Dichotomy of the Two Quantum Correlation Paradigms

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Outline

- Introduction to quantum correlations
- Quantum discord
- Dichotomy :
    - Monogamy of entanglement & discord
    - Statistical behavior of two paradigms

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- Introduction to quantum correlations
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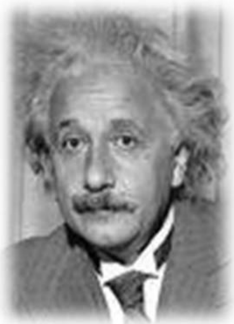
EPR


Albert Einstein


Boris Podolsky


Nathan Rosen

Phys. Rev. **47**, 777 (1935)

Quantum theory

COMPLETE THEORY?

Enter Schrödinger

Erwin Schrödinger

Proceedings of the Cambridge Philosophical Society, 555 **31** (1935); 446 **32** (1936)

Schrödinger coined the term '*entanglement*' for the first time

'Spooky action at a distance'
-A. Einstein

up to 1960's ☹

Can quantum correlations be created in well controlled environments between distinct quantum systems?

→ inconceivable question

Enter Bell

J S Bell

Early attempt to quantify quantum correlations was through Bell

Physics **1** 195 (1965)

"John Bell's questioning orthodox quantum mechanics was just his *hobby*, and it is this hobby *John Bell* is most famous for."

Quantum [un]speakables: from Bell to quantum information
- R. A. Bertlmann & A. Zeilinger

Present time ☺

Present time ☺

Entanglement as resource

- Teleportation ⇒ Quantum state transfer
- Cryptography ⇒ Secure communication
- Quantum dense coding ⇒ Classical information transfer via quantum state

Several quantum phenomena have been discovered in which entanglement is absent

- Entanglement in quantum computation: Not clear
- "quantum nonlocality without entanglement" – locally indistinguishable orthogonal product states
- Deterministic quantum computation with one quantum bit

Quantify quantum correlations independent of the entanglement-separability paradigm

Quantify quantum correlations independent of the entanglement-separability paradigm

Information-theoretic measures

- Quantum discord

L. Henderson and V. Vedral, J. Phys. A **34**, 6899 (2001)
H. Ollivier and W.H. Zurek, Phys. Rev. Lett. **88**, 017901 (2002)

- Quantum deficit

J. Oppenheim, M, P, R, Horodecki,Phys. Rev. Lett. 89, 180402 (2002)
M, P, R, Horodecki, J. Oppenheim, A, U, Sen, and B. Synak-Radtke, Phys. Rev. A 71, 062307 (2005)
A.K.Rajagopal and R.W.Rendell, Phys. Rev. A 66,022104 (2002)

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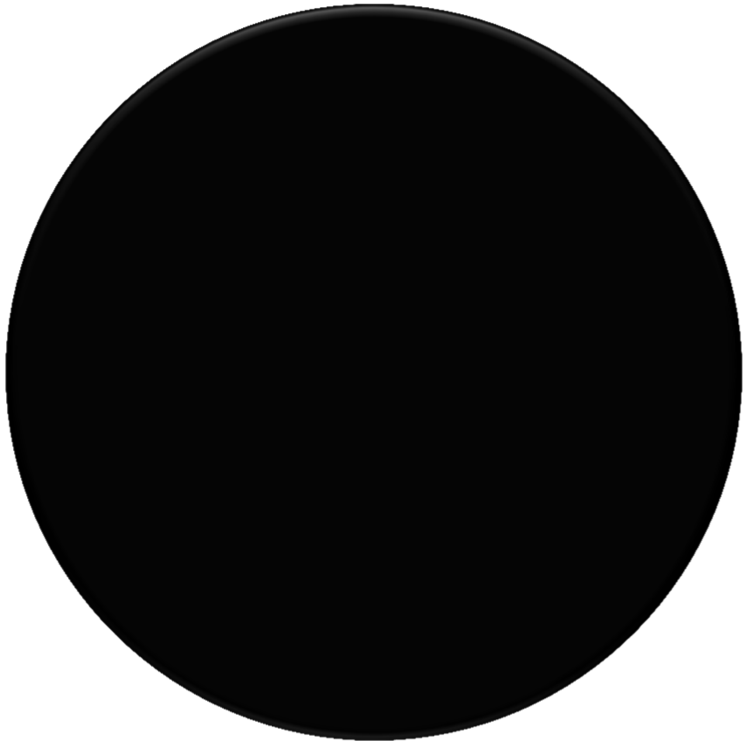
What is quantum discord?

What is quantum discord?

Classical



$H(X)$

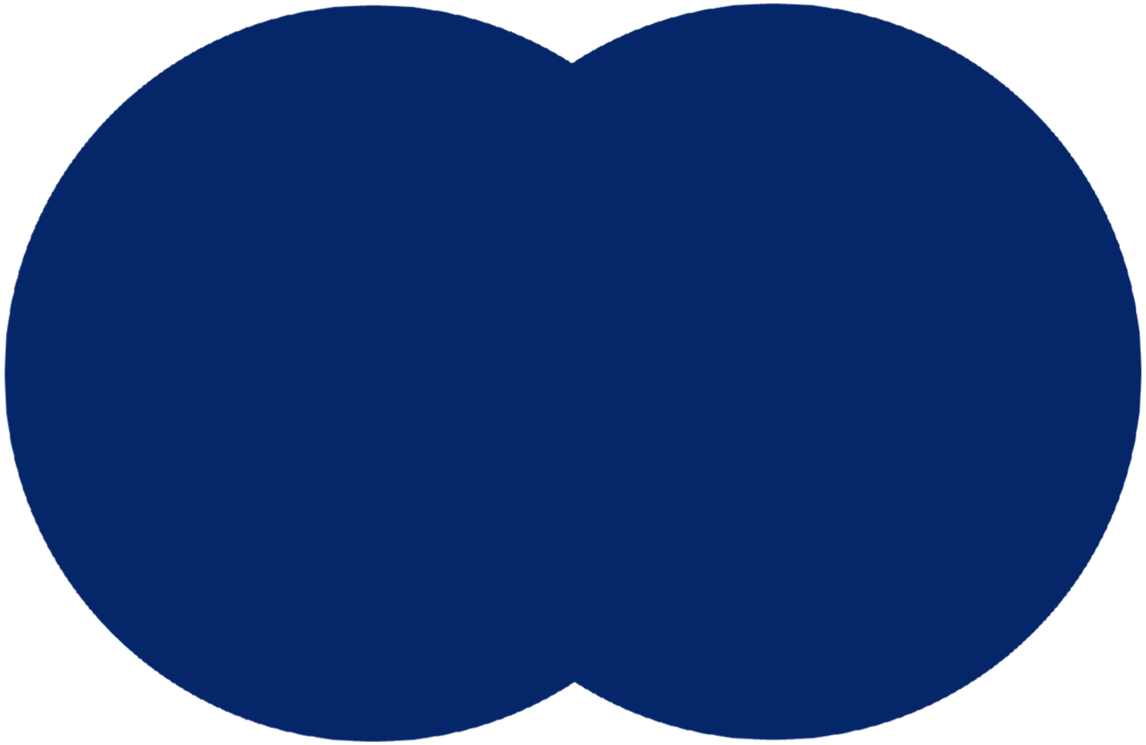
$$H(\{p_i\}) = -\sum p_i \log_2 p_i$$

What is quantum discord?

Classical

What is quantum discord?

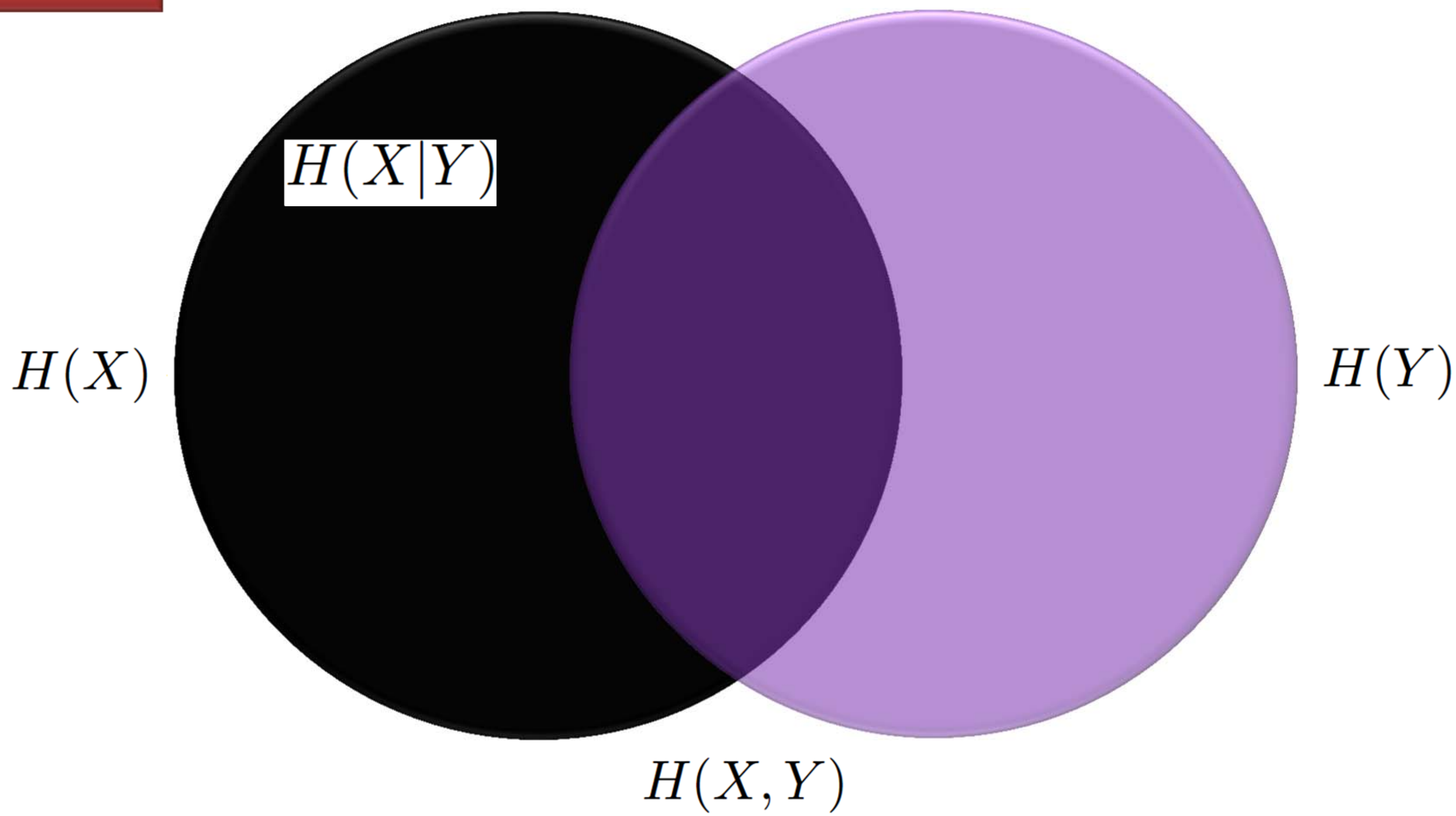
Classical



$H(X,Y)$

What is quantum discord?

Classical



What is quantum discord?

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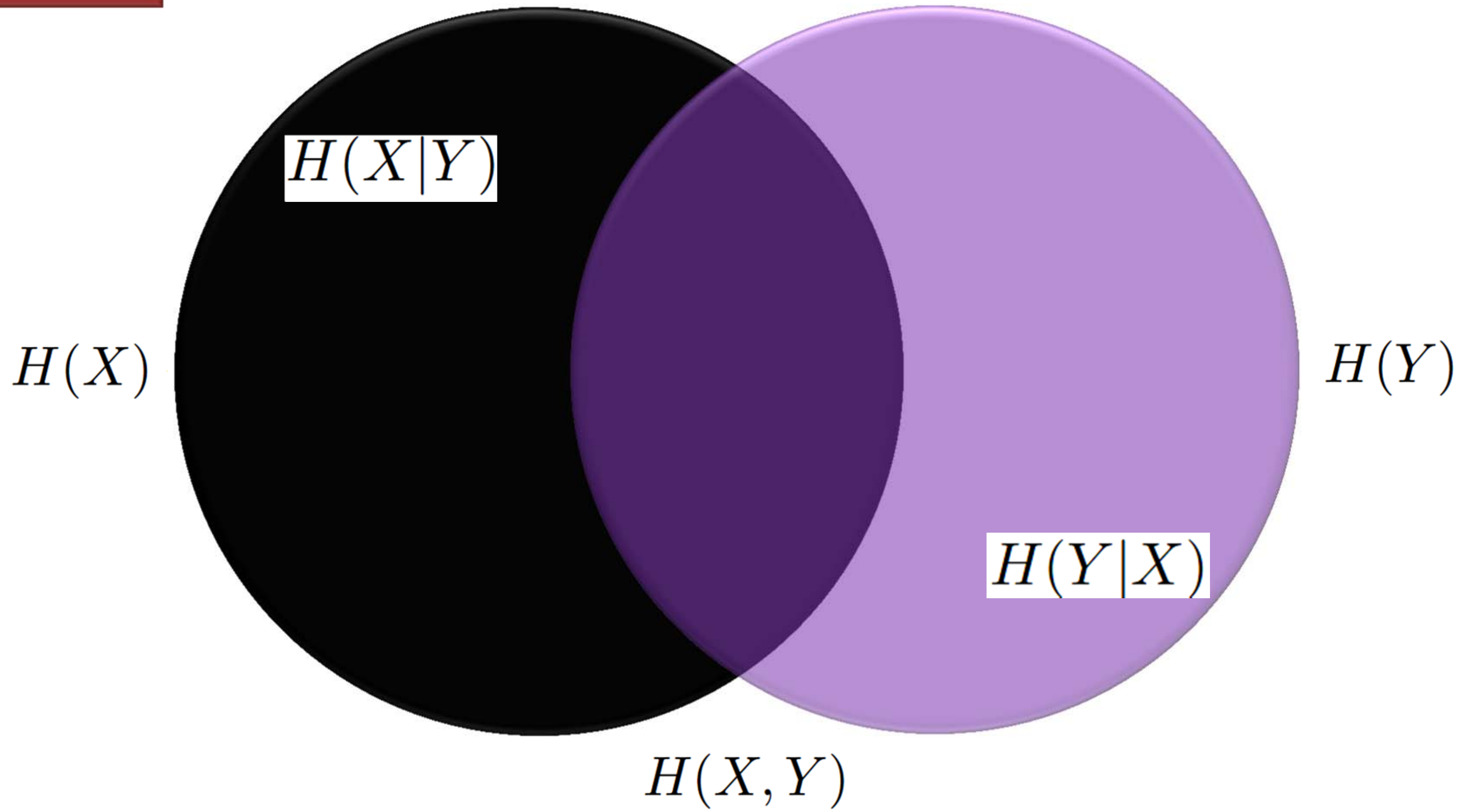

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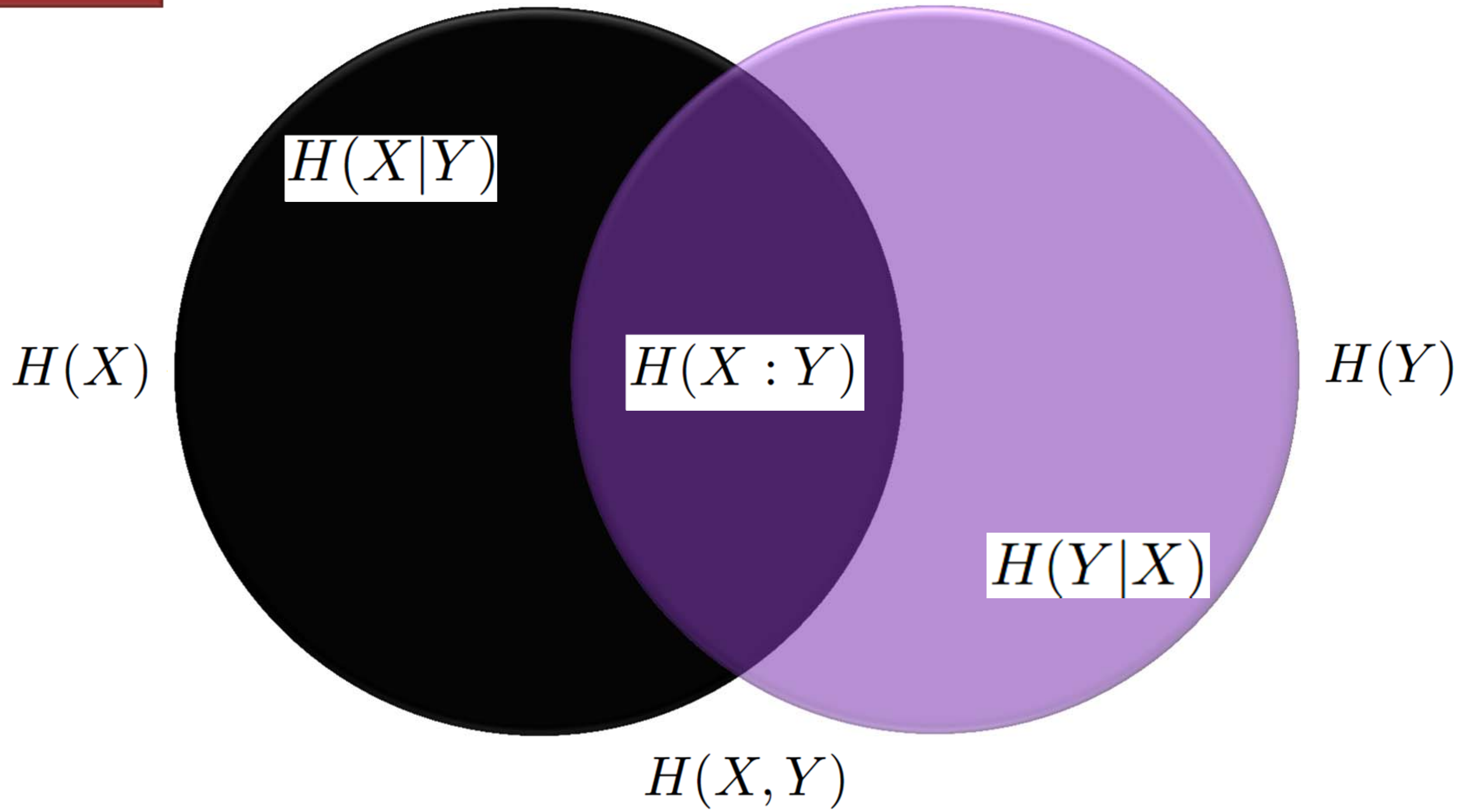

What is quantum discord?

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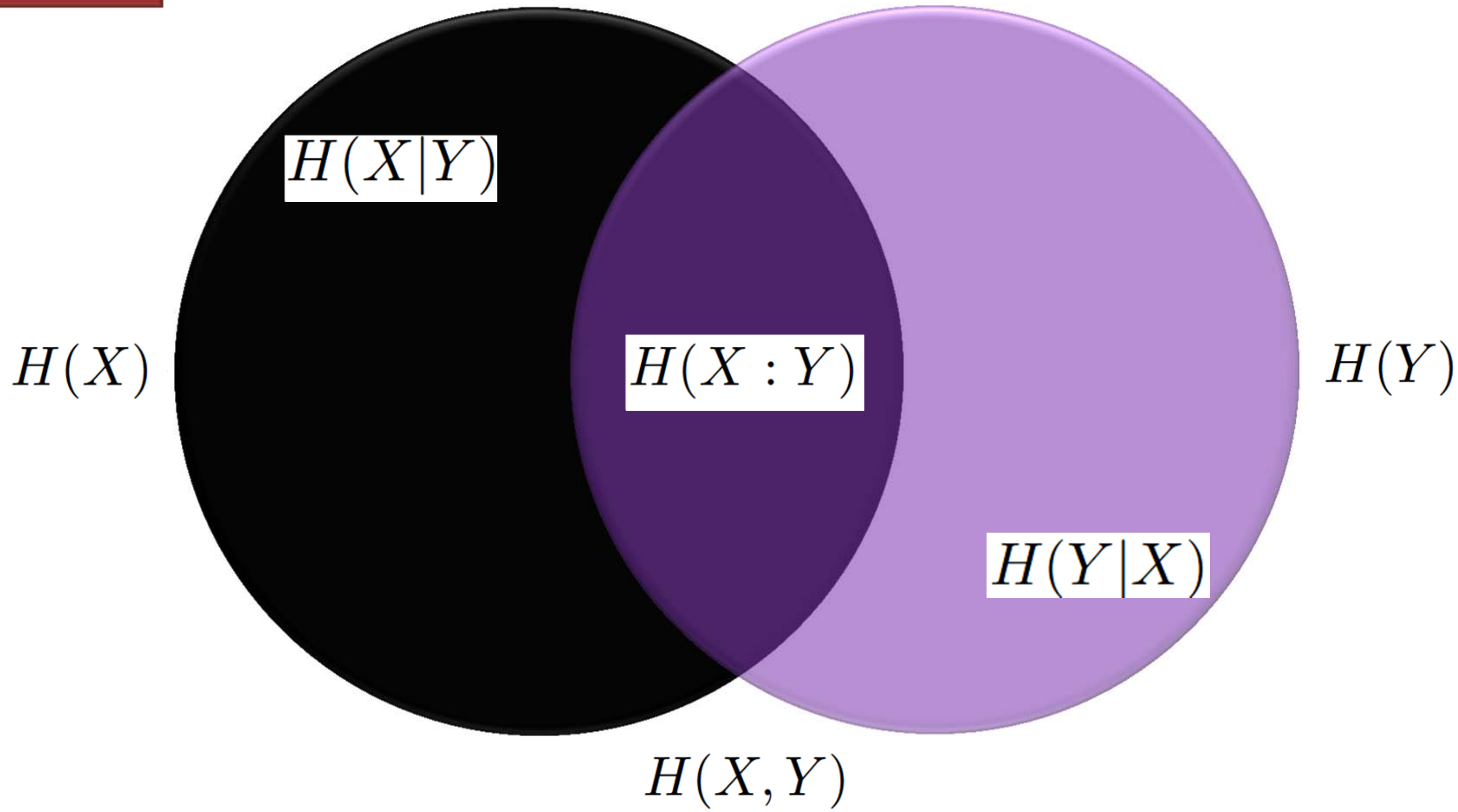


$$H(X:Y) \begin{cases} H(X) + H(Y) - H(X,Y) \\ H(X) - H(X|Y) \end{cases}$$

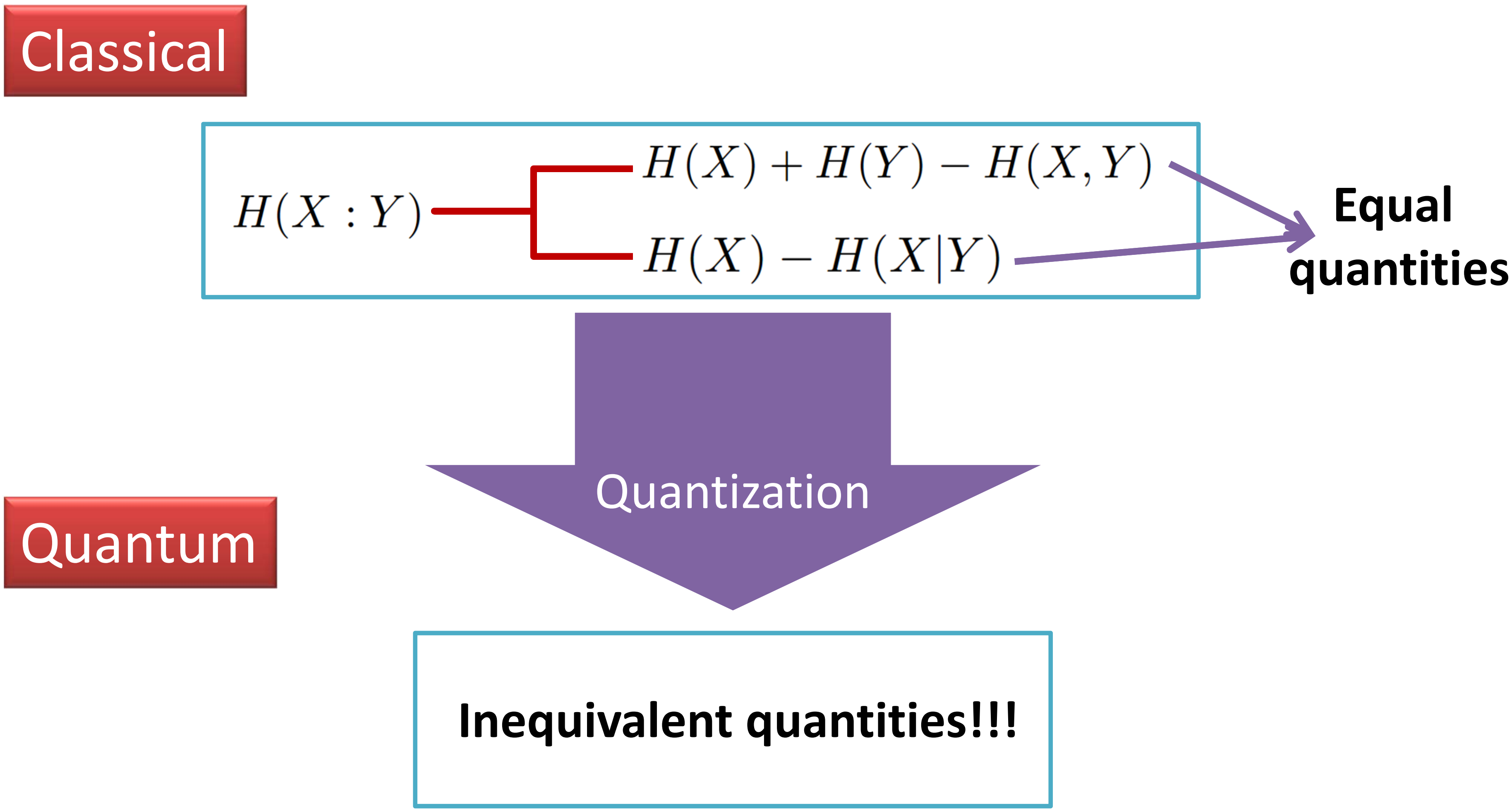
What is quantum discord?

Classical

$$H(X:Y) \begin{cases} H(X)+H(Y)-H(X,Y) \\ H(X)-H(X|Y) \end{cases}$$

Equal quantities

What is quantum discord?



What is quantum discord?

Classical

$$H(X:Y) \begin{cases} H(X) + H(Y) - H(X,Y) \\ H(X) - H(X|Y) \end{cases}$$

Equal quantities

Quantization

Quantum

Inequivalent quantities!!!

Difference is quantum discord

What is quantum discord?

Classical

$$H(X:Y) \begin{cases} H(X)+H(Y)-H(X,Y) \\ H(X)-H(X|Y) \end{cases}$$

Equal quantities

Quantization

Quantum

Inequivalent quantitie

Difference is quantum discord

CLASSICAL

One form:

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

CLASSICAL

One form:

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

QUANTUM

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

CLASSICAL

One form:

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

Second form:

$$H(X:Y) = H(X) - H(X|Y)$$

QUANTUM

$$I(\rho_{AB}) \;=\; S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

CLASSICAL

One form:

$$H(X:Y) = H(X) + H(Y) - H(X,Y)$$

Second form:

$$H(X:Y) = H(X) - H(X|Y)$$

QUANTUM

$$I(\rho_{AB}) \;=\; S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Physical quantity that can be negative for some quantum states!

What next then?

Fixing second form:

Classical correlation: the difference between the von Neumann entropies before and after a complete measurement on the subsystem B

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

Fixing second form:

Classical correlation: the difference between the von Neumann entropies before and after a complete measurement on the subsystem B

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

where quantum conditional entropy

$$S(\rho_{A|B}) = \min_{\{\Pi_i^B\}} \sum_i p_i S(\rho_{A|i})$$

minimization being over all projection-valued measurements performed on subsystem B

$$\rho_{A|i} = \frac{1}{p_i} \mathrm{tr}_B(\mathbb{I}_A \otimes \Pi_i^B \rho \mathbb{I}_A \otimes \Pi_i^B)$$

What is quantum discord?

Classical

$$H(X:Y) \begin{cases} H(X) + H(Y) - H(X,Y) \\ H(X) - H(X|Y) \end{cases}$$

Equal quantities

Quantization

Quantum

$$I(\rho_{AB}) \;=\; S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

$$J(\rho_{AB}) = S(\rho_A) - S(\rho_{A|B})$$

Difference is quantum discord

What is quantum discord?

Classical

$H(X:Y)$ —
- $H(X)+H(Y)-H(X,Y)$
- $H(X)-H(X|Y)$

Equal quantities

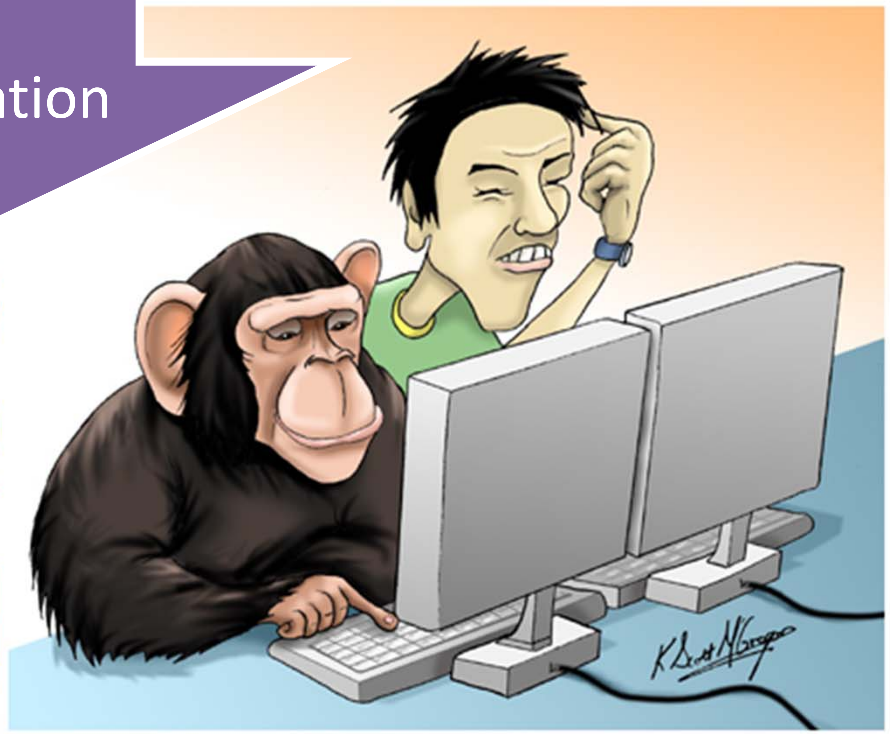
Quantization

Quantum

$I(\rho_{AB}) \;=\; S(\rho_A)+S$

$J(\rho_{AB})=S(\rho_A)-S($

Difference is qua



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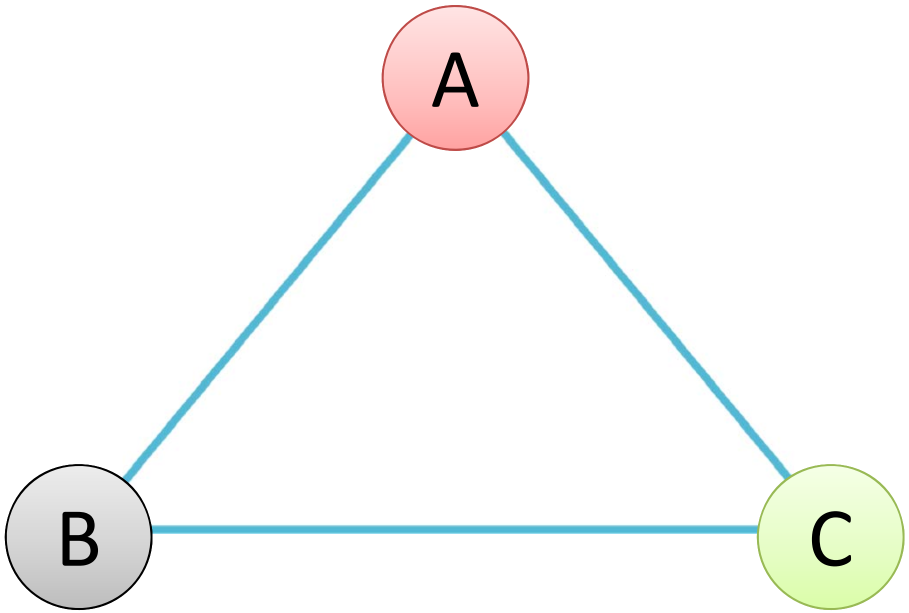
Monogamy of Entanglement

In a multipartite setting, sharing entanglement between several parties is restricted by the monogamy of entanglement

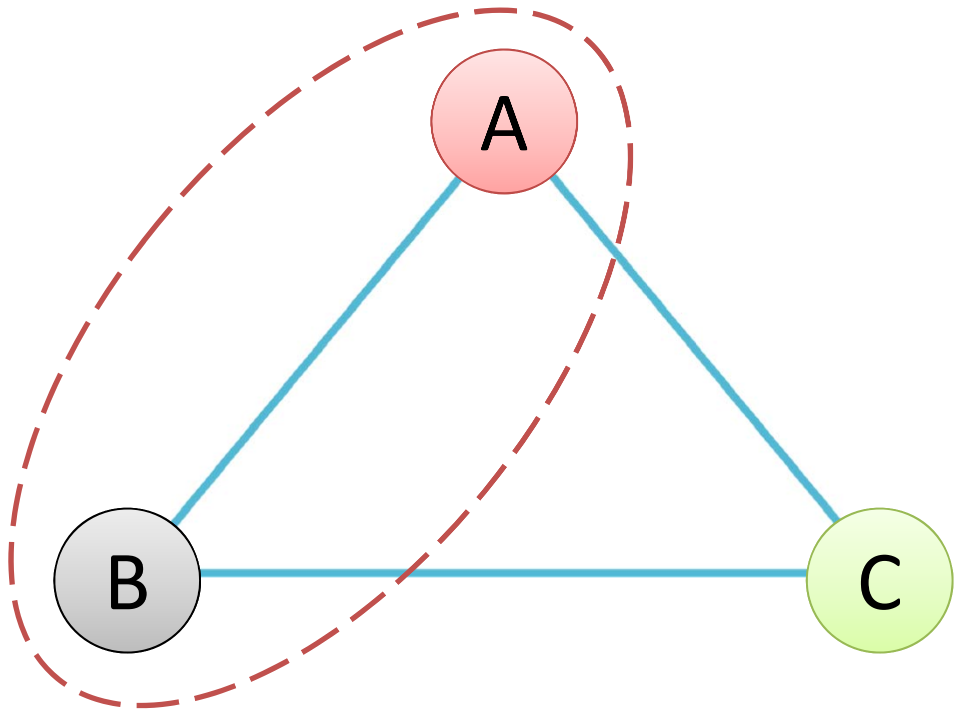
C.H. Bennett, et al., Phys. Rev. A **53**, 2046 (1996)

V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000)

Monogamy of Entanglement

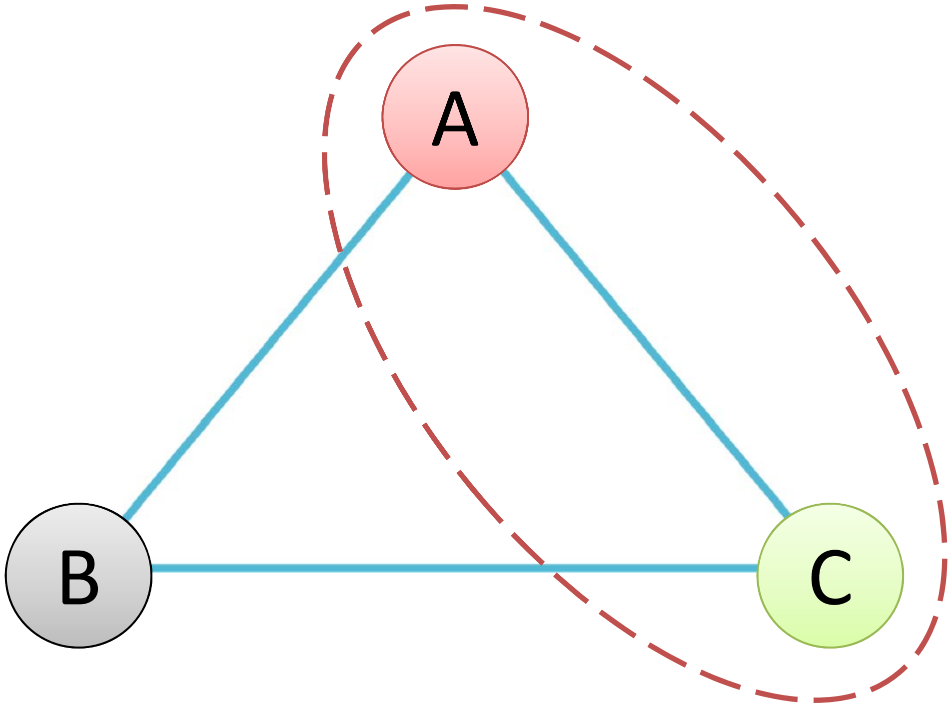


Monogamy of Entanglement



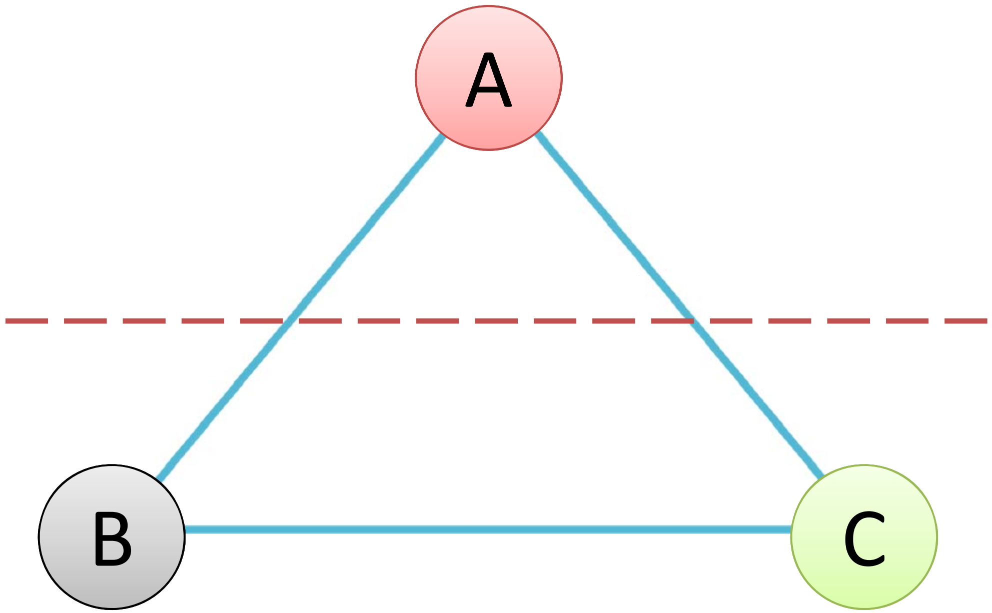
$$E_{AB}$$

Monogamy of Entanglement



$$E_{AC}$$

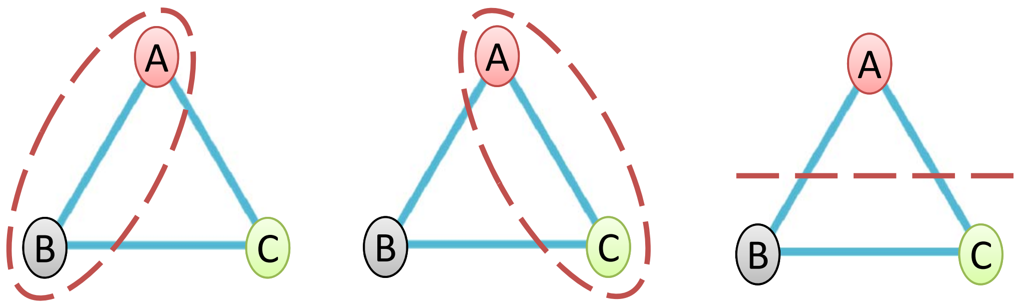
Monogamy of Entanglement



$$E_{A:BC}$$

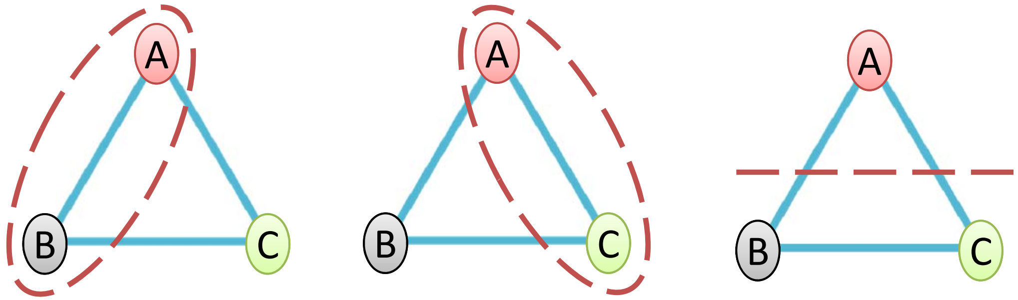
Monogamy of Entanglement

$$E_{AB} + E_{AC} \leq E_{A:BC}$$



Monogamy of Entanglement

$$E_{AB} + E_{AC} \leq E_{A:BC}$$



Example:

$$\mathcal{C}^2_{AB} + \mathcal{C}^2_{AC} \leq \mathcal{C}^2_{A:BC}$$

V. Coffman, J. Kundu, and W.K. Wootters, Phys. Rev. A **61**, 052306 (2000)

Questions

- Does quantum discord satisfy monogamy relation?
- Does the sharing of quantum discord follow the same broad guidelines that are followed by entanglement?

Monogamy of Discord

For tripartite state ρ_{ABC}

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$

?

Monogamy of Discord

For tripartite state ρ_{ABC}

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$

Monogamy relation → satisfied → Monogamous

Monogamy relation → violated → Polygamous

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$

OR

$$\boxed{\delta_M = D(\rho_{AB}) + D(\rho_{AC}) - D(\rho_{A:BC})}$$

$$D(\rho_{AB}) + D(\rho_{AC}) \leq D(\rho_{A:BC})$$

OR

$$\delta_M = D(\rho_{AB}) + D(\rho_{AC}) - D(\rho_{A:BC})$$

$\delta_M \leq 0$ → QC measure → Monogamous

$\delta_M > 0$ → QC measure → Polygamous

Interaction information

The difference between the information shared by the subsystem AB when C is present, and when C is not present (traced out)

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The difference between the information shared by the subsystem AB when C is present, and when C is not present (traced out)

For tripartite state ρ_{ABC}

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

$$\mathcal{S}(\rho_{A|C}) - \mathcal{S}(\rho_{A|BC})$$

(Conditional mutual information)

Interaction information

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

McGill W J (1954)

Han T S (1980)

R. W. Yeung (1991)

H. Matsuda (2000)

A. Jakulin (2003)

Useful in biophysics, medicine, etc.

Generalization of mutual info to 3 parties

Went out of favor as it can be both positive and negative

But it naturally fits into the discord monogamy scenario

Interaction information (2 types)

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

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$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

$\{\Pi_k^B, \Pi_i^C, \Pi_j^{BC}\}$ $\{\Pi_i^C, \Pi_j^{BC}\}$ $\{\Pi_k^B\}$

Interaction information (2 types)

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

$\{\Pi_k^B, \Pi_i^C, \Pi_j^{BC}\}$ $\{\Pi_i^C, \Pi_j^{BC}\}$ $\{\Pi_k^B\}$

Interrogated interaction information

Interaction information (2 types)

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

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Interrogated interaction information

$$I(\rho_{ABC}) = I(\rho_{A:B|C}) - I(\rho_{AB})$$

Unmeasured interaction information

Monogamy of Discord

Interaction information

Theorem:

$$I(\cancel{\rho_{ABC}}) \leq I(\rho_{ABC}) \quad \Longleftrightarrow \quad \text{MONOGAMY OF DISCORD}$$

Monogamy of Discord

Interaction information

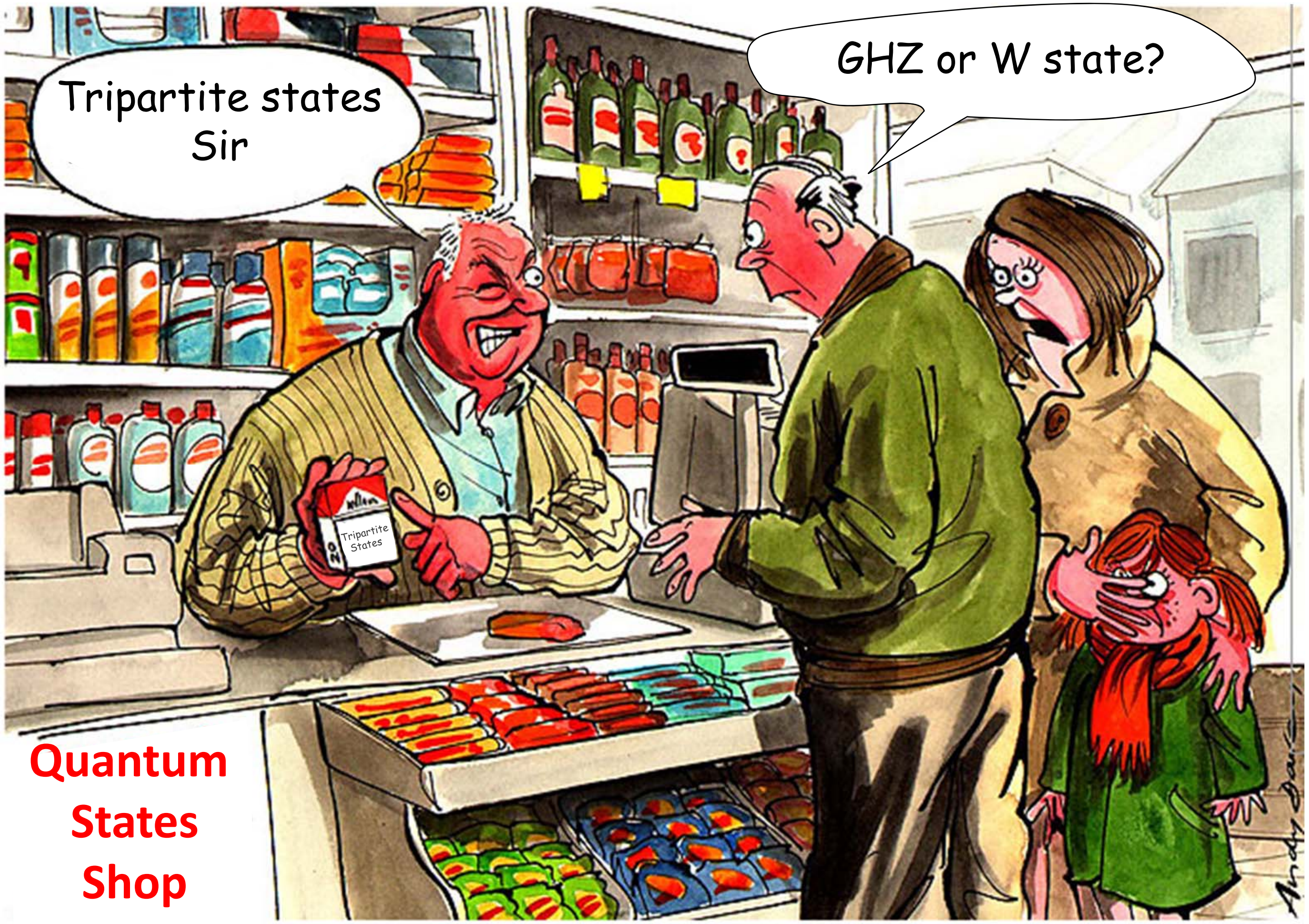
Theorem:

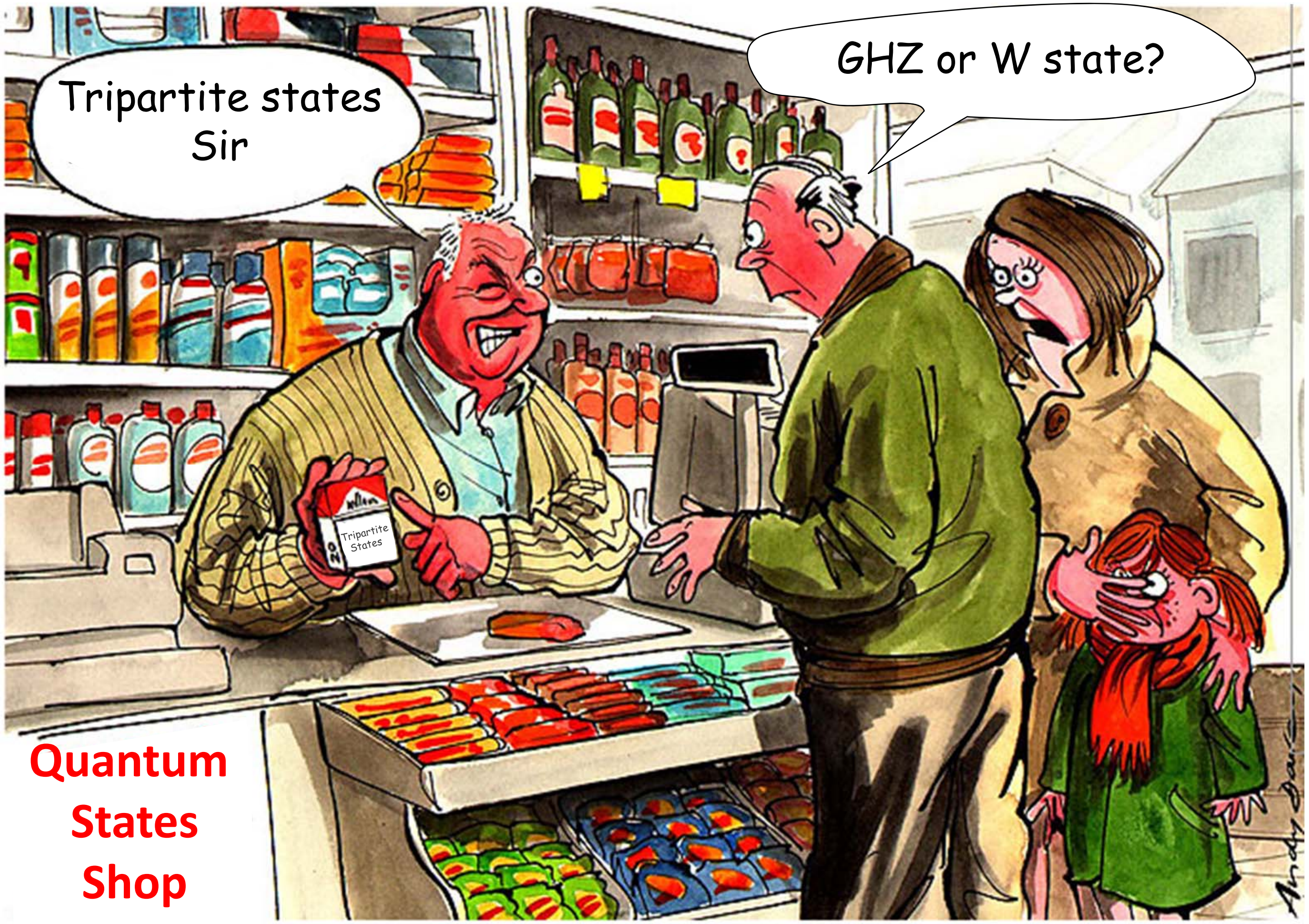
$$I(\rho_{ABC}) < 0$$

$$I(\rho_{ABC}) > 0$$

Polygamous

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168


Tripartite states Sir
GHZ or W state?
Tripartite States
Quantum States Shop



Discord monogamy can be a test?

Generalized GHZ states

$$|\psi_{GHZ}\rangle_{ABC} = \cos\Phi|000\rangle + \sin\Phi|111\rangle$$

δ_M <0 → Monogamous

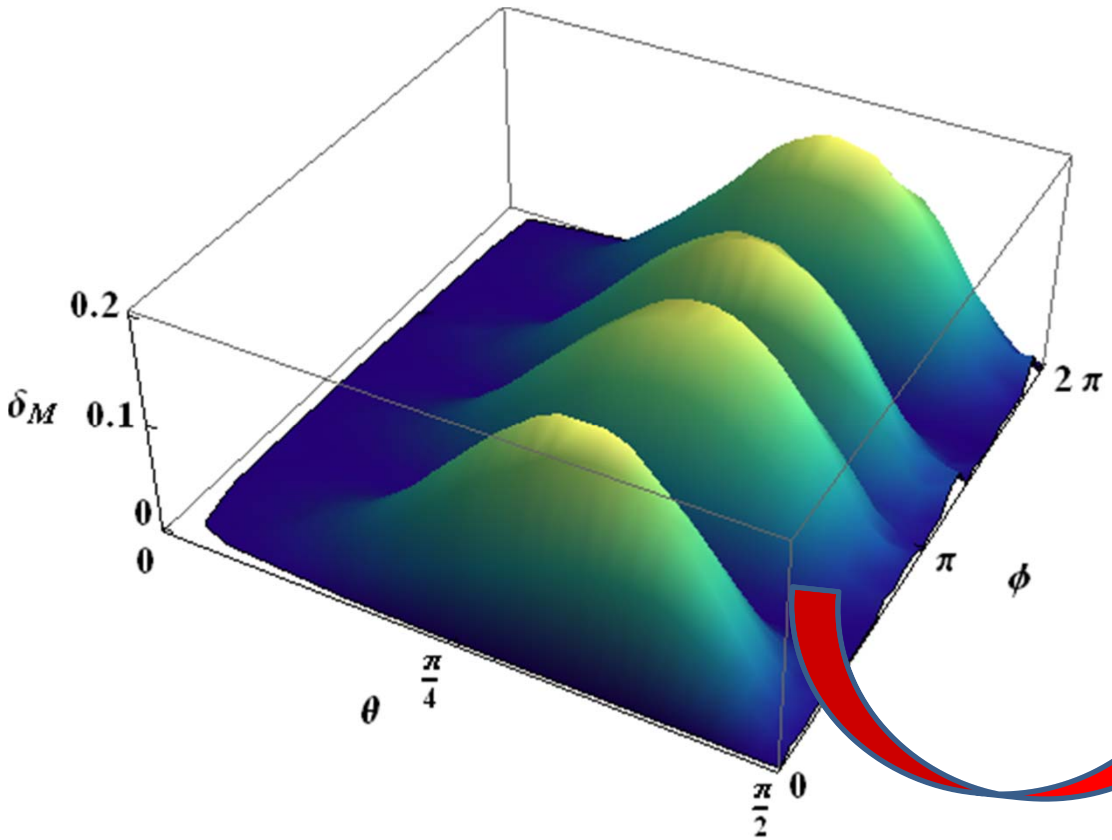
Generalized W states

$$|\psi_W\rangle_{ABC} = b|011\rangle + c|101\rangle + d|110\rangle$$

here $b = \sin\theta\cos\phi$

$c = \sin\theta\sin\phi$

$d = \cos\theta$



Polygamous

$\delta_M > 0$

Results for discord monogamy test

Tripartite states	Discord Monogamy Deficit	Monogamy test result
Gen GHZ	< 0	Satisfy
Gen W	> 0	Violate

GHZ class

$$|\psi_G\rangle = \cos\frac{\theta}{2}|000\rangle + |\psi_1\rangle|\psi_2\rangle|\psi_3\rangle$$

where $|\psi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$

$$i = 1, 2, 3$$

W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000)

75% → Monogamous

25% → Polygamous

W class

$$|\psi_{\mathrm{W}}\rangle = |a_1\rangle|b_1\rangle|c_1\rangle + |a_2\rangle|\phi_{BC}\rangle$$

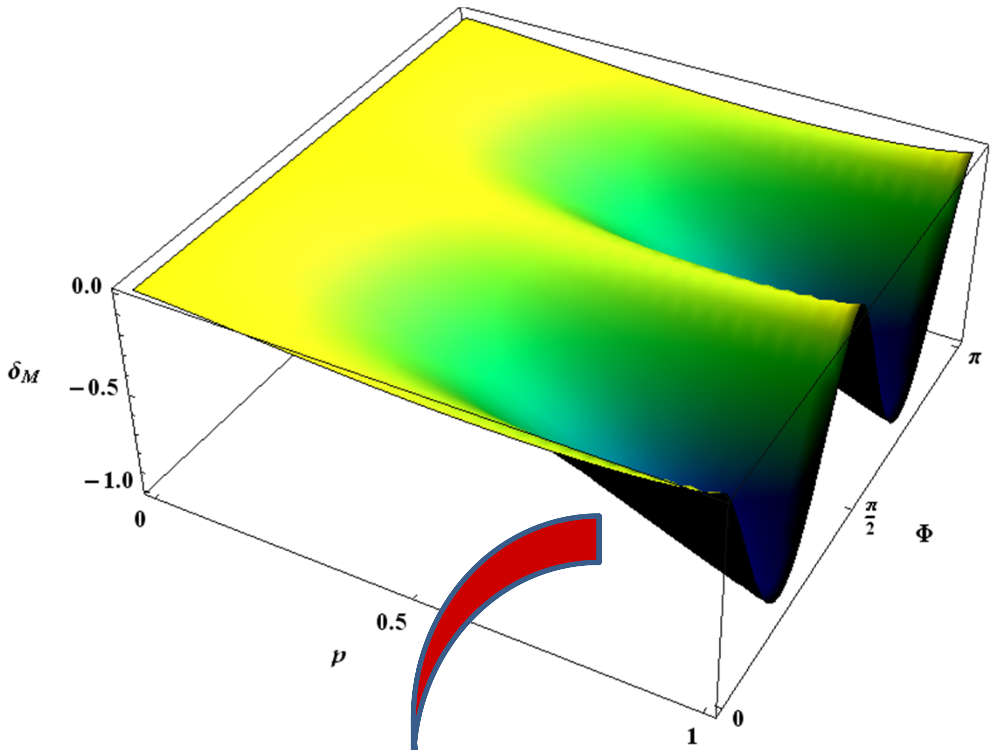
W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314 (2000)

Numerical results confirm that these states always violate monogamy

100 % → Polygamous

Mixed state: Generalized GHZ states

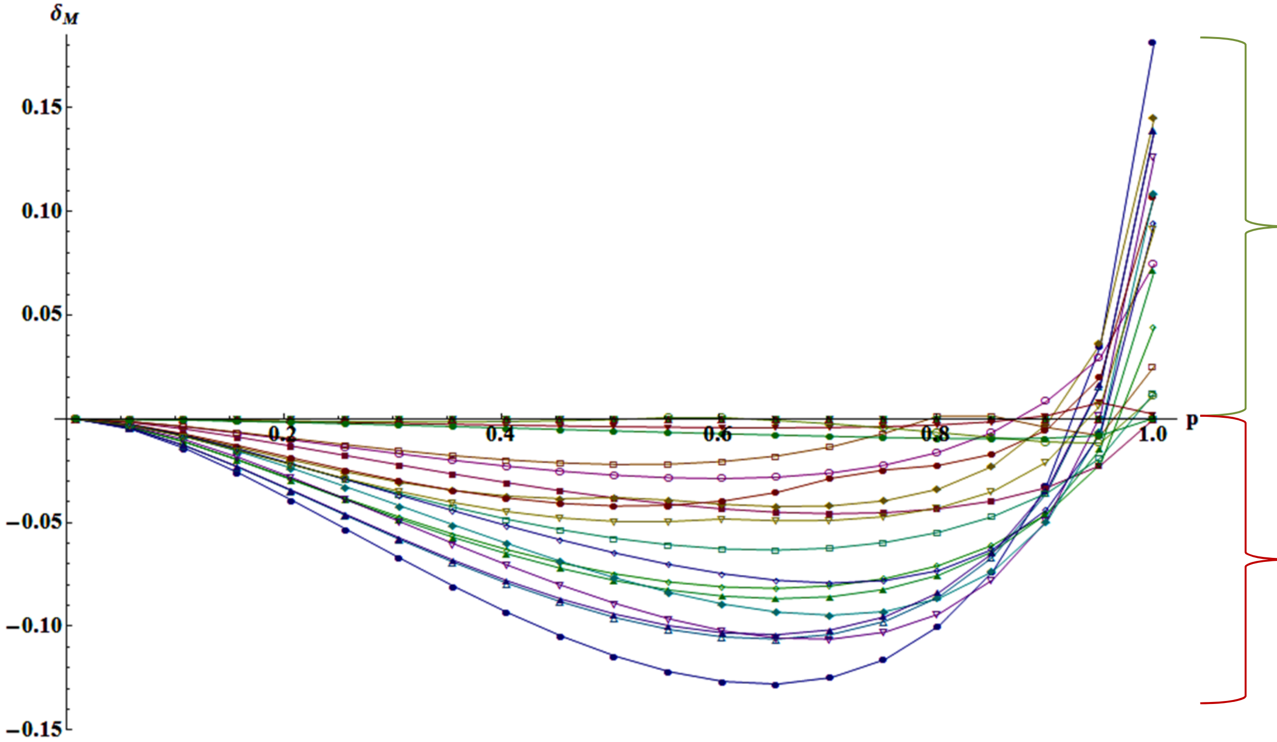
$$\rho_{GHZ} = (1-p)\mathbb{I}/8 + p|\psi_{GHZ}\rangle\langle\psi_{GHZ}|$$



$\delta_M \leq 0$ → Monogamous

Mixed state: Generalized W states

$$\rho_W = (1-p)\mathbb{I}/8 + p|\psi_W\rangle\langle\psi_W|$$



$\delta_M > 0$

Polygamous

$\delta_M \leq 0$

Monogamous

- Does quantum discord satisfy monogamy relation?

- Does the sharing of quantum discord follow the same broad guidelines that are followed by entanglement?

Rich !

Dichotomy of Quantum correlations

Entanglement → Monogamous

Discord → Polygamous

R. P, A.K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168

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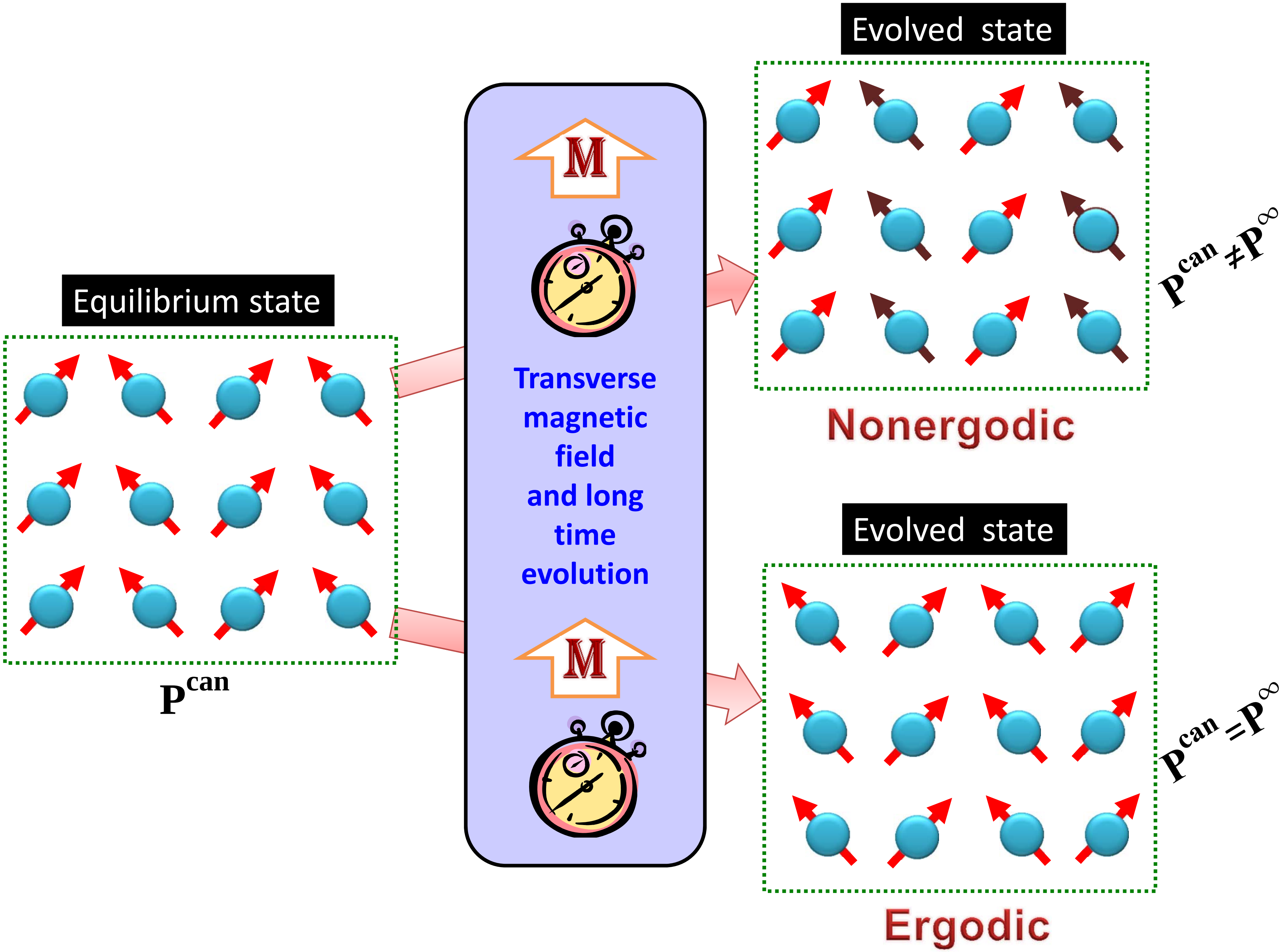
Dichotomy of Quantum correlations

Statistical mechanical behaviors of the quantum correlations measures from entanglement-separability paradigm and information-theoretic one are opposite !

Ergodicity


Equilibrium state
P^can
M
Transverse magnetic field and long time evolution
M
Evolved state
P^can ≠ P^∞
Nonergodic
Evolved state
P^can = P^∞
Ergodic

1d *XY* MODEL IN THE TRANSVERSE FIELD

$$H = J\sum_i \left[(1+\gamma)S_i^x S_{i+1}^x + (1-\gamma)S_i^y S_{i+1}^y\right] - h(t)\sum_i S_i^z$$

Anisotropic XY model
in transverse field

Transverse applied field

$$h(t) = \begin{cases} a, & t \leq 0 \\ 0, & t > 0 \end{cases}$$

- This model is exactly solvable
- Jordan-Wigner transformation

Analytical results

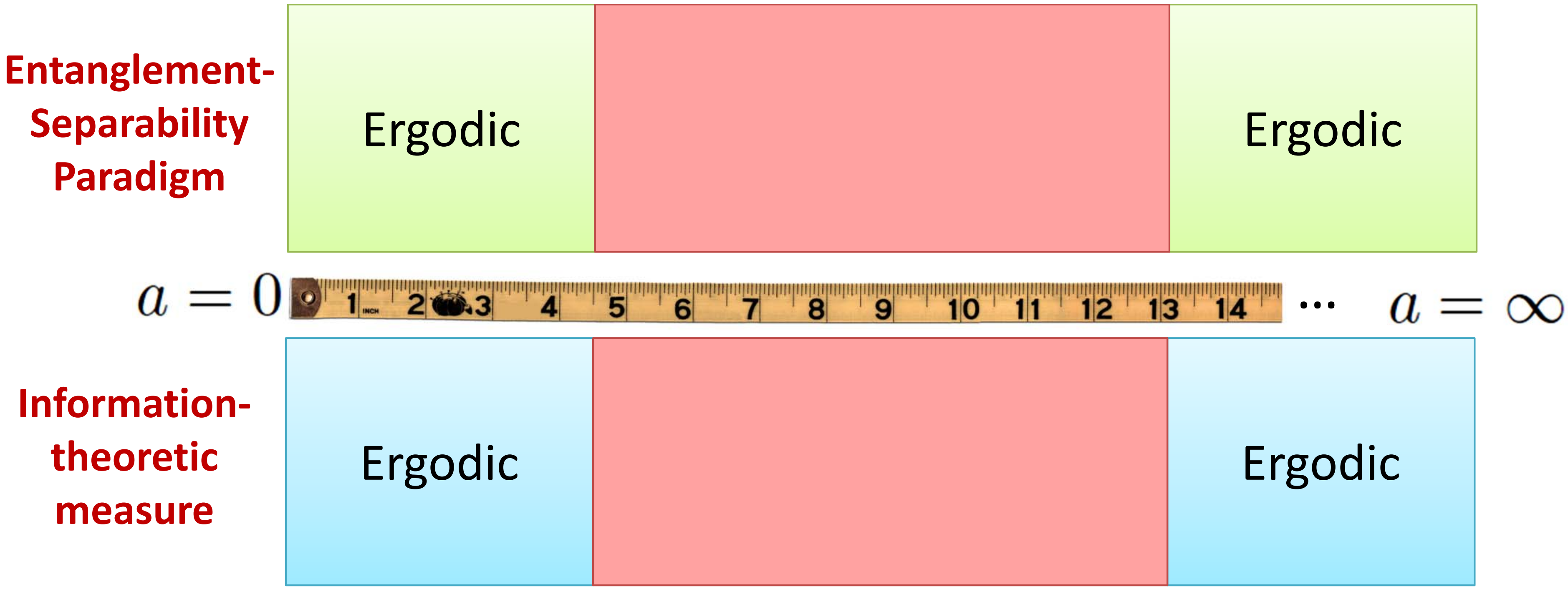
at low fields (near $a = 0$)

All quantum correlations are ergodic in any dimension

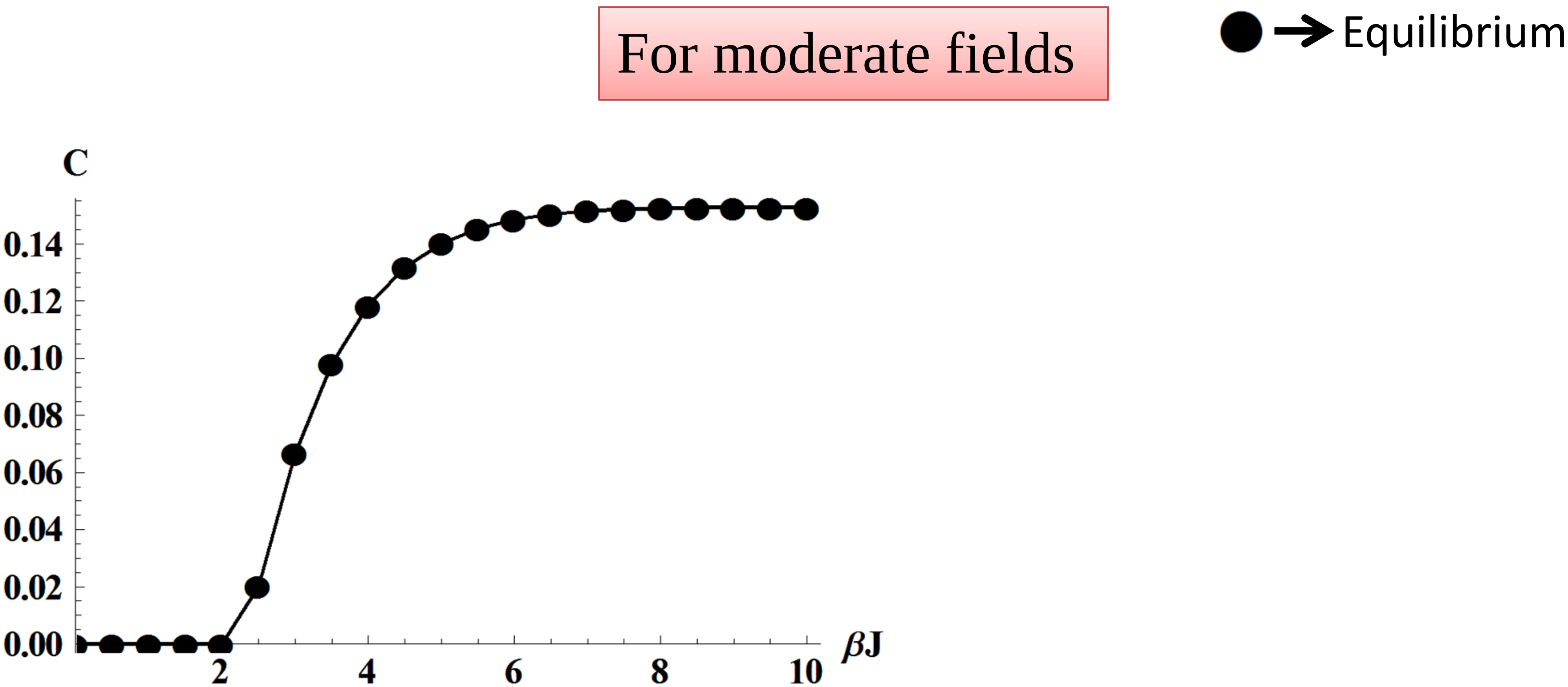
at high fields (near $a \to \infty$)

All quantum correlations are ergodic in infinite XY spin chain

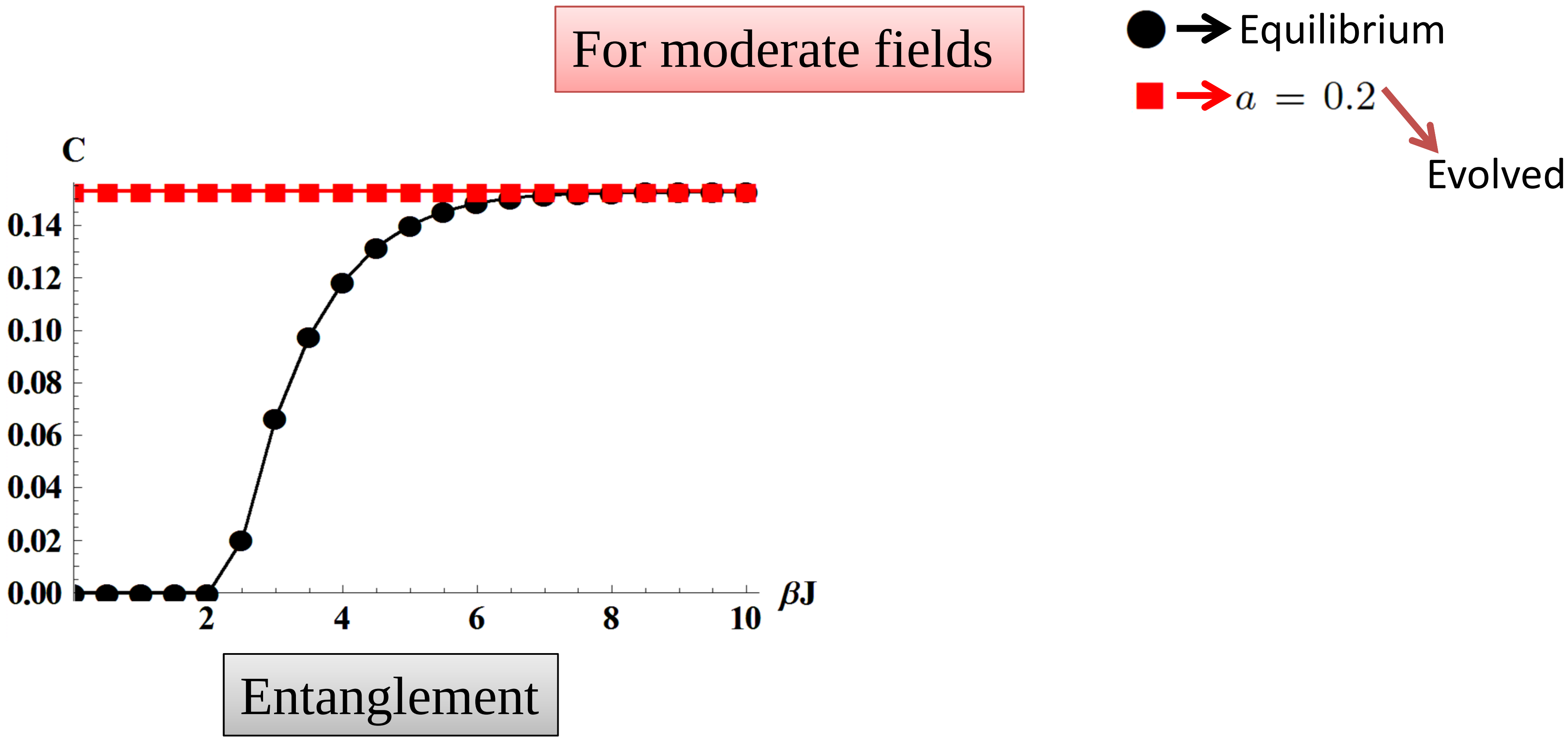
For infinite XY spin chain



For moderate fields

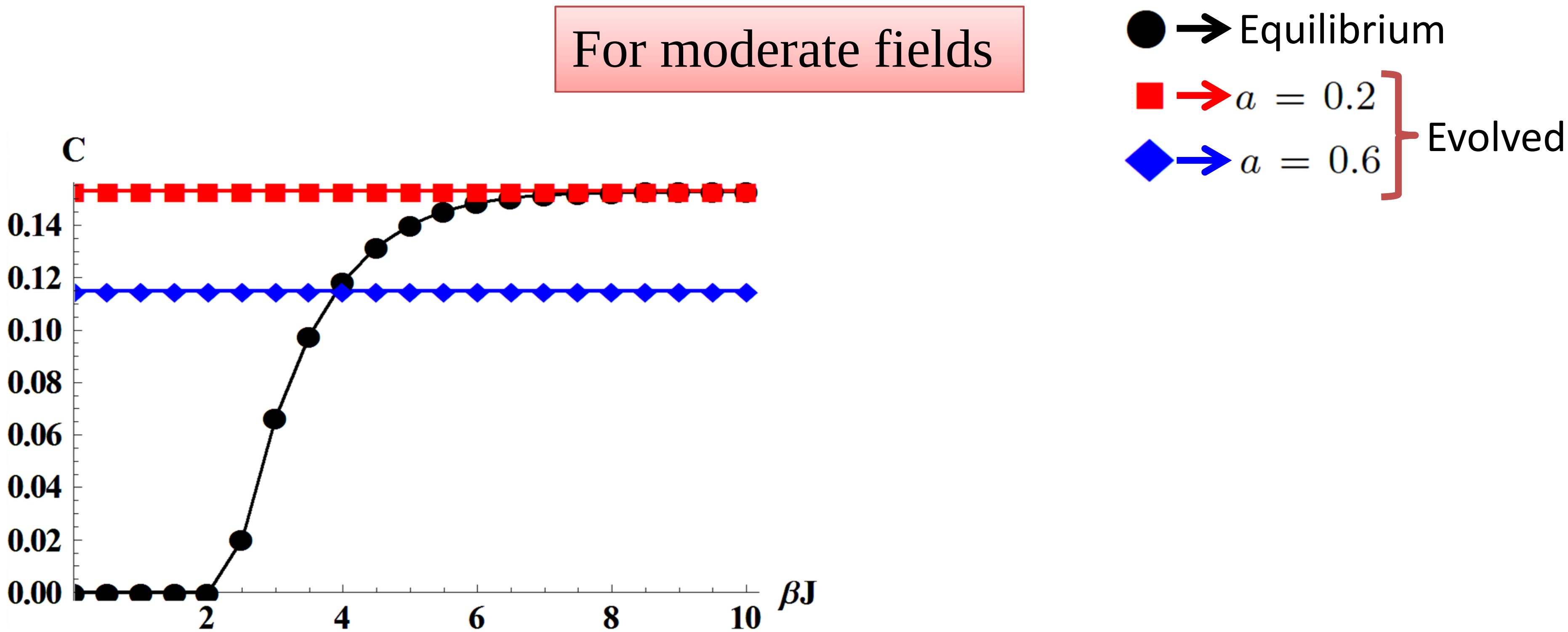


Entanglement

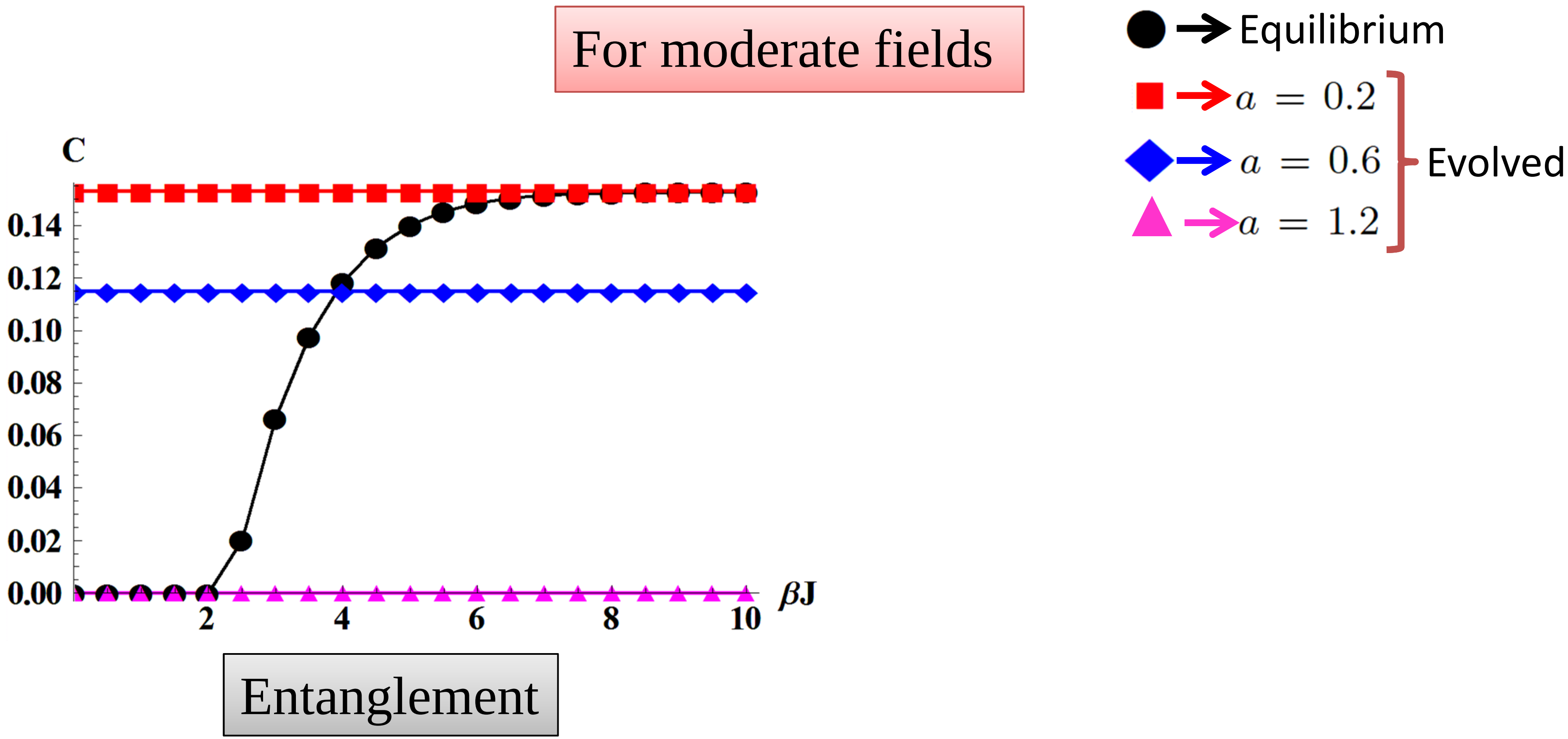


For moderate fields
Equilibrium
$a = 0.2$
Evolved
C
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
2
4
6
8
10
βJ
Entanglement

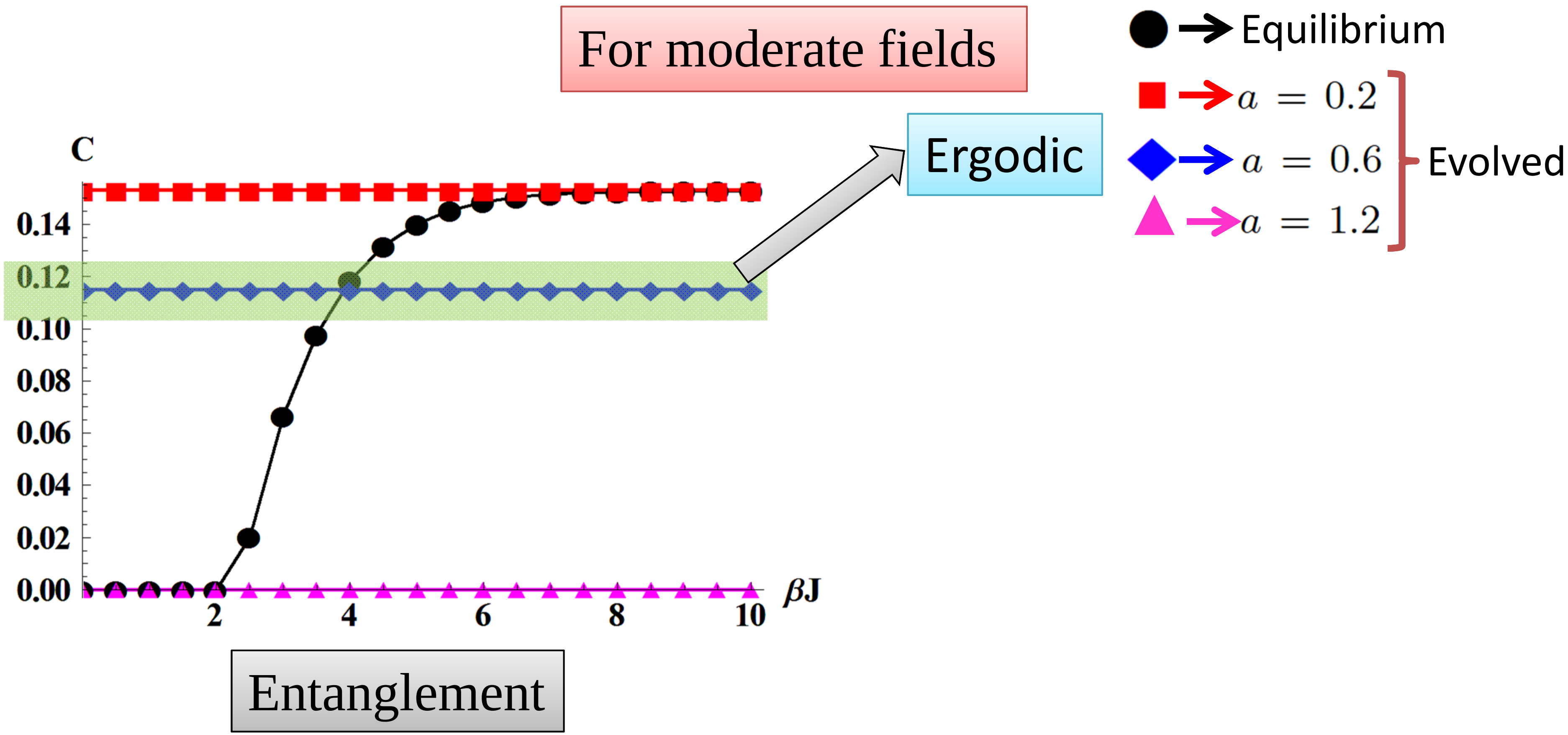
For moderate fields



Entanglement



For moderate fields
C
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
2
4
6
8
10
βJ
Entanglement
Equilibrium
a = 0.2
a = 0.6
a = 1.2
Evolved


For moderate fields
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Evolved
Ergodic
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0.14
0.12
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0.08
0.06
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0.02
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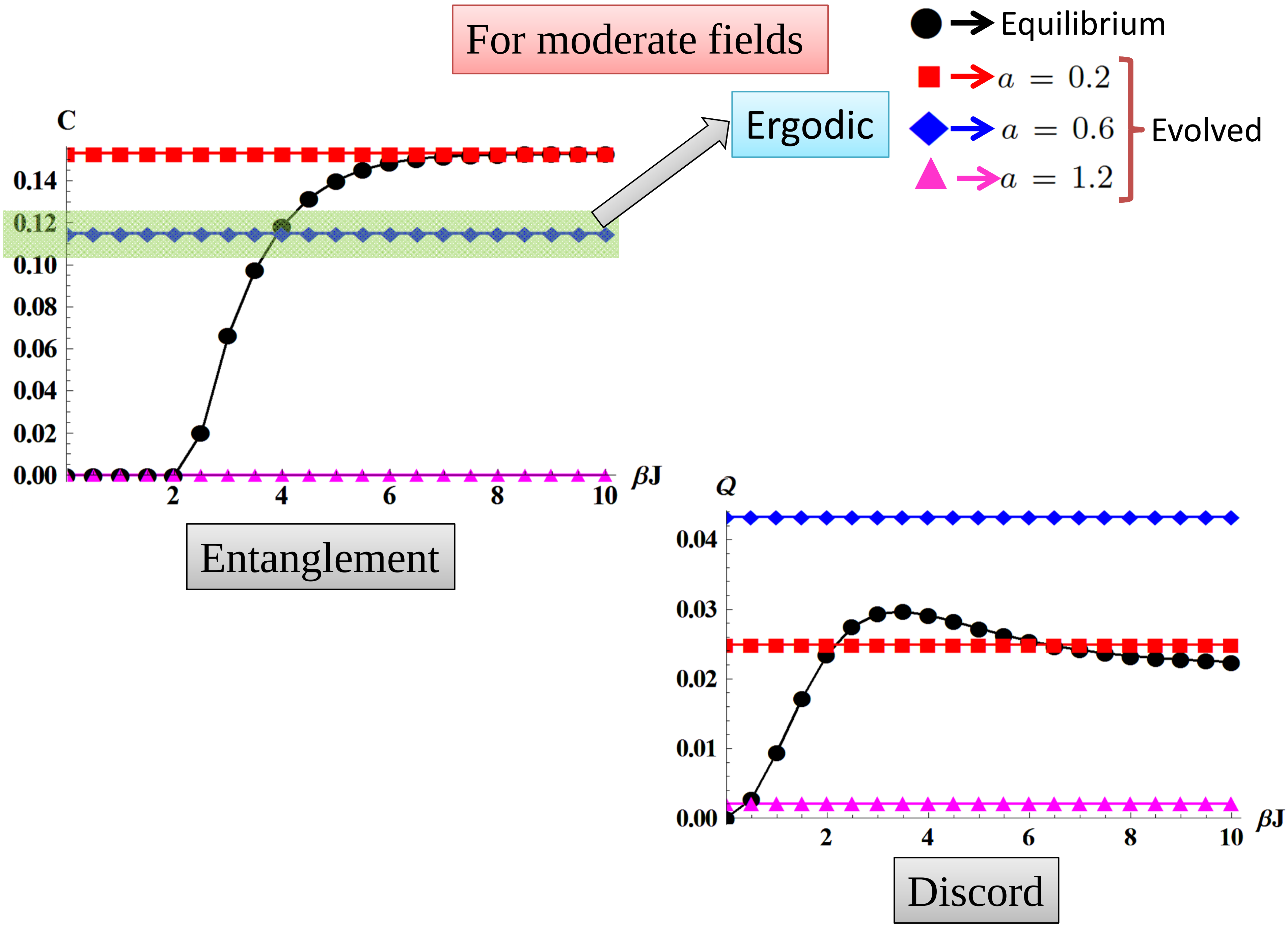
10
βJ
Entanglement



For moderate fields
Equilibrium
a = 0.2
a = 0.6
a = 1.2
Evolved
Ergodic
C
0.14
0.12
0.10
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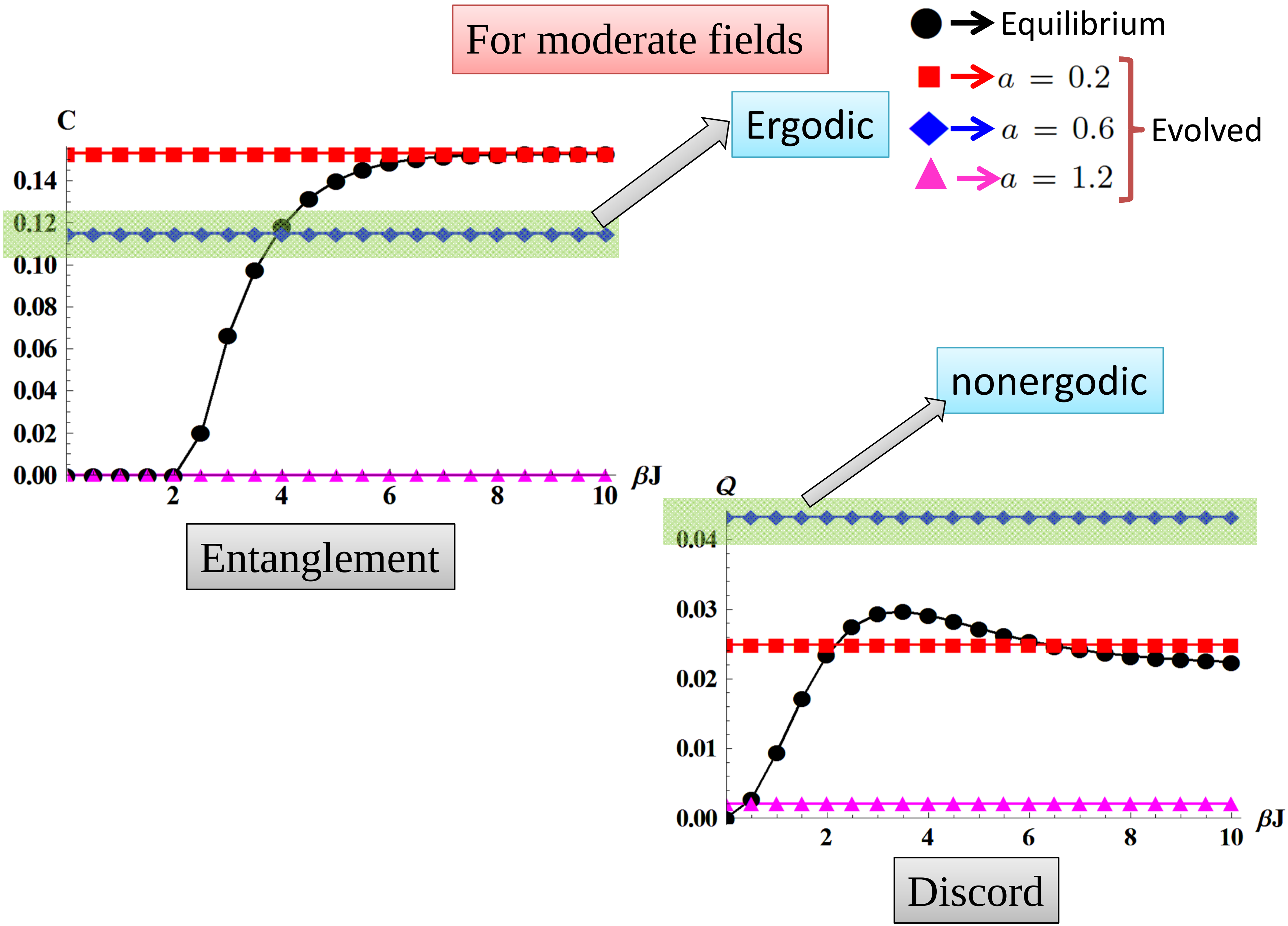
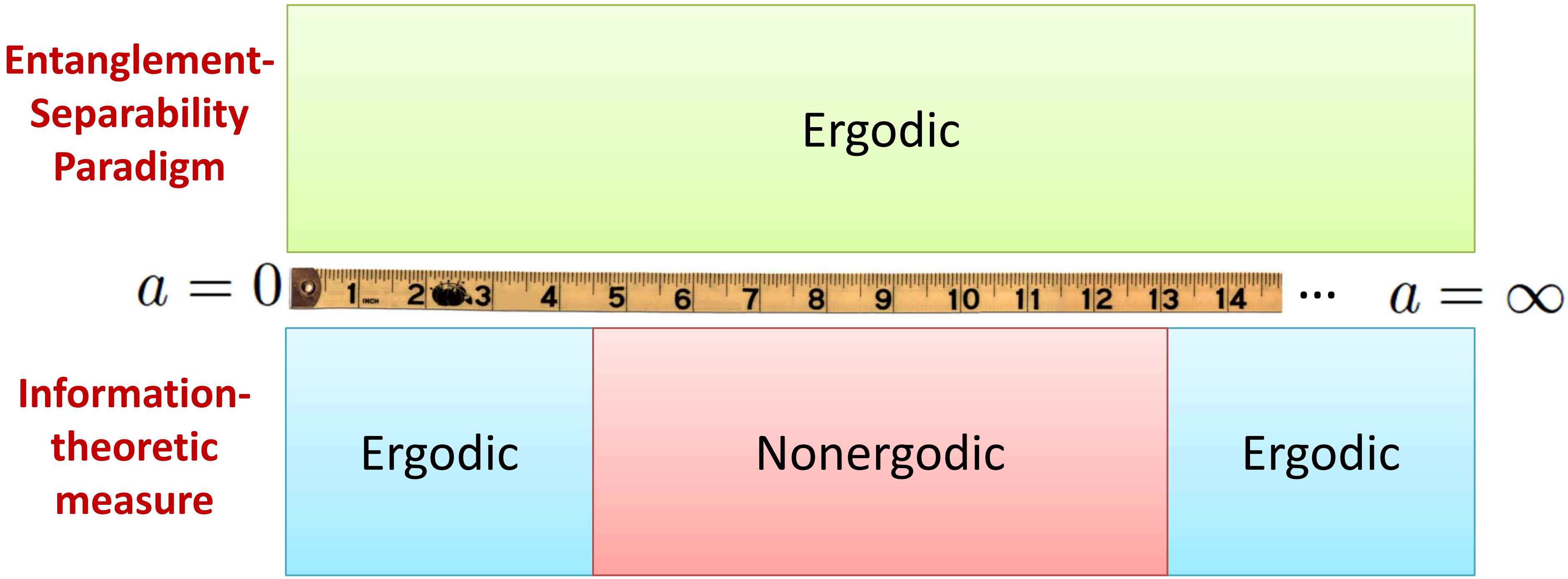
0.00
2
4
6
8
10
βJ
Entanglement
Q
0.04
0.03
0.02
0.01
0.00
2
4
6
8
10
βJ
Discord



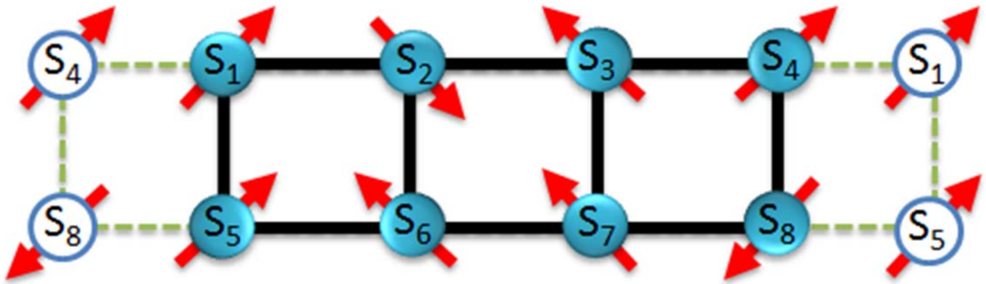
For moderate fields
Equilibrium
a = 0.2
a = 0.6
a = 1.2
Evolved
Ergodic
nonergodic
C
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
2
4
6
8
10
βJ
Entanglement
Q
0.04
0.03
0.02
0.01
0.00
2
4
6
8
10
βJ
Discord

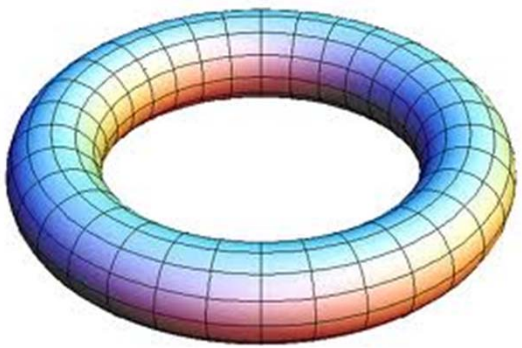
For infinite XY spin chain:

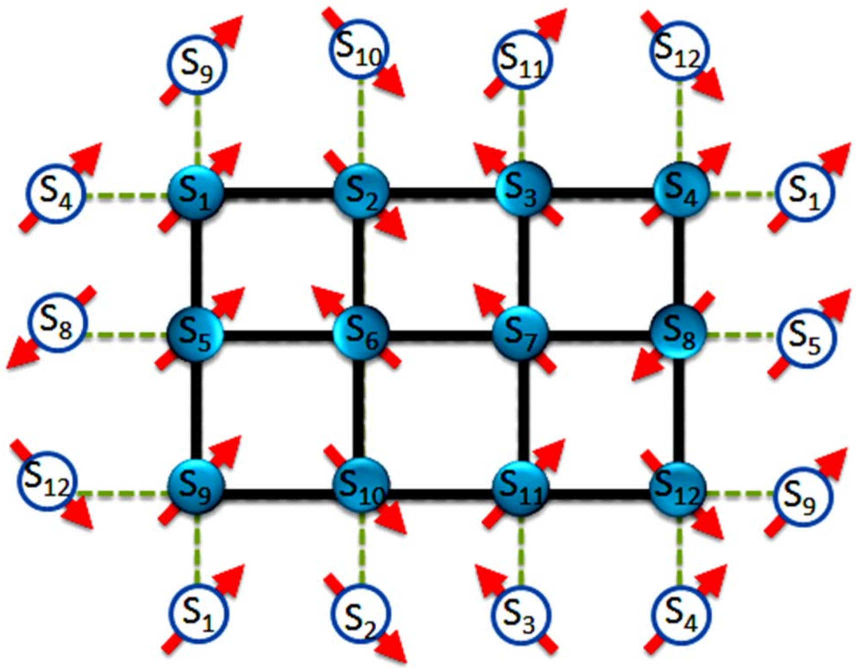


R. P, A. Sen(De), and U. Sen, arXiv:1112.1856

Generic for 1D, Ladder & Two-dimension







Conclusion

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Quantum correlation measures

Entanglement-Separability paradigm measures

Information-theoretic paradigm measures

Conclusion

Quantum correlation measures

Entanglement-Separability paradigm measures

Information-theoretic paradigm measures

Dichotomy of quantum correlations

Entanglement is monogamous but discord is not

Entanglement is ergodic but discord is not

THANK YOU

for your attention