Classical correlations and separability perceived via generalized measurements

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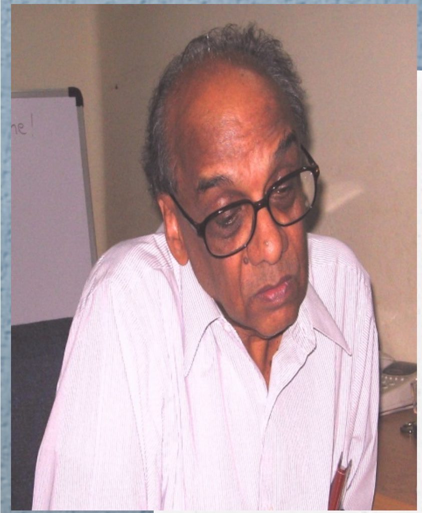

Collaborators:

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Inspire Institute, Alexandria, VA, USA

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Kuvempu University, Shankaraghatta, India.

Quantum correlations other than entanglement?

Separability and quantumness of correlations

A. R. Usha Devi and A. K. Rajagopal, Phys. Rev. Lett. **100** 140502 (2008)

A R Usha Devi, A K Rajagopal and Sudha, arXiv:1105.4115
International Journal of Quantum Information **9,** 1757 (2011)

Classical Probability notions:

Two random variables A and B are correlated if their joint probability distributions cannot be expressed as a mere product of the marginal probabilities:

$$P(a,b) \neq P(a)P(b) \quad \Longrightarrow \quad \text{correlated}$$

Shannon information entropies

$$H(A,B) = -\sum_b P(a,b) \log P(a,b)$$

$$H(A) = -\sum_a P(a) \log P(a)$$

$$H(B) = -\sum_b P(b) \log P(b)$$

$$\sum_b P(a,b) = P(a), \quad \sum_a P(a,b) = P(b)$$

Shannon Mutual information entropy quantifies correlations

$$H(A:B) = H(A) + H(B) - H(A,B)$$

$$= -\sum_a P(a)\log P(a) - \sum_b P(b)\log P(b) + \sum_{a,b} P(a,b)\log P(a,b)$$

$$H(A:B) = 0 \quad \text{iff} \quad P(a,b) = P(a)P(b)$$

Quantum description:

$$P(a,b) \rightarrow \rho_{AB}$$

$$P(a) \rightarrow \rho_A = Tr_B\ \rho_{AB}$$

$$P(b) \rightarrow \rho_B = Tr_A\ \rho_{AB}$$

Bipartite density matrix shared by Alice and Bob

Subsystem density matrices

Natural extension of the concept of correlation

$$\rho_{AB} \neq \rho_A \otimes \rho_B \Longrightarrow \text{correlated}$$

von Neumann mutual information quantifies correlations

$$S(A:B) = S(A) + S(B) - S(A,B)$$

$$= S\left(\rho_{AB} \,\|\, \rho_A \otimes \rho_B\right) = -Tr \;\; \rho_A \log \rho_A - Tr \;\; \rho_B \log \rho_B$$

$$+ Tr \;\; \rho_{AB} \log \rho_{AB}$$

$$S(A:B) = 0 \quad \text{iff} \quad \rho_{AB} \neq \rho_A \otimes \rho_B$$

Notion of correlation *per se* does not set a borderline between classical and quantum descriptions.

How do we distinguish between classical and quantum correlations in a bipartite quantum state?

Can we express

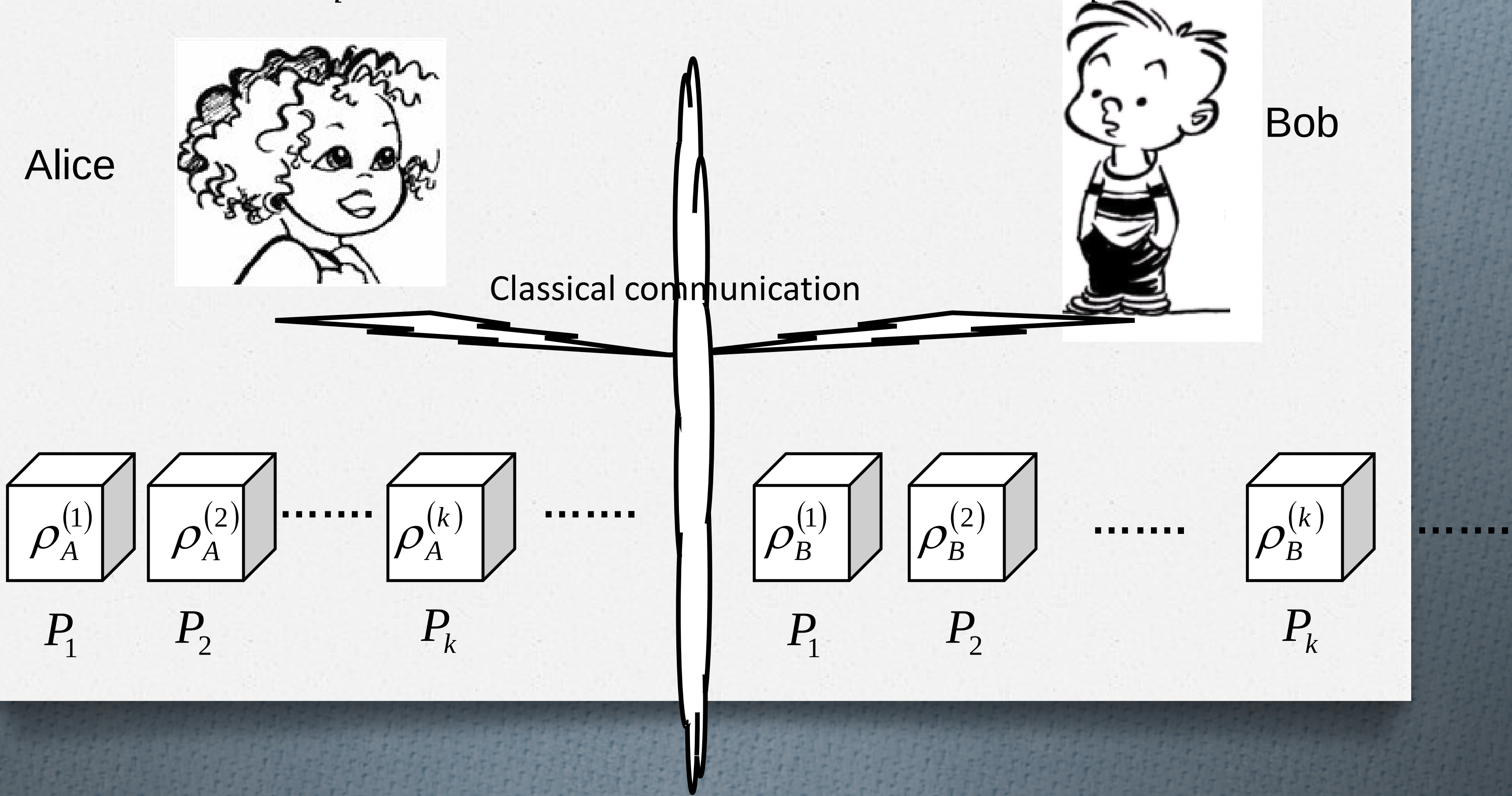
$S(A:B) =$ Contribution from classical correlations **+** Contribution from quantum correlations ?

L. Henderson and V. Vedral: J. Phys. A: Math. Gen. **34,** 6899 (2001)

R. F. Werner, Phys. Rev. A **40**, 4277 (1989)

A bipartite density operator ρ_{AB} is **classically** correlated (separable) if it admits a convex combination of product states:

$$\rho_{AB}^{(sep)} = \sum_i P_i\, \rho_A^{(i)} \otimes \rho_B^{(i)}; \quad 0 \le P_i \le 1, \quad \sum_i P_i = 1$$



Separable states respect Bell inequalities

Measurements on one part of the quantum system distinguishing classical and quantum correlation:

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88,** 017901 (2001)

Measurements on one end disturbs the **quantum** correlated state in general:

$$\rho_{AB} \xrightarrow[\text{on A}]{\text{measurement}} \rho'_{AB} \neq \rho_{AB}$$

If an optimal measurement scheme (on one part) exists such that $\rho'_{AB} = \rho_{AB}$ the state is **classically correlated**

Classically correlated state remains insensitive to an optimal measurement at one end of the system

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88,** 017901 (2001)

Are the set of all separable states classical?

(Do separable states remain insensitive to an optimal measurements at one end of the composite system?)

OZ approach

Projective measurements on *A* $\left\{\Pi_\alpha^A \otimes I_B\right\}$

$$\sum_\alpha \Pi_\alpha^A = I_A$$

$$\Pi_\alpha^A \Pi_{\alpha'}^A = \Pi_\alpha^A \delta_{\alpha\alpha'}$$

Completeness

Orthogonality

The **conditional density operator** of subsystem *B* – when measurement $\Pi_{\alpha}^{A} \otimes I_{B}$ is known to have led to a value α -
is given by,

$$\rho_{B|A}^{\alpha} = \frac{\Pi_{\alpha}^{A} \otimes I_{B} \quad \rho_{AB} \quad \Pi_{\alpha}^{A} \otimes I_{B}}{Tr\ [(\Pi_{\alpha}^{A} \otimes I_{B})\ \rho_{AB}]}$$

$$\mathrm{Tr}\left[\left(\Pi_{\alpha}^{A} \otimes I_{B}\right)\rho_{AB}\right] = P_{\alpha}, \qquad \sum_{\alpha} P_{\alpha} = 1$$

Given the results of the complete measurements $\left\{\Pi_\alpha^A \otimes I_B\right\}$ the **conditional information entropy** is given by,

$$S\left(B \mid A_{\left\{\Pi_\alpha^A\right\}}\right) = \sum_\alpha P_\alpha \, S\left(\rho_{B|A}^{(\alpha)}\right)$$

exhibits inherent dependence on measurements

positive

Another form of conditional entropy ⇒ Realized by a structural generalization of Shannon conditional entropy

$$H(B|A) = H(A,B) - H(A)$$

$$-\sum_{a,b} P(a,b)\log P(b|a)$$

This is a consequence of the **Bayes' rule**

$$P(b|a) = \frac{P(a,b)}{P(a)}$$

gives von Neumann conditional entropy

$$S(B|A) = S(A,B) - S(B)$$

uncritical extension of Shannon form

Not necessarily a positive definite !!!!!

Quantum discord (OZ): Optimal difference of two classically identical expressions for conditional (mutual) information entropies:

$$\delta(A,B) = \min_{\{\Pi_\alpha^A\}} S\left(B \mid A_{\{\Pi_\alpha^A\}}\right) - S(B \mid A)$$

$$= S(\rho_{AB} \parallel \rho_A \otimes \rho_B) - \left(S(B) - \min_{\{\Pi_\alpha^A\}} S\left(B \mid A_{\{\Pi_\alpha^A\}}\right)\right)$$

$$\delta(A,B)=0 \quad \text{iff} \quad \rho'_{AB}=\sum_{\alpha}\left(\Pi_{\alpha}^{A}\otimes I_B \;\; \rho_{AB} \;\; \Pi_{\alpha}^{A}\otimes I_B\right)=\rho_{AB}$$

Discord vanishes, only when the state is left insensitive as a result of optimal projective measurement on one part of the system

- Optimal measurement $\{\Pi_{\alpha}^{A}\}$ leaves the overall state with least disturbance.

- Minimum disturbance quantified by Quantum Discord $\delta(A,B)$

- Bipartite states, which are in conformity with the Bayes’ Rule ⇒ $\delta(A,B)=0$

  ⇒ Classically correlated

$$\delta(A,B)=0 \quad \text{iff} \quad \rho'_{AB}=\sum_{\alpha}\left(\Pi_{\alpha}^{A}\otimes I_{B}\ \rho_{AB}\ \Pi_{\alpha}^{A}\otimes I_{B}\right)=\rho_{AB}$$

➢The quantum state is insensitive to an optimal projective measurement on one part of the system if its subsystems are classically correlated

Quantum Discord DOES NOT VANISH FOR ALL SEPARABLE STATES !!!!

Separability is not synonymous with classicality!

Quantum bipartite states with vanishing quantum discord:

$$\rho_{AB}^{(classical)} = \sum_a P_a \; \Pi_a^A \otimes \rho_{B|\Pi_a^A}$$

Classical:

$$P(a,b) = P(a)\,P(b|a)$$

Quantum:

$$\rho_{AB}^{(\text{classical})} = \sum_a P(a)\,\Pi_a^A \otimes \rho_{B|\Pi_a}$$

$$\Pi_a^A = |a\rangle\langle a|$$

Examples of two qubit separable states with non-zero quantum discord:

$$\rho_{AB} = P\left|0_A,0_B\right\rangle\left\langle 0_A,0_B\right| + (1-P)\left|+_A,+_B\right\rangle\left\langle +_A,+_B\right|$$

$$\left|\pm\right\rangle = \frac{1}{\sqrt{2}}\left(\left|0\right\rangle \pm \left|1\right\rangle\right); \quad 0 \le \mathrm{P} \le 1$$

S. Hamieh, R. Kobes and H. Zaraket, Phy. Rev. A **70,** 052325 (2004)

Werner state with $0 < \mathrm{p} \le 1/3$:

$$\rho_W = \frac{(1-p)}{4} I \otimes I + p\left|\Psi_{AB}^-\right\rangle\left\langle \Psi_{AB}^-\right|; \quad 0 < \mathrm{p} \le 1/3$$

$$\left|\Psi_{AB}^-\right\rangle = \frac{1}{\sqrt{2}}[\left|0_A,1_B\right\rangle - \left|1_A,0_B\right\rangle]$$

Another non-zero discord separable state: A. Datta, A. Shaji, C. Caves, Phys. Rev. Lett. **100**, 050502 (2008)

$$\rho_{AB} = \frac{1}{4}[|+,0\rangle\langle+,0| + |-,1\rangle\langle-,1| + |+,0\rangle\langle+,0|$$
$$+|0,-\rangle\langle0,-| + |1,+\rangle\langle1,+|]$$

A. K. Rajagopal and R. W. Rendell, Phys. Rev. A 66, 022104 (2002)

Quantum Deficit: $\mathrm{D}_{\mathrm{AB}} = S\left(\rho_{AB} \| \rho_{AB}^{d}\right)$

- Another equivalent measure of quantumness of correlations

$\rho_{AB}^{(d)}$: classical decohered counterpart of ρ_{AB}

Measurement Induced Disturbance (MID)

S. Luo, Phys. Rev. A **77**, 022301 (2008)

$$\rho_{AB} = \sum_{\alpha,\beta} \rho_{\alpha'\beta';\alpha\beta} \, |\alpha'\rangle\langle\alpha| \otimes |\beta'\rangle\langle\beta|$$

$$\rho_{AB}^{(d)} = \sum_{\alpha,\beta} \rho_{\alpha\beta;\alpha\beta} \, \Pi_\alpha^{(A)} \otimes \Pi_\beta^{(B)}$$

$$= \sum_\alpha P_\alpha \Pi_\alpha^{(A)} \otimes \left[\frac{\sum_\beta \rho_{\alpha\beta;\alpha\beta} \Pi_\beta^{(B)}}{P_\alpha} \right]; \quad P_\alpha = \sum_\beta \rho_{\alpha\beta;\alpha\beta}$$

$$= \sum_\alpha P_\alpha \Pi_\alpha^{(A)} \otimes \rho_\alpha^{(B)}$$

⇒ Classically correlated

Subsystem eigen basis

$$\Pi_\alpha^{(A)} = |\alpha\rangle\langle\alpha|$$

$$\Pi_\beta^{(B)} = |\beta\rangle\langle\beta|$$

$$\Pi_\alpha^{(A)} \Pi_{\alpha'}^{(A)} = \Pi_\alpha^{(A)} \delta_{\alpha'\alpha}$$

$$\sum_\alpha \Pi_\alpha^{(A)} = I_A \text{ etc...}$$

L. Henderson and V. Vedral: J. Phys. A: Math. Gen. **34,** 6899 (2001)

Classical correlation: $C_A(\rho_{AB}) = \max_{\{V_i^A\}} S(\rho_B) - \sum_i P_i S(\rho^i_{B|A})$

Residual information entropy of B after carrying out a POVM measurement $\{V_i^A\}$ on the subsystem *A*

$$\rho^i_{B|A} = \frac{1}{P_i} Tr_A \left[V_i^A \otimes I_B \; \rho_{AB} \; V_i^{A^\dagger} \otimes I_B \right];$$

$$P_i = Tr_{AB} \left[V_i^A \otimes I_B \; \rho_{AB} \; V_i^{A^\dagger} \otimes I_B \right]$$

Classical and entangled correlations do not add up to give total correlations!

$$C_A(\rho_{AB}) + E_{RE}(\rho_{AB}) \leq S(A:B)$$

“Are different types of correlations not additive?”

The one-way information deficit is defined as the minimal increase of entropy after a projective measurement $\{\hat{\Pi}_\alpha^A\}$ on subsystem A is done:

$$\Delta^{\rightarrow}(\hat{\rho}_{AB}) = \min_{\{\hat{\Pi}_\alpha^A\}} S\left(\sum_\alpha \hat{\Pi}_\alpha^A \, \hat{\rho}_{AB} \, \hat{\Pi}_\alpha^A\right) - S(\hat{\rho}_{AB})$$

(M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen(De), U. Sen, and B. Synak-Radtke, Phys. Rev. A **71**, 062307 (2005)) The one-way information deficit vanishes only on states with zero quantum discord.

- An unified approach to understand quantum and classical correlations – in terms of relative entropy (K. Modi, T. Paterek, W. Son, V. Vedral, M. Williamson , Phys. Rev. Lett. 104, 080501 (2010); K. Modi, V. Vedral, AIP Conf. Proc. 1384, 69-75 (2011))

Non-classical correlation in terms of distance between the given state with its closest classical state -- “relative entropy of discord” Measures of dissonance, entanglement – follow in an analogous manner in terms of relative entropy.

- Quantum features of discord – brought out in terms of entanglement consumption in state merging protocol (D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, A. Winter, Phys. Rev. A 83, 032324(2011); V. Madhok, A. Datta, Phys. Rev. A, 83, 032323 (2011))

Measurements play a crucial role in distinguishing and quantifying correlations as **classical** and **quantum**

Do measurements at one end distinguish correlations as **classical** and **quantum?**

What kind of measurements (at one end of the composite system) leave the set of all separable states insensitive?

Generalized measurements

A. R. Usha Devi and A. K. Rajagopal, Phys. Rev. Lett. **100** 140502 (2008)

A R Usha Devi, A K Rajagopal and Sudha, arXiv:1105.4115
International Journal of Quantum Information **9,** 1757 (2011)

Classical:

$$P(c,a,b) = P(c,a)\,P(b|ca)$$

Quantum:

$$\rho_{ABC}^{(\text{classical})} = \sum_{\alpha} P(\alpha)\,\Pi_{\alpha}^{CA} \otimes \rho_{B|\Pi_{\alpha}^{CA}}, \quad \alpha \to \{ca\}$$

This corresponds to:

$$\rho_{AB}^{(\text{separable})} = \mathrm{Tr}[\rho_{ABC}^{(\text{classical})}] \sum_{\alpha} P(\alpha) \; = \rho_{\alpha}^{A} \otimes \rho_{B|\Pi_{\alpha}^{CA}}$$

where $\rho_{\alpha}^{A} = \mathrm{Tr}_C[\Pi_{\alpha}^{CA}]$

Separable states are zero discord states in an extended space

- Consider all tripartite extensions ρ_{CAB} of the state ρ_{AB} such that

$$Tr_C\left[\rho_{CAB}\right] = \rho_{AB}$$

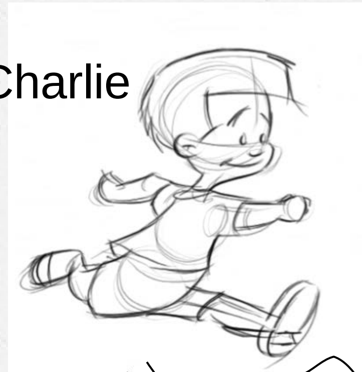
State under investigation

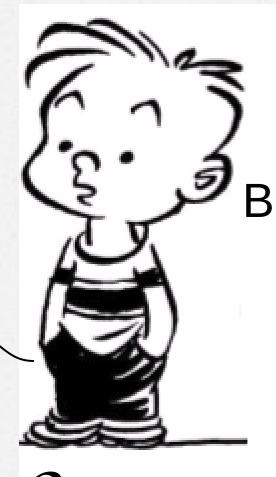
- Perform generalized projective measurements $\left\{\Pi_i^{(CA)} \otimes I_B\right\}$ on **one part** (at the Alice-Charlie end) of the system.

$$\rho'_{AB} = Tr_C\left[\sum_i \Pi_i^{(CA)} \otimes I_B \rho_{CAB} \Pi_i^{(CA)} \otimes I_B\right] = Tr_C(\rho'_{CAB})$$

state left after generalized measurement

Charlie



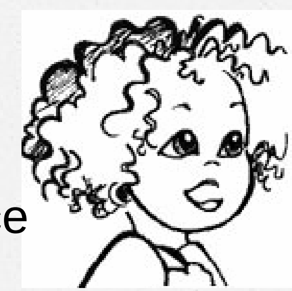


Bob

Optimal projective measurement by CA

$$\rho_{CAB}$$

$$Tr_C\left(\rho_{CAB}\right) = \rho_{AB}$$



Alice

$$\rho'_{AB} = \rho_{AB} \Rightarrow$$ Classically correlated

$$\rho'_{AB} \neq \rho_{AB} \Rightarrow$$ Quantum correlated

Def: Quantumness of correlations

$$\mathrm{Q}_{\mathrm{AB}} = \min_{\left\{\Pi_i^{(CA)} \otimes I_B \,,\, \rho_{CAB}\right\}} S\left(\rho_{AB} \,\|\, \rho_{AB}^{'}\right)$$

Minimization is over the set of all tripartite extensions and the set of all projective measurements at the Alice-Charlie end

$$\mathrm{Q}_{AB} = 0 \quad \text{iff} \quad \rho_{\mathrm{AB}} = \rho_{AB}^{'}$$

Optimizing quantumness

$$\mathrm{Q}_{\mathrm{AB}} = \min_{\left\{\Pi_i^{CA} \otimes I_B,\, \rho_{CAB}\right\}} S\left(\rho_{AB} \,||\, \rho'_{AB}\right)$$

over all possible three party extended states ρ_{CAB}
and also over all projective measurements $\left\{\Pi_i^{CA}\right\}$?

Optimal generalized measurement projects a bipartite state to its closest separable state.

$$\rho'_{CAB} = \sum_i \Pi_i^{(CA)} \otimes I_B \; \rho_{CAB} \; \Pi_i^{(CA)} \otimes I_B$$

$$= \sum_{i,b',b} \langle i\, b' | \rho_{CAB} | i\, b \rangle \; |i\rangle\langle i| \otimes |b'\rangle\langle b|$$

$$= \sum_i \; \Pi_i^{(CA)} \otimes \sum_{b',b} \langle i\, b' | \rho_{CAB} | i\, b \rangle |b'\rangle\langle b|$$

$$= \sum_i P_i \; \Pi_i^{(CA)} \otimes \rho_i^{(B)}$$

$$\Pi_i^{(CA)} = |i\rangle\langle i|; \qquad \rho_i^{(B)} = \sum_{b',b} \frac{\langle i\, b' | \rho_{CAB} | i\, b \rangle}{P_i} \; |b'\rangle\langle b|$$

$$P_i = Tr_{CAB}\left[\Pi_i^{(CA)} \otimes I_B \ \rho_{CAB}\right] = \sum_b \langle i\, b | \rho_{CAB} | i\, b \rangle$$

$$Tr_{CA}\,[\rho'_{CAB}] = \sum_i P_i \ \rho_i^{(B)} = \sum_{i,b',b} \langle i\, b' | \rho_{CAB} | i\, b \rangle \ \ |b'\rangle\langle b| = \rho_B$$

The bipartite quantum state after measurement is a separable state!

$$\rho'_{AB} = \mathrm{Tr}_C\ \rho'_{CAB} = \sum_i P_i\ \rho_i^{(A)} \otimes \rho_i^{(B)}$$

Optimal generalized measurement

$\Longrightarrow$ closest separable state.

Minimization is over a set of all separable states (with same marginal for one of the subsystems) :

$$\mathrm{Q_{AB}} = \min_{\left\{\Pi_i^{CA} \otimes I_B,\, \rho_{CAB}\right\}} S\left(\rho_{AB} \,\|\, \rho'_{AB}\right)$$

$$= \min_{\left\{\rho_{AB}^{(sep)}\right\}} S\left(\rho_{AB} \,\|\, \rho_{AB}^{(sep)}\right)$$

$$\text{with} \quad \mathrm{Tr_A}\left[\rho_{AB}^{(sep)}\right] = \rho_B$$

- Entangled states get projected to their closest separable states (with same marginal for one of the subsystems) by an optimal generalized projective measurement on one part

- Quantumness $Q_{AB} \neq 0$ for all entangled states

Generalized measurements are NOT necessarily POVMs

Connection with Not Completely Positive (NCP) Projective Measurement maps

A R Usha Devi, A K Rajagopal and Sudha, arXiv:1105.4115
International Journal of Quantum Information **9,** 1757 (2011)

A re-examination of the two qubit example:

$$\rho_{AB} = P\left|0_A,0_B\right\rangle\left\langle 0_A,0_B\right| + \left(1-P\right)\left|+_A,+_B\right\rangle\left\langle +_A,+_B\right|$$

$$\left|\pm\right\rangle = \tfrac{1}{\sqrt{2}}\left(\left|0\right\rangle \pm \left|1\right\rangle\right); \quad 0 \leq \mathrm{P} \leq 1$$

Three qubit extended state

$$\rho_{CAB} = P\left|1_C,0_A,0_B\right\rangle\left\langle 1_C,0_A,0_B\right| + (1-P)\left|0_C,+_A,+_B\right\rangle\left\langle 0_C,+_A,+_B\right|$$

$$\mathrm{Tr}_{\mathrm{C}}[\rho_{CAB}] = P\left|0_A,0_B\right\rangle\left\langle 0_A,0_B\right| + (1-P)\left|+_A,+_B\right\rangle\left\langle +_A,+_B\right|$$

An optimal measurement at Alice Charlie end*:*

$$\begin{cases} \Pi_1^{(CA)} = |0_C, +_A\rangle\langle 0_C, +_A| \\ \Pi_2^{(A'A)} = |0_C, -_A\rangle\langle 0_C, -_A| \\ \Pi_3^{(CA)} = |1_C, 0_A\rangle\langle 1_C, 0_A| \\ \Pi_4^{(CA)} = |1_C, 1_A\rangle\langle 1_C, 1_A| \end{cases}$$

This leaves the overall state unperturbed:

$$\sum_{i=1}^{4} \Pi_i^{(CA)} \otimes I_B \;\; \rho_{CAB} \;\; \Pi_i^{(CA)} \otimes I_B = \rho_{CAB}$$

$$\rho'_{AB} = \rho_{AB} \quad \Longrightarrow \quad \mathrm{Q}_{AB} = S(\rho_{AB} \,||\, \rho'_{AB}) = 0$$

Linear Maps associated with generalized measurements

$$\rho_A \xrightarrow{\text{measurement map}} \rho'_A$$

Trace, Hermiticity, Positivity preserving linear map

$$\left(\rho'_A\right)_{ij} = \sum A_{ij;kl} \left(\rho_A\right)_{kl}$$

$$= \sum B_{ik;jl} \left(\rho_A\right)_{kl}$$

Realigned B matrix is Hermitian and positive for a completely positive map.

(Linear A and B maps: E. C. G. Sudarshan, P. Mathews, J. Rau, Phys. Rev. 121 (1961) 920; T. F. Jordan, E. C. G. Sudarshan, J. Math. Phys. 2 (1961) 772.

Spectral decomposition of B $\Longrightarrow$

$$(\rho'_A)_{ij} = \sum_{k,l} B_{ik;\,jl}\ (\rho_A)_{kl}$$

$$= \sum_{\mu,k,l} \lambda_\mu (M_\mu)_{ik} (M^*{}_\mu)_{jl}\ (\rho_A)_{kl}$$

Or $\quad \rho'_A = \sum_\mu \lambda_\mu\ M_\mu\ \rho_A\ M_\mu{}^+$

$\lambda_\mu \geq 0 \Rightarrow$ measurement map is completely positive (CP)

$\{\sqrt{\lambda_\mu} M_\mu\}$ $\Longrightarrow$ POVM

Generalized projective maps:

$$\rho'_A = \mathrm{Tr}_C[\rho'_{CA}] = \mathrm{Tr}_c\left[\sum_\mu \Pi_\mu^{(CA)} \rho_{CA} \Pi_\mu^{(CA)}\right]$$

$$= \mathrm{Tr}_c\left[\sum_\mu P_\mu \, \Pi_\mu^{(CA)}\right]$$

$$= \sum_\mu P_\mu \, \rho_\mu^{(A)}$$

$$P_\mu = \mathrm{Tr}[\Pi_\mu^{(CA)} \rho_{CA} \Pi_\mu^{(CA)}]$$

$$\rho_\mu^{(A)} = \mathrm{Tr}_C[\Pi_\mu^{(CA)}]$$

If $\rho_{CA} = \rho_C \otimes \rho_A$ then the projective measurement is POVM . (CP map) … otherwise, CP map is not guaranteed!

Assignment Map

(C. A. Rodriguez-Rosario, K. Modi, Phys. Rev. A 81 (2010) 012313)

$$\mathrm{Tr}[\Pi_\mu^{(CA)}\rho_{CA}] = \mathrm{Tr}[\Pi_\mu^{(CA)}\overline{A}(\rho_A)] = \mathrm{T}\bullet\Pi\bullet\overline{A}$$

$$\overline{A}(\rho_A) = \rho_{CA}$$

$$\overline{A}(P_\alpha^A) = \tau_\alpha^C \otimes P_\alpha^A \quad \Rightarrow \quad \overline{A}(\sum_\alpha p_\alpha P_\alpha^A) = \sum_\alpha p_\alpha \tau_\alpha^C \otimes P_\alpha^A$$

linearly independent states

Hermitian operators

$$\{Q_\beta\} \quad \rightarrow \quad \mathrm{Tr}[P^A_\alpha Q_\beta] = \delta_{\alpha\beta}, \quad \sum_\beta Q_\beta = I$$

$$\Rightarrow \quad \overline{A} = \tau^C_\alpha \otimes P^A_\alpha \otimes \tilde{Q}_\alpha$$

$$B = \sum_\alpha \eta^A_\alpha \otimes \tilde{Q}_\alpha, \qquad \eta^A_\alpha = \mathrm{Tr}[\sum_\mu \mathrm{q}_{\mu\alpha} \rho^{(A)}_\mu]$$

$$\mathrm{q}_{\mu\alpha} = \mathrm{Tr}[\Pi^{(CA)}_\mu (\tau^C_\alpha \otimes P^A_\alpha)]$$

B matrix associated with the specific example:

$$\mathcal{B} = P_1 \otimes Q_1 + \frac{I}{2} \otimes Q_2 + P_3 \otimes Q_3 + P_4 \otimes Q_4$$

$$= \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 \end{pmatrix}.$$

Negative eigenvalues ⟹ NCP map

A generalized measurement corresponds to a quantum operation mapping a density matrix to another density matrix, preserving its positivity, hermiticity and traceclass. The Positive Operator Valued Measure (POVM) correspond to completely positive (CP) maps. The other class, the not completely positive (NCP) measurement maps when included in the optimization over measurements, resolve the dichotomy of separability vs classicality.

$$\rho_A \xrightarrow{\text{measurement map}} \rho'_A$$

Measurements correspond to entanglement breaking maps

Corresponding B matrix is Hermitian, but not necessarily positive – i.e., generalized measurements include NCP maps too.

Quantumness of correlations:

$$\mathrm{Q_{AB}} = \min_{\left\{\Pi_i^{CA} \otimes I_B,\, \rho_{CAB}\right\}} S\left(\rho_{AB} \,\|\, \rho'_{AB}\right)$$

$$= \min_{\left\{\rho_{AB}^{(sep)}\right\}} S\left(\rho_{AB} \,\|\, \rho_{AB}^{(sep)}\right)$$

Equivalently,

$$\mathrm{Q_{AB}} \equiv \min_{\{CP, NCP\}} S\left(\rho_{AB} \,\|\, \rho'_{AB}\right)$$

- Importance of generalized extended projective measurements in discerning quantumness of correlations.
- Entangled states get projected to their closest separable states (with same marginal for one of the subsystems) by an optimal generalized projective measurement on one part.
- **Quantumness of correlations** is the minimum entropic distance of the bipartite state with its closest separable state
- Including NCP measurements – alongwith CP (POVM) resolves the separability vs classicality dichotomy

Search towards further understanding of correlations other than entanglement continues…….

All non-classical correlations can be activated into distillable entanglement

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We devise a protocol in which general non-classical multipartite correlations produce a physically relevant effect, leading to the creation of bipartite entanglement. In particular, we show that the relative entropy of quantumness, which measures all non-classical correlations among subsystems of a quantum system, is equivalent to and can be operationally interpreted as the minimum distillable entanglement generated between the system and local ancillae in our protocol. We emphasize the key role of state mixedness in maximizing non-classicality: Mixed entangled states can be arbitrarily more non-classical than separable and pure entangled states.

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Are general quantum correlations monogamous?

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All measures of quantum correlations (other than entanglement) must satisfy the following basic properties:

- **Positivity**: $Q^{A|B}(\rho_{AB}) \geq 0$
- **Invariance under local unitaries** $U_A \otimes V_B$: $Q^{A|B}(\rho_{AB})\rho_{AB}U_A^\dagger \otimes V_B^\dagger)$
- **Non-increase upon attaching an ancilla**: $Q^{A|B}(\rho_{AB}) \geq Q^{A|BC}(\rho_{AB} \otimes |0\rangle_C\langle 0|)$.

In addition, if *monogamy of correlations* viz.,

$$Q^{A|BC}(\rho_{ABC}) \geq Q^{A|B}(\rho_{AB}) + Q^{A|C}(\rho_{AC})$$

is imposed, then the measure of correlations vanishes for the set of all separable states.

A Review on quantum discord, MID and other related measures of correlations: arXiv:1112.6238, Kavan Modi, A. Brodutch, H. Cable, T. Paterek, V. Vedral


Thank You!