Entangled state discrimination, generation and error correction

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Plan of the talk

- Deterministic Bell State Discrimination
- NMR implementation
  - NDD of Bell States by NMR
  - Preparation of PPS
  - Preparation of Bell states
  - Encoding Phase and Parity Information
- Natural entaglement
  - Effects of entanglement in susceptibitlity and g-factors
  - Heisenberg model
  - Exchange coupled pair model (Dimer)
  - Analysis: Susceptibility as an Entanglement witness
  - Thermal Entanglement (intermediate temp)
  - Concurrence

Plan of the talk

- Entangled channels, perfect teleportation, multi-electron quantum dots
  - Teleportation of 1-qubit states
  - Generation of the $|W_3\rangle$ state
  - A N-qubit teleporting channel with one magnon
  - Generation from Hamiltonian dynamics
  - Decoherence from nuclear spin environment
- Exchange coupled pair model (Tetramer)
  - Coupling dependent entanglement
  - Generation of a 4 particle - 2 magnon state

Plan of the talk

- Common bath decoherence
  - Decoherence free Teleportation
  - Average Fidelity of Teleportation
  - Measurement in partially entangled basis
  - Effect of initial polarizations on the Fidelity
- Conclusions

Deterministic Bell State Discrimination

- Local operations with 2 Ancilla bits to deterministically distinguish all 4 Bell states, without affecting the quantum channel

- Never failing Bell measurement is impossible: Both theory and experiments

- Bell states into disentangled basis states. However, in the process of measurement the entangled state is vandalized

- A scheme which discriminates all the four Bell states deterministically and is able to preserve these states for further use: Two Ancilla bits instead of LOCC with unitary operations

- The Bell states:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad |\psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \tag{1}$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{2}$$

- Under Hadamard/Pauli: Transforms into each other $\rightarrow$ Distinguishing (Fig.)

M. Gupta and PKP, quant-ph/0504183v1

Deterministic Bell State Discrimination

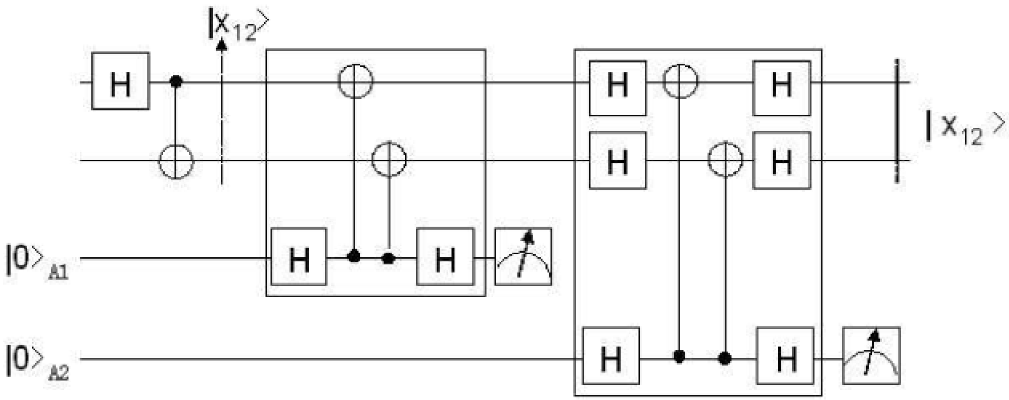


Diagram depicting the circuit for Bell state discriminator

- In the end, measurement is taken on these Ancilla bits to know with certainty, the type of Bell state that exists in the channel
  - Measurement on the first Ancilla will differentiate the four Bell states into two pairs: $|\psi^+\rangle/|\phi^+\rangle$ or $|\psi^-\rangle/|\phi^-\rangle$
  - Second Ancilla measurement differentiates the Bell states.
- Bell states in first two quantum channels retain their initial states, even after being discriminated

Deterministic Bell State Discrimination

- Before measurement the states can be explicitly written as:

$$|R_{A1}\rangle = [I_2 \otimes I_2 \otimes H] * [(x_1 \oplus A_1) \otimes (x_2 \oplus A_1) \otimes I_2] * [I_2 \otimes I_2 \otimes H] * [|x_{12}\rangle \otimes |A_1\rangle]$$

and $$|R_{A2}\rangle = [H^{\otimes^3}] * [(x_1 \oplus A_2) \otimes (x_1 \oplus A_2) \otimes I_2] * [H^{\otimes^3}] * [|x_{12}\rangle \otimes |A_2\rangle]$$

Table:

Bell State	**Measurement** A_1	**Measurement** A_2
$\|\psi^+\rangle$	0	0
$\|\psi^-\rangle$	1	0
$\|\phi^+\rangle$	0	1
$\|\phi^-\rangle$	1	1

NMR implementation

- The Bell state is determined by its parity and relative phase
- Two Ancilla qubits are measured in the circuit
  - In the first circuit, measurement on first Ancilla determines the relative phases of $|x\rangle$ and $|\bar{x}\rangle$ giving $|0\rangle$ for $\frac{1}{\sqrt{2}}(|x\rangle + |\bar{x}\rangle)$ and $|1\rangle$ for $\frac{1}{\sqrt{2}}(|x\rangle - |\bar{x}\rangle)$.
  - Measurement on second Ancilla: Exchange parity between two qubits

Bell state	First measurement	Second measurement
$\|\phi^+\rangle$	$\|0\rangle$	$\|0\rangle$
$\|\phi^-\rangle$	$\|1\rangle$	$\|0\rangle$
$\|\psi^+\rangle$	$\|0\rangle$	$\|1\rangle$
$\|\psi^-\rangle$	$\|1\rangle$	$\|1\rangle$

J. R. Samal, M. Gupta, PKP and A. Kumar, J. Phys. B: At. Mol. Opt. Phys. **43** (2010) 095508

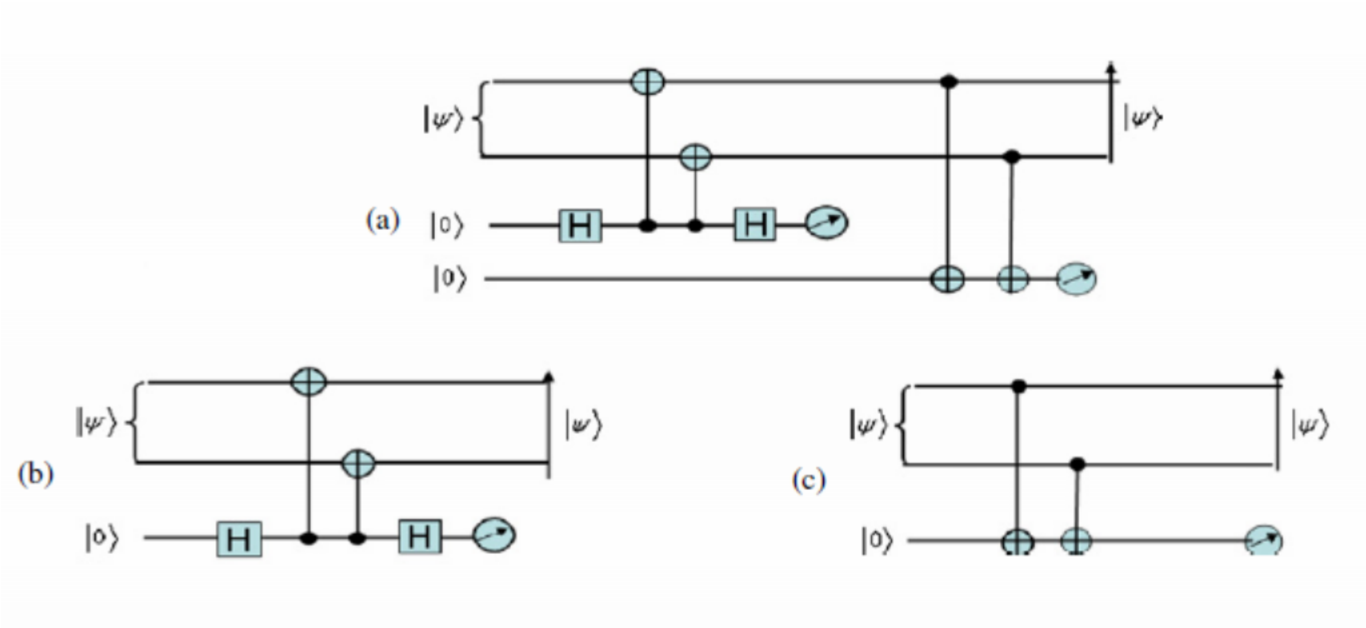
NDD of Bell States by NMR

- Single Ancilla qubit is used : Two separate measurements with three qubit NMR system [J. R. Samal *et al.*, At. Mol. Opt. Phys. **43** (2010) 095508]

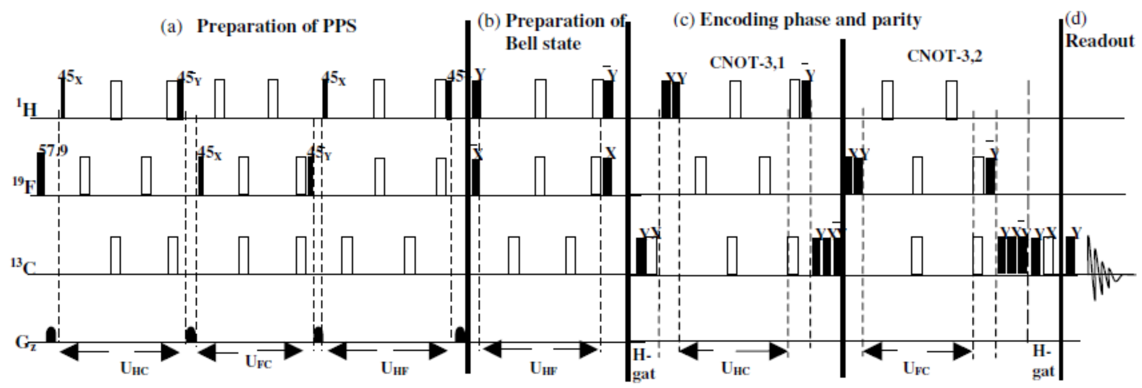
- The Bell states:

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right), \qquad |\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right), \tag{3}$$

  i. e. in general $|\chi^{\pm_x}\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle \pm |\bar{x}\rangle\right)$



NMR implementation



- The spin system chosen for the protocol is ^{13}C labeled $^{13}CHFBr_2$, ^1H, ^{19}F and ^{13}C being three qubits
- Four steps:
  - a)Preparation of $|000\rangle$ Pseudo Pure State (PPS)
  - b)EPR pair/Bell states from PPS
  - c)Encoding phase and parity information
  - d)Measurement on Ancilla qubit

NMR implementation

- The Bell state is determined by its parity and relative phase
- Two Ancilla qubits are measured in the circuit
  - In the first circuit, measurement on first Ancilla determines the relative phases of $|x\rangle$ and $|\bar{x}\rangle$ giving $|0\rangle$ for $\frac{1}{\sqrt{2}}(|x\rangle + |\bar{x}\rangle)$ and $|1\rangle$ for $\frac{1}{\sqrt{2}}(|x\rangle - |\bar{x}\rangle)$.
  - Measurement on second Ancilla: Exchange parity between two qubits

Bell state	First measurement	Second measurement
$|\phi^+\rangle$	$|0\rangle$	$|0\rangle$
$|\phi^-\rangle$	$|1\rangle$	$|0\rangle$
$|\psi^+\rangle$	$|0\rangle$	$|1\rangle$
$|\psi^-\rangle$	$|1\rangle$	$|1\rangle$

NMR implementation

- The NMR Hamiltonian for a weakly coupled 3 spin system:

$$H = -\sum_{i=1}^{3} \omega_i I_z^i + 2\pi \sum_{i<j=1}^{3} J_{ij} I_z^i I_z^i \tag{4}$$

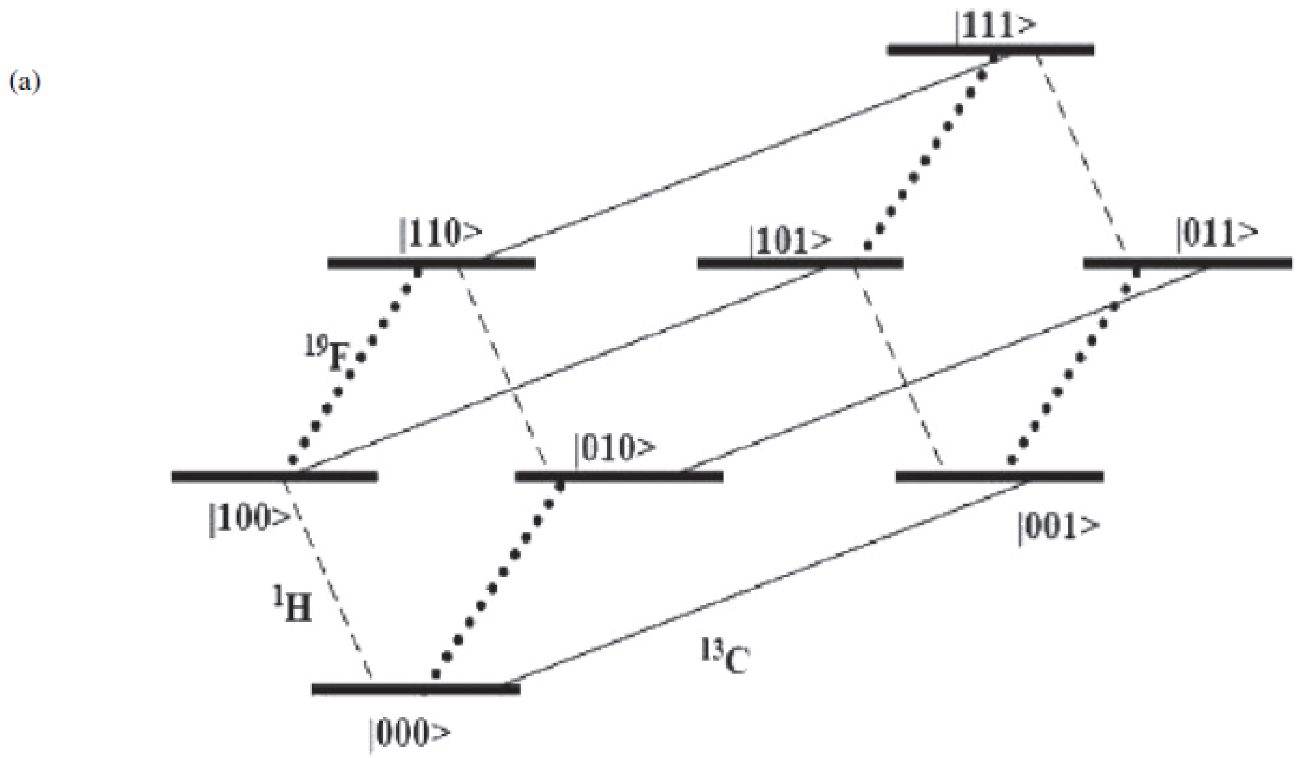
- The equilibrium density matrix under high temperature and high field approximation In a highly mixed state:

$$\rho_{eq} = \frac{1}{N}\left(I - \beta \nabla \rho_{eq}\right) \tag{5}$$

where,

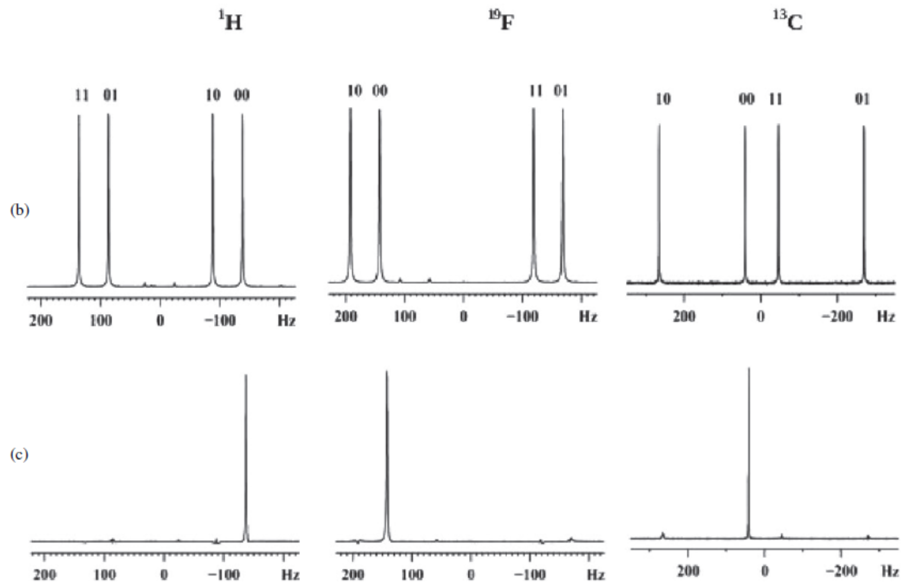
$$\nabla \rho_{eq} = \gamma_H I_z^H + \gamma_C I_z^C = \gamma_H \left(I_z^H + 0.94 I_z^F + 0.25 I_z^C\right) \tag{6}$$

NMR implementation



a) Schematic energy-level diagram of the three-qubit system. The horizontal bars depict the eight energy levels. The dashed lines (- - - - -), the dotted lines (. . . . . . .) and the solid lines (————-) respectively represent the first (^{1}H), second (^{19}F) and the third qubit (^{13}C) transitions

NMR implementation



b) Equilibrium spectra of ^1H, ^{19}F and ^{13}C of $^{13}CHFBr_2$. The labels on each transition of a qubit represent the state of the other two qubits in the transition
c)The population spectra of ^1H, ^{19}F and ^{13}C after the preparation of the $|000\rangle$ pseudo-pure state. Population spectra obtained by using measuring pulses of 90^0 on each spin individually after a gradient pulse to kill any unwanted coherence created by imperfections in the PPS sequence. Exclusive presence of the 00 transition for each qubit confirms the creation of the $|000\rangle$ PPS.

Preparation of PPS

- In liquid state NMR, one prepares PPS mimicking pure states. They are prepared from the equilibrium density matrix by spatial averaging

  $[57.9]_x^F \to G_z \to [\pi/4]_x^H \to [1/2J]^{HC} \to [\pi/4]_{-y}^H \to G_z \to [\pi/4]_x^F \to [1/2J]^{FC} \to [\pi/4]_{-y}^F \to G_z \to [\pi/4]_x^H \to [1/2J]^{HF} \to [\pi/4]_{-y}^H \to G_z$

  $\nabla\rho_{eq} = \gamma_H \left(I_z^H + \frac{1}{2}I_z^F + \frac{1}{4}I_z^C\right)$

- The $[\pi/4]$ pulses before and after the J-evolution and the $[\pi]$ pulses in the middle of the evolution period transfer the density matrix into the $|000\rangle$ PPS

$$\nabla\rho_{000} = \frac{\gamma_H}{4}\left(I_z^H + I_z^C + 2I_z^H I_z^C + 2I_z^H I_z^F + 2I_z^F I_z^C + 4I_z^H I_z^F I_z^C\right) \quad (7)$$

- The $|000\rangle$ PPS is used for preparation of Bell states $\left[|00\rangle + |11\rangle\right]/\sqrt{2}$ and $\left[|00\rangle - |11\rangle\right]/\sqrt{2}$

- For preparation of Bell states $\left[|01\rangle + |10\rangle\right]/\sqrt{2}$ and $\left[|01\rangle - |10\rangle\right]/\sqrt{2}$, one needs to start from $|000\rangle$ PPS

$$\nabla\rho_{000} = \frac{\gamma_H}{4}\left(-I_z^H + I_z^C - 2I_z^H I_z^C - 2I_z^H I_z^F + 2I_z^F I_z^C - 4I_z^H I_z^F I_z^C\right) \quad (8)$$

Preparation of Bell states

- Four Bell states are prepared from first two qubits (^1H and ^{19}F from $|000\rangle$ and $|001\rangle$ PPS respectively, by unitary transformation: $U^{\pm} = exp\left(\mp i I_{x^H} I_{y^F} \pi\right)$

  $[\pi/2]^F_{\mp x} \rightarrow [\pi/2]^H_y \rightarrow [1/2J]^{HF} \rightarrow [\pi/2]^H_{-y} \rightarrow [\pi/2]^F_{\pm x}$

- Under $U^{\pm}$, the PPS density matrix transforms:

$$\Delta\rho_{|\phi^{\pm}\rangle|0\rangle} = (\gamma_H/4)\big[I_z^C \pm 2I_x^H I_x^F \mp 2I_y^H I_y^F + 2I_z^H I_z^F \pm 4I_x^H I_x^F I_z^C \mp 4I_y^H I_y^F I_z^C + 4I_z^H I_z^F I_z^C\big],$$

$$\Delta\rho_{|\psi^{\pm}\rangle|0\rangle} = (\gamma_H/4)\big[I_z^C \pm 2I_x^H I_x^F \pm 2I_y^H I_y^F - 2I_z^H I_z^F \pm g4I_x^H I_x^F I_z^C \pm 4I_y{}^H I_y^F I_z^C - 4I_z^H I_z^F I_z^C\big].$$

Preparation of Bell states

- These density matrices correspond respectively to the states:

$$|\phi^{\pm}\rangle_{HF}|0\rangle_C = \frac{1}{\sqrt{2}}[|00\rangle \pm |11\rangle]_{HF}|0\rangle_C,$$
$$|\psi^{\pm}\rangle_{HF}|0\rangle_C = \frac{1}{\sqrt{2}}[|01\rangle \pm |10\rangle]_{HF}|0\rangle_C,$$

where the first two qubits combine to form Bell states and the third qubit is in the state $|0\rangle$

- Average absolute deviation:

$$\langle|\nabla x|\rangle = \frac{1}{N^2}\sum_{i,j=1}^{N}|x_{ij}^T - x_{ij}^E| \quad (9)$$

- Maximum absolute deviation:

$$\nabla x_{max} = Max|x_{ij}^T - x_{ij}^E|, \qquad \forall\ i,j\epsilon\{1,N\} \quad (10)$$

where x_{ij}^T, x_{ij}^T are theoretical and experimental elements respectively

Preparation of Bell states

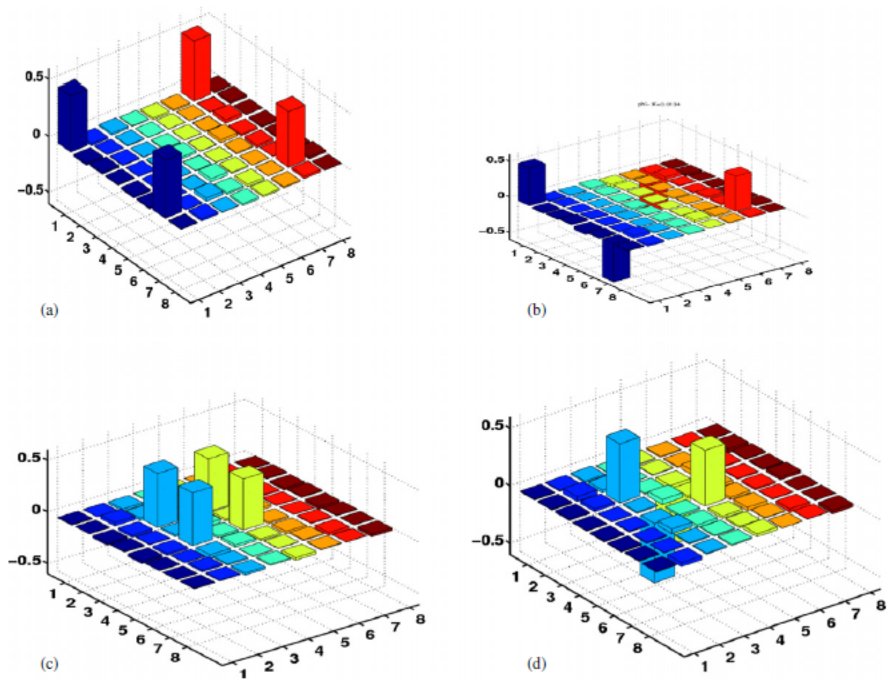


Figure 4. Tomography of the real parts of experimentally obtained density matrices after the preparation of four Bell states. The numbers 1–8 along the *x*- and the *y*-axis represent the states $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$ and $|111\rangle$ respectively. (a) $|\phi^{+}\rangle_{\mathrm{HF}}|0\rangle_{C} = (1/\sqrt{2})[|000\rangle + |110\rangle]_{\mathrm{HFC}}$. The density matrix contains two diagonal elements corresponding to the $|000\rangle$ and $|110\rangle$ states and DQ coherences corresponding to the ^{1}H and ^{19}F qubits, all are of equal intensity and same phase. (b) $|\phi^{-}\rangle_{\mathrm{HF}}|0\rangle_{C} = (1/\sqrt{2})[|000\rangle - |110\rangle]_{\mathrm{HFC}}$. The density matrix contains two diagonal elements corresponding to the $|000\rangle$ and $|110\rangle$ states and DQ coherences corresponding to the ^{1}H and ^{19}F qubits, all are of equal intensity; the diagonal elements and DQ coherences are of opposite phase. (c) $|\psi^{+}\rangle_{\mathrm{HF}}|0\rangle_{C} = (1/\sqrt{2})[|010\rangle + |100\rangle]_{\mathrm{HFC}}$. The density matrix contains two diagonal elements corresponding to the $|010\rangle$ and $|100\rangle$ states and ZQ coherences corresponding to the ^{1}H and ^{19}F qubits, all are of equal intensity and same phase. (d) $|\psi^{-}\rangle_{\mathrm{HF}}|0\rangle_{C} = (1/\sqrt{2})[|010\rangle - |100\rangle]_{\mathrm{HFC}}$. The density matrix contains two diagonal elements corresponding to the $|010\rangle$ and $|100\rangle$ states and ZQ coherences corresponding to the ^{1}H and ^{19}F qubits, all are of equal intensity; the diagonal elements and ZQ coherences are of opposite phase.

Encoding Phase and Parity Information

- By the use of Hadamard gates and controlled-NOT (CNOT) gates:

Hadamard $\longrightarrow$ $([\pi/2]_y \rightarrow [\pi]_x)$

CNOT-i,j: $\longrightarrow$ $\big([\pi/2]^j_x \rightarrow \{[\pi/2]^j_y \rightarrow [1/2J]^{ij} \rightarrow [\pi/2]^j_{-y}\} \rightarrow \{[\pi/2]^i_y \rightarrow [\pi/2]^i_x \rightarrow [\pi/2]^i_{-y}\big)$

- The deviation density matrices after the phase and parity measurements are:

$$\Delta\rho_{|\phi^+\rangle|0\rangle} \xrightarrow{\text{phase measurement}} (\gamma_H/4)\big[I^C_z + 2\,I^H_x I^F_x - 2\,I^H_y I^F_y + 2\,I^H_z I^F_z + 4\,I^H_x I^F_x I^C_z - 4\,I^H_y I^F_y I^C_z + 4\,I^H_z I^F_z I^C_z\big] = \Delta\rho_{|\phi^+\rangle|0\rangle}$$

$$\Delta\rho_{|\phi^-\rangle|0\rangle} \xrightarrow{\text{phase measurement}} (\gamma_H/4)\big[-I^C_z - 2\,I^H_x I^F_x + 2\,I^H_y I^F_y + 2\,I^H_z I^F_z + 4\,I^H_x I^F_x I^C_z - 4\,I^H_y I^F_y I^C_z - 4\,I^H_z I^F_z I^C_z\big] = \Delta\rho_{|\phi^+\rangle|1\rangle}$$

Encoding Phase and Parity Information

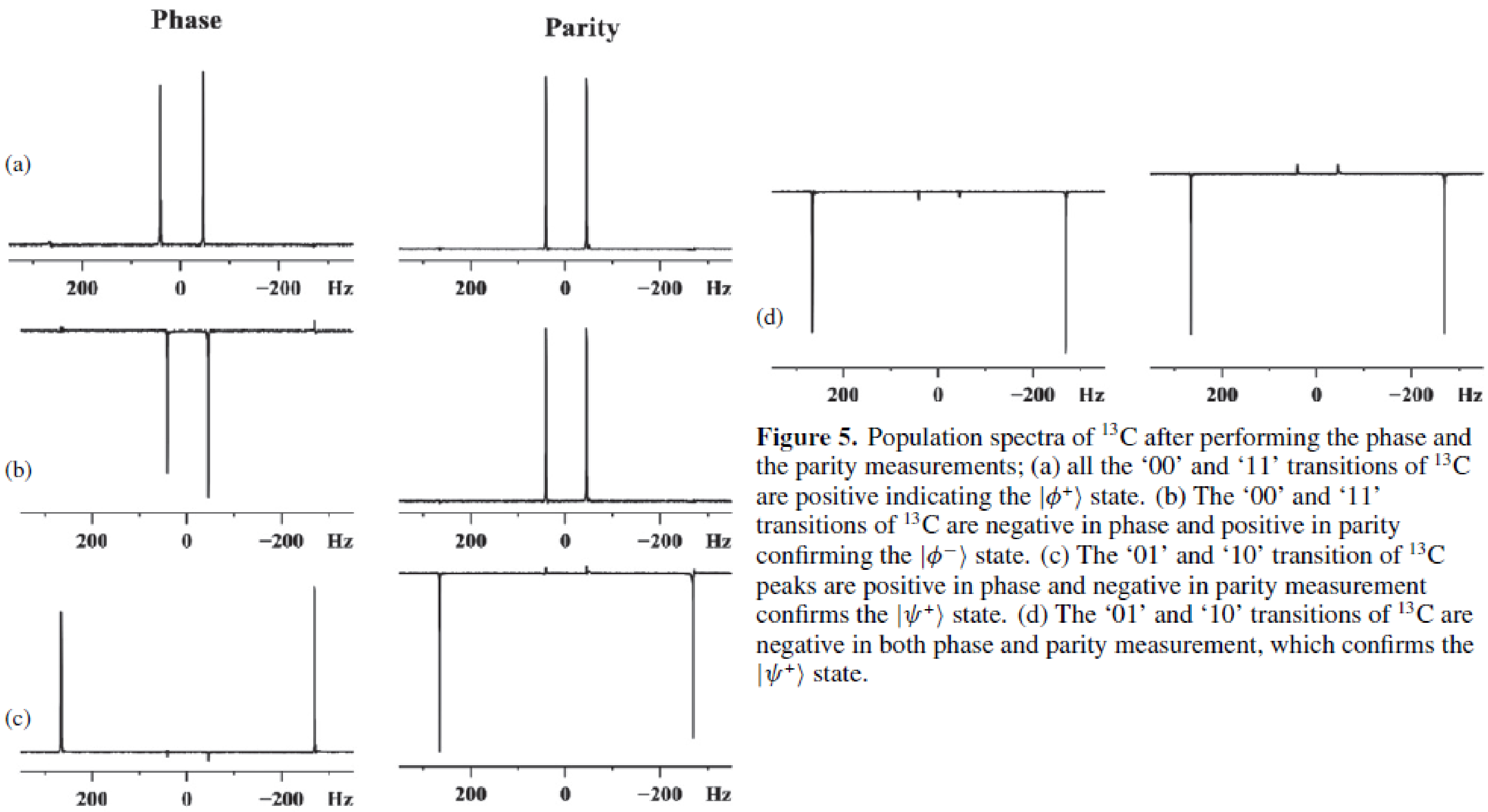
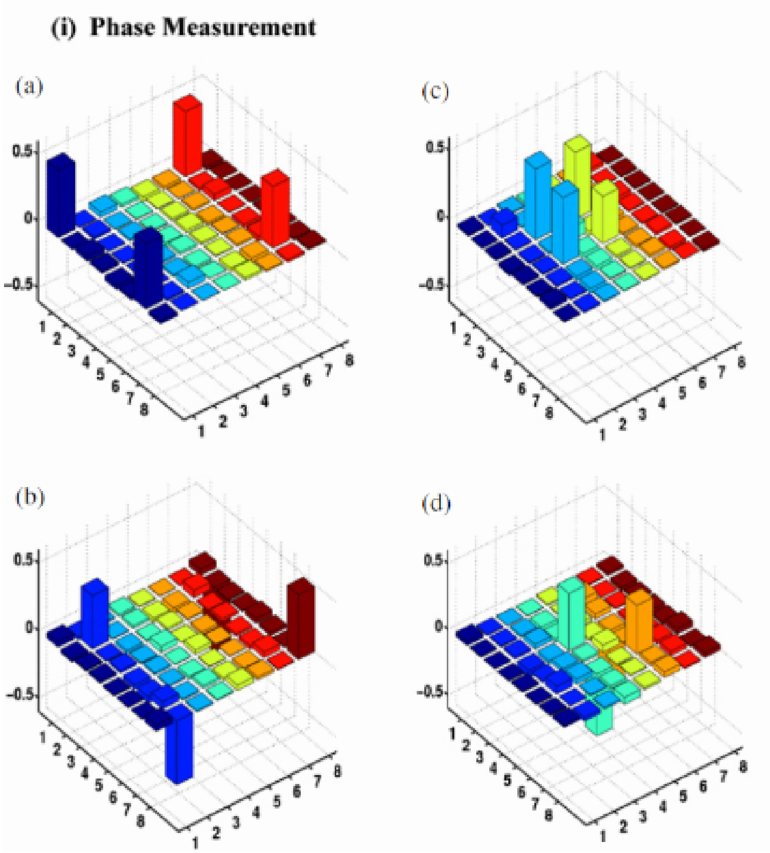


Figure 5. Population spectra of ^{13}C after performing the phase and the parity measurements; (a) all the '00' and '11' transitions of ^{13}C are positive indicating the $|\phi^+\rangle$ state. (b) The '00' and '11' transitions of ^{13}C are negative in phase and positive in parity confirming the $|\phi^-\rangle$ state. (c) The '01' and '10' transition of ^{13}C peaks are positive in phase and negative in parity measurement confirms the $|\psi^+\rangle$ state. (d) The '01' and '10' transitions of ^{13}C are negative in both phase and parity measurement, which confirms the $|\psi^+\rangle$ state.

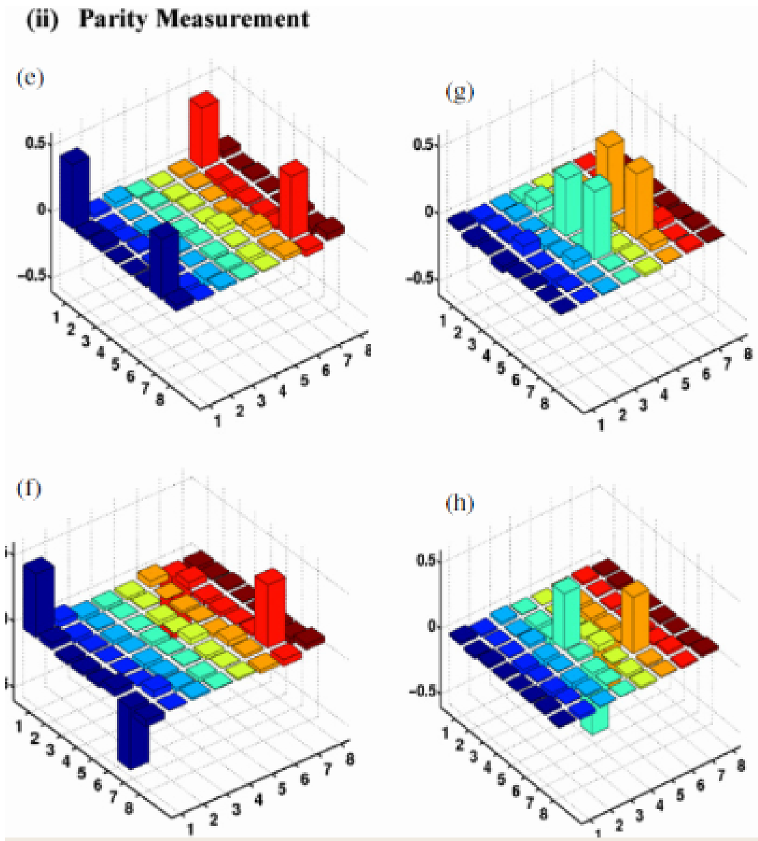
Encoding Phase and Parity Information



Encoding Phase and Parity Information

Figure 6. Tomography of the real parts of the experimentally obtained density matrices after the implementation of phase and parity measurements. The numbers 1–8 along the x- and the y-axis represent the states $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$ and $|111\rangle$ respectively. (i) Phase measurement, (a) for the initial state $|\phi^+\rangle_{\mathrm{HF}}|0\rangle_C$, the theoretically predicted final state is $|\phi^+\rangle_{\mathrm{HF}}|0\rangle_C = (1/\sqrt{2})[|000\rangle + |110\rangle]_{\mathrm{HFC}}$. (b) For the initial state $|\phi^-\rangle_{\mathrm{HF}}|0\rangle_C$, the theoretically predicted final state is $|\phi^-\rangle_{\mathrm{HF}}|1\rangle_C = (1/\sqrt{2})[|001\rangle - |111\rangle]_{\mathrm{HFC}}$. (c) For the initial state $|\psi^+\rangle_{\mathrm{HF}}|0\rangle_C$,the theoretically predicted final state is $|\psi^+\rangle_{\mathrm{HF}}|0\rangle_C = (1/\sqrt{2})[|010\rangle + |100\rangle]_{\mathrm{HFC}}$. (d) For the initial state $|\psi^-\rangle_{HF}|0\rangle_C$the theoretically predicted final state is $|\psi^-\rangle_{\mathrm{HF}}|1\rangle_C = (1/\sqrt{2})[|011\rangle - |101\rangle]_{\mathrm{HFC}}$. (ii) Parity measurement (e) for the initial state $|\phi^+\rangle_{\mathrm{HF}}|0\rangle_C$, the theoretically predicted final state is $|\phi^+\rangle_{\mathrm{HF}}|0\rangle_C = (1/\sqrt{2})[|000\rangle + |110\rangle]_{\mathrm{HFC}}$. (f) For the initial state $|\phi^-\rangle_{\mathrm{HF}}|0\rangle_C$, the theoretically predicted final state is $|\phi^-\rangle_{\mathrm{HF}}|0\rangle_C = (1/\sqrt{2})[|000\rangle - |110\rangle]_{\mathrm{HFC}}$. (g) For the initial state $|\psi^+\rangle_{\mathrm{HF}}|0\rangle_C$,the theoretically predicted final state is $|\psi^+\rangle_{\mathrm{HF}}|1\rangle_C = (1/\sqrt{2})[|011\rangle + |101\rangle]_{\mathrm{HFC}}$. (h) For the initial state $|\psi^-\rangle_{\mathrm{HF}}|0\rangle_C$,the theoretically predicted final state is $|\psi^-\rangle_{\mathrm{HF}}|1\rangle_C = (1/\sqrt{2})[|011\rangle - |101\rangle]_{\mathrm{HFC}}$.

Encoding Phase and Parity Information



Natural entaglement

- Entanglement that is present ‘naturally’ in easily accessible states of certain systems (for example, in ground states or in thermal equilibrium)

- Natural question to ask:
  - How much is there? Can we quantify it?
  - How is it distributed in space?
  - Can we use it for anything?

Natural entaglement

Ising model:

$$H = -\sum_{\langle ij \rangle} J \sigma_z^i \sigma_z^j + B_z \sum_i \sigma_z^i \tag{11}$$

Transverse field Ising model

$$\mathcal{H} = -J \sum_i \sigma_z^i \sigma_z^{i+1} + B \sum_i \sigma_x^i \tag{12}$$

No non-trivial quantum dynamics:

$$[H, \sigma_z^i] = 0 \tag{13}$$

Each spin seperately a constant of the motion.

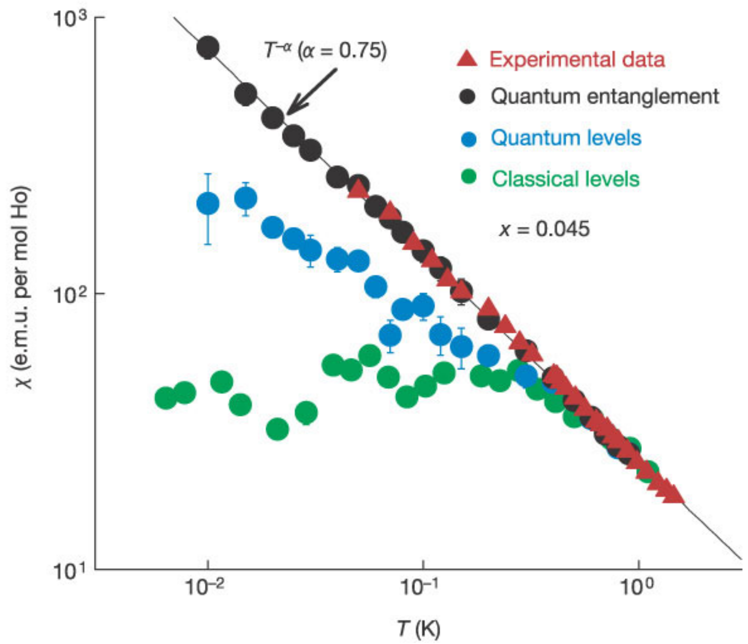
eg. B in x-direction:

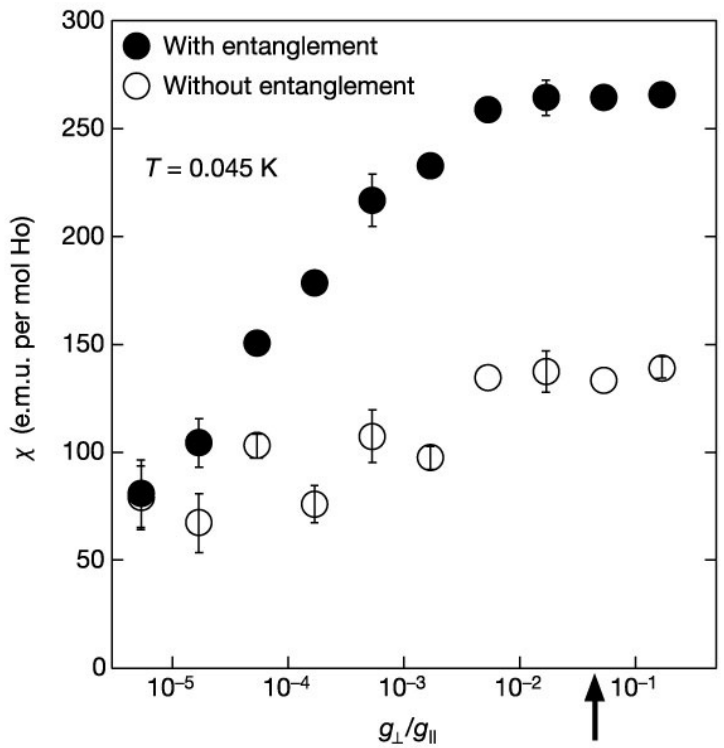
Low B: ground state ferromagnetic in z direction

Large B: ground state aligned in x direction

Two different domains seperated by a *quantum phase transition*

Effects of entanglement in susceptibitlity and g-factors





Ghosh, Rosenbaum, Aeppli, Coppersmith, *Nature*, 425, 48 (2003)

Heisenberg model

$$\mathcal{H} = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1} + B\sum_i S_z^i \quad (14)$$

Singlet (AF): $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Triplet:

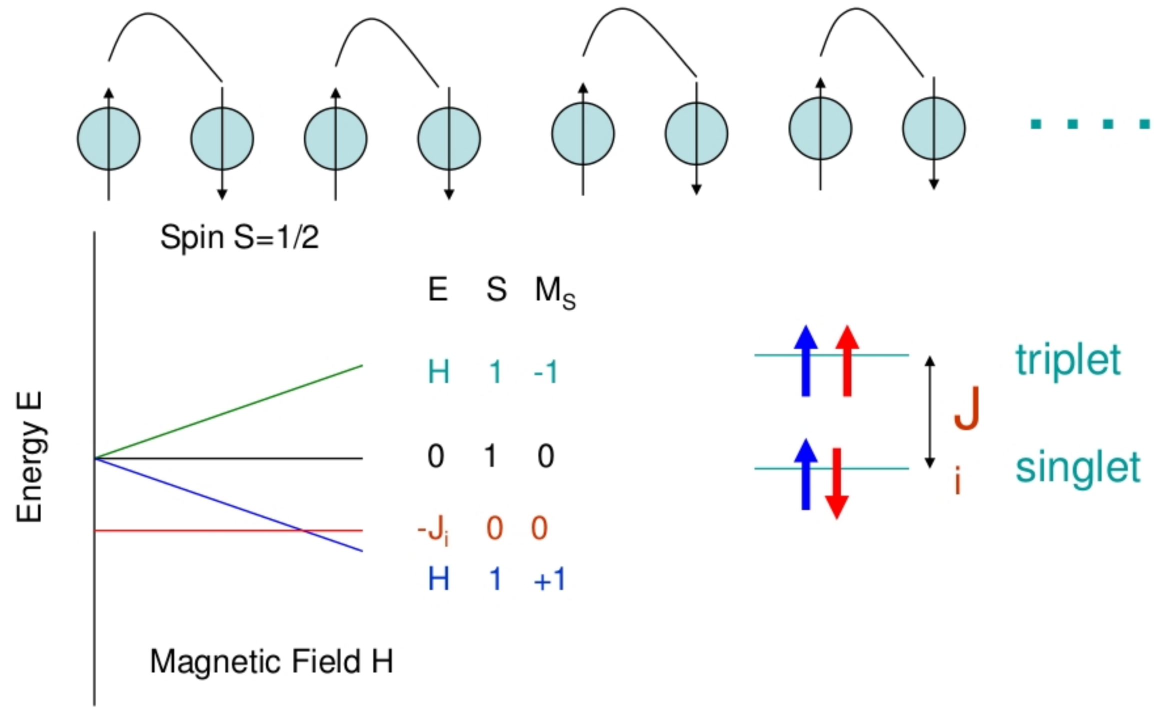
$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

$$|\uparrow\uparrow\rangle$$

$$|\downarrow\downarrow\rangle$$

No entanglement for ferromagnetic ground state!!

Exchange coupled pair model (Dimer)

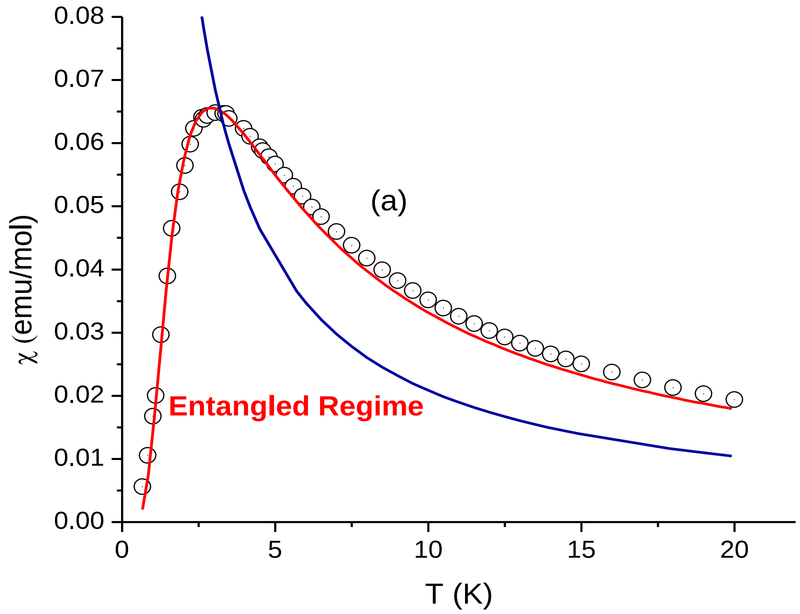


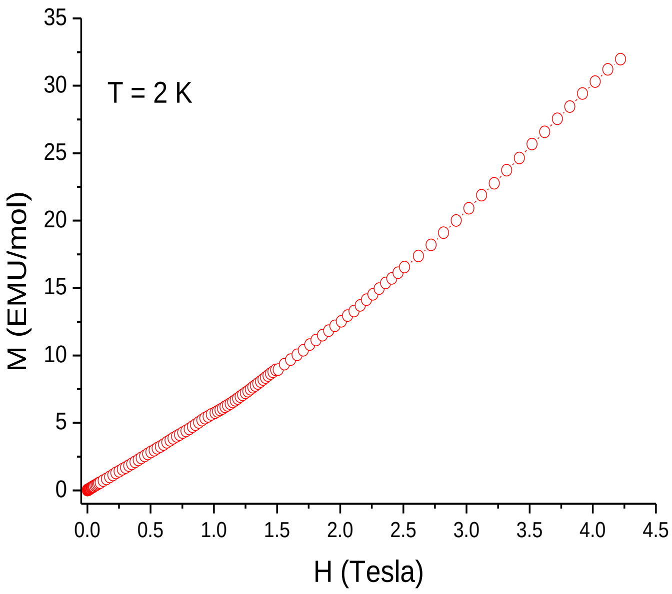
$$H = 2\sum_i J_i \vec{S}_i \cdot \vec{S}_{i+1} + g\mu B H \sum_i \vec{S}_i$$

Copper Nitrate (CN): a spin half Anti-Ferromagnet (Spin Chain) *$Cu(NO_3)_2 . 2.5H_2O$*

(Berger et al Phys Rev **132**, 1057 (1963))

Neel Temp, T_N= 4K





Panigrahi and Mitra, Jour Indian Institute of Science, **89**, 333-350(2009)

Analysis: Susceptibility as an Entanglement witness
(Bipartite systems)

$$\mathcal{H} = J\vec{S}_1 \cdot \vec{S}_2 + B(S_1^z + S_2^z)$$

$$S_i = \frac{\sigma_i}{2}; i = 1, 2, 3$$

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{Eigenvectors of } \sigma_3 \; are \qquad |\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\uparrow\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad |\downarrow\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad |\uparrow\downarrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The Hamiltonian for two qubit is written as:

$$\mathcal{H} = J\vec{S}_1 \cdot \vec{S}_2 + B(S_1^z + S_2^z)$$

$$= \frac{J}{4}(\sigma_1^x \cdot \sigma_2^x + \sigma_1^y \cdot \sigma_2^y + \sigma_1^z \cdot \sigma_2^z) + \frac{B}{2}(\sigma_1^z + \sigma_2^z)$$

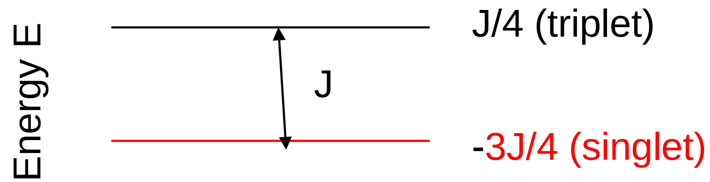
$$= \frac{J}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \frac{B}{2}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{J}{4}+B & 0 & 0 & 0 \\ 0 & -\frac{J}{4} & \frac{J}{2} & 0 \\ 0 & \frac{J}{2} & -\frac{J}{4} & 0 \\ 0 & 0 & 0 & \frac{J}{4}-B \end{bmatrix}$$

$$|\uparrow\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \frac{J}{4}+B \qquad |\downarrow\downarrow\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \frac{J}{4}-B$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}; \text{J/4} \qquad |\phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}; \text{-3J/4}$$

In the ground state (at low temperatures) the system is in the **pure** state and is in the state

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} ;\text{-3J}/4$$



$$\rho = \rho_{AB} = |\phi^-\rangle\langle\phi^-| = 1/2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho_A = Tr_B(\rho) = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Longrightarrow$$

Maximum Mixing

At finite temperatures the system is in a mixed state

$$\rho = \frac{1}{Z}\{|\phi^-\rangle\langle\phi^-|e^{\frac{3J}{4}\beta} + |\phi^+\rangle\langle\phi^+|e^{-\frac{J}{4}\beta} +$$
$$|\uparrow\uparrow\rangle\langle\uparrow\uparrow|e^{-(\frac{J}{4}-B)\beta} + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|e^{-(\frac{J}{4}+B)\beta}\}$$

$$\mathrm{z}{=}\mathrm{Tr}(\rho)$$

$$|\uparrow\uparrow\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}; \qquad |\downarrow\downarrow\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}; \quad |\phi^+\rangle = \tfrac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}; \qquad |\phi^-\rangle = \tfrac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix};$$

$$\rho = \frac{1}{Z}\begin{pmatrix} e^{-(\frac{J}{4}-B)\beta} & 0 & 0 & 0 \\ 0 & e^{-\frac{J}{4}\beta} + e^{\frac{3J}{4}\beta} & e^{-\frac{J}{4}\beta} - e^{\frac{3J}{4}\beta} & 0 \\ 0 & e^{-\frac{J}{4}\beta} - e^{\frac{3J}{4}\beta} & e^{-\frac{J}{4}\beta} + e^{\frac{3J}{4}\beta} & 0 \\ 0 & 0 & 0 & e^{-(\frac{J}{4}+B)\beta} \end{pmatrix}$$

At very high temperatures, $\beta \longrightarrow 0$, the density matrix, Reduces to

$$\rho = \rho_{AB} = \tfrac{1}{4}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} = \tfrac{1}{4}\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \tfrac{1}{4}\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$
$$+ \tfrac{1}{4}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix} + \tfrac{1}{4}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix} \Longrightarrow separable$$

$$\rho = \frac{1}{4}\begin{pmatrix}1&0\\0&0\end{pmatrix} \otimes \begin{pmatrix}1&0\\0&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&0\\0&1\end{pmatrix} \otimes \begin{pmatrix}1&0\\0&0\end{pmatrix}$$
$$+\frac{1}{4}\begin{pmatrix}1&0\\0&0\end{pmatrix} \otimes \begin{pmatrix}0&0\\0&1\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&0\\0&1\end{pmatrix} \otimes \begin{pmatrix}0&0\\0&1\end{pmatrix}$$

Panigrahi and Mitra, Jour Indian Institute of Science, **89**, 333-350(2009)

ρ is separable if it can be expressed as a convex sum of tensor product states of the two subsystems

There exists $\mathrm{p}_k \geq 0$, such that

$$\rho = \sum_k p_k \rho_1^k \otimes \rho_2^k, \text{ for } \sum_k p_k = 1, \{\rho_1^k\} \text{ and } \{\rho_2^k\}$$

Hence the system is perfectly separable

$$\langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle = \langle \sigma_x^1 \sigma_x^2 \rangle + \langle \sigma_y^1 \sigma_y^2 \rangle + \langle \sigma_z^1 \sigma_z^2 \rangle =$$
$$\langle \sigma_x^1 \rangle \langle \sigma_x^2 \rangle + \langle \sigma_y^1 \rangle \langle \sigma_y^2 \rangle + \langle \sigma_z^1 \rangle \langle \sigma_z^2 \rangle$$

(goes to zero, since the Pauli matrices are traceless)

Thermal Entanglement (intermediate temp)

$$\rho = \sum_i p_i \rho_i^1 \otimes \rho_i^2 \otimes \cdots \otimes \rho_i^N$$

$$U = \langle \mathcal{H} \rangle \qquad M = \sum_{i=1}^{N} \langle \sigma_z^i \rangle$$

$$|\langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle| = |\langle \sigma_x^1 \sigma_x^2 \rangle + \langle \sigma_y^1 \sigma_y^2 \rangle + \langle \sigma_z^1 \sigma_z^2 \rangle| =$$
$$|\langle \sigma_x^1 \rangle \langle \sigma_x^2 \rangle + \langle \sigma_y^1 \rangle \langle \sigma_y^2 \rangle + \langle \sigma_z^1 \rangle \langle \sigma_z^2 \rangle|$$
$$\leq |\vec{\sigma}_1||\vec{\sigma}_2| \leq 1$$

$$\frac{U-BM}{NJ} = \frac{1}{N} |\sum_{i=1}^{N} (\langle \sigma_x^i \sigma_x^{i+1} \rangle + \langle \sigma_y^i \sigma_y^{i+1} \rangle + \langle \sigma_z^i \sigma_z^{i+1} \rangle)| \leq 1$$

$$\frac{|U-BM|}{N|J|} > 1 \implies \text{entangled state}$$

Concurrence

$$\tilde{\rho}_{12} = \sigma_2 \otimes \sigma_2 \rho^*_{12} \sigma_2 \otimes \sigma_2$$

$$\mathcal{C} = max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})$$

[W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)]

In Ferromagnet it is zero

$$\vec{\sigma_i} \cdot \vec{\sigma_{i+1}} = {\sigma_i}^+ \cdot {\sigma_{i+1}}^- + {\sigma_i}^- \cdot {\sigma_{i+1}}^+ + 2\sigma_3$$

$$\langle {\sigma_i}^+ \cdot {\sigma_{i+1}}^- \rangle = \langle \uparrow\uparrow | \, {\sigma_i}^+ \cdot {\sigma_{i+1}}^- \, | \uparrow\uparrow \rangle = 0$$

For an Antiferromagnet $\frac{|U|}{N|J|} > 1 \implies \text{entangled state}$

$$\mathcal{C} = \frac{1}{2} \text{ max } [0, \frac{|U|}{NJ} - 1]$$

$$\frac{U}{NJ} = \quad |\langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle| = |\langle \sigma_x^1 \sigma_x^2 \rangle + \langle \sigma_y^1 \sigma_y^2 \rangle + \langle \sigma_z^1 \sigma_z^2 \rangle| =$$
$$|\langle \sigma_x^1 \rangle \langle \sigma_x^2 \rangle + \langle \sigma_y^1 \rangle \langle \sigma_y^2 \rangle + \langle \sigma_z^1 \rangle \langle \sigma_z^2 \rangle|$$
$$\leq |\vec{\sigma}_1||\vec{\sigma}_2| \leq 1$$

Isotropic system

$$\langle \sigma_x^1 \sigma_x^2 \rangle = \langle \sigma_y^1 \sigma_y^2 \rangle = \langle \sigma_z^1 \sigma_z^2 \rangle$$

$$\mathcal{C} = 2 \max [0, -\langle \vec{S}^1 \cdot \vec{S}^2 \rangle - (1/4)]$$

$$\langle \vec{S}^1 \cdot \vec{S}^2 \rangle = (\tfrac{-3}{4}) \tfrac{1-e^{-\beta J}}{1+3e^{-\beta J}}$$

B = 0 limit

$$\mathcal{C} = 2 \max [0, \tfrac{1-e^{-\beta J}}{1+3e^{-\beta J}}]$$

Susceptibility as an Entanglement Witness

$$\langle \vec{S}^1 \cdot \vec{S}^2 \rangle = (\frac{-3}{4}) \frac{1-e^{-\beta J}}{1+3e^{-\beta J}}$$

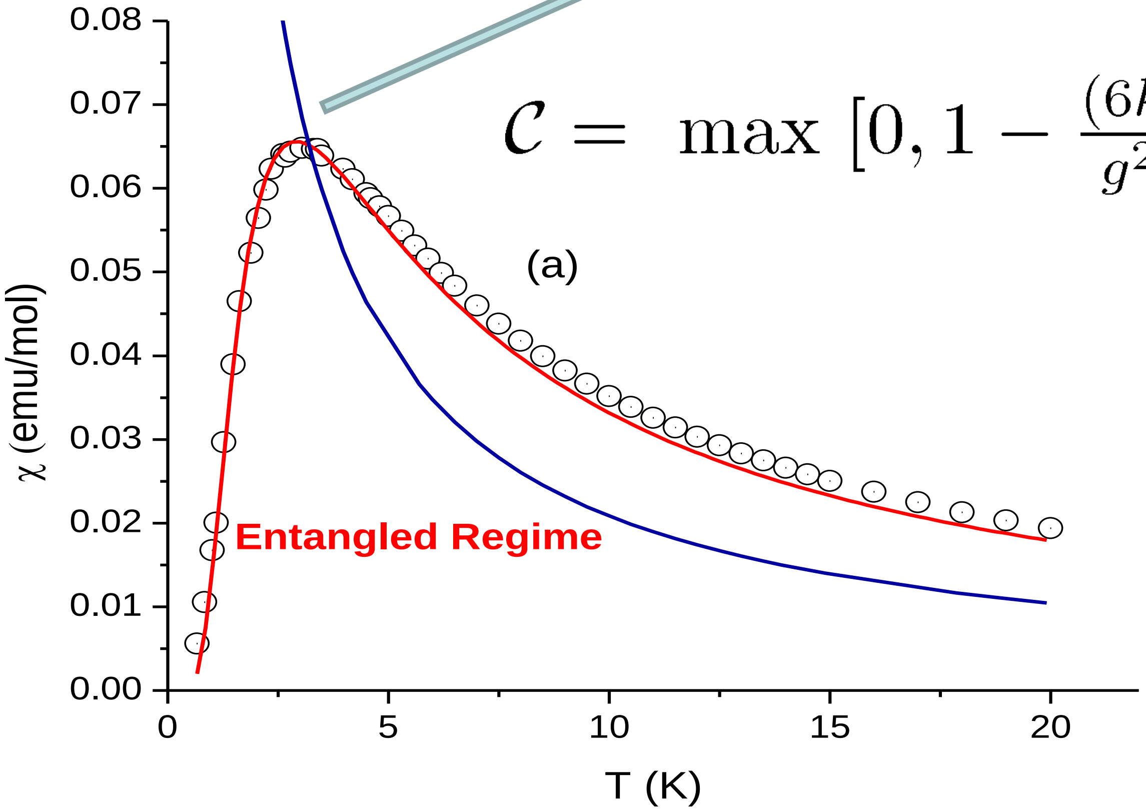
$$\chi = (\frac{\delta M}{\delta B}) = \frac{g^2 \mu_B^2 N}{k_B T} [\frac{1}{4} + (\langle \vec{S}^1 \cdot \vec{S}^2 \rangle)/3]$$

$$\mathcal{C} = \frac{1}{2} \max [0, \frac{|U|}{NJ} - 1]$$

$$\mathcal{C} = \max [0, 1 - \frac{(6 k_B T \chi)}{g^2 \mu_B^2 N}]$$

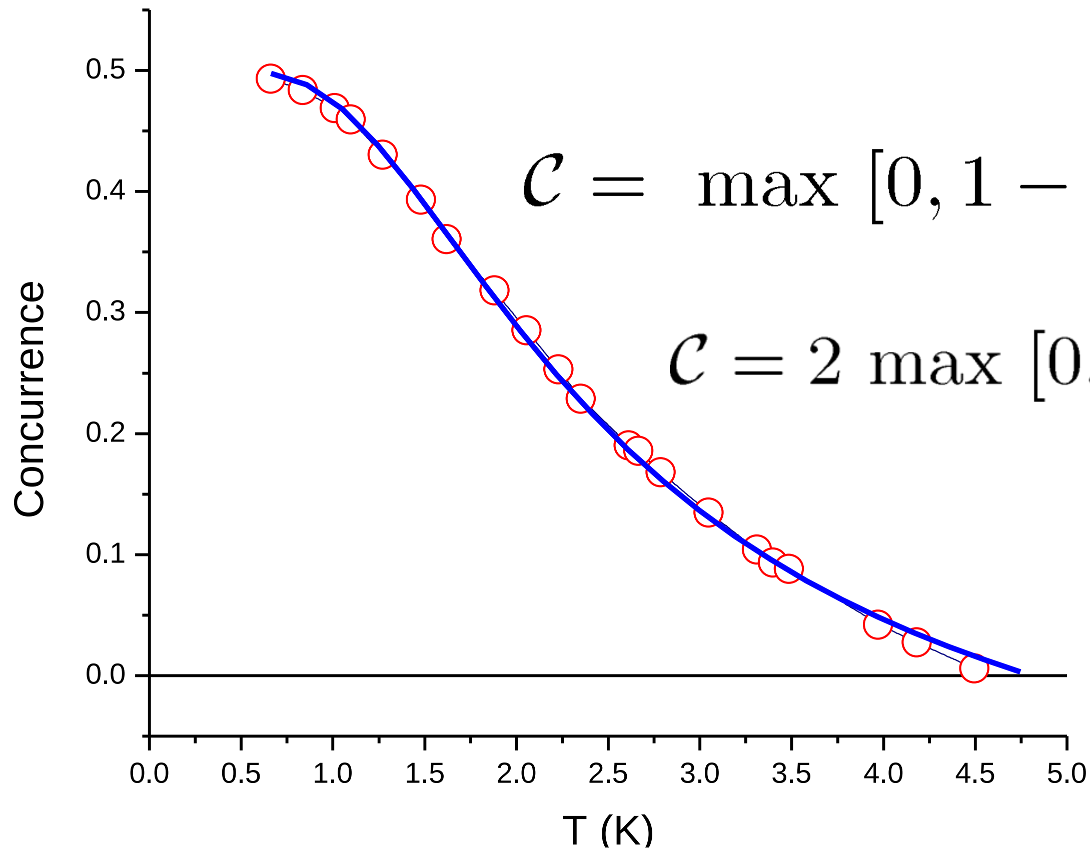
$$\chi \geq \left(\frac{1}{6}\right)\frac{g^2\mu_B^2 N}{k_B T} \implies \text{separable region}$$

$$\mathcal{C} = \max\left[0, 1 - \frac{(6k_B T\chi)}{g^2\mu_B^2 N}\right]$$



ArXiv: 1109.1640v2; Das, Chakraborty, Singh, Mitra

Concurrence in Copper Nitrate

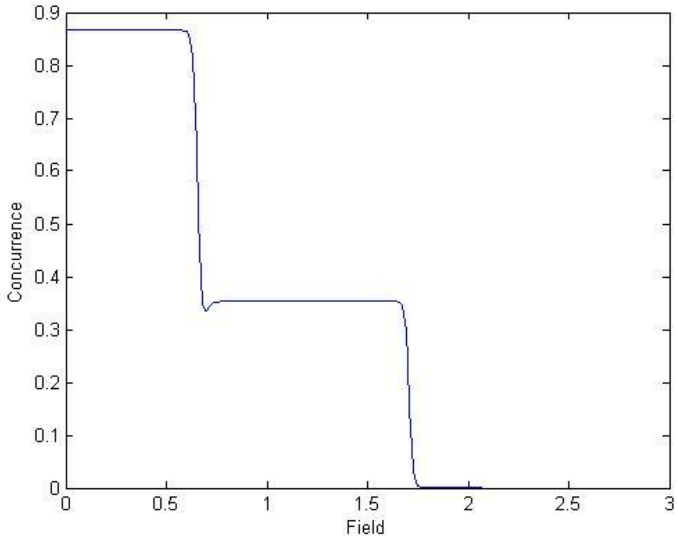


$$\mathcal{C} = \max\left[0, 1 - \frac{(6k_B T\chi)}{g^2\mu_B^2 N}\right]$$

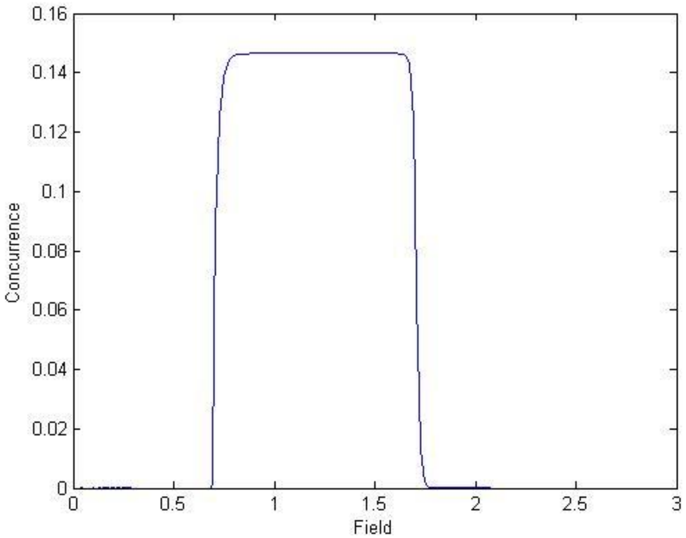
$$\mathcal{C} = 2\max\left[0, \frac{1-e^{-\beta J}}{1+3e^{-\beta J}}\right]$$

Panigrahi and Mitra, Jour Indian Institute of Science, **89**, 333-350(2009)

Heisenberg 1-2

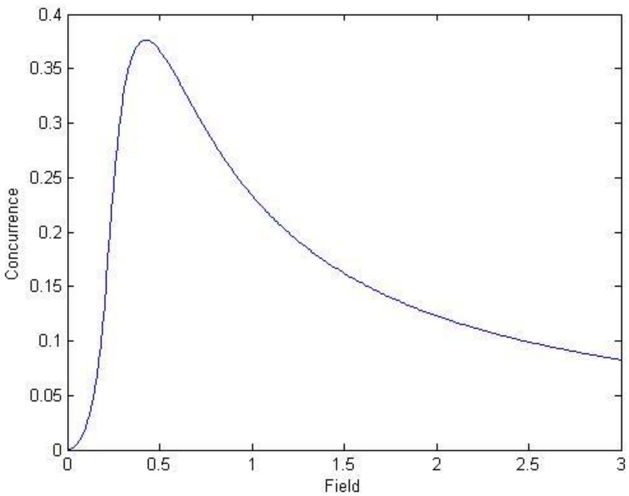


Heisenberg 1-4

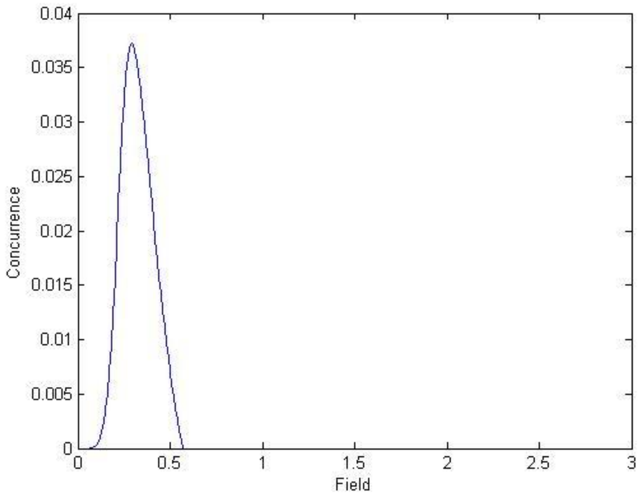


$$\mathcal{H} = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + B \sum_i S_i^z$$

Ising 1-2

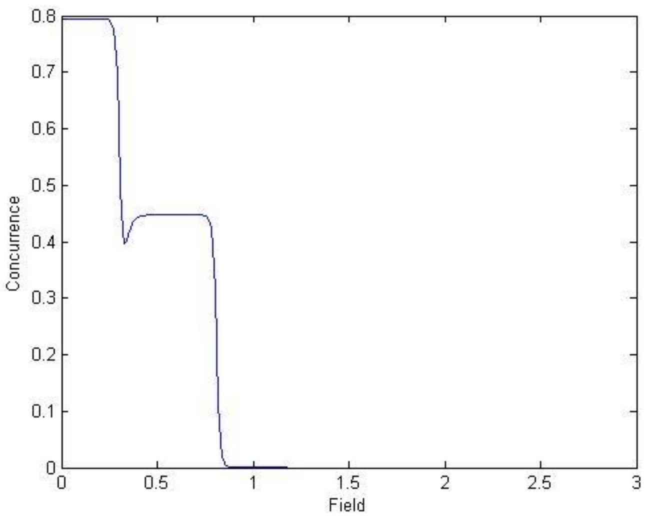


Ising 1-4

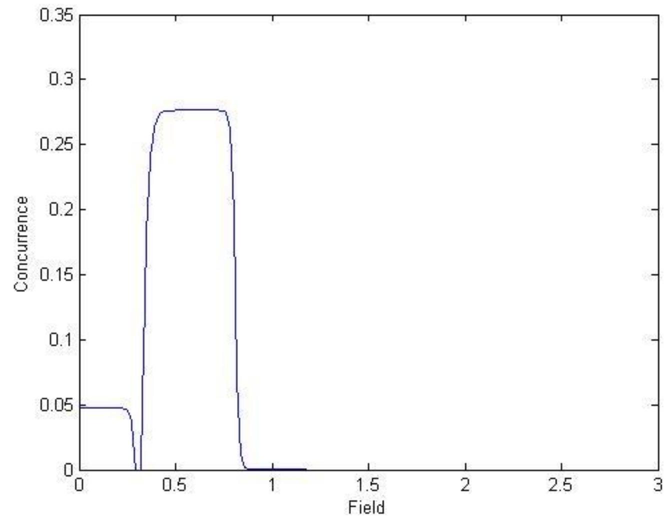


$$H = J\sum_i S_i^z . S_{i+1}^z + B\sum_i S_i^x$$

Heisenberg XY 1-2

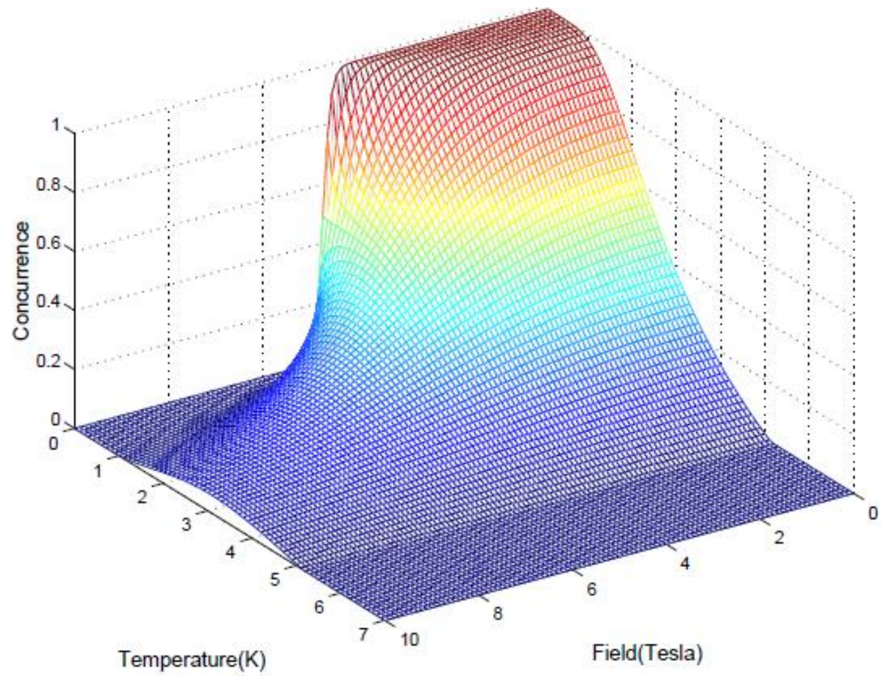


Heisenberg XY 1-4



$$H = J_x \sum_i S_i^x . S_{i+1}^x + J_y \sum_i S_i^y . S_{i+1}^y + B \sum_i S_i^z$$

Heisenberg XXX 1-2



Entangled channels, perfect teleportation, multi-electron quantum dots

- Generating N qubit entangled states teleporting an unknown state perfectly
- Desired states through suitable exchange interaction
- A multi electron quantum dot can be a possible realization for generating such N qubit states with high fidelity
- Effect of the nuclear spin environment on the fidelity of teleportation for a general N qubit entangled channel

D. D. B. Rao, S. Ghosh and PKP, Phys. Rev. A **78**, 042328 (2008)

Teleportation of 1-qubit states

- Teleportation of an unknown state through multi-particle entangled states
- In addition to the N-qubit GHZ states, W states form another class of entangled states
- W states from N-qubit interactions conserving z-component of the total spin of qubits
- Symmetric three qubit W state: $|\psi\rangle = \frac{1}{\sqrt{3}}\Big[|100\rangle + |010\rangle + |001\rangle\Big]$ fails to teleport the unknown state perfectly to Bob
- Modification of W-states $\longrightarrow$ Agarwal and Pati *et al.*, but not for quantum state sharing
- The modified W states can be connected to GHZ states by performing entangling operations on any two qubits
- Thus the modified W-state can be a key source for quantum protocols as different tasks can be performed as required

Teleportation of 1-qubit states

- We propose an experimental way of generating these states using exchange interaction between the qubits in quantum dot systems, and further generalize to N qubits.

- 3-qubit state:

$$|W_3\rangle = \alpha_1|100\rangle + \alpha_2|010\rangle + \alpha_3|001\rangle, \quad |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1 \tag{15}$$

- Bob has 1 qubit, Alice has 2. Alice to Bob: $|\psi\rangle = a|0\rangle + b|1\rangle$, with $|\alpha_3|^2 = |\alpha_1|^2 + |\alpha_2|^2$
Example:

$$\begin{aligned} |W_3\rangle &= \frac{1}{2}\Big[|100\rangle + |010\rangle + \sqrt{2}|001\rangle\Big] \\ \alpha_1 &= \frac{1}{\sqrt{2}}, \quad \alpha_2 = \frac{1}{\sqrt{2}}\sin\phi e^{\iota\chi_1}, \quad \alpha_3 = \frac{1}{\sqrt{2}}\cos\phi e^{\iota\chi_1}, \end{aligned}$$

with $0 \le \phi \le 2\pi$ and $0 \le \chi_1, \chi_2 \le 2\pi$

- Bob with the first qubit: $|\alpha_1|^2 = |\alpha_2|^2 + |\alpha_3|^2$ as different weights of the basis states

- A W-like state :

$$|\tilde{W}_3\rangle = \frac{1}{2}\Big[|100\rangle + |010\rangle + |001\rangle + |111\rangle\Big], \tag{16}$$

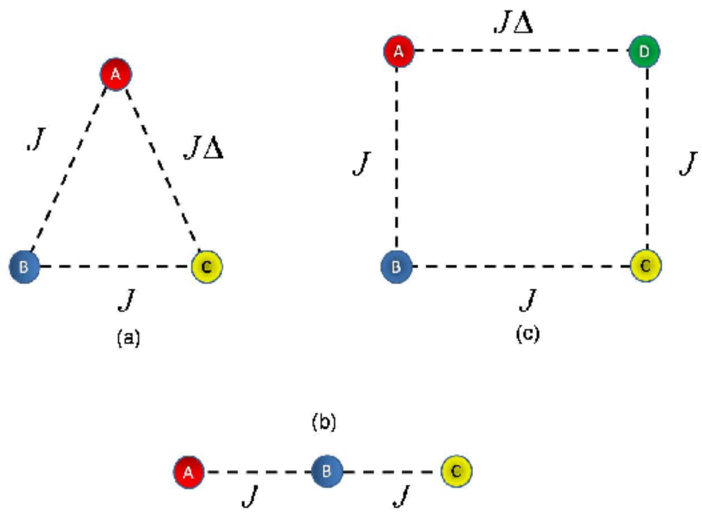
but does not conserve total $\hat{S}_z$

Generation of the $|W_3\rangle$ state

- Hamiltonian:

$$\mathcal{H} = J\vec{S}_A.\vec{S}_B + J\vec{S}_B.\vec{S}_C + J\Delta\vec{S}_A.\vec{S}_C, \quad J \sim 0.01eV \tag{17}$$

- $\nabla = 1 \rightarrow$ closed chain and $\nabla = 0 \rightarrow$ open chain: Different $|\tilde{W}_3\rangle$s



Representation of closed and open chains for 3 and 4 qubits interacting through Heisenberg exchange interaction. For $\Delta = 1$ the chain is perfectly closed and $\Delta = 0$ the chain is open.

Generation of the $|W_3\rangle$ state

- Initial 3-qubit state: $|\psi(0)\rangle = |100\rangle$ —itemAs $\mathcal{H}$ conserves $\hat{z}$ component of the total spin:

$$
\begin{aligned}
|\psi(t)\rangle &= \alpha_1(t)|100\rangle + \alpha_2(t)|010\rangle + \alpha_3(t)|001\rangle, \\
\alpha_1(t) &= \frac{1}{6}\left[2e^{-itE_1} + 3e^{-itE_2} + e^{-itE_3}\right], \\
\alpha_2(t) &= \frac{1}{3}\left[e^{-itE_1} - e^{-itE_3}\right], \\
\alpha_3(t) &= \frac{1}{6}\left[2e^{-itE_1} - e^{-itE_2} + e^{-itE_3}\right]
\end{aligned}
$$

- Eigenvalues: $E_1 = J(2+\Delta)/4, E_2 =_3 J/4, E_3 = (\Delta - 4)/4$

- Condition for perfect teleportation with Bob having the first qubit:

$$
3\cos(Jt\frac{1+2\Delta}{2}) + \cos\frac{3Jt}{2} + \frac{3}{2}\cos((1-\Delta)Jt) - 1 = 0, \qquad (18)
$$

- For $J = 0$ the 3-qubit state can perfectly teleport

- The roots of Eq.18 can specifically be found for open and closed chains

Generation of the $|W_3\rangle$ state

- For $\nabla = 0$, Solution: $Jt = \frac{2}{3}\cos^{-1}\left(-\frac{1}{8}\right)$ with periodicity of $4\pi/3$

  Also, $|\alpha_1(t)^2| = \frac{1}{2}$ and $|\alpha_2(t)|^2 = |\alpha_3(t)|^2 = \frac{1}{4}$

- Finally:

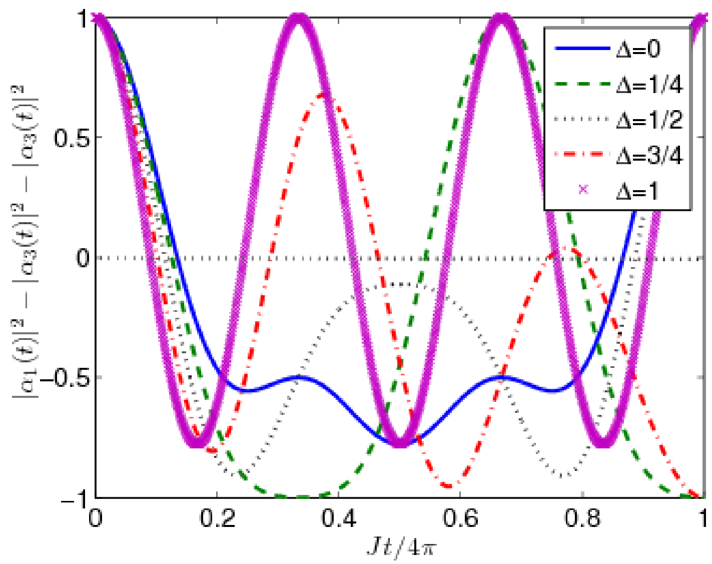
$$|\psi(t=\tau)\rangle = \frac{1}{2}\left[\sqrt{2}e^{\iota\phi_1}|100\rangle + e^{\iota\phi_2}|010\rangle + e^{\iota\phi_2}|001\rangle\right], \quad (19)$$

  where $\tau = \frac{2}{3}[2n\pi + \cos^{-1}(-\frac{1}{8})]$, and $\phi_1 = \tan^{-1}(-\sqrt{2})$ and $\phi_2 = \tan^{-1}(\frac{\sqrt{2}}{3})$

- First qubit to Bob $\longrightarrow |100\rangle$,

  last qubit to Bob $\longrightarrow |001\rangle$

Generation of the $|W_3\rangle$ state



We have plotted Eq. (18) as a function of time. The times at which the curves intersect with the horizontal line are the solutions to the non-linear equation given in Eq. (18). The asymmetry parameter Δ is varied from 0 (open) to 1 (perfectly closed). The times at which one can find solutions to Eq. (18) increases with decreasing Δ

- As $|\alpha_1|^2 = 1$, for any value of ∇,

$$|\psi(t = \tau_\Delta)\rangle = \frac{1}{2}\left[\sqrt{2}\mathrm{e}^{\iota\phi_1(\Delta)}|100\rangle + \mathrm{e}^{\iota\phi_2(\Delta)}|010\rangle + \mathrm{e}^{\iota\phi_3(\Delta)}|001\rangle\right] \tag{20}$$

A N-qubit teleporting channel with one magnon

- In teleporting a single qubit, states with at least one magnon excitation are required

- n magnon state:

$$|N;n\rangle = \sum_{i_1 i_2 \ldots\ldots i_n=1}^{N} C_{i_1 i_2 \cdots i_n} |i_1 \cdots i_7 \cdots i_n\rangle, \qquad \sum_{i_1 i_2 \ldots\ldots i_n=1}^{N} |C_{i_1 i_2 \cdots i_n}|^2 = 1 \tag{21}$$

- We shall consider all the complex coefficients to be some phase factors only

- One qubit to Bob and keep the remaining $N1$ qubits with Alice. Initially:

$$|\psi\rangle_i = (\alpha|0\rangle + \beta|1\rangle) \otimes \sum_{i=1}^{N} C_i |i\rangle. \tag{22}$$

- Alice and Bob qubits: $\sum_{i=1}^{N} C_i |i\rangle = \sum_{i=1}^{N-1} C_i |i\rangle |0\rangle + C_N |00 \cdots 0\rangle |1\rangle$

A N-qubit teleporting channel with one magnon

- Now:

$$|\psi\rangle_i = \frac{1}{2}\Big[(|\xi_1\rangle + |\xi_4\rangle)(\alpha|0\rangle + \beta|1\rangle) + (|\xi_1\rangle - |\xi_4\rangle)(\alpha|0\rangle - \beta|1\rangle)$$
$$+ (|\xi_2\rangle + |\xi_3\rangle)(\beta|0\rangle + \alpha|1\rangle) + (|\xi_2\rangle - |\xi_4\rangle)(-\beta|0\rangle + \alpha|1\rangle)\Big],$$
$$|\xi_1\rangle = |0\rangle \sum_{i=1}^{N-1} C_i|i\rangle, \quad |\xi_4\rangle = |1\rangle|00\cdots 0\rangle, \quad |\xi_3\rangle = |1\rangle \sum_{i=1}^{N-1} C_i|i\rangle \tag{23}$$

- Orthonormal basis for Alice: $|\xi_1\rangle \pm |\xi_4\rangle$ and $|\xi_1\rangle \pm |\xi_4\rangle \quad \longrightarrow \sum_{i=1}^{N-1} |C_i|^2 = |C_N|^2$

- Then the single qubit teleporting state is:

$$|\psi\rangle_C = \frac{1}{\sqrt{2(N-1)}} \sum_{j=1}^{N-1} e^{i2\pi j/N}|j\rangle|0\rangle \pm \frac{1}{\sqrt{2}}|00\cdots 0\rangle|1\rangle. \tag{24}$$

- For N = 2 this is the usual Bell state and for $N = 3$ this is the modified W state

- Deterministic teleportation using the above class of states can be possible only for $N = 2$, as Alice has the complete orthonormal basis for measurement

A N-qubit teleporting channel with one magnon

- A generalization:

$$|\psi\rangle_C = \frac{1}{\sqrt{2\left[{}^{N-1}C_n\right]}}\left[\sum_{j_1 j_2 .. j_n=1}^{N-1} |j_1 j_2 \cdots j_n\rangle|0\rangle \pm \sqrt{\frac{N-n}{n}} \sum_{j_1 j_2 .. j_{n-1}=1}^{N-1} |j_1 j_2 \cdots j_{n-1}\rangle|1\rangle\right] \quad (25)$$

- ${}^{N-1}C_n = (N-1)!/n!(N-n-1)!$

- Only when N = 2, Alice can perform a complete orthonormal basis measurement

Generation from Hamiltonian dynamics

- By switching the exchange interaction on between the qubits
- Interaction Hamiltonian:

$$\mathcal{H} = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}, \quad N+1 = 1 \tag{26}$$

with 1-magnon eigenstate $|k\rangle = \sum_n \mathrm{e}^{ikn}|n\rangle$, where $k = 2\pi\lambda/N$, $\lambda = 0, 1, 2 \cdots N-1$ and $|n\rangle$ represents the site number at which the spin is flipped
Eigenvalues: $E_k = J(1 - \cos k)$

- For $|\psi(0)\rangle = |100 \cdots 0\rangle$,

$$|\psi(t)\rangle = \sum_k \sum_n \mathrm{e}^{iE_k t} e^{ik(n-1)}|n\rangle \tag{27}$$

- For perfect teleportation:

$$\left| \sum_k \mathrm{e}^{iE_k t} \right|^2 - \sum_{n=2}^{N} \left| \sum_k \mathrm{e}^{iE_k t} e^{ik(n-1)} \right|^2 = 0. \tag{28}$$

Generation from Hamiltonian dynamics

- For $\left|\sum_k \mathrm{e}^{iE_k(t=\tau)}\right|^2 = 1/2$, for finite N: $\tau = \frac{2}{3J}\cos^{-1}\left(-\frac{1}{8}\right)$

- For large N:

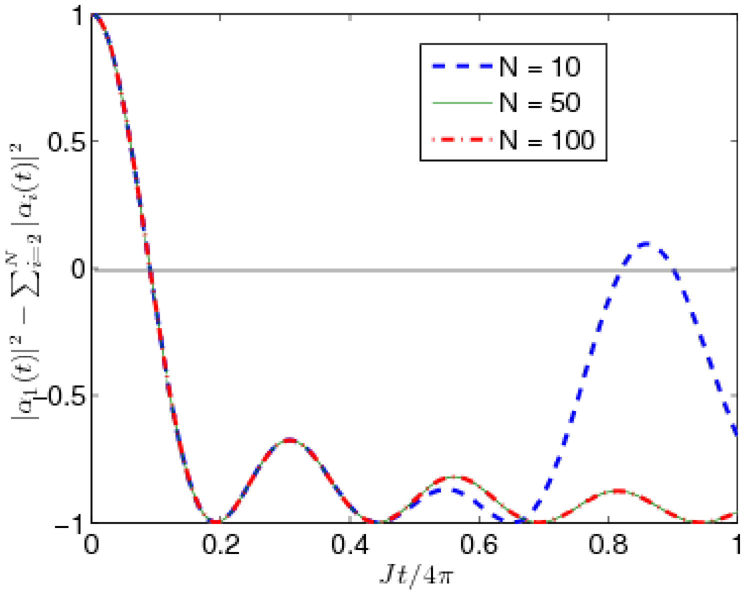
$$\sum_{p=0}^{N-1} \mathrm{e}^{iJt\cos(2\pi p/N)} \approx \frac{N}{2\pi}\int_0^{2\pi} dx \mathrm{e}^{iJt\cos x} = N\mathcal{J}_0(Jt) \tag{29}$$

- Existence of solution to Eq.27: $\mathcal{J}_0(Jt) = 1/\sqrt{2}$

- There will no revival of $|\alpha_1(t)|^2$ close to 1/2 as N becomes large: Only one solution at large N

- A more general condition of generating the N qubit entangled channels to teleport one qubit perfectly across a chain of non interacting quantum dots

Generation from Hamiltonian dynamics



We have plotted Eq. 28 as a function of time for different values of N. The times at which the curves intersect with the horizontal line are the solutions to the non-linear equation given in the above equation. In large N limit, there are no strong revivals indicating that there can exist only one solution corresponding to the first intersection with the horizontal line. Though for finite N there will always be revivals, the revival time increase drastically with N

DECOHERENCE FROM NUCLEAR SPIN ENVIRONMENTS

- In quantum dot systems, interaction of the electron spin with nuclear spin bath gives the dominant contribution to its decoherence

- N qubit entangled states are generated at the order of few pico seconds and then separated, only the effects of individual environments for each spin need to be considered

- The system bath interaction:

$$\mathcal{H}_{SE} = \sum_{ij} K_i \vec{S}_i \cdot \vec{I}_i, \quad \vec{I}_i = \sum_k \vec{I}_{i,k} \tag{30}$$

- For the initial state of the bath to be completely unpolarized, the time evolution of each of the N qubit teleporting channels:

$$\begin{aligned} \rho_N(t) \quad = \quad & \mathrm{Tr}_{I_1, I_2 \cdots I_N} \sum_{i,j} \Big\{ \Big(a_i(t) + b_i(t) \vec{S}_i \cdot \vec{I}_i \Big) \\ & |\psi_N\rangle\langle\psi_N| \otimes \rho_B(0) \ \Big(a_j^*(t) + b_j^*(t) \vec{S}_j \cdot \vec{I}_j \Big) \Big\}, \end{aligned}$$

where $a_i = \cos \Lambda_i t + i K_i \sin \Lambda_i t / 2\Lambda_i$ and $b_i = 2 i K_i \sin \Lambda_i t / \Lambda_i$, where $2\Lambda_i = K_i (I_i + 1/2)$

DECOHERENCE FROM NUCLEAR SPIN ENVIRONMENTS

- The state $|\psi\rangle = \frac{1}{2}\Big[\sqrt{2}|100\rangle + e^{\iota\phi}|010\rangle + e^{\iota\phi}|001\rangle\Big]$ can be rewritten in terms of the spin operators for each qubit as:

$$\begin{aligned}\rho \;=\; & \frac{1}{8}\Big[\mathcal{I} - (S_B^z + S_C^z)(\mathcal{I} + 2S_A^z) - \sqrt{2}e^{i\phi}(S_A^+ S_B^- + S_A^+ S_C^-) \\ & + S_B^+ S_C^- - 2\sqrt{2}e^{i\phi}(S_A^+ + S_B^- S_C^z + S_A^+ S_B^z S_C^-) \\ & - 2S_A^z S_B^+ S_C^- + 8S_A^z S_B^z S_C^z + h.c\Big], \qquad S^{\pm} = S^x \pm iS^y \end{aligned} \tag{31}$$

- The state at any later time obtained after tracing out the bath degrees of freedom:

$$\vec{S}_i(t) = \vec{S}_i(0)\left\{\frac{1}{3} + \frac{2}{3}\left(1 - \frac{t^2}{\tau_i^2}\right)e^{-t^2/2\tau_i^2}\right\}, \qquad \tau_i = \frac{2}{K_i\sqrt{N_i}} \tag{32}$$

- $S_A^z S_b^x S_C^y$ decay much faster (decay rate $\tau_1 + \tau_2 + \tau_3$) than $S_A^x S_B^y$ (decay rate $\tau_1 + \tau_2$) $\longrightarrow$ The state still can be entangled

- The GHZ state with three particle correlations, becomes completely mixed quickly

- The modified W states can teleport unknown states with better fidelity than GHZ states

- The higher order correlations decay faster: Need control schemes with longer entanglement preserving of multi-qubit states

Exchange coupled pair model (Tetramer)

The Heisenberg chain for four particles is given by

$$\mathcal{H} = \sum_{i=1}^{3} \Delta_a \vec{S}_i \cdot \vec{S}_{i+1} + \Delta_b \vec{S}_4 \cdot \vec{S}_1$$

The eigenspectrum and the eigenstates of the system are:

$$\begin{aligned}
\lambda_a &\equiv \lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{2}(-\Delta_a + \Delta_b) \\
\lambda_b &\equiv \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = \frac{1}{2}(3\Delta_a + \Delta_b) \\
\lambda_c &\equiv \lambda_9 = \frac{1}{2}(-3\Delta_a - \Delta_b - 2p) \\
\lambda_d &\equiv \lambda_{10} = \frac{1}{2}(-3\Delta_a - \Delta_b + 2p) \\
\lambda_e &\equiv \lambda_{11} = \lambda_{12} = \lambda_{13} = \frac{1}{2}(-\Delta_a - \Delta_b - 2q) \\
\lambda_f &\equiv \lambda_{14} = \lambda_{15} = \lambda_{16} = \frac{1}{2}(-\Delta_a - \Delta_b + 2q)
\end{aligned}$$

Exchange coupled pair model (Tetramer)

Here $p = \sqrt{3\Delta_a^2 + \Delta_b^2}$ and $q = \sqrt{2\Delta_a^2 + 2\Delta_a\Delta_b + \Delta_b^2}$. The eigenvectors are given by:

$$
\begin{aligned}
|1\rangle &= |0001\rangle - |0010\rangle - |0100\rangle + |1000\rangle \\
|2\rangle &= |0110\rangle - |1001\rangle \\
|3\rangle &= |0111\rangle - |1011\rangle - |1101\rangle + |1110\rangle \\
|4\rangle &= |0000\rangle \\
|5\rangle &= |0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle \\
|6\rangle &= |0011\rangle + |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle \\
|7\rangle &= |0111\rangle + |1011\rangle + |1101\rangle + |1110\rangle \\
|8\rangle &= |1111\rangle \\
|9\rangle &= |0011\rangle - \frac{2\Delta_a + p}{\Delta_a + \Delta_b}|0101\rangle + \frac{\Delta_a - \Delta_b + p}{\Delta_a + \Delta_b}|0110\rangle + \frac{\Delta_a - \Delta_b + p}{\Delta_a + \Delta_b}|1001\rangle + \frac{2\Delta_a + p}{\Delta_a + \Delta_b}|1010\rangle \\
&\quad + |1100\rangle
\end{aligned}
$$

Exchange coupled pair model (Tetramer)

$$
\begin{aligned}
|10\rangle &= |0011\rangle - \frac{2\Delta_a + p}{\Delta_a + \Delta_b}|0101\rangle - \frac{-\Delta_a - \Delta_b + p}{\Delta_a + \Delta_b}|0110\rangle - \frac{-\Delta_a - \Delta_b + p}{\Delta_a + \Delta_b}|1001\rangle - \frac{2\Delta_a + p}{\Delta_a + \Delta_b}|1010\rangle \\
&\quad + |1100\rangle \\
|11\rangle &= |0001\rangle - \frac{\Delta_a + \Delta_b + q}{\Delta_a}|0010\rangle + \frac{\Delta_a - \Delta_b + q}{\Delta_a}|0100\rangle - |1000\rangle \\
|12\rangle &= |0011\rangle - \frac{\Delta_a + q}{\Delta_a - \Delta_b}|0101\rangle + \frac{\Delta_a + q}{\Delta_a - \Delta_b}|1010\rangle - |1100\rangle \\
|13\rangle &= |0111\rangle - \frac{\Delta_a - \Delta_b + q}{\Delta_a}|1011\rangle + \frac{\Delta_a - \Delta_b + q}{\Delta_a}|1101\rangle - |1110\rangle \\
|14\rangle &= |0001\rangle + \frac{-\Delta_a + \Delta_b + q}{\Delta_a}|0010\rangle - \frac{-\Delta_a + \Delta_b + q}{\Delta_a}|0100\rangle - |1000\rangle \\
|15\rangle &= |0011\rangle + \frac{-\Delta_a + q}{\Delta_a - \Delta_b}|0101\rangle + \frac{\Delta_a - q}{\Delta_a - \Delta_b}|1010\rangle - |1100\rangle \\
|16\rangle &= |0111\rangle + \frac{-\Delta_a + \Delta_b + q}{\Delta_a}|1011\rangle - \frac{-\Delta_a + \Delta_b + q}{\Delta_a}|1101\rangle - |1110\rangle
\end{aligned}
$$

Coupling dependent entanglement

Consider a general four particles - two magnon state:

$$|\psi\rangle = \frac{1}{N_\psi}\Big[a\,|0011\rangle + b\,|0101\rangle + c\,|0110\rangle + d\,|1001\rangle + e\,|1010\rangle + f\,|1100\rangle\Big].$$

where, $N_\psi = \sqrt{|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |f|^2}$ is the normalization. The two particle reduced density matrix ρ_{12} for the partition 12|34 will be:

$$\rho_{12} = \frac{1}{N_\psi^2}\begin{pmatrix} |a|^2 & 0 & 0 & 0 \\ 0 & |b|^2 + |c|^2 & bd^* + ce^* & 0 \\ 0 & b^*d + c^*e & |d|^2 + |e|^2 & 0 \\ 0 & 0 & 0 & |f|^2 \end{pmatrix}$$

which is of the form:

$$\rho' = \frac{1}{N_\psi^2}\begin{pmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & w_3^* & 0 \\ 0 & w_3 & w_4 & 0 \\ 0 & 0 & 0 & w_5 \end{pmatrix}.$$

The von-Neumann entropy of ρ_{12} is given by

$$S(\rho_{12}) = -\sum_{i=1}^{4} \lambda_i \log \lambda_i.$$

where, λ are the eigenvalues of ρ_{12} and are given as

$$\begin{aligned}
\lambda_1 &= \left(|a|^2\right)/N_\psi^2 \\
\lambda_2 &= \left(|b|^2+|c|^2-\left(|bd|^2+b^*cde^*+bc^*d^*e+|ce|^2\right)\right)/N_\psi^2 \\
\lambda_3 &= \left(|d|^2+|e|^2-\left(|bd|^2+b^*cde^*+bc^*d^*e+|ce|^2\right)\right)/N_\psi^2 \\
\lambda_1 &= \left(|f|^2\right)/N_\psi^2
\end{aligned}$$

The concurrence is given by

$$C = \max\left(0, \sqrt{\Lambda_1}-\sqrt{\Lambda_2}-\sqrt{\Lambda_3}-\sqrt{\Lambda_4}\right)$$

where, Λ are the eigenvalues of

$$\tilde{\rho}_{12} = \rho_{12}\sigma_y\otimes\sigma_y\rho_{12}^*\sigma_y\otimes\sigma_y = \frac{1}{N_\psi^4}\begin{pmatrix} |f|^2 & 0 & 0 & 0 \\ 0 & |d|^2+|e|^2 & b^*d+c^*e & 0 \\ 0 & bd^*+ce^* & |b|^2+|c|^2 & 0 \\ 0 & 0 & 0 & |a|^2 \end{pmatrix},$$

such that $\Lambda_1 \geq \Lambda_2 \geq \Lambda_3 \geq \Lambda_4$. The eigenvalues of $\tilde{\rho}_{12}$ are thus given by

$$\begin{aligned}
\Lambda_1 &= \left(|a|^2\right)/N_\psi^4 \\
\Lambda_2 &= \left(|b|^2+|c|^2-\left(|bd|^2+b^*cde^*+bc^*d^*e+|ce|^2\right)\right)/N_\psi^4 \\
\Lambda_3 &= \left(|d|^2+|e|^2-\left(|bd|^2+b^*cde^*+bc^*d^*e+|ce|^2\right)\right)/N_\psi^4 \\
\Lambda_1 &= \left(|f|^2\right)/N_\psi^4
\end{aligned}$$

Generation of a 4 particle - 2 magnon state

The time evolution of an initial state $|\phi\rangle(0)$ under the Hamiltonian is given as,

$$\begin{aligned} |\phi(t)\rangle &= a_4(t)\,|0011\rangle + a_6(t)\,|0101\rangle + a_7(t)\,|0110\rangle \\ &\quad + a_{10}(t)\,|1001\rangle + a_{11}(t)\,|1010\rangle + a_{13}(t)\,|1100\rangle \end{aligned}$$

where,

$$\begin{aligned} a_4(t) &= \left\{e^{\imath tD_1}a_4(0) - e^{\imath tD_3}a_7(0) - e^{\imath tD_4}a_{10}(0) + e^{\imath tD_5}a_{11}(0) + e^{\imath tD_6}a_{13}(0)\right\} \\ a_6(t) &= \left\{e^{\imath tD_1}a_4(0) + M^+e^{\imath tD_3}a_7(0) + M^-e^{\imath tD_4}a_{10}(0) + P^-e^{\imath tD_5}a_{11}(0) \right. \\ &\quad \left. + P^+e^{\imath tD_6}a_{13}(0)\right\} \\ a_7(t) &= \left\{e^{\imath tD_1}a_4(0) - e^{\imath tD_2}a_6(0) + Q^-e^{\imath tD_5}a_{11}(0) + Q^+e^{\imath tD_6}a_{13}(0)\right\} \\ a_{10}(t) &= \left\{e^{\imath tD_1}a_4(0) + e^{\imath tD_2}a_6(0) + Q^-e^{\imath tD_5}a_{11}(0) + Q^+e^{\imath tD_6}a_{13}(0)\right\} \\ a_{11}(t) &= \left\{e^{\imath tD_1}a_4(0) - M^+e^{\imath tD_3}a_7(0) - M^-e^{\imath tD_4}a_{10}(0) + P^-e^{\imath tD_6}a_{11}(0) \right. \\ &\quad \left. + P^+e^{\imath tD_7}a_{13}(0)\right\} \\ a_{13}(t) &= \left\{e^{\imath tD_1}a_4(0) + e^{\imath tD_3}a_7(0) + e^{\imath tD_4}a_{10}(0) + e^{\imath tD_5}a_{11}(0) + e^{\imath tD_6}a_{13}(0)\right\} \end{aligned}$$

where,

$$
\begin{aligned}
M^{\pm} &= \frac{\frac{\Delta_a \mp \Delta_b}{2} \mp \sqrt{2\Delta_a^2 - 2\Delta_a\Delta_b + \Delta_b^2}}{\Delta_a - \Delta_b} \pm \frac{1}{2} \\
P^{\pm} &= \frac{-\frac{\Delta_a - \Delta_b}{2} - \frac{3\Delta_a + \Delta_b}{2} \pm \sqrt{3\Delta_a^2 + \Delta_b^2}}{\Delta_a + \Delta_b} \\
Q^{\pm} &= \frac{-\frac{\Delta_a - 3\Delta_b}{2} + \frac{3\Delta_a + \Delta_b}{2} \pm \sqrt{3\Delta_a^2 + \Delta_b^2}}{\Delta_a + \Delta_b} \\
D_1 &= \frac{3\Delta_a + \Delta_b}{2} \\
D_2 &= \frac{\Delta_b - \Delta_a}{2} \\
D_3 &= \sqrt{2\Delta_a^2 - 2\Delta_a\Delta_b + \Delta_b^2} - \frac{\Delta_b + \Delta_a}{2} \\
D_4 &= -\sqrt{2\Delta_a^2 - 2\Delta_a\Delta_b + \Delta_b^2} - \frac{\Delta_b + \Delta_a}{2} \\
D_5 &= \sqrt{3\Delta_a^2 + \Delta_b^2} - \frac{\Delta_b + 3\Delta_a}{2} \\
D_6 &= -\sqrt{3\Delta_a^2 + \Delta_b^2} - \frac{\Delta_b + 3\Delta_a}{2}
\end{aligned}
$$

Common bath decoherence

- Central spin model to study the effect of spin bath on the qubits used for teleportation
- nuclear spins interacting through homogeneous Heisenberg interaction with system spins
- The Hamiltonian: $H = K_a \vec{S}_a \cdot \vec{I}_{\mathcal{E}a} + K_A \vec{S}_A \cdot \vec{I}_{\mathcal{E}A} + K_B \vec{S}_B \cdot \vec{I}_{\mathcal{E}B}, \qquad \vec{I}_{\mathcal{E}} = \sum_k \vec{I}_{\mathcal{E},k}$
- Common spin bath or Separate baths
- Common bath can induce entanglement between two initially unentangled qubits
- The most general representation of a two-qubit Bell state:

$$\rho_{AB} = \frac{1}{4}\hat{\mathcal{I}} + \sum_k D_k S_A^k S_B^k, \tag{33}$$

- Correlation vector: $D_k \equiv \mathrm{Tr}(\rho_{AB} S_A^k S_B^k)$
- For Bell states: $\vec{D}_{S_0} = [-1,-1,-1]$, $\vec{D}_{T_0} = [1,1,-1]$, $\vec{D}_{T_+} = [1,-1,1]$, and $\vec{D}_{T_-} = [-1,1,1]$
- The states are: $|S_0\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow - \downarrow\uparrow\rangle]$, $|T_0\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow + \downarrow\uparrow\rangle]$, $|T_+\rangle = \frac{1}{\sqrt{2}}[|\uparrow\uparrow + \downarrow\downarrow\rangle]$ and $|T_-\rangle = \frac{1}{\sqrt{2}}[|\uparrow\uparrow - \downarrow\downarrow\rangle]$

D. D. B. Rao, PKP and C. Mitra, Phys. Rev. A **78**, 022336 (2008)

Decoherence free Teleportation

- The initial state:

$$\rho_{aAB} = \rho_a \otimes \rho_{AB} = \frac{1}{2}\left[\hat{\mathcal{I}} + 2\vec{P}_a \cdot \vec{S}_a\right] \otimes \frac{1}{4}\left[\hat{\mathcal{I}} + 4\sum_k D_k S_A^k S_B^k\right], \quad \vec{P}_a = \mathrm{Tr}\rho_a \vec{S}_a \qquad (34)$$

- Alice's measurement: $\mathrm{Tr}_{aA}[\rho_{aA} \otimes \hat{\mathcal{I}}_B \rho_{aAB}]$, where $\rho_{aA} = \frac{1}{4}\hat{\mathcal{I}} + \sum_k M_k S_a^k S_A^k$

- M has the information of the Bell measurement made by Alice

- Bob gets $\rho_B = \frac{1}{2}\left[\hat{\mathcal{I}} + 2\vec{P}_B \cdot \vec{S}_B\right]$ with probability 1/4

- Polarizations are related as: $P_B^i = D_i M_i P_a^i$

- $\vec{D} = \vec{M}$: Bob needs to do nothing
  $(\vec{D} \times \vec{M}) \cdot \hat{n} \neq 0$: Bob has to do a rotation along $\hat{n} \longrightarrow$ multiplying $D_i M_i$ to P_B^i in Eq.**??**

- $\rho_B = \rho_a$: Bob is the unknown state that Alice wishes to teleport

Common bath decoherence for Alice's qubits

- The Hamiltonian reduces to:

$$H = (K_a\vec{S}_a + K_A\vec{S}_A)\cdot\vec{I}_{\mathcal{E}}, \tag{35}$$

  where $\vec{I}_{\mathcal{E}}$ represents the total bath spin

- Interaction with the bath can result in an indirect coupling between the two: Entagle-ment between Alices qubits

- Initial state of the Alice-Bob system:

$$\rho_{aAB}(0) = \frac{1}{8}\hat{\mathcal{I}} + \frac{1}{2}\vec{P}_a\cdot\vec{S}_a + \frac{1}{2}\vec{\mathbf{P}}_A\cdot\vec{S}_A + \sum_{m,n=1}^{3}\mathbf{D}_{mn}S_a^m S_A^n, \tag{36}$$

  where $\mathbf{P}_A^i = D_i S_B^i$, and $\mathbf{D}^{mn} = P_a^m D^n S_B^n$

- Before Alice's measurement:

$$\rho_{aAB}(t) = \mathrm{Tr}_{\mathcal{E}}\left(U_H(t)\rho_{aAB}(0)\otimes\rho_{\mathcal{E}}U_H^\dagger(t)\right) \tag{37}$$

COMMON BATH DECOHERENCE FOR ALICES QUBITS

- The unitary operator:

$$\begin{aligned} U_H &= \left[a_1(t) + a_2(t)(\vec{S}_a - \vec{S}_A)\cdot\vec{I}_{\mathcal{E}}\right](1 - \frac{\hat{S}^2_{aA}}{2}) \\ &+ \left[a_3(t) + a_4(t)\vec{S}_{aA}\cdot\vec{I}_{\mathcal{E}} + a_5(t)(\vec{S}_{aA}\cdot\vec{I}_{\mathcal{E}})^2\right. \\ &\left. + a_6(t)(\vec{S}_a - \vec{S}_A)\cdot\vec{I}_{\mathcal{E}} + a_7(t)(\vec{S}_a\times\vec{S}_A)\cdot\vec{I}_{\mathcal{E}}\right]\frac{\hat{S}^2_{aA}}{2}, \\ \vec{S}_{aA} &= \vec{S}_a + \vec{S}_A \end{aligned} \tag{38}$$

- On tracing out bath degrees of freedom:

$$\rho_{aAB}(t) = \frac{1}{8}\hat{\mathcal{I}} + \frac{1}{2}\vec{P}_a(t)\cdot\vec{S}_a + \frac{1}{2}\vec{\mathbf{P}}_A(t)\cdot\vec{S}_A + \sum_{m,n=1}^{3}\mathbf{D}_{mn}(t)S_a^mS_A^n.$$

COMMON BATH DECOHERENCE FOR ALICES QUBITS

- After Alice makes the Bell measurement, Bob gets $\rho_B(t) = \frac{1}{2}\hat{\mathcal{I}} + \sum_k M_k \mathbf{D}_{kk}(t)$ with 1/4 probability

$$\begin{aligned} \mathbf{D}_{kk}(t) &= f(t)\mathbf{D}_{kk}(0) + g(t)\text{Tr}[\mathbf{D}(t)], \\ &= f(t)P_a^k D_k S_B^k + g(t)\sum_m P_a^m D_m S_B^m. \end{aligned} \tag{39}$$

- The final state of Bob: $\rho_B(t) = \frac{1}{2}\hat{\mathcal{I}} + \vec{P}_B(t)\cdot\vec{S}_B$, with $P_B^i = f(t)P_a^i + g(t)M_i \text{Tr} M P_a^i$

- If system bath interaction is zero we get perfect teleportation

- In the presence of the bath Bob's final state depends on $\vec{M}$ from which he can know about the measurement made by Alice

- If the qubits of Alice see separate environments there will be no such dependence of Alice measurement.

Average Fidelity of Teleportation

- Fidelity: How close the teleported state to the unknown state,

$$\mathcal{F}(t) \equiv \frac{1}{2}\left[1+\vec{P}_a\cdot\vec{P}_B(t)\right], \\ = \frac{1}{2}\left[1+f(t)|\vec{P}_a|^2+g(t)\mathrm{Tr}M\sum_k (P_a^k)^2 M_k\right]. \quad (40)$$

and on averaging over all pure states of qubit a,

$$\mathcal{F}_{av}(t) = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \mathcal{F}(t), \\ = \frac{1}{2}\left[1+f(t)+\frac{1}{3}g(t)(\mathrm{Tr}M)^2\right]. \quad (41)$$

- Upto leading order:

$$f(t) \approx 1-\frac{1}{3}\{\langle\hat{I}_{\mathcal{E}}^2\rangle(K_a^2+K_A^2+K_aK_A)\}t^2, \\ g(t) \approx \frac{1}{3}\langle\hat{I}_{\mathcal{E}}^2\rangle K_aK_At^2. \quad (42)$$

If $K_a = K_A$ then $f(t)+3g(t)=1 \longrightarrow$ Perfect teleportation of the unknown state

Average Fidelity of Teleportation

- Time dependence:

$$\mathcal{F}_{av}(t) = 1 - \frac{t^2}{12}\langle \widehat{I}_{\mathcal{E}}^2\rangle (K_a^2 + K_A^2)\left[1 + \Delta(1 - (\mathrm{Tr}M)^2/3\right]$$
$$\Delta = 2K_A K_a/(K_A^2 + K_a^2), \quad \langle \widehat{I}_{\mathcal{E}}^2\rangle = \sum_{I_\mathcal{E}} \lambda_{I_\mathcal{E}} I_\mathcal{E}(I_\mathcal{E}+1) \qquad (43)$$

- For completely unpolarized baths i.e., $\rho_\mathcal{E} = \frac{1}{2^N}\widehat{\mathcal{I}}$, $\langle \widehat{I}_{\mathcal{E}}^2\rangle = 3N/4$, where N is number of bath spins

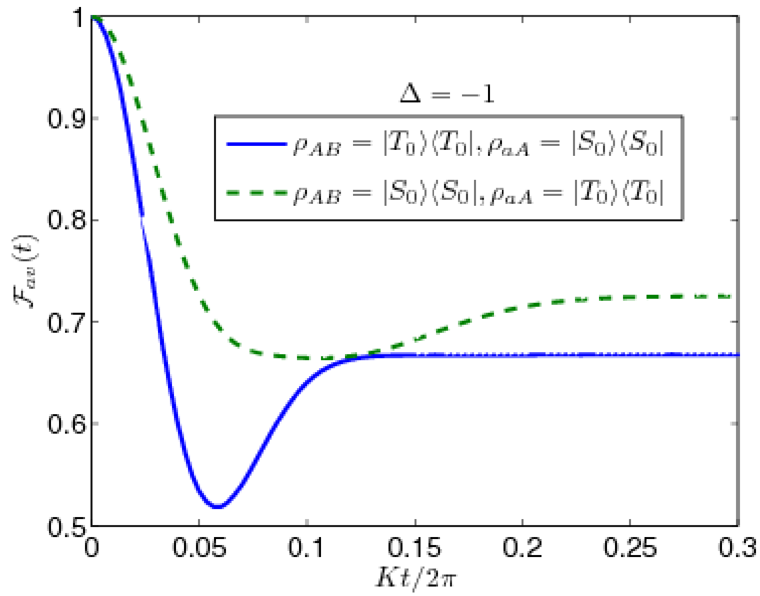
- For Bell states, Tr$M = -3$ for singlet and Tr$M = 1$ otherwise

- The average fidelity has an initial Gaussian decay, $\mathcal{F}_{av}(t) = \exp(-t^2/\tau^2)$, with two different decoherence time scales:

$$\left(\frac{1}{\tau^2}\right)_{S_0} = \frac{1}{6}\langle \widehat{I}_{\mathcal{E}}^2\rangle (K_a^2 + K_A^2)(1-\Delta),$$
$$\left(\frac{1}{\tau^2}\right)_{T_0,T_+,T_-} = \frac{1}{6}\langle \widehat{I}_{\mathcal{E}}^2\rangle (K_a^2 + K_A^2)\left(1+\frac{\Delta}{3}\right). \qquad (44)$$

- The sign of the interaction with the bath can decide which particular measurement of Alice can give Bob a less decohered state.

Average Fidelity of Teleportation



Average fidelity of teleporting the unknown state given to Alice as function of time. When the Bell state shared by Alice and Bob belongs to the triplet sector and measurement made by Alice is in the singlet sector and the converse. One of the Alice's qubits interact ferro-magnetically and the other anti-ferromagnetically. Here, $\Delta = -1$, S_0 and T_0 with $N = 22$ and $K^2 = K_a^2 + K_A^2$

Measurement in partially entangled basis

- Rao *et al.*: One can find values of Δ, where non-maximally entangled states have larger decoherence time scale in comparison to the maximally entangled states
- Whether measurement on the partial entangled basis by Alice can improve the average fidelity
- One-parameter class of states as basis:

$$
\begin{aligned}
|S_0^r\rangle &= \frac{1}{\sqrt{1+r^2}}[|\uparrow\downarrow - r\downarrow\uparrow\rangle], \; |T_0^r\rangle = \frac{1}{\sqrt{1+r^2}}[r|\uparrow\downarrow + \downarrow\uparrow\rangle], \\
|T_+^r\rangle &= \frac{1}{\sqrt{1+r^2}}[r|\uparrow\uparrow + \downarrow\downarrow\rangle], \; |T_-^r\rangle = \frac{1}{\sqrt{1+r^2}}[|\uparrow\uparrow - r\downarrow\downarrow\rangle]
\end{aligned}
\tag{45}
$$

- The density matrix:

$$
\rho_{aA} = \frac{\widehat{\mathcal{I}}}{4} + \frac{1}{2}P_a^z(r)S_a^z - \frac{1}{2}P_A^z(r)S_A^z + \sum_{m=1}^{3} \Pi_{mm}(r) S_a^m S_A^m \tag{46}
$$

Measurement in partially entangled basis

- Similarly as before:

$$\begin{aligned}
\mathcal{F}_{av}^{S_0^r}(t) &= \frac{1}{2} + \frac{(1+r)^2+2r}{6(1+r^2)}\left[1 - \frac{t^2}{\tau_0^2}(1-\Delta)\right], \\
\mathcal{F}_{av}^{T_0^r}(t) &= \frac{1}{2} + \frac{(1+r)^2+2r}{6(1+r^2)}\left[1 - \frac{t^2}{\tau_0^2}\left(1 + \frac{\Delta(1+r^2)}{(1+r)^2+2r}\right)\right], \\
\mathcal{F}_{av}^{T_\pm^r}(t) &= \frac{1}{2} + \frac{(1+r)^2+2r}{6(1+r^2)}\left[1 - \frac{t^2}{\tau_0^2}\left(1 + \frac{2\Delta r}{(1+r)^2+2r}\right)\right], \\
1/\tau_0^2 &= \frac{1}{3}\langle \hat{I}_{\mathcal{E}}^2\rangle (K_a^2 + K_A^2) \qquad (47)
\end{aligned}$$

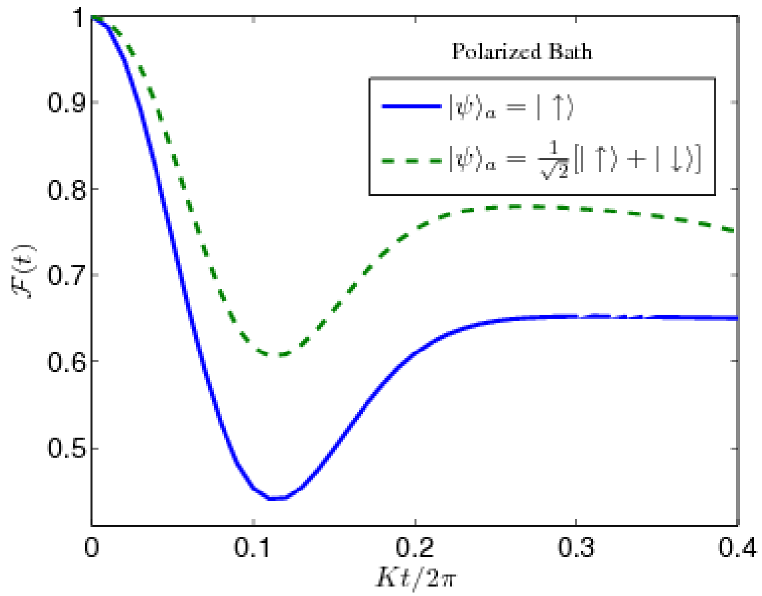
- The average fidelity of teleportation in the case of partial entangled measurement is always less than the Bell-state measurement: $\mathcal{F}_{av}^{S_0}(t) \geq \mathcal{F}_{av}^{S_0^r}(t)$

- T_0^r giving one value of $\mathcal{F}_{av}(t)$ and T_+^r, T_-^r different.

- Individual polarizations of qubits P_a and P_A though non-zero, did not contribute to $\mathcal{F}_{av}(t)$ as averaging was performed on the surface of Bloch sphere.

Effect of initial polarizations on the fidelity

- If the bath is polarized there will be a preferred direction chosen by the bath because of which the fidelity of various state which Alice wishes to teleport can be different.



Fidelity as a function of time. Two different states to be teleported in presence of a polarized bath. Initial: $\rho_{\mathcal{E}} = \sum_I \lambda_I |I;I\rangle\langle I;I|$, where $\lambda_I \sim I^2 \exp(-2I^2/N)$, and in $|I;I\rangle$ the first index gives spin value and the second value yields $\hat{z}$ component of the spin. $N = 22$ and $K^2 = K_a^2 + K_A^2$

Conclusions

- The Bell states can be discriminated deterministically without affecting the quantum channel
- NDD of Bell states are experimentally achieved by NMR, with encoding phase and parity informations
- Different models of natural entanglement have been studied
- Perfact teleportation can be achieved with multi-electron quantum dots, using magnon states
- Working on coupled pair model with 2-magnon states
- Decoherence-free teleportation is achieved through common bath

Thanks for such a patient listening