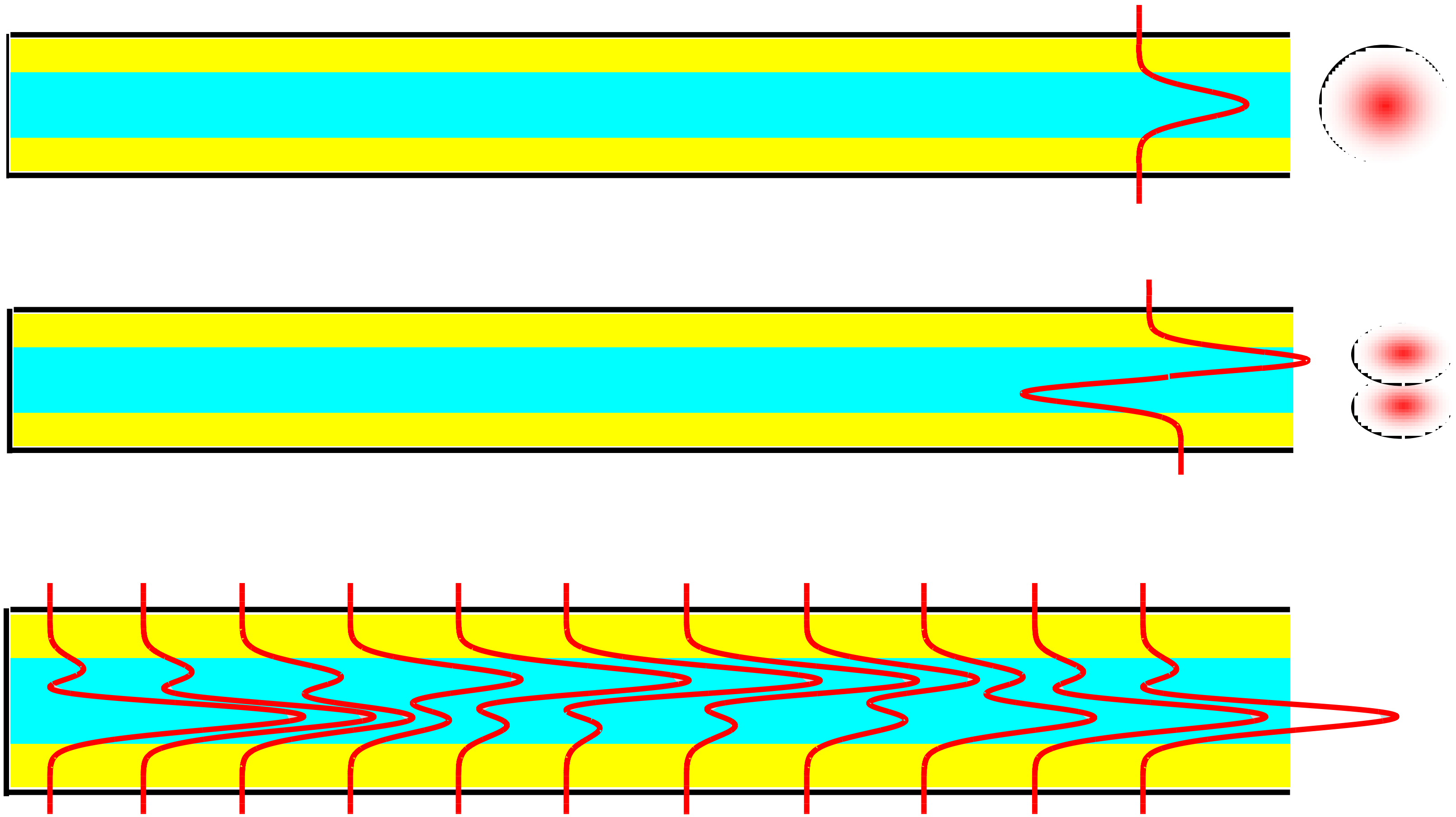
Modal field

$$E(x,y,z,t)=\frac{1}{2}\left[A(z)\,\psi(x,y)\,e^{i\left(\omega t-\beta z\right)}+cc\right]$$

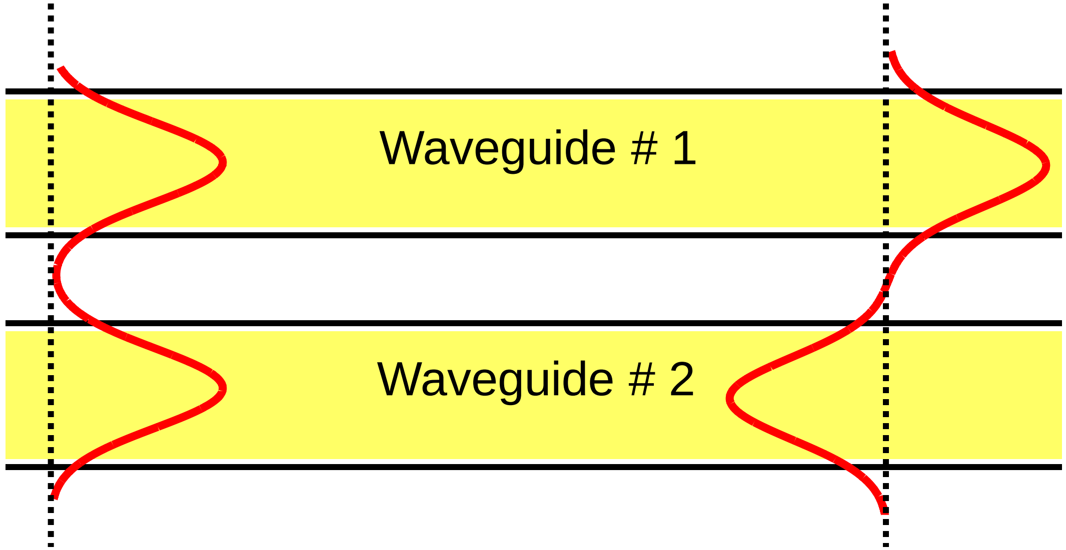
A:	Amplitude of the mode
$\psi(x,y)$:	Transverse modal field distribution
β:	Propagation constant of the mode

- Discrete number of guided modes
- Single mode waveguide: only a single guided mode is possible
- Each guided mode characterized by a different modal field pattern and propagation constant

Modes of propagation in a waveguide



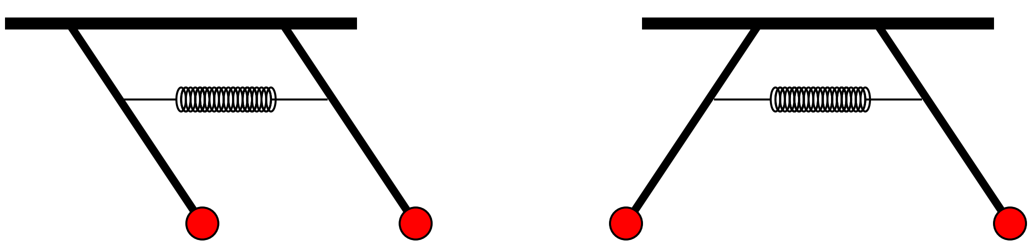
Modes of coupled waveguides



Modes have different velocities

Symmetric mode

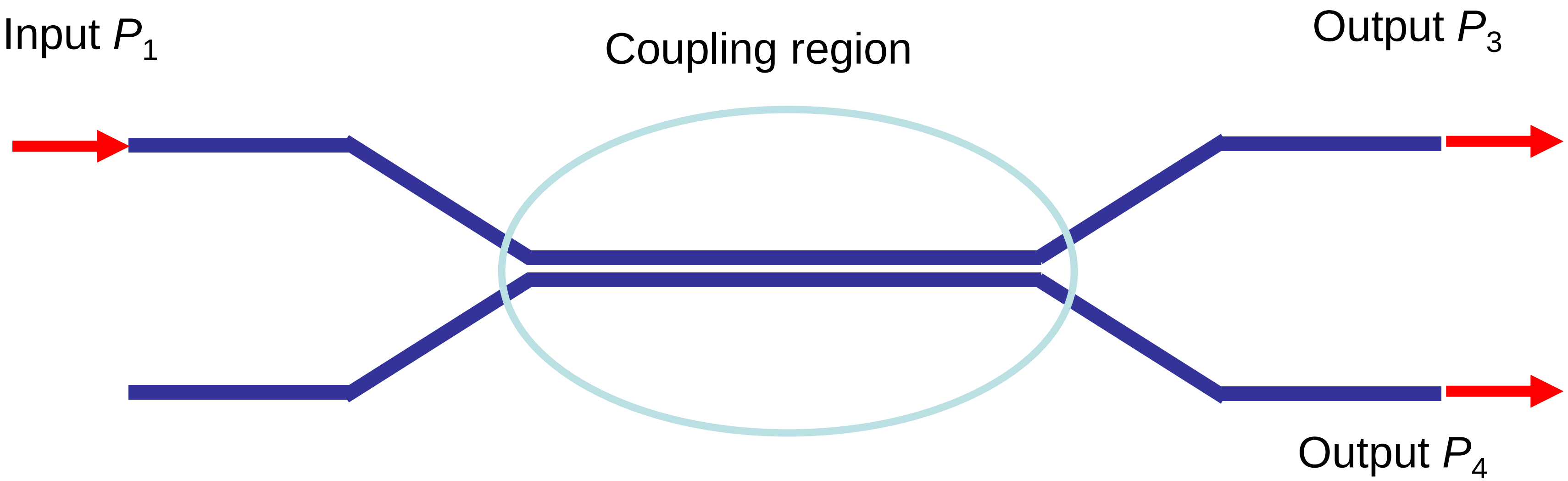
Anti-symmetric mode



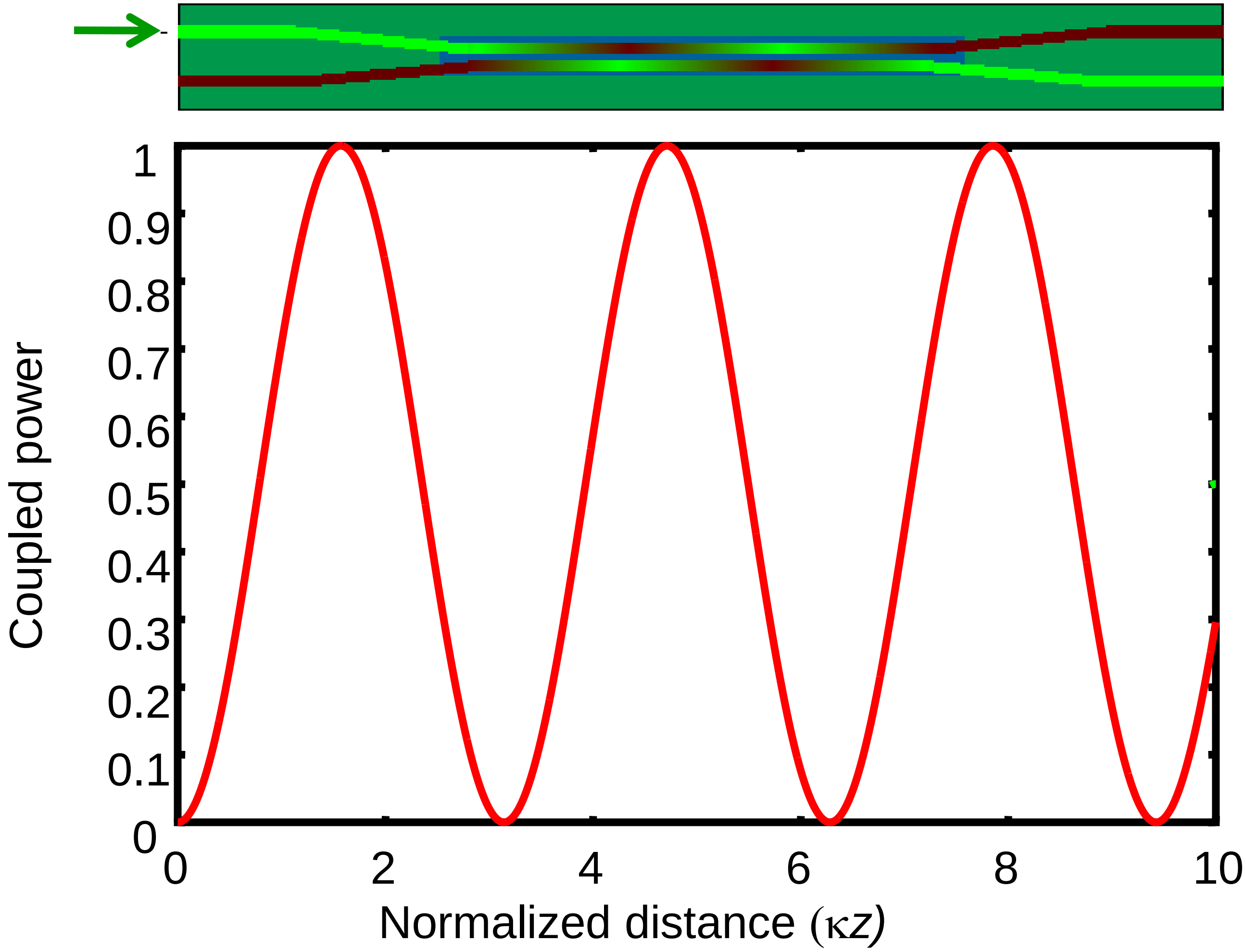
Modes have different oscillation frequencies

Directional coupler

Used to split or combine optical signals

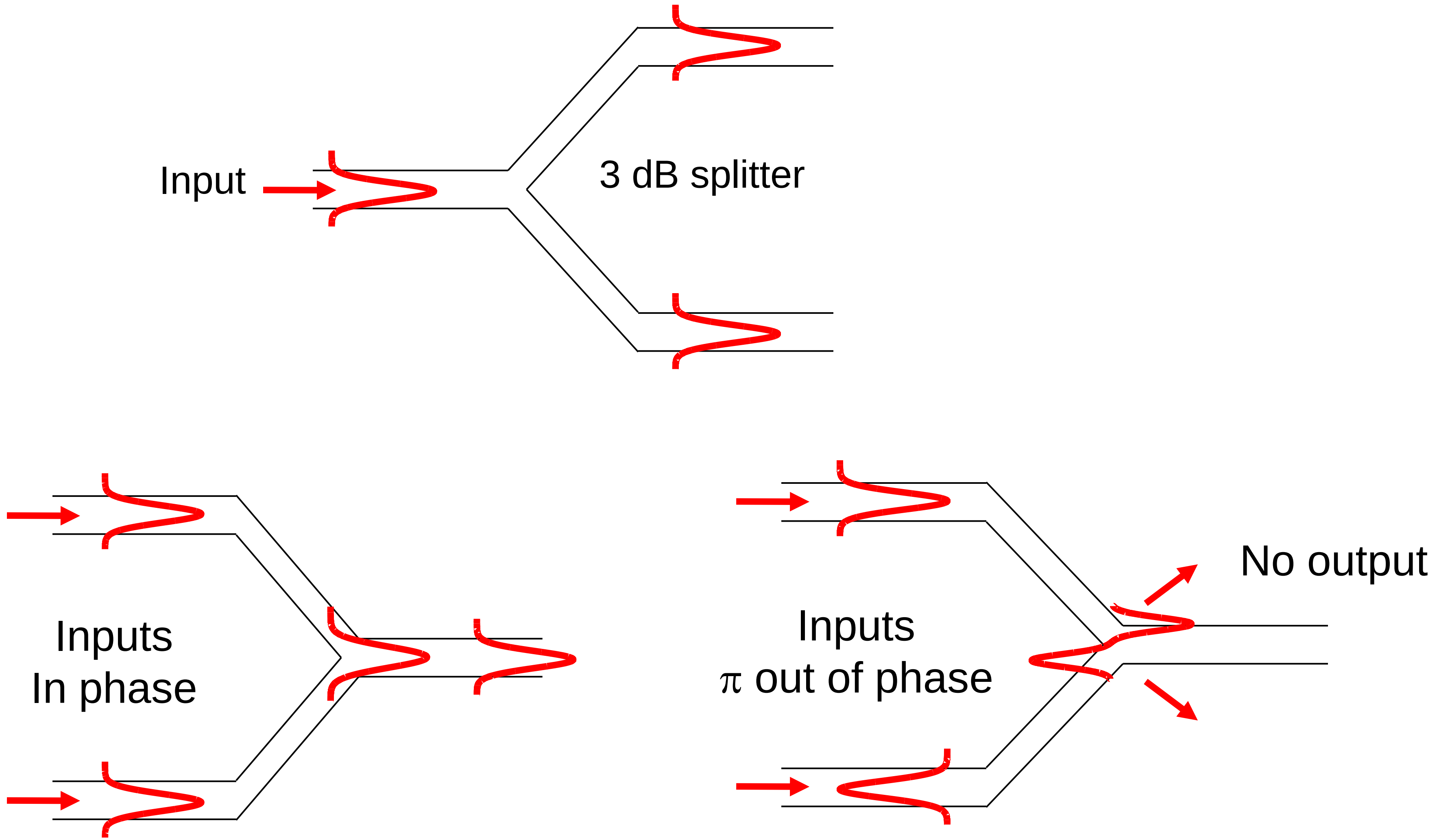


Power coupling in a directional coupler



Ref: *Introduction to fiber optics*, A Ghatak and K Thyagarajan, Cambridge Univ Press, 1998

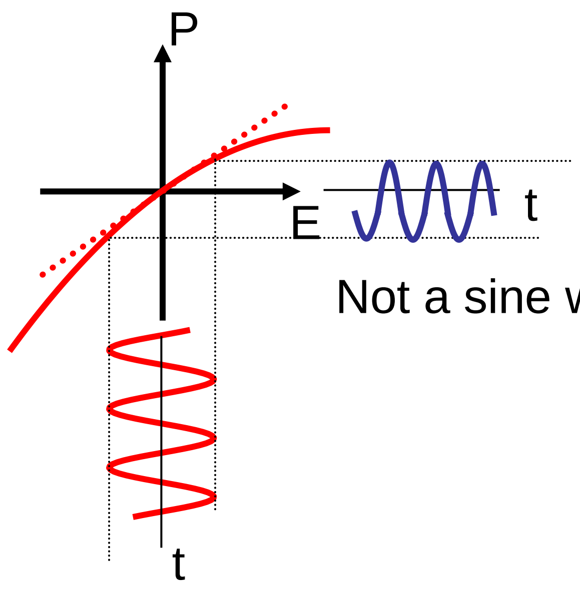
Y-branch with single mode guides



Nonlinear polarization

For light waves with high intensity, the electric fields are high and then polarization is a nonlinear function of electric field

$$\wp = \varepsilon_0 \chi \mathcal{E} + 2 \varepsilon_0 d \mathcal{E}^2 + \varepsilon_0 \chi^{(3)} \mathcal{E}^3 \ldots$$



Not a sine wave

Second order nonlinearity

$$\wp_{NL} = 2\varepsilon_0 d\,\mathcal{E}^2$$

If we consider a plane em wave incident in the medium

$$\mathcal{E} = A\cos(\omega t - kz)$$

Then

$$\wp_{NL} = 2\varepsilon_0 d\,A^2\cos^2(\omega t - kz)$$

$$= \varepsilon_0 d\,A^2 + \varepsilon_0 d\,A^2\cos[2(\omega t - kz)]$$

- The 2ω term responsible for the generation of second harmonic electromagnetic field

Sum and difference frequency generation

Input electric field:

$$\mathcal{E} = A_1 \cos(\omega_1 t - k_1 z) + A_2 \cos(\omega_2 t - k_2 z)$$

Nonlinear polarization gets generated at the following frequencies:

$$0, \underbrace{2\omega_1, 2\omega_2}_{\text{SHG}}, \underbrace{\omega_1 + \omega_2}_{\text{SFG}}, \underbrace{\omega_1 - \omega_2}_{\text{DFG}}$$

SHG: Second harmonic generation
SFG: Sum frequency generation
DFG: Difference frequency generation

Phase matching

In general the velocity of nonlinear polarization is not equal to the velocity of the electromagnetic wave at the same frequency that it is trying to generate

For efficient generation, these two velocities have to be equal

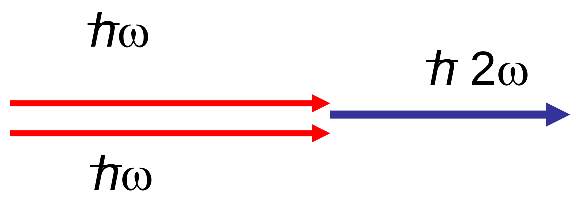
PHASE MATCHING CONDITION

$$k_2 = 2k_1 \quad \text{or} \quad n(2\omega) = n(\omega)$$

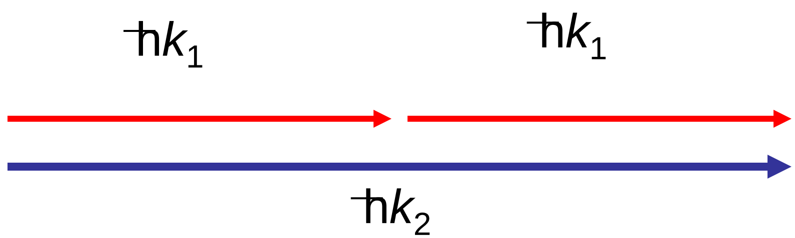
- Due to dispersion this is normally not possible
- Use birefringence of the crystal
- Use periodic interaction to compensate for mismatch

Photon picture

SHG can be considered as a fusion of two photons at frequency ω to form one photon at frequency 2 ω



For efficient interaction, we need to conserve momentum



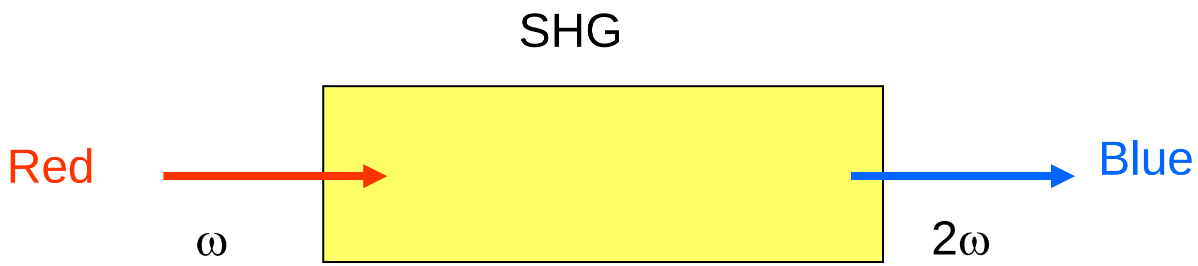
Vector diagram

$$k_2 = 2\, k_1$$

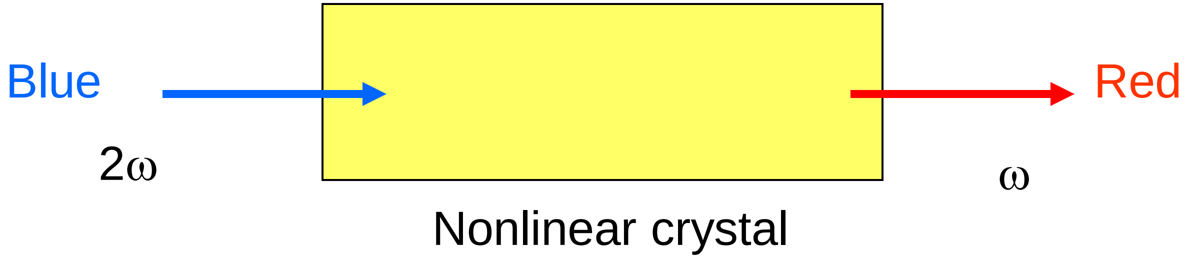
Phase matching condition

Sub harmonic generation

Second harmonic generation



Is the following possible?

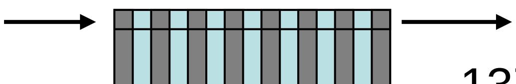


CLASSICALLY PROHIBITED PROCESS

Parametric fluorescence

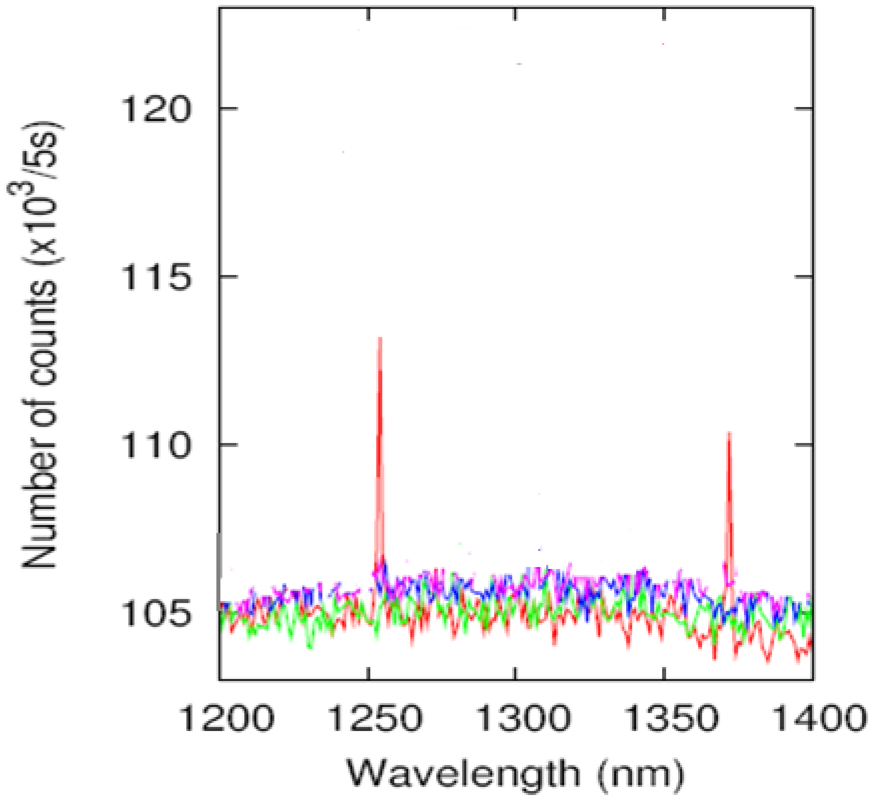
- Incident photon at one frequency spontaneously generates a pair of photons at lower frequencies

655 nm



1255 nm

1370 nm



Ref: Martin et al., Opt Exp 17 (2009) 1033

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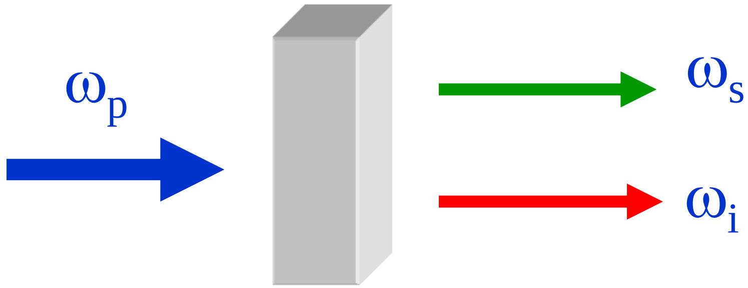
Spontaneous parametric down conversion (SPDC)

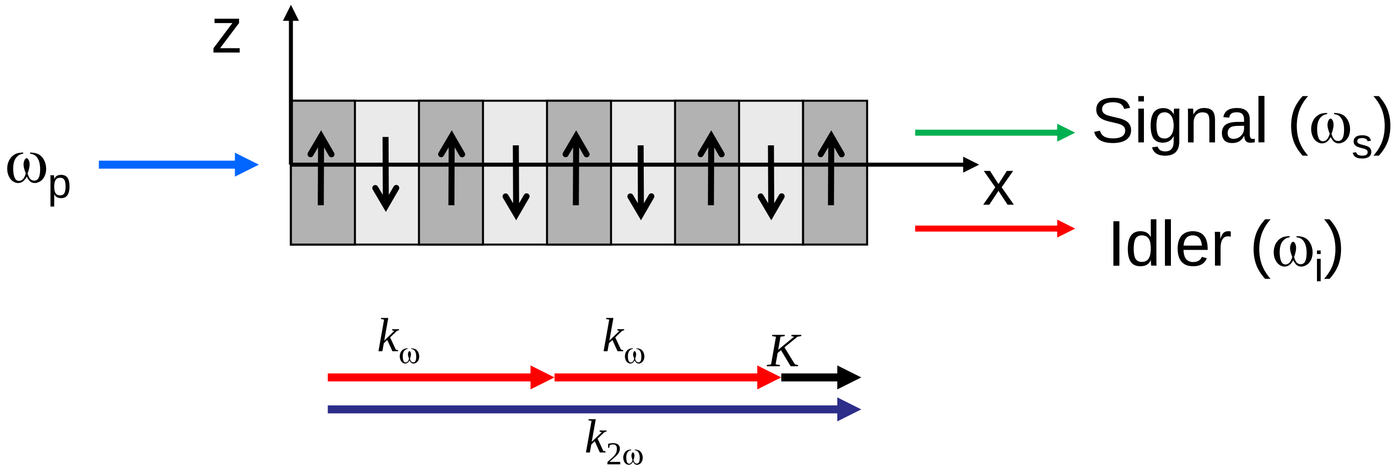


- One photon at ω_p splits *spontaneously* into one photon at ω_s and another at ω_i
- Explanation for the process is quantum mechanical
- For efficient down conversion
  - Energy conservation
  - Momentum conservation

$$\omega_p = \omega_s + \omega_i$$

$$k_p = k_s + k_i$$

SPDC using Quasi Phase Matching

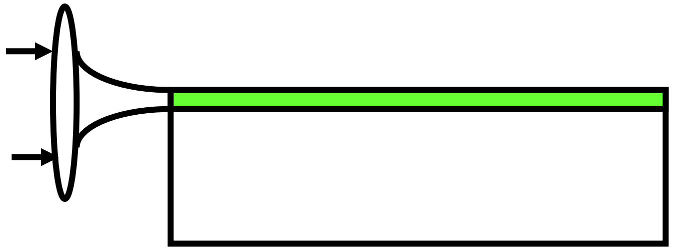


- Periodic variation in the nonlinear coefficient
  - Spatial frequency K chosen to compensate for phase mismatch

$$k_p = k_s + k_i + K$$

- Most widely used technique for SPDC
  - Can be applied to any pair of signal and idler λs
  - Use highest nonlinear coefficient tensor element

Bulk vs. waveguide configuration



Waveguide

t ~ 2 μm

$$\frac{A(waveguide)}{A(bulk)} \sim 10^{-4}$$

- Lightwaves get guided through the device
- Photons generated in well defined spatial modes
  - Ease of collection and further processing
  - Due to restricted modal structure, much higher probability of emission into distinct modes
  - Effective decoupling of spectral and spatial degrees of freedom
- Novel configurations and integrated geometry

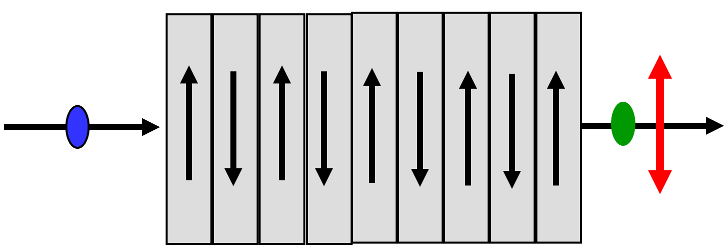
Entangled photons via SPDC

- Entanglement in different degrees of freedom
  - Polarization
  - Mode or path
- Generation SPDC using $\chi^{(2)}$ in waveguides
- Many existing schemes for polarization entanglement
  - need extra experimental steps to entangle signal and idler photon pairs
- Direct generation of non degenerate entangled signal and idler photons interesting

Type II quasi phase matching in $LiNbO_3$

- Using different QPM periods one can downconvert an o-polarized pump to either
  - An o-polarized signal and an e-polarized idler or
  - An e-polarized signal and an o-polarized idler
- In both cases the polarization states of output are well defined

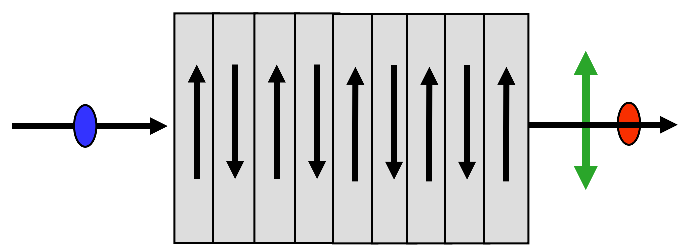
QPM period Λ_1



o-signal
e-idler

$$\frac{1}{\Lambda_1} = \left(\frac{n_{po}}{\lambda_p} - \frac{n_{so}}{\lambda_s} - \frac{n_{ie}}{\lambda_i} \right)$$

QPM period Λ_2

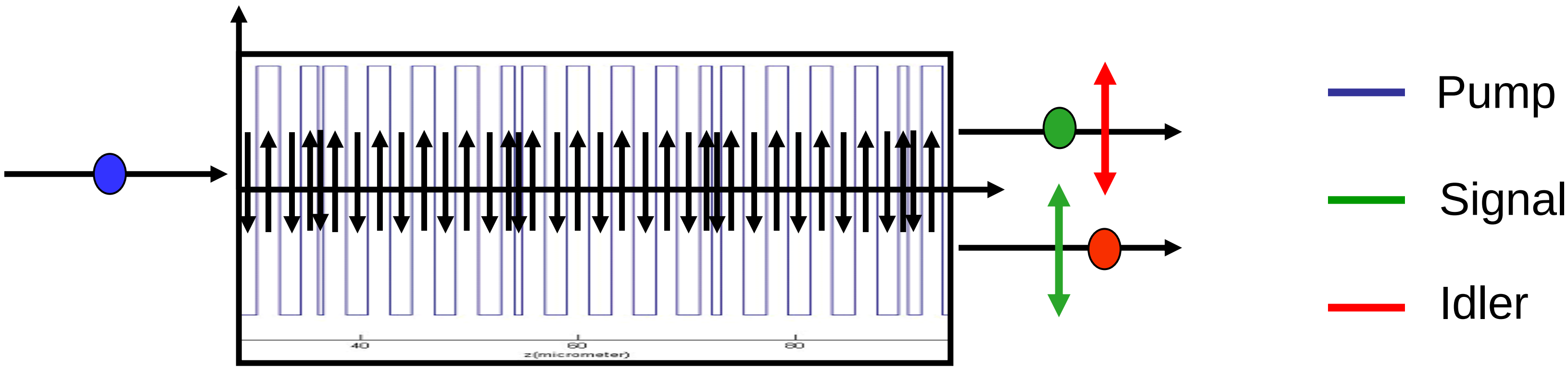


e-signal
o-idler

$$\frac{1}{\Lambda_2} = \left(\frac{n_{po}}{\lambda_p} - \frac{n_{se}}{\lambda_s} - \frac{n_{io}}{\lambda_i} \right)$$

Doubly periodic poling

- It is possible to satisfy both QPM conditions *simultaneously*
- Variation of nonlinear coefficient *d* along propagation direction is doubly periodic
- The grating contains two spatial frequencies required to phase match both the processes simultaneously



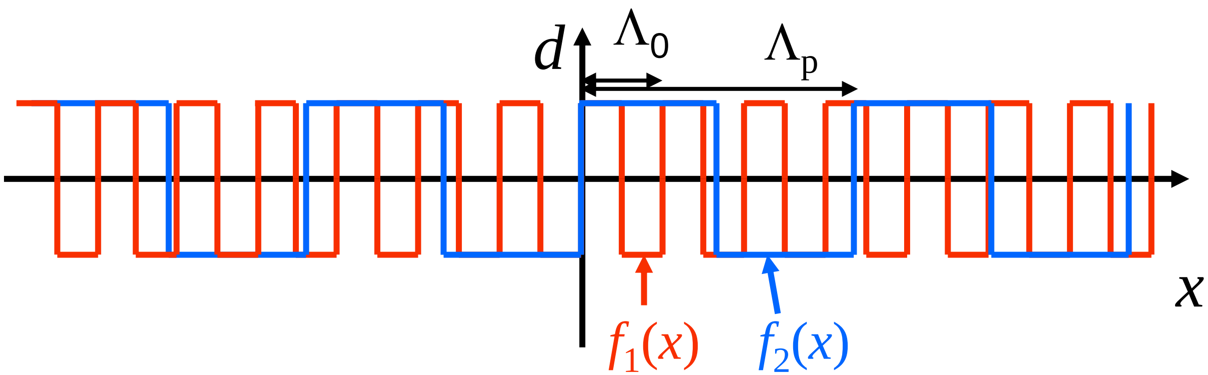
Domain engineered crystal

Ref: Thyagarajan, Lugani, Ghosh, Martin, Ostrowsky, Alibart, Tanzilli, PHYSICAL REVIEW A 80, 052321 (2009)

Doubly periodic poling

- x-variation of nonlinear coefficient

$$\bar{d} = d_{24} f_1(x) f_2(x)$$



$$\bar{d} = -\frac{4d_{24}}{\pi^2}\left(e^{iK_1x} + e^{-iK_1x} - e^{iK_2x} - e^{-iK_2x}\right) + ...$$

$$K_1 = K_0 + K_p$$

$$K_2 = K_0 - K_p$$

By choosing appropriate values of K_0 and K_p, it is possible to achieve phase matching for both o->o+e and o->e+o processes

Polarization state of output photons

- Classically we would say that the output is either of the following:
  - Signal is H polarized and idler is V polarized or
  - Signal is V polarized and idler is H polarized
- According to quantum mechanics
  - *Polarization state of individual signal and idler photons are undefined but are orthogonal to each other*
  - *The output signal and idler photons are entangled in polarization*

Fields at pump, signal and idler

- Pump is taken to be a classical wave and signal and idler are treated quantum mechanical.

Pump(o) $$\vec{E}_{po} = \frac{1}{2} e_{po}(r) E_{po} \left(e^{(ik_p x - \omega_p t)} + e^{-(ik_p x - \omega_p t)} \right) \hat{y}$$

Signal(o) $$\hat{E}_{so} = i \int d\omega_s e_{so}(\vec{r}) \sqrt{\frac{\hbar \omega_s}{2\varepsilon_{so} L_{\text{int}}}} \left(\hat{a}_{so} e^{ik_{so} x} - \hat{a}_{so}^{+} e^{-ik_{so} x} \right) \hat{y}$$

Signal(e) $$\hat{E}_{se} = i \int d\omega_s e_{se}(\vec{r}) \sqrt{\frac{\hbar \omega_s}{2\varepsilon_{se} L_{\text{int}}}} \left(\hat{a}_{se} e^{ik_{se} x} - \hat{a}_{se}^{+} e^{-ik_{se} x} \right) \hat{z}$$

Idler(o) $$\hat{E}_{io} = i \int d\omega_i e_{io}(\vec{r}) \sqrt{\frac{\hbar \omega_i}{2\varepsilon_{io} L_{\text{int}}}} \left(\hat{a}_{io} e^{ik_{io} x} - \hat{a}_{io}^{+} e^{-ik_{io} x} \right) \hat{y}$$

Idler(e) $$\hat{E}_{ie} = i \int d\omega_i e_{ie}(\vec{r}) \sqrt{\frac{\hbar \omega_i}{2\varepsilon_{ie} L_{\text{int}}}} \left(\hat{a}_{ie} e^{ik_{ie} x} - \hat{a}_{ie}^{+} e^{-ik_{ie} x} \right) \hat{z}$$

Interaction Hamiltonian

$$\hat{H}_{\text{int}} = \int d\omega_s \left[C_{oe}^{(1)} \left(\hat{a}_{so}^{+} \hat{a}_{ie}^{+} e^{-i\omega_p t} + \hat{a}_{so} \hat{a}_{ie} e^{i\omega_p t} \right) + C_{eo}^{(1)} \left(\hat{a}_{se}^{+} \hat{a}_{io}^{+} e^{-i\omega_p t} + \hat{a}_{se} \hat{a}_{io} e^{i\omega_p t} \right) \right]$$

$$C_{oe}^{(1)} = -\left(\frac{4 d_{24} E_{p0} \hbar \sqrt{\omega_s \omega_i}\, I_{oe}}{\pi^2 n_{so} n_{ie}} \right) \exp\left(\frac{-i \Delta k_{oe} L_{\text{int}}}{2} \right) \sin c\left(\Delta k_{oe} \frac{L_{\text{int}}}{2} \right)$$

$$C_{eo}^{(1)} = -\left(\frac{4 d_{24} E_{p0} \hbar \sqrt{\omega_s \omega_i}\, I_{eo}}{\pi^2 n_{se} n_{io}} \right) \exp\left(\frac{-i \Delta k_{eo} L_{\text{int}}}{2} \right) \sin c\left(\Delta k_{eo} \frac{L_{\text{int}}}{2} \right)$$

$$\Delta k_{oe} = \frac{2\pi}{\Lambda_1} - 2\pi \left(\frac{n_{po}}{\lambda_p} - \frac{n_{so}}{\lambda_s} - \frac{n_{ie}}{\lambda_i} \right), \qquad \Delta k_{eo} = \frac{2\pi}{\Lambda_1} - 2\pi \left(\frac{n_{po}}{\lambda_p} - \frac{n_{se}}{\lambda_s} - \frac{n_{io}}{\lambda_i} \right)$$

Output state

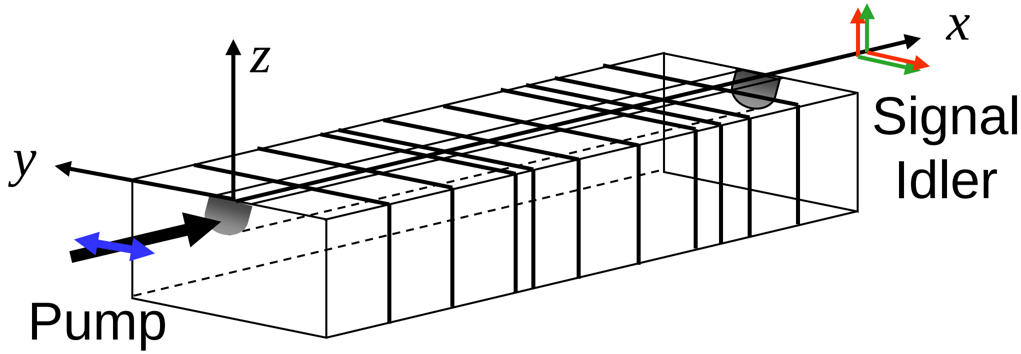
Entangled in polarization

$$\left|\psi\right\rangle = i\int d\omega_s \left(C_{oe}\left|s_o, i_e\right\rangle + C_{eo}\left|s_e, i_o\right\rangle\right)$$

- The relative magnitudes of the C coefficients would determine if the output is maximally entangled or not.
- We have defined the degree of entanglement as

$$\gamma = \frac{\min\left(C_{oe}, C_{eo}\right)}{\max\left(C_{oe}, C_{eo}\right)} \qquad 0 < \gamma < 1$$

Ti:LiNbO$_3$ waveguide



- Refractive index profile:

$$n^2(y,z) = n_b^2 + 2n_b \Delta n e^{-y^2/w^2} e^{-z^2/h^2}; \qquad z < 0$$
$$= n_c^2; \qquad z > 0$$

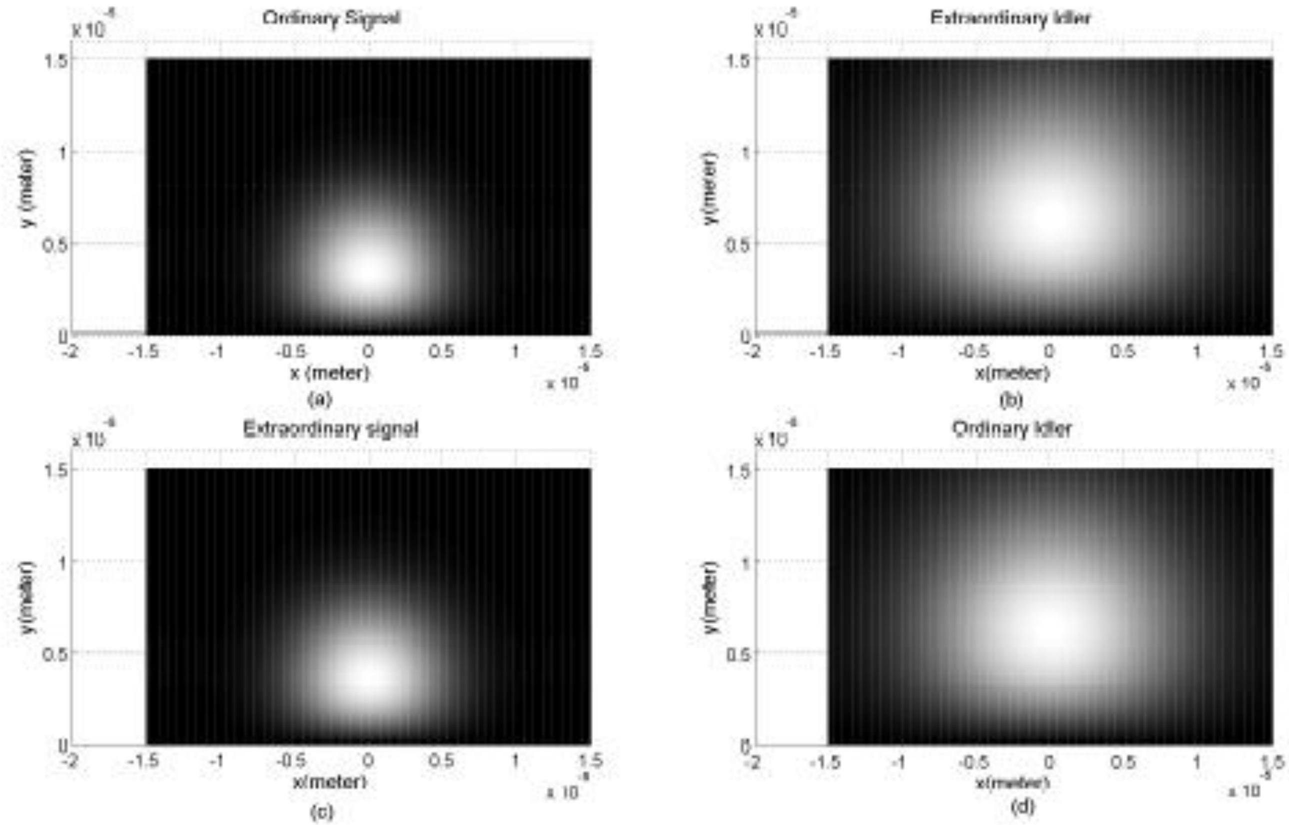
- Use standard variational analysis to calculate
  - Effective indices at pump, signal and idler wavelengths
  - Mode field profiles of the interacting modes

Transverse mode distributions

Signal Idler

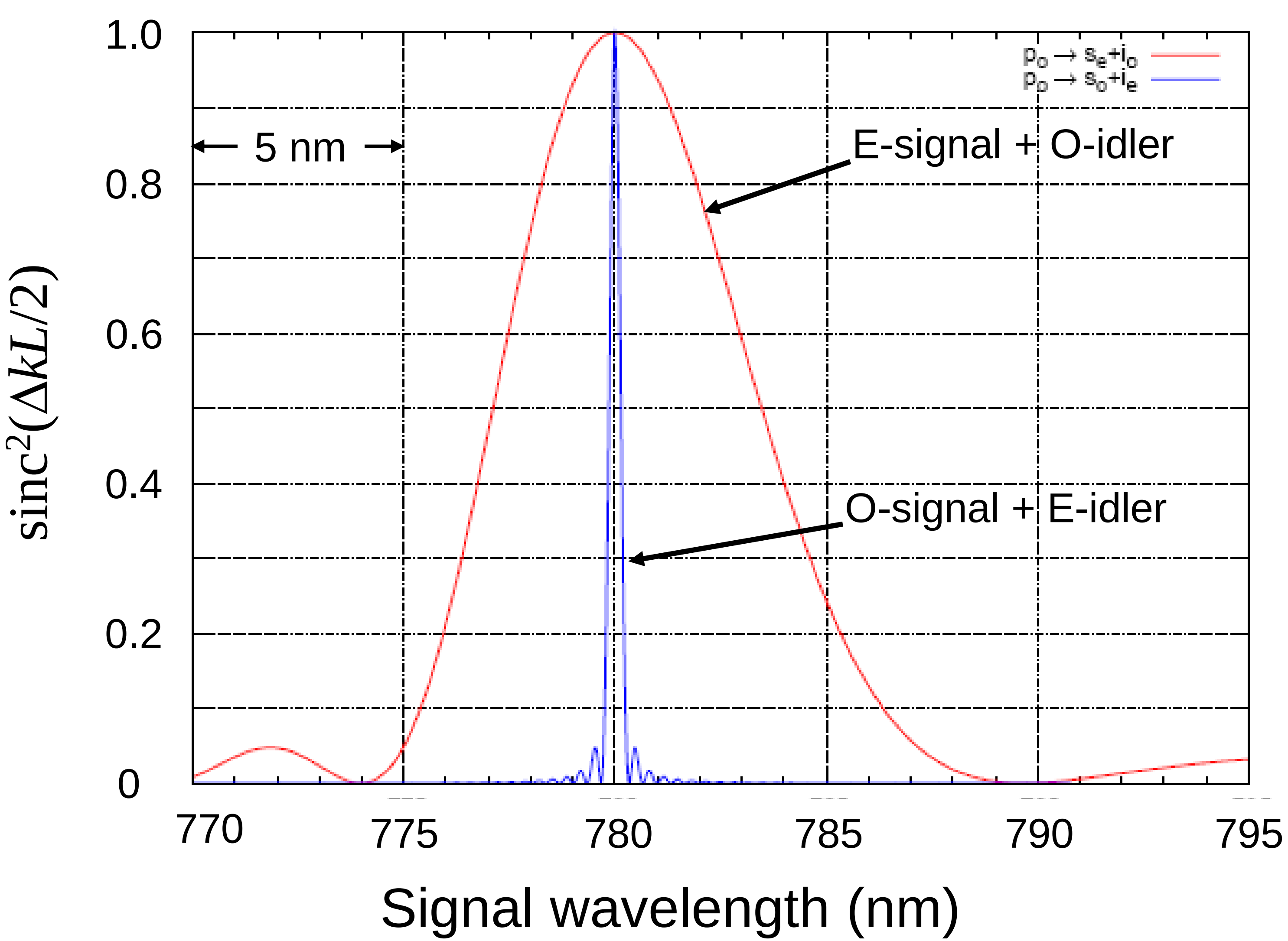
o-signal e-idler

e-signal o-idler

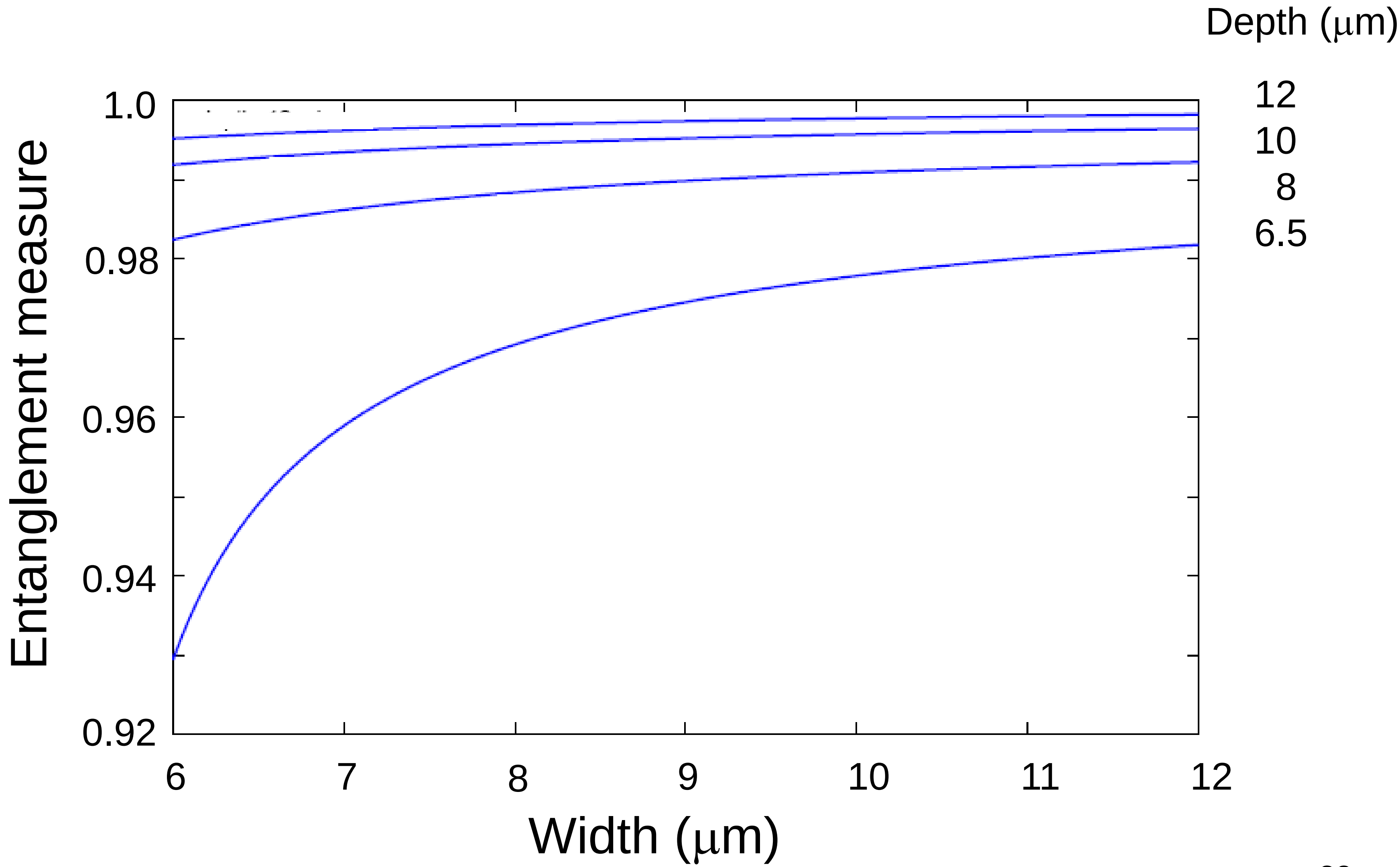


Good modal overlap for both the processes

Spectrum of the two processes

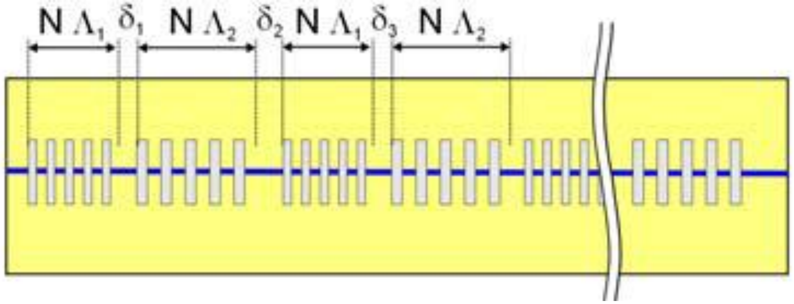


Entanglement vs. width

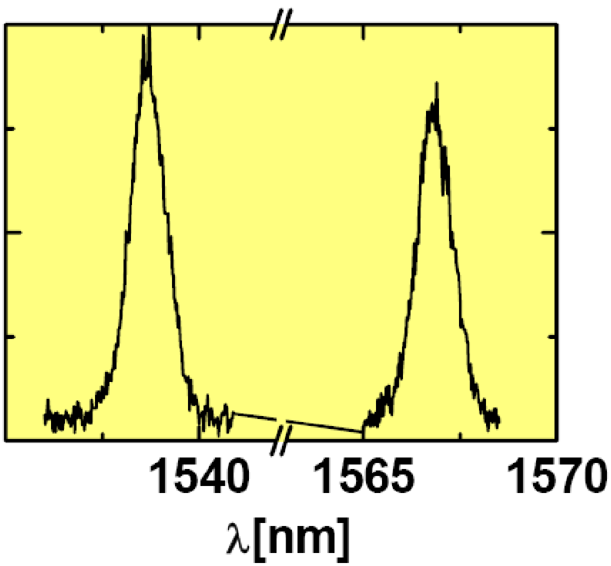


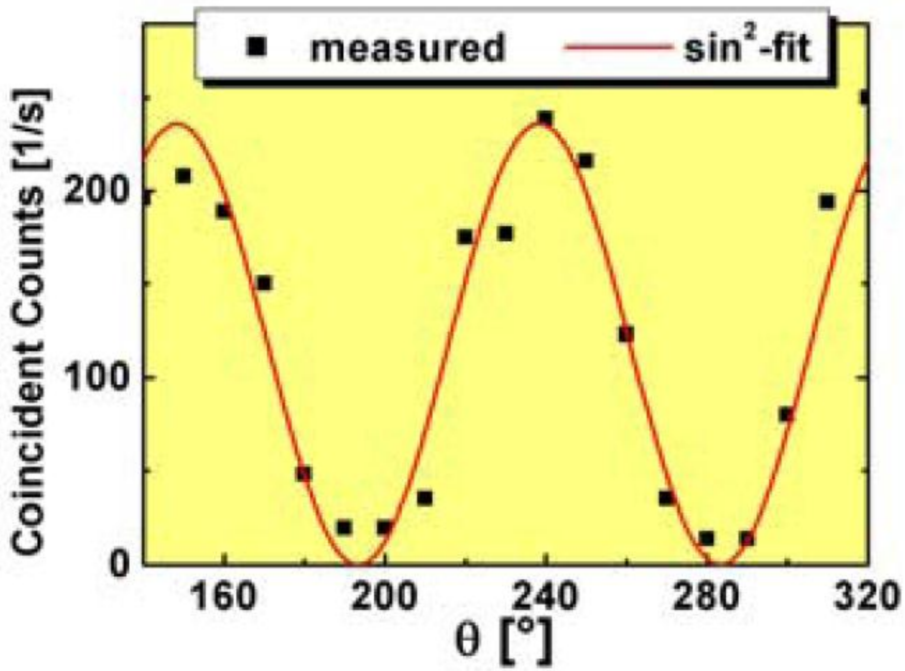
Experimental verification

Interlaced domain structures



SPDC



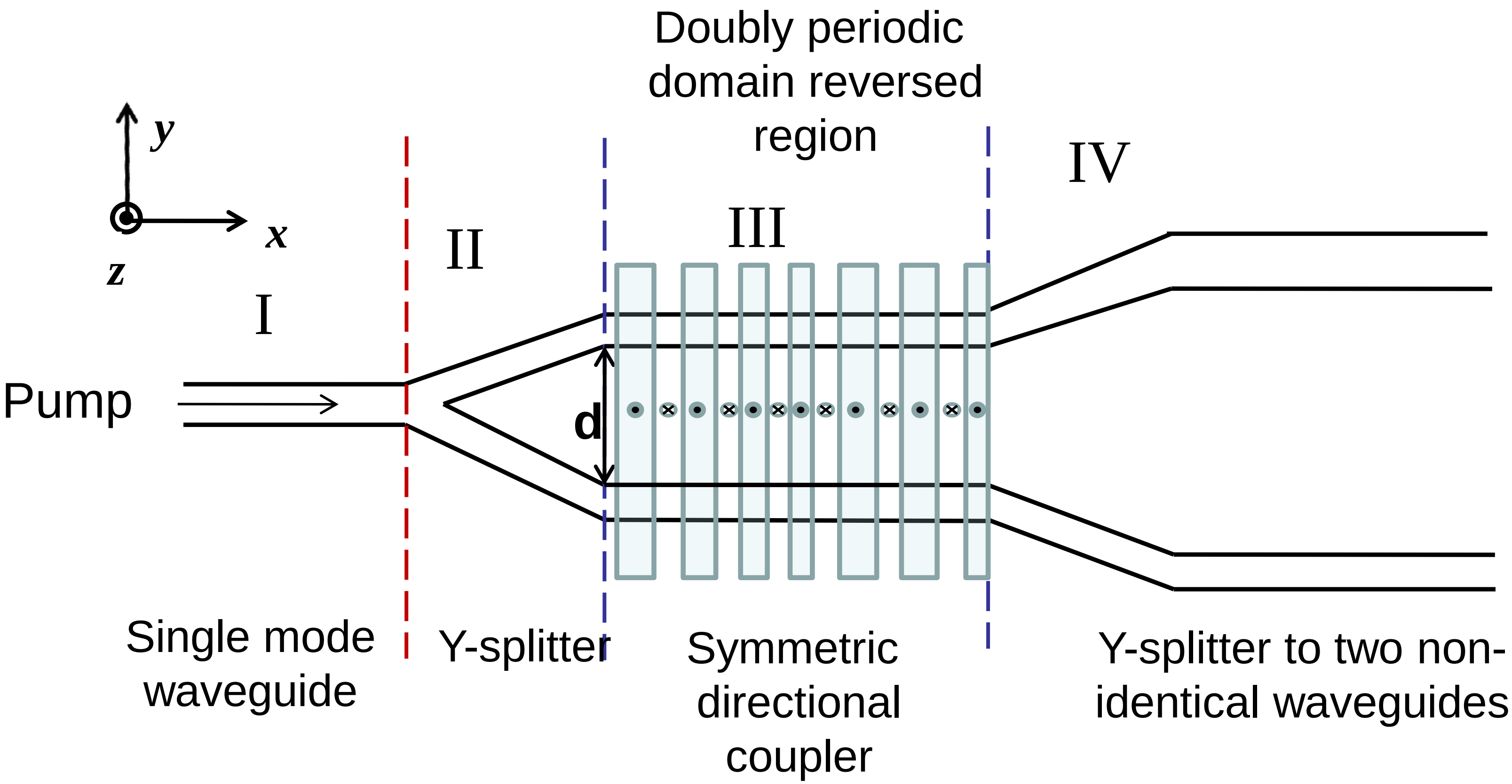


Ref: Thomas, Herrmann and Sohler, ECIO, Cambridge (2010), ThC4

Modal and path entangled photons

- Modal and path entangled photons can find applications in quantum information processing, lithography etc.
- Waveguide device supporting two spatial modes
  - Incident pump generates signal and idler photons entangled in modal degree of freedom
  - Modal entanglement can be converted to path entanglement using waveguide device

Waveguide device geometry



Ref: Jasleen Lugani, Sankalpa Ghosh, and K. Thyagarajan, Phys. Rev. A 83 (2011) 062333

Mode Entangled photons

- Generating non-degenerate, co-polarised mode entangled photons.
- Satisfying QPM conditions of two different SPDC processes, simultaneously, leading to mode entangled pairs of photons.
- All waves have e-polarization and use d_{33} coefficient

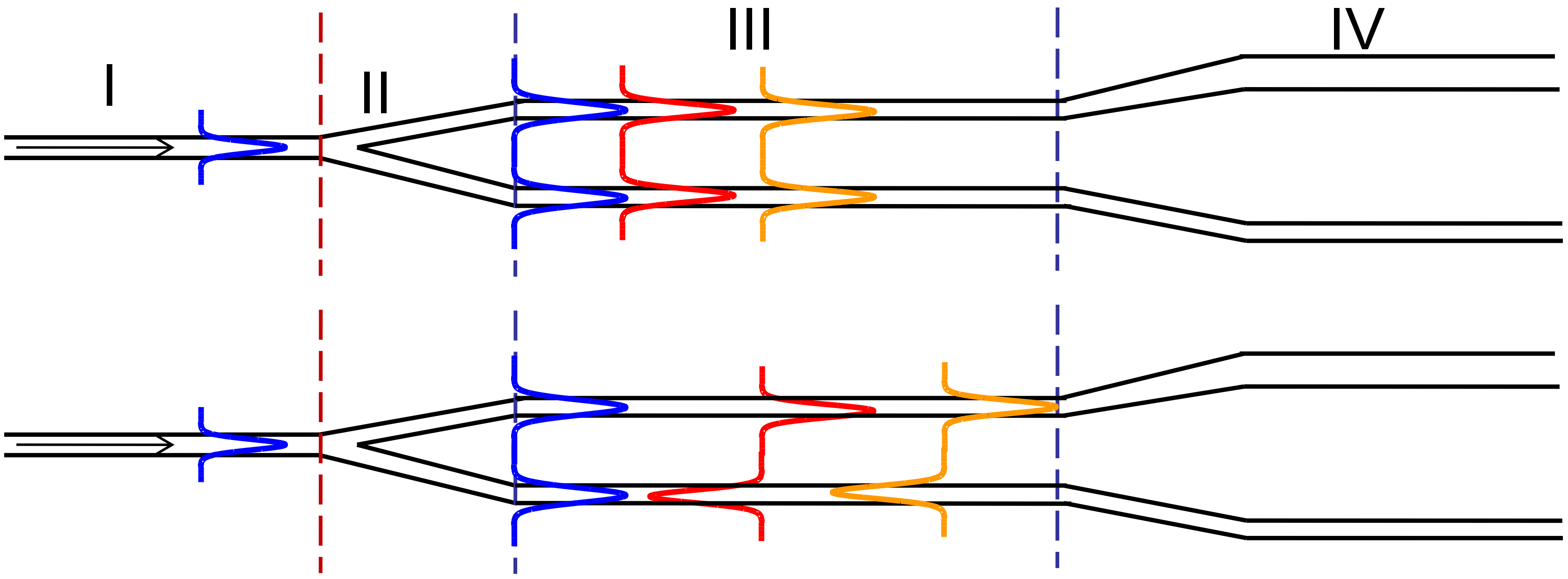
a. pump (0) -> signal(0) + idler(0)

$$\boxed{K_1 = \frac{2\pi}{\Lambda_1} = 2\pi\left(\frac{n_{p0}}{\lambda_p} - \frac{n_{s0}}{\lambda_s} - \frac{n_{i0}}{\lambda_i}\right)}$$

b. pump(0) -> signal(1) + idler(1)

$$\boxed{K_2 = \frac{2\pi}{\Lambda_2} = 2\pi\left(\frac{n_{p0}}{\lambda_p} - \frac{n_{s1}}{\lambda_s} - \frac{n_{i1}}{\lambda_i}\right)}$$

Domain engineering



- Incident pump photon down converts
  - Either into symmetric signal and symmetric idler modes
  - Or into antisymmetric signal and antisymmetric idler modes
- Both processes almost equally efficient

Quantum mechanical analysis

- Pump is assumed to be classical and signal and idler treated quantum mechanically.
- The interaction Hamiltonian in the interaction picture under energy conservation and RWA is

$$\hat{H}_{\text{int}} = \int d\omega_s \left[C_0 \left(\hat{a}_{s0}^{+} \hat{a}_{i0}^{+} + \hat{a}_{s0} \hat{a}_{i0} \right) + C_1 \left(\hat{a}_{s1}^{+} \hat{a}_{i1}^{+} + \hat{a}_{s1} \hat{a}_{i1} \right) \right]$$

$$C_0 = \left(\frac{4 d_{33} E_{p0} t \sqrt{\omega_s \omega_i} I_o}{\pi^2 n_{s0} n_{i0}} \right) e^{-i\Delta k_0 L/2} \text{sinc} \left(\frac{\Delta k_0 L}{2} \right)$$

$$I_0 = \iint e_{p0}(y,z) e_{s0}(y,z) e_{i0}(y,z) dydz$$

$$C_1 = \left(\frac{4 d_{33} E_{p0} t \sqrt{\omega_s \omega_i} I_1}{\pi^2 n_{s1} n_{i1}} \right) e^{-i\Delta k_1 L/2} \text{sinc} \left(\frac{\Delta k_1 L}{2} \right)$$

$$I_1 = \iint e_{p0}(y,z) e_{s1}(y,z) e_{i1}(y,z) dydz$$

Output state

- Output state at the end of Region III:

$$|\psi\rangle = i\int d\omega_s \left(C_0 |s_0, i_0\rangle + C_1 |s_1, i_1\rangle\right)$$

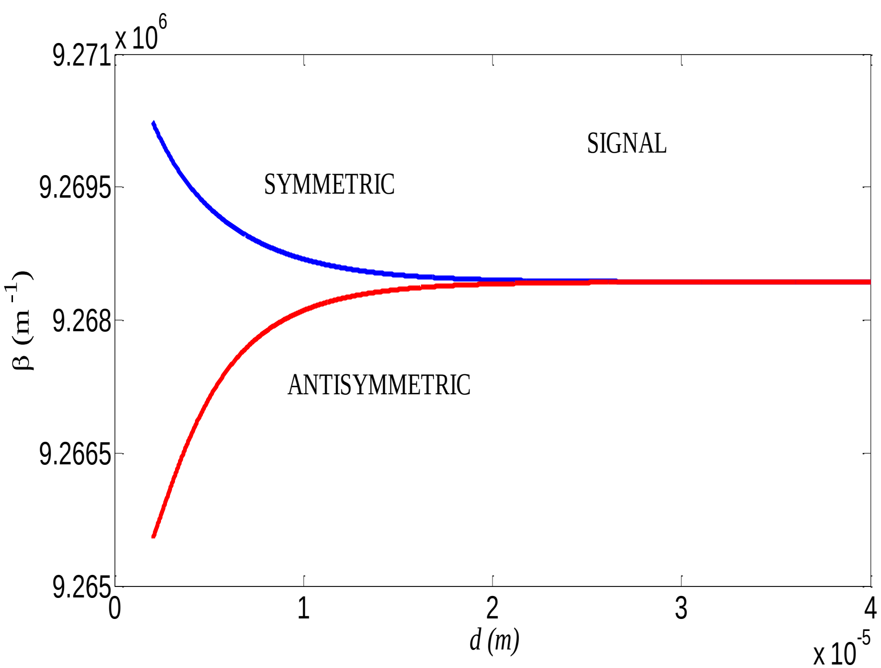
- Output represents a mode entangled state
  - 0: symmetric mode
  - 1: antisymmetric mode

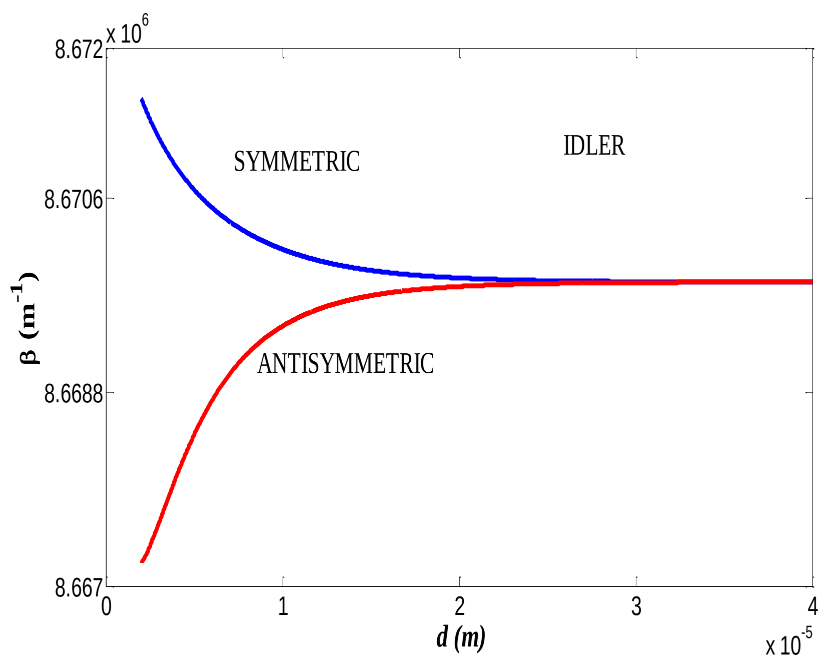
Results

- We have carried out simulations for the planar domain engineered LN waveguide.

Wavelength (nm)	∆ne
750	.0033
1452	.0026
1550	.0025

- The propagation constants vary with core separation

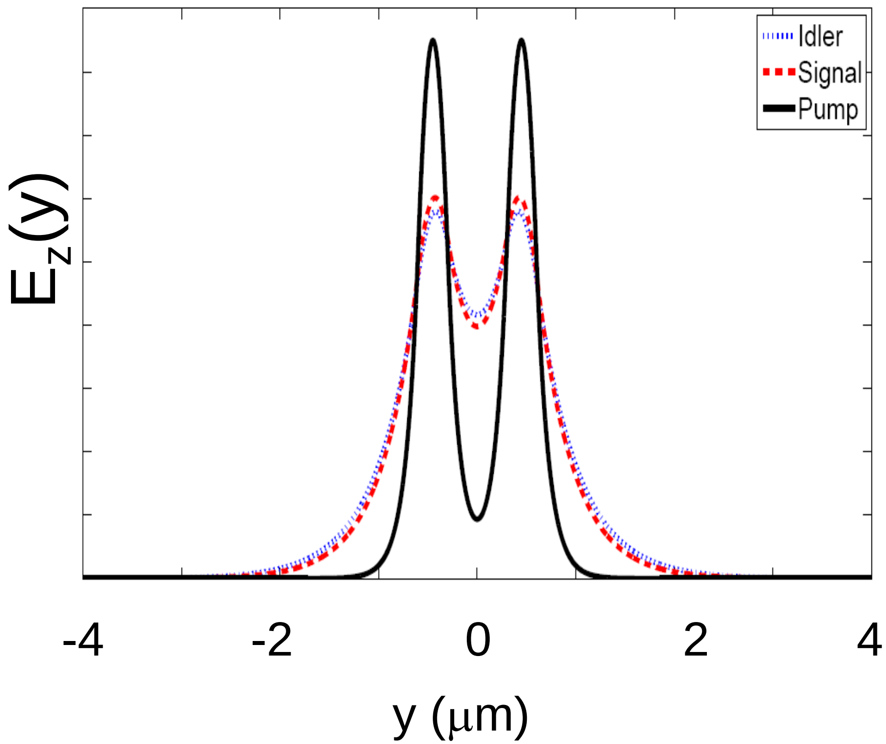




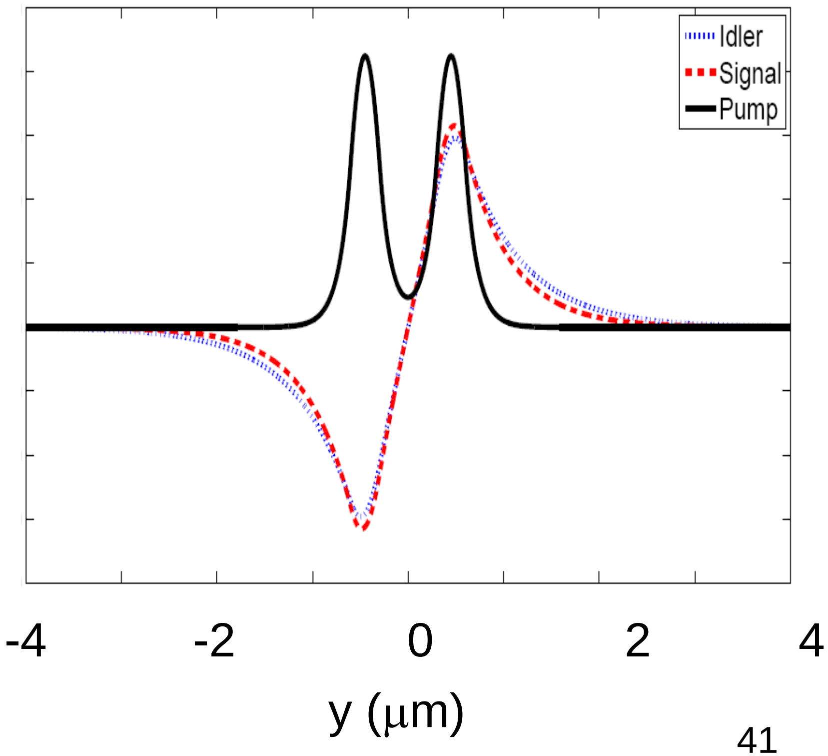
Modal patterns

- Titanium indiffused waveguides in lithium niobate
- The modes have very good modal overlap
- This leads to maximal modal entanglement

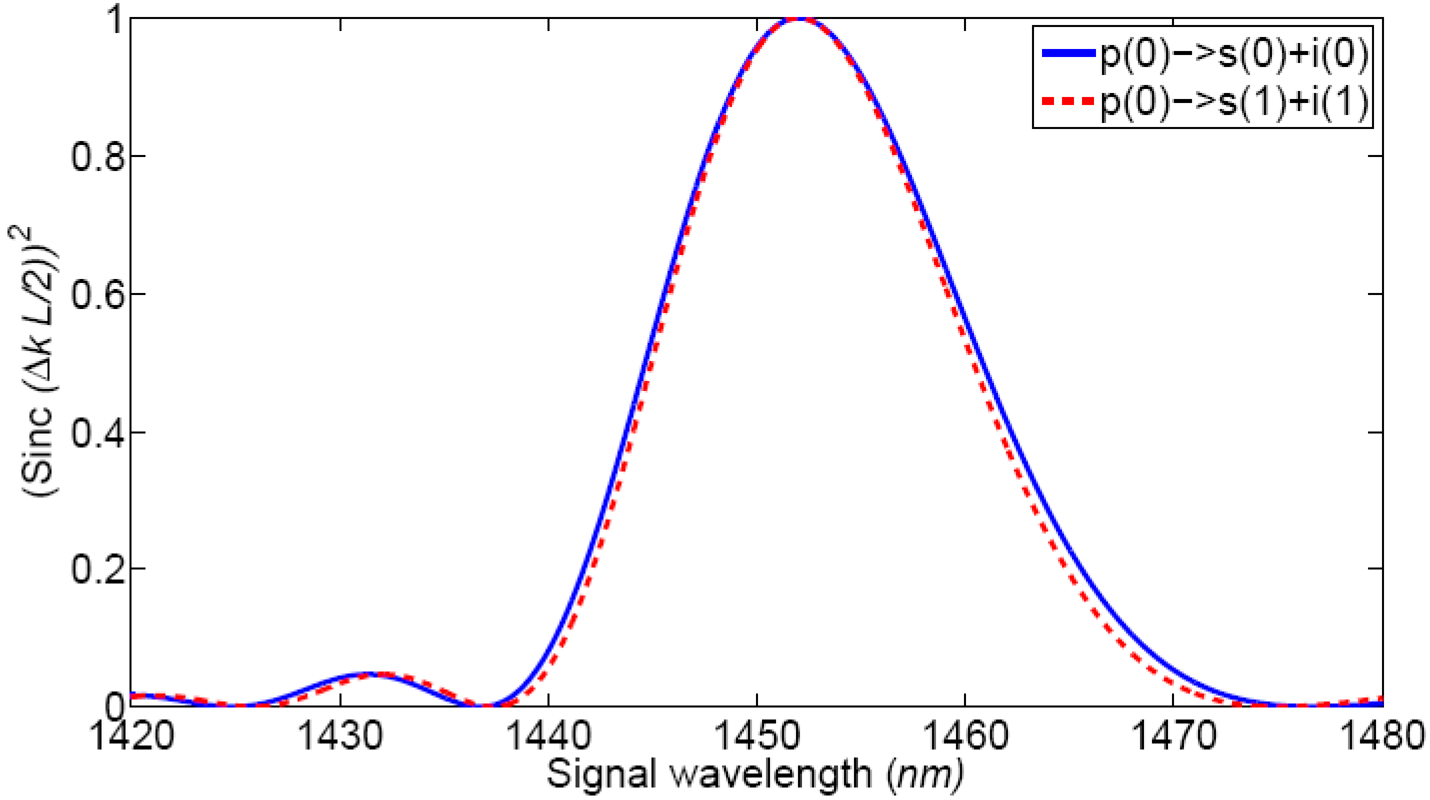
pump (0) -> signal(0) + idler(0)



pump(0) -> signal(1) + idler(1)

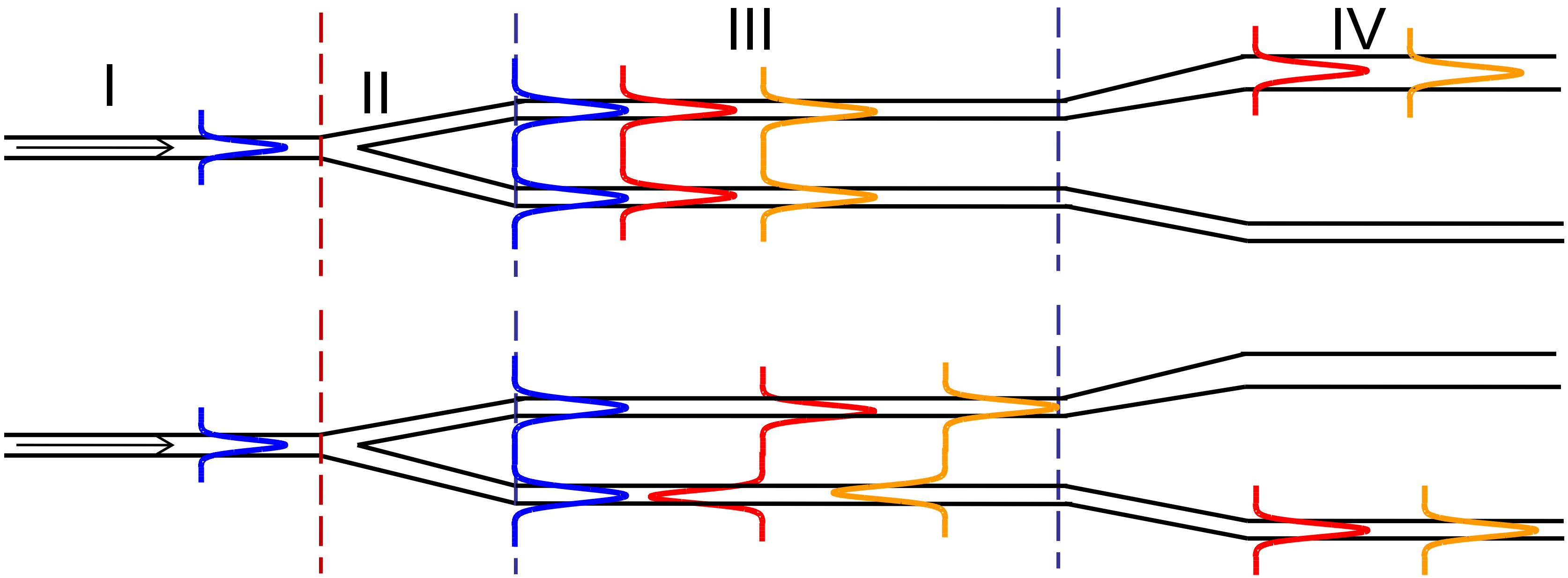


Bandwidth of the two processes



Bandwidths for both the processes are almost identical (16 nm)

Modal entanglement to path entanglement



- Fundamental symmetric modes exit from the upper waveguide having higher propagation constant
- First order antisymmetric modes exit from the lower waveguide with smaller propagation constant
- Output state is path entangled $\left|\Psi\right\rangle = i\int d\omega_s \left(C_0\left|s_u, i_u\right\rangle + C_1\left|s_l, i_l\right\rangle\right)$

SPDC with increased pump bandwidth

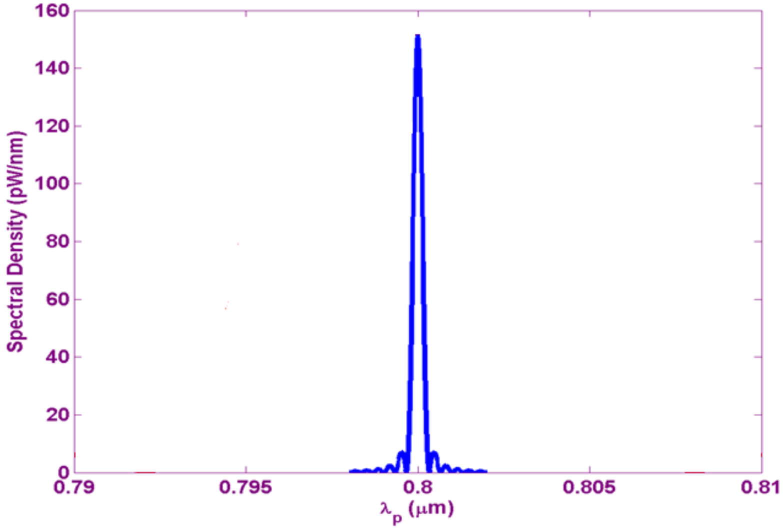
- SPDC source with
  - Signal photon at the telecom wavelength of 1550 nm
- Essential requirements:
  - High efficiency generation
  - Increased pump acceptance bandwidth so that femtosecond pump could be used
  - Narrow signal bandwidth so that signal photons can be used as flying qubits

Idea

- Efficiency of the down conversion varies as $\mathrm{sinc}2\left(\frac{\Delta\beta L}{2}\right)$

  L: Length of interaction

  $\Delta\beta = \beta_p - \beta_s - \beta_i - K = 0$



- Now

$$\frac{d\beta_s}{d\lambda_p} = \frac{d}{d\lambda_p}\left(\beta_p - \beta_i\right)$$

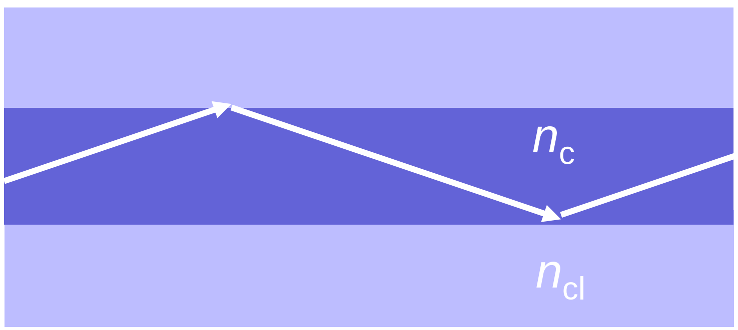
- If waveguide design is such that $(\beta_p - \beta_i)$ exhibits a minimum at a specific λ_p, then
  - As λ_p changes, λ_s will remain fixed
  - This will lead to
    - large pump acceptance bandwidth and
    - narrow signal bandwidth

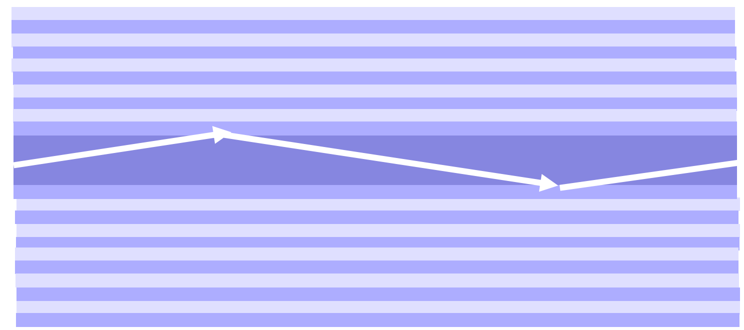
Bragg reflection waveguides

- Variation of the propagation constants at different wavelengths depends on
  - Material dispersion
  - Waveguide dispersion
- We need to counter the strong material dispersion at pump wavelength by a strong waveguide dispersion at the idler wavelength
- Bragg reflection waveguides (BRW) can provide such a possibility

Ref: Thyagarajan, Das, Alibart, de Micheli, Ostrowsky, Tanzilli, Optics Express, 16 (2008) 3577.

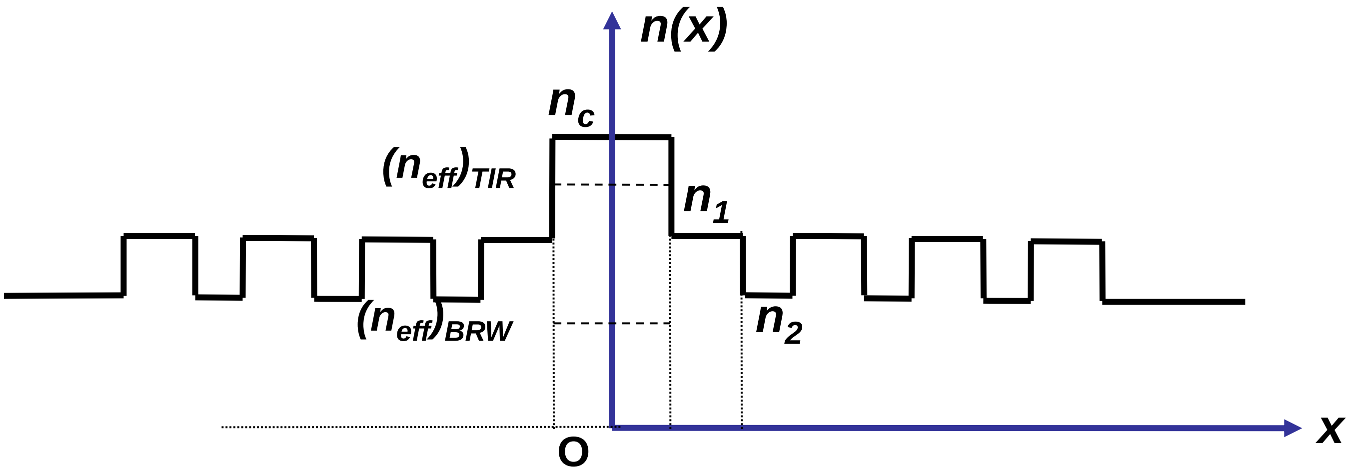
Total internal reflection and Bragg reflection





- Total internal reflection
  - Reflection total
  - $n_c > n_{cl}$
- Bragg reflection
  - Reflection partial
  - No restrictions on n_c

TIR and Bragg modes

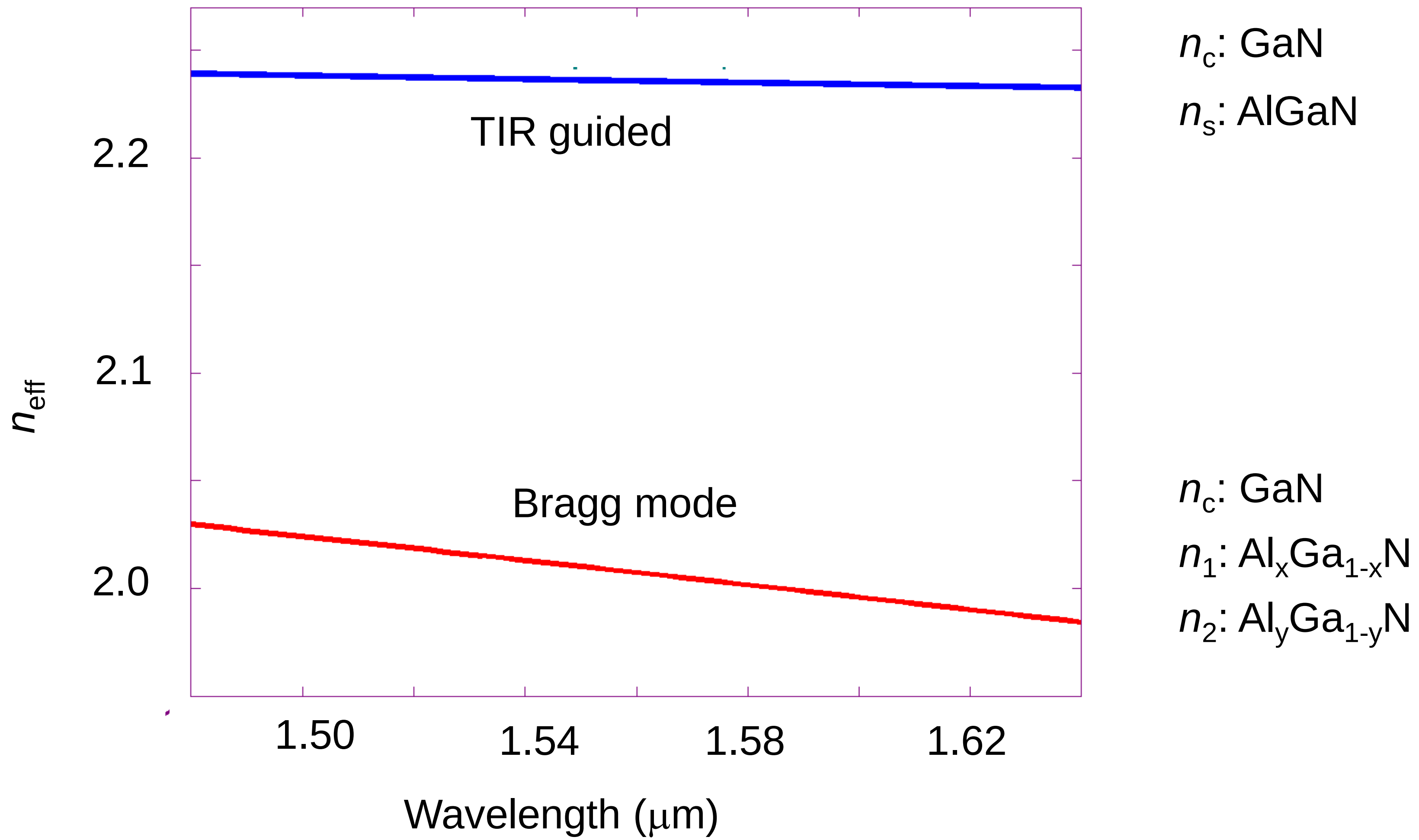


TIR modes: $n_1 < n_{\mathrm{eff}} < n_{\mathrm{c}}$

Bragg modes: $n_{\mathrm{eff}} < n_2$

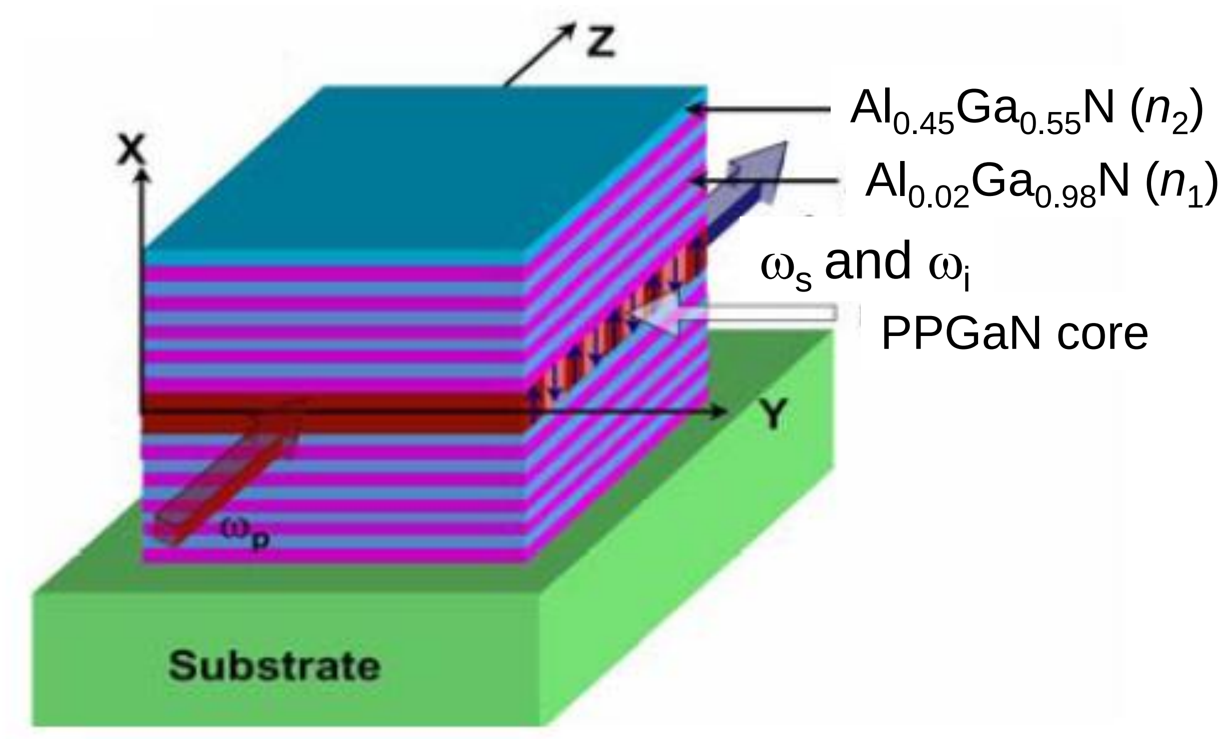
- Dispersion of TIR and Bragg modes can be very different
- Large design space for dispersion of Bragg modes

Modal dispersion



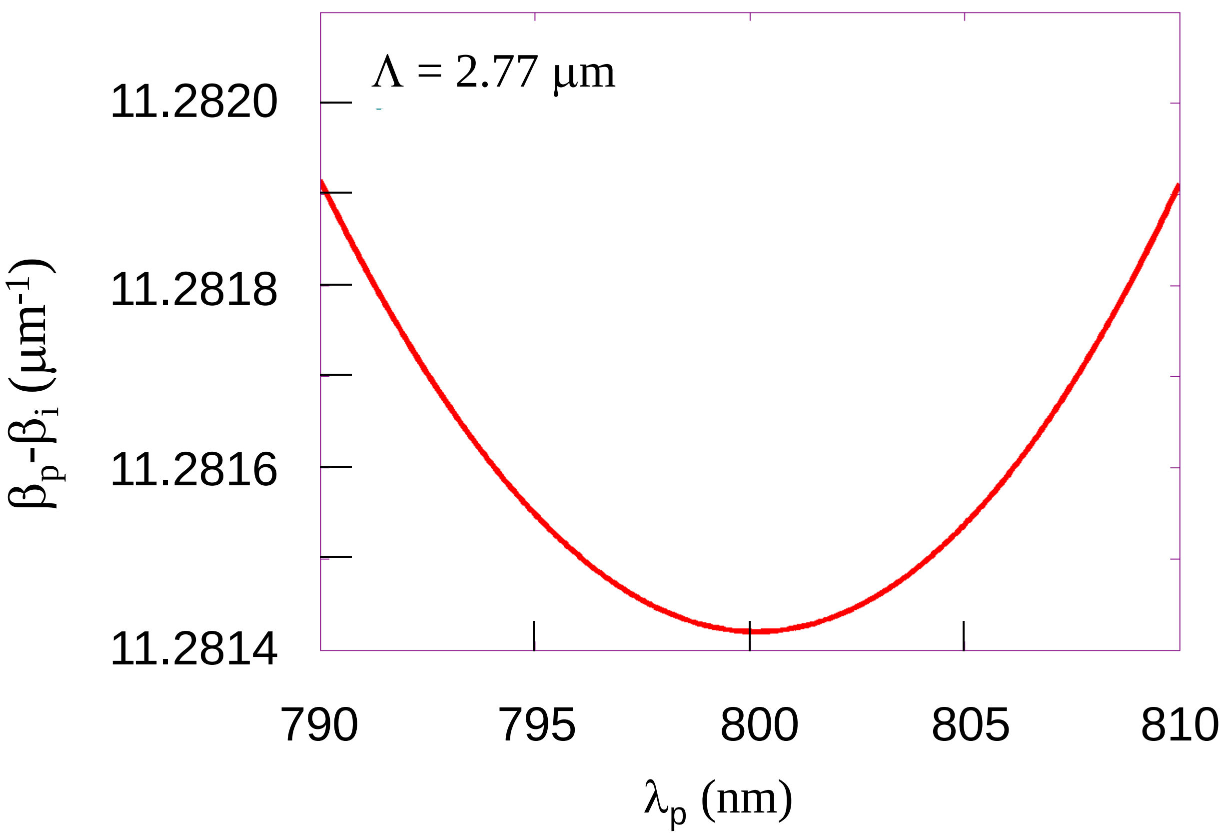
Bragg modes exhibit very strong dispersion

SPDC with BRW



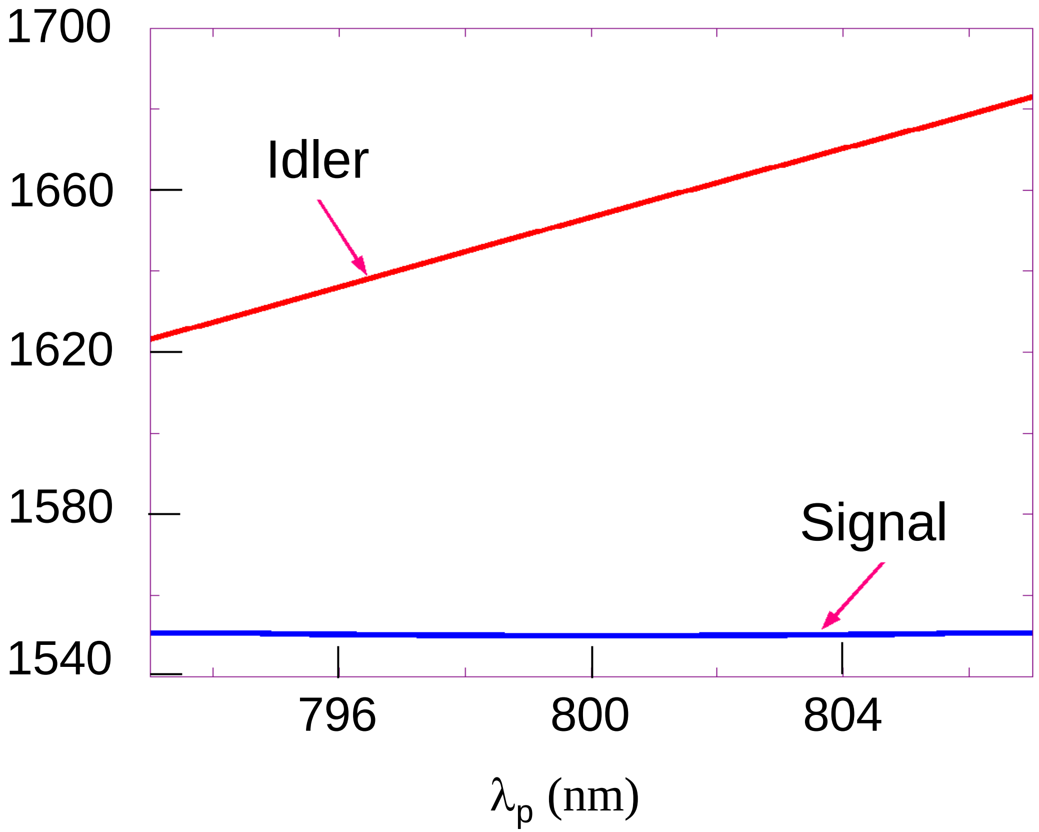
- Pump (~0.8 μm): TIR mode
- Signal (~1.55 μm): TIR mode
- Idler (~ 1.653 μm): BRW mode

Phase matching



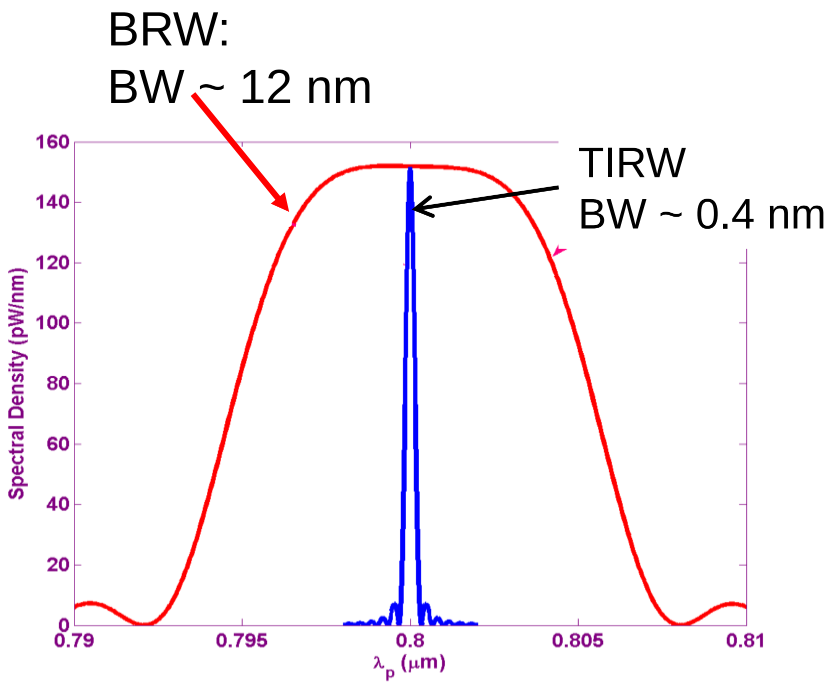
- Appearance of a minimum implies that as λ_p changes
  - Phase matching condition will continue to be satisfied

Wavelength variation of signal and idler

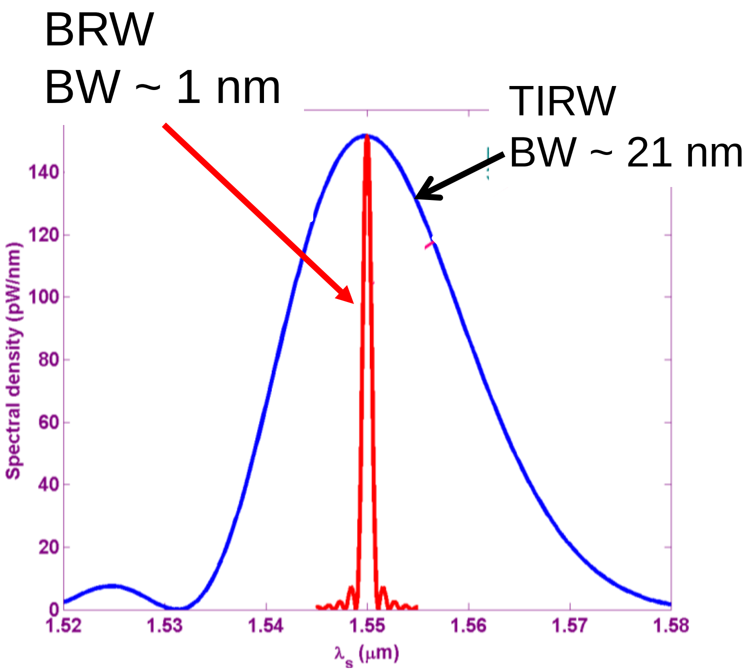


- Modal dispersion designed so that as λ_p changes, λ_s remains the same

Bandwidth



Pump bandwidth



Signal bandwidth

- Pump acceptance bandwidth increased 30 times compared to conventional geometry
- Very small signal bandwidth; useful in quantum communication systems

Separable states

- The idea proposed also leads to separable signal-idler pair states
  - Useful for various applications like heralding identical single photon states etc.

$$\frac{d}{d\lambda_p}\left(\beta_p - \beta_i\right) = 0$$

- implies

$$\frac{d\beta_p}{d\omega_p} = \frac{d\beta_i}{d\omega_i}$$

- This condition leads to the generation of separable state

Conclusions

- Using waveguide geometries
  - Provides with additional degrees of freedom
  - Output photons in well defined discrete spatial modes
  - Higher efficiency
- Domain engineering
  - Direct generation of polarization, mode or path entangled photon pair
- BRWs have very interesting dispersion behaviour
  - Designs to achieve large pump acceptance bandwidth and small signal bandwidth or separable states
- Integrated quantum optical circuits should play a very important role in the future

Acknowledgement

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