GEOMETRIC PHASE : A DIAGNOSTIC TOOL FOR ENTANGLEMENT ?

Sandhya S. N
I. I. T. Kanpur

Collaborator: Subhashish Banerjee

Why geometric phase ?

- A connection between GP and entanglement is possible: both are in the realm of quantum kinematics.

- Existing measures require tomography.

- Need for an operational measure.

- Physical feasibility: may have an edge over Tomography.

- Both GP and topological phases may be designed to realize fault tolerance (Knill Nature 2005) and resilience to decoherence (Wu et al, PRL 2005, Oreshkov et al PRL 2009). But GP is easier to implement.

- Quantum gates with GP have been demonstrated in the NMR set up (Jones et al Nature 2000).

- For photon states interferometric set up helps measure GP.

Plan

- Evaluate the distribution of entanglement in the parameter space.
- Next, the distribution of the GP.
- Examine whether there is a correlation
  analytically - pure states without decoherence
  numerically - mixed states (real system).

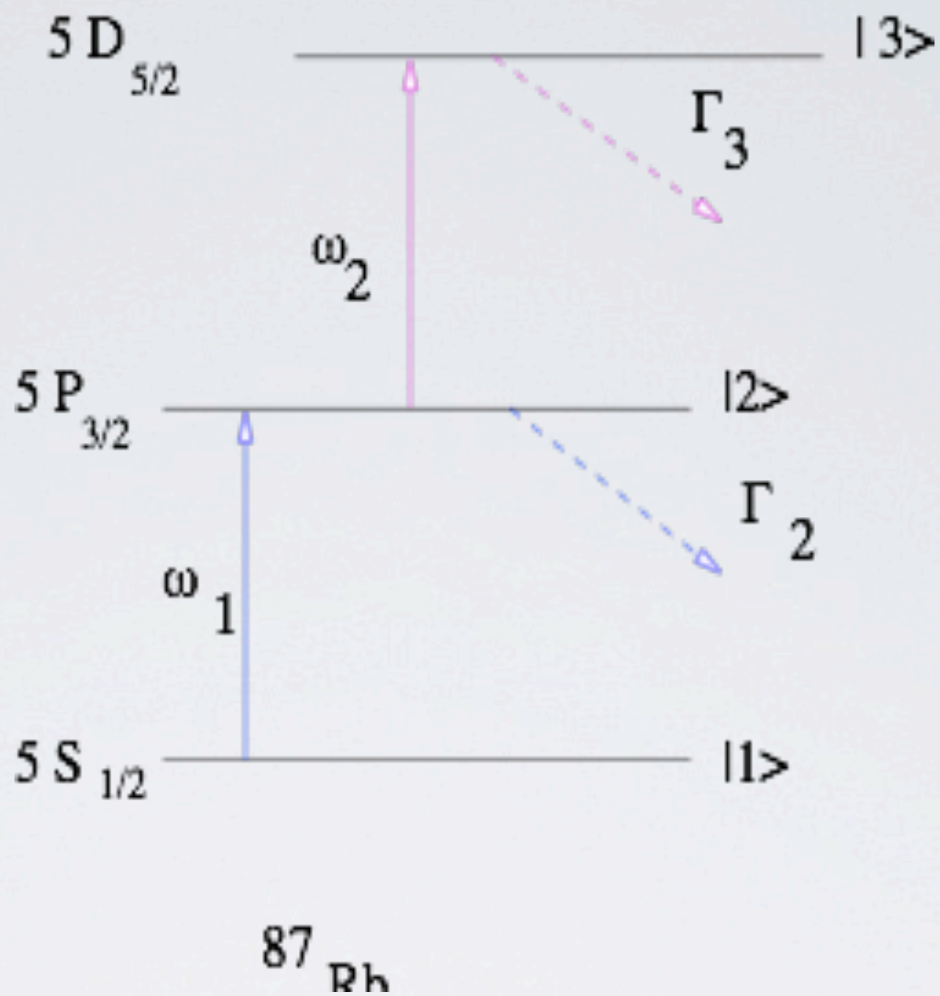
System

- Photons emitted by Atomic systems.
- Atom-Photon interactions offers good control for tailoring the photon states.
- Better than Parametric down conversion:
  Larger cross-section,

- Larger number of control parameters
- availability of mixed states
- Also easy to identify parameters for controlling separability/ entanglement and purity/mixedness.

Limitations of our scheme

- Experimental implementation involves null measurement.
- May not be suitable for other quantum information protocols.
- However, provides a test ground for understanding entanglement.



Three level cascade system: Two photon states show non-classical properties (Clauser PRD 1974)

TWO-PHOTON STATE

- The two photon state $|n\rangle = |n_1\rangle |n_2\rangle$ is in the Fock space and corresponds to the two modes emitted by the two transitions.

- The modes are centered around the frequency of transition.

- For simplicity we assume that $n_i = \{0,1\}$ (weak excitation regime < atomic excited state lifetime ~6MHz).

- The density matrix is of rank three (follows from Schmidt decomposition).

- This class of states is in a smaller subspace of dimension $(N+1)^2-1$ rather than 4^N.

HOW TO DETERMINE THE TWO PHOTON STATE

- There is no master equation for emitted photons.
- Equivalence in the weak excitation regime.

atom DM $\Leftrightarrow$ photon DM

$t-r/c$ $\qquad$ t

master equation exists

Sandhya, Ravishankar PRA 2011

Density matrix of the atomic system

Hamiltonian in the interaction representation and in the RWA

$$H = \frac{\hbar}{2}(\omega_1\sigma_1^z + \omega_2\sigma_2^z) + \hbar\Omega_1(e^{-i\omega_1 t}\sigma_1^+ + h.c.) + \hbar\Omega_2(e^{-i\omega_2 t}\sigma_2^+ + h.c.)$$

Steady state solution of the equation

$$i\hbar\dot{\rho}^A = [H, \rho^A] - \frac{i\hbar}{2}\{\Gamma, \rho^A\}$$

Parameter Space:

driving field strength Ω_i, $i=1,2$;

detuning, $\Delta_i = (\omega_L - \omega_i)$; $i=1,2$.

two-photon detuning $\Delta=\Delta_1+\Delta_2$.

Lifetime of the excited state Γ_i, $i=2,3$.

- Effect of these parameters on the two-photon state:

— Excited state life time:

infinitely long lived ⇒ no decoherence.

Real system -metastable (~1 MHz) ⇒ some decoherence.

- Two-photon detuning Δ

$\Delta=0 \Rightarrow$ pure state in the absence of decoherence

$\Rightarrow$ small admixture to the pure state in the presence of decoherence.

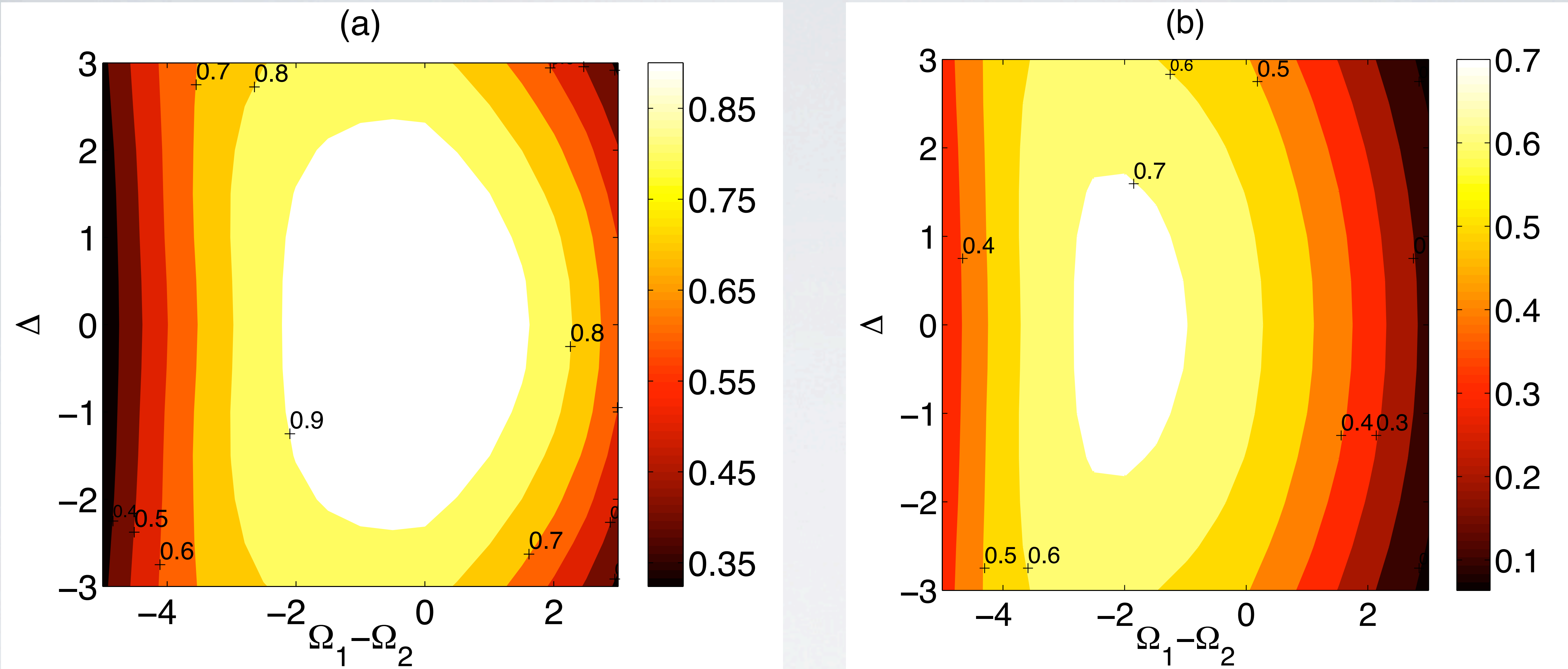
- Driving field strengths Ω_1, Ω_2

$\Omega_1 \approx \Omega_2$, $\Delta=0 \Rightarrow$ pure Bell states (no decoherence)

$\Rightarrow$ maximally entangled states (with decoherence).

$\Omega_1 << \Omega_2 \Rightarrow$ separable states.

$\Omega_2 << \Omega_1 \Rightarrow$ mixed states (with decoherence).



Distribution of concurrence in the parameter space in a) the absence of decoherence and b) presence of decoherence.

- In the absence of decoherence the pure state at $\Delta=0$

$$|\Psi> = -\ Sin\ (X)\ |00> + Cos(X)\ |11>$$

where $X = Tan^{-1}(\Omega_1/\Omega_2)$ is a dimensionless parameter.

Concurrence is 2 Sin(X)Cos(X) and is maximally entangled for $X=\pi/4$.

GEOMETRIC PHASE

Quick Review:

- Pancharatnam phase for pure states:

$$\alpha(t)=\mathrm{Arg}(\langle\Psi(0)|\Psi(t)\rangle).$$

- Berry showed that if there is a Hamiltonian which depends on slowly varying external parameters, in the adiabatic approximation the solution of the Schrodinger equation gains an additional phase apart from the dynamical phase.
- Aharnov Anandan- generalization to all non-adiabatic cyclic
- Samuel Bhandari - non-cyclic evolution.
- Mukunda and Simon - quantum kinematic approach
- Chaturvedi et al -Berry's phase for coherent states
- Arun Pati et al - unitary evolution of mixed states.
- Tong et al - non-unitary evolution of mixed states.

- We are interested in the evolution of the GP in the parameter space $\{\Delta, X\}$.

- To span the entire parameter space, we need to consider a non-adiabatic, non-cyclic, non-unitary evolution.

- For mixed states, using purification by the introduction of an ancilla, it has been shown that the total GP is the weighted sum of the individual phases of the pure eigenstates.

- Mixed states: Purification by introducing an ancilla

$$|\Psi(t)\rangle = \sum_k \sqrt{\lambda(k)}|\phi_k(t)\rangle \otimes |a_k\rangle; t \in [0,\tau].$$

- Impose the parallel transport condition on each of the eigenstates $|\Phi_k>$

$$\gamma_g(\tau) = Arg[\sum_k \sqrt{\lambda_k(\tau)\lambda_k(0)}\langle\phi_k(0)|\phi_k(\tau)\rangle \times e^{-\int_0^\tau \langle\phi_k(t')|\dot{\phi}_k(t')\rangle dt'}]$$

Strategy

- For pure state (no decoherence) we analytically show the relation between GP and entanglement.
- For mixed state (real system) we illustrate it numerically.

A. K. Pati and collaborators PRL 2003 , Tong et al, PRL 2004

Pure states

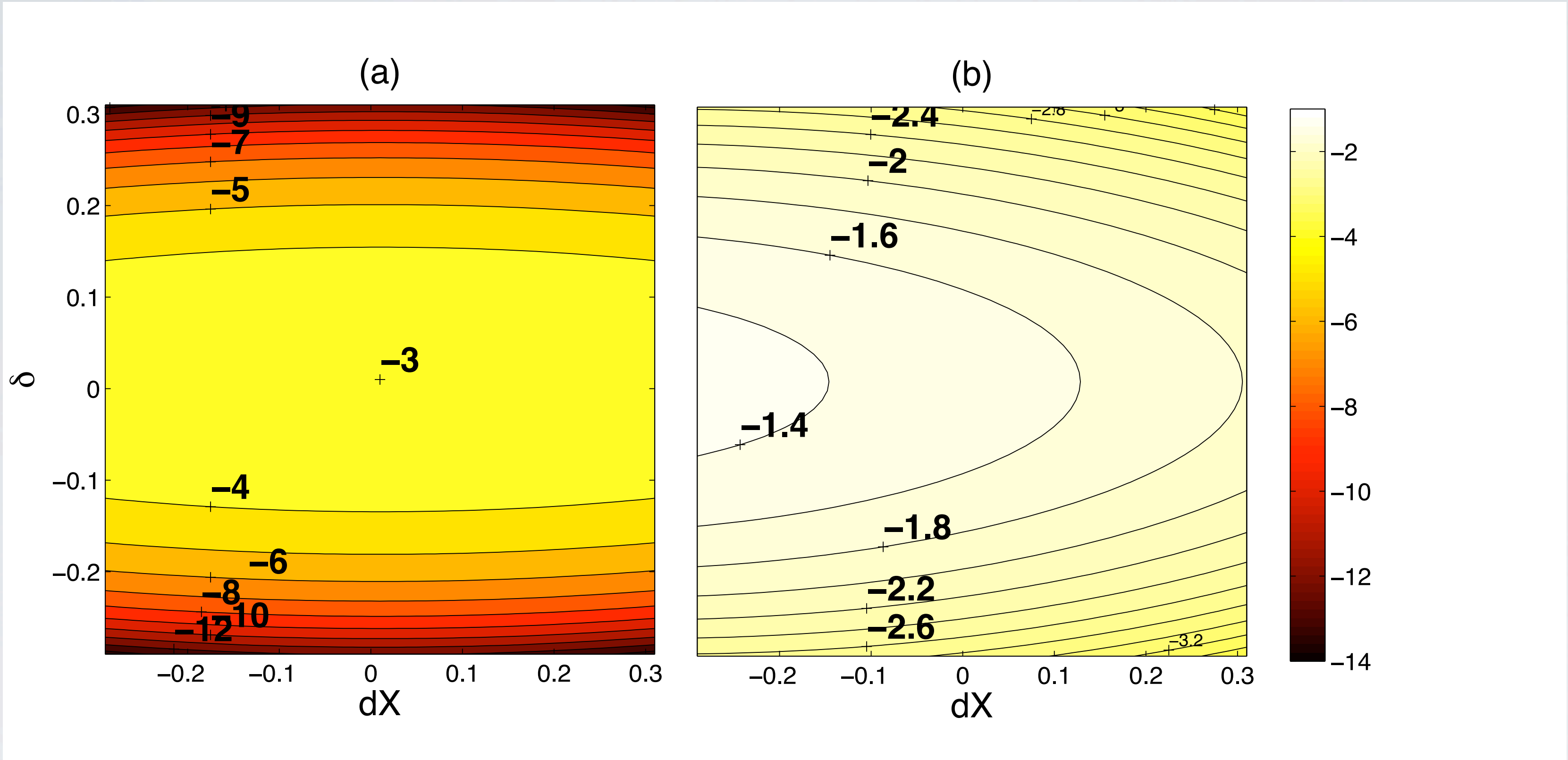
Geometric phase evaluated in the parameter space $\{\Delta, X\}$ with no decoherence.

GP at a point $\chi=(\delta, X+dX)$ in the neighborhood of $\chi_0=(0, X)$

$$\gamma_g(\chi) = Arg[1 - \frac{dX^2}{2} + \beta\,\delta^2 - i\gamma_{21}\mathrm{Cos}(X)^2\delta - \frac{i\gamma_{21}\mathrm{Cos}(X)\mathrm{Sin}(X)\;\delta\;dX}{2}]$$

$$=$$

$$Arctan[\frac{-i\gamma_{21}\mathrm{Cos}(X)^2\delta - i\gamma_{21}C\;\delta\;dX/4}{1-\frac{dX^2}{2}+\beta\,\delta^2}]$$



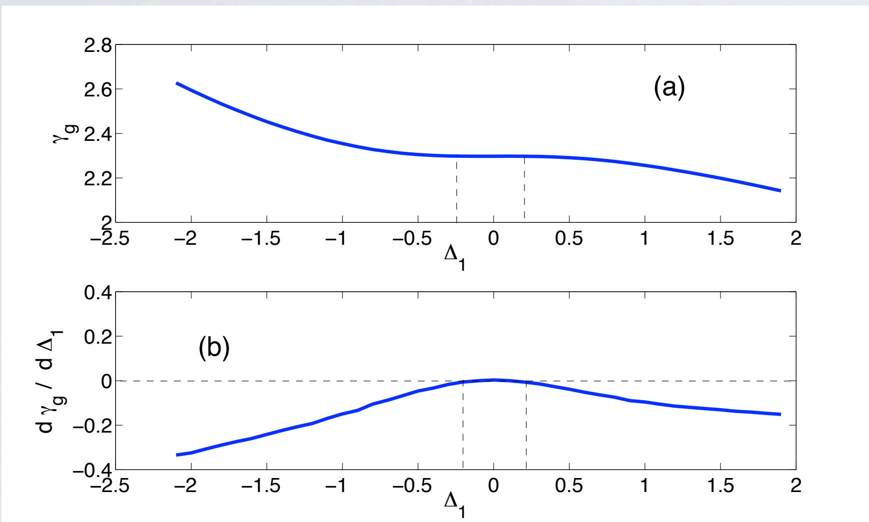
Variation of the derivative of the GP in the neighborhood of $\chi_0 = (0, X)$.

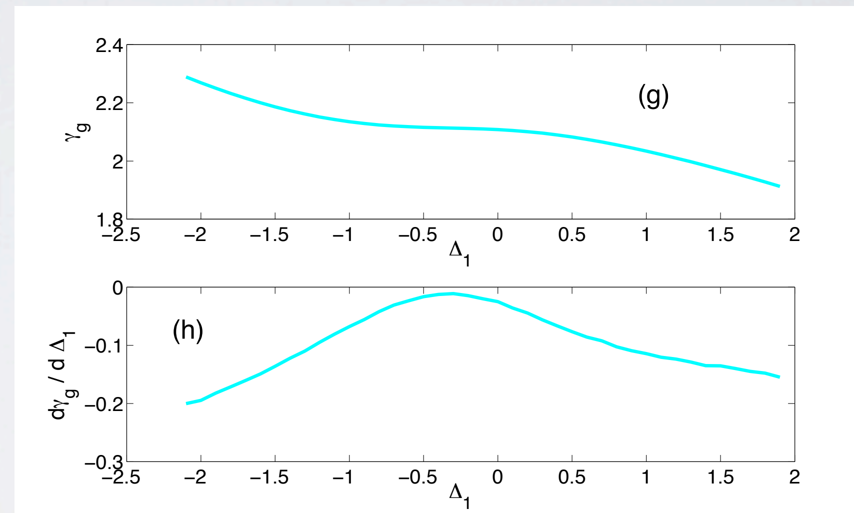
a) X=0, concurrence=0 and

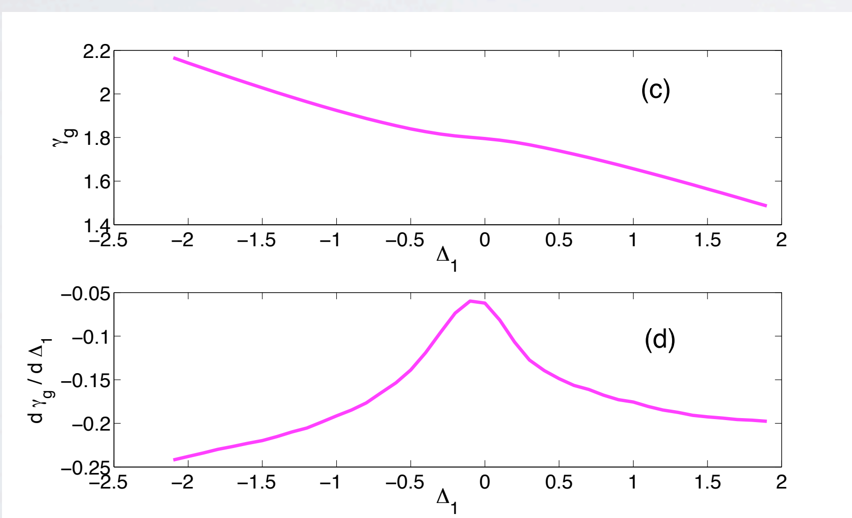
b) X= π/4, concurrence=1.

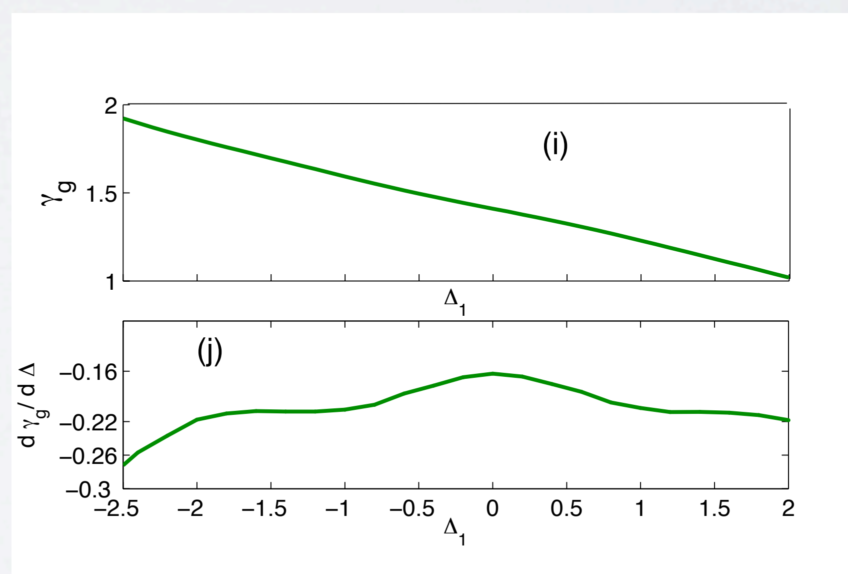
Variation of the GP and its derivative in the real system

pure+admixture (~20%), entangled states



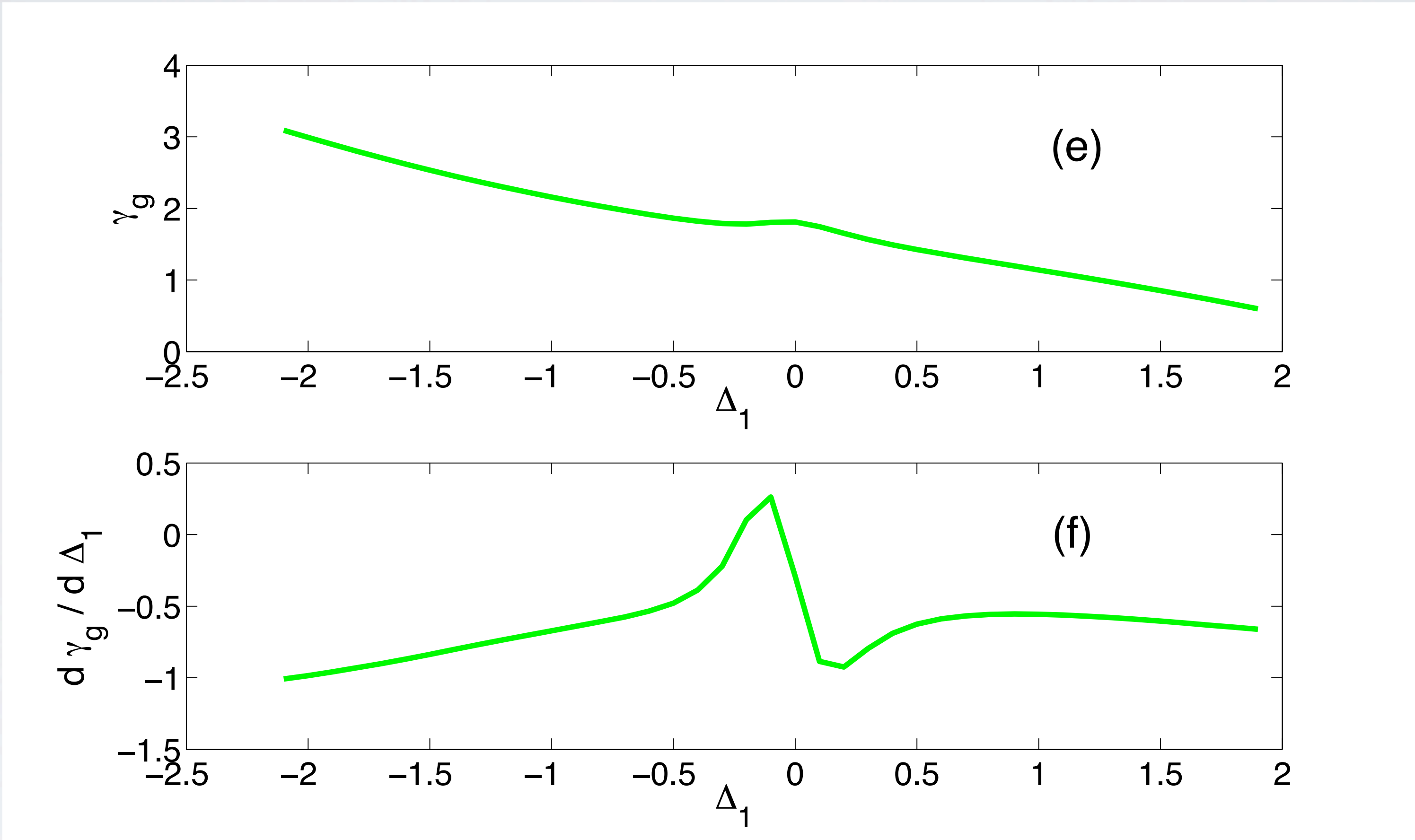






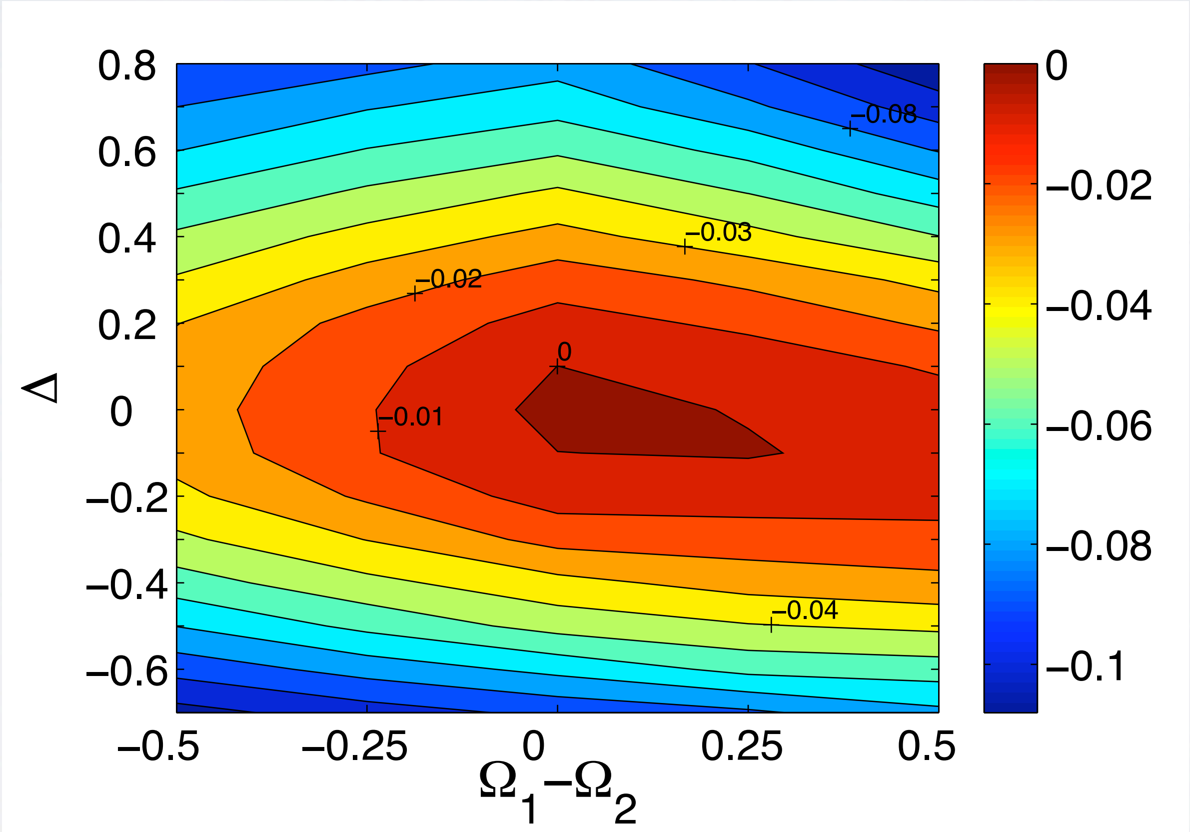
pure state+admixture (~10%), weak entgl

mixed state



System parameters are rendered dimensionless by scaling with atomic lifetime ~1MHz

Stability of GP for small fluctuations of the parameters



Summary

- We study the relation between GP and entanglement in a Hamiltonian system.
- We identify the control parameters for purity and entanglement.
- We obtain the distribution of entanglement in the parameter space.
- We show the dependence of the GP on the concurrence for the pure state.
- For the mixed state, we show that the derivative of GP is sensitive to both entanglement and purity.