Effective field theory of axion monodromy inflation

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Work in progress with Nemanja Kaloper, AL, and Lorenzo Sorbo

Related work in progress with Sergei Dubovsky, AL, and Matthew Roberts
and with Sergei Dubovsky, Raphael Flauger, and AL

I. Introduction

II. 4d models of axion monodromy

III. Quantum corrections

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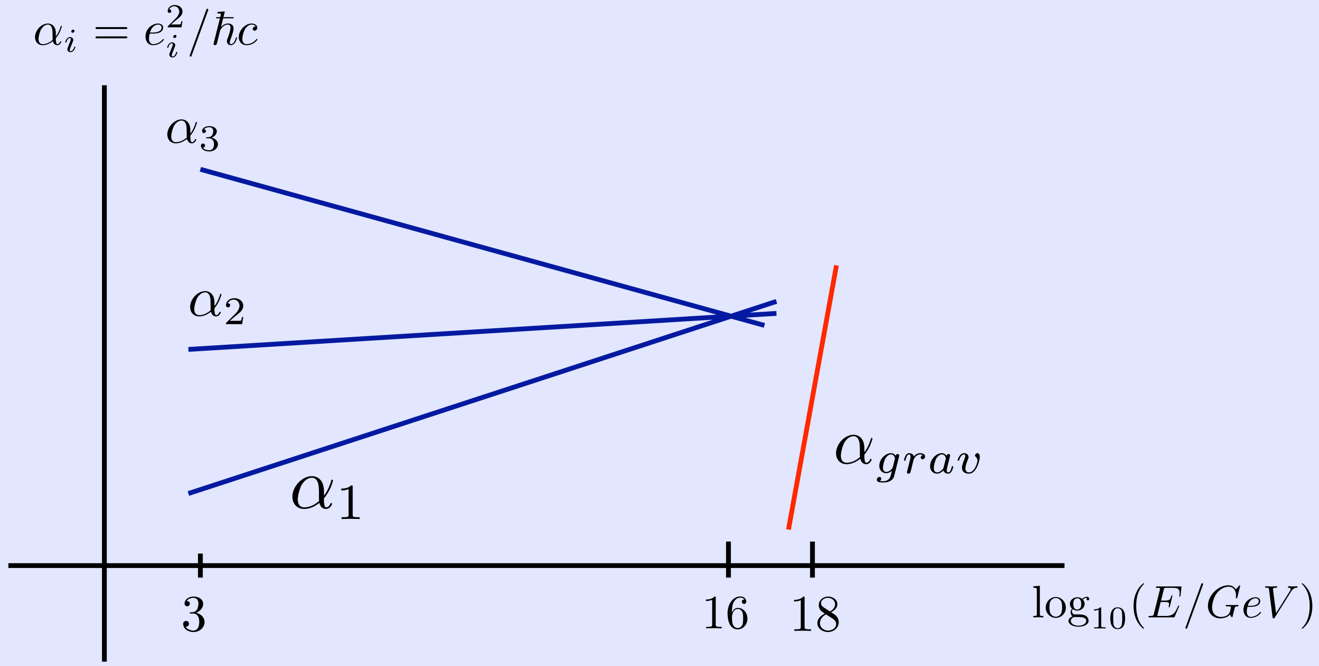
I. Introduction

Scale of inflation

ϕ -- inflaton $\quad V(\phi)$ -- potential energy driving inflation

Observational upper bound on primordial GW: $V \lesssim 10^{16}\ GeV \sim M_{GUT}$

Close to "unification scale"



Couplings unify (assuming MSSM above 1 TeV) at approximately 10^{16} GeV. Graph not to scale.

See also:

- p decay
- ν mass

Detectable gravitational radiation requires large fields

Lyth, hep-ph/9606387

- $\left(\frac{\delta\rho}{\rho}\right) = c\frac{V^{1/2}}{m_{pl}^3}\frac{V}{V'} \sim 10^{-5}$ from observations

upper bound on V $\Rightarrow$ upper bound on V'/V

- $N_e = \int dt H = \int \frac{d\phi}{\dot\phi} H = \frac{3}{m_{pl}^2}\int d\phi \frac{V}{V'} \gtrsim 60$

to match observed flatness

upper bound on $\frac{d\phi}{dN}, \Delta\phi$ during inflation

$$\Rightarrow \quad \Delta\varphi \gg m_{pl}$$

Effective field theory and large ϕ

Effective field theory: expansion in 1/M for some UV scale M

$$V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$$

generically

- $g_n \sim 1$ unless forbidden by symmetry
- $M \lesssim m_{pl}$

A sort of measure of "tastefulness"

Expansion breaks down for $\phi > M$

- New degrees of freedom could become light
- Relevant d.o.f. very different

Inflation is a highly nongeneric theory

Consider $V \sim m^2\phi^2$ or $V \sim \lambda\phi^4$

$$\delta\rho/\rho \sim 10^{-5},\ N_e \gtrsim 60 \quad \Rightarrow$$

- $\frac{m^2}{m_{pl}^2} \sim 10^{-12}$
- $\lambda \sim 10^{-14}$

Corrections $\delta V = \sum_n g_n \frac{\phi^n}{M^{n-4}}$

all g_n must be small: infinite fine tuning!

else e.g. $\eta = m_{pl}^2 \frac{V''}{V} \geq 1$

Slow roll inflation requires approximate shift symmetry

$$\phi \rightarrow \phi + a$$

UV completions make slow roll difficult to maintain

Continuous global symmetries like $\phi \rightarrow \phi + a$ are always (we think) broken

- Gravity breaks continuous global symmetries (Hawking radiation/virtual black holes, wormholes,...)

  Holman et al; Kamionkowski and March-Russell; Barr and Seckel; Lusignoli and Roncadelli; Kallosh, Linde, and Susskind

- String theory: continuous global symmetries tend to be gauged, anomalous

- Anomalous symmetries broken by nonperturbative effects (e.g. Peccei-Quinn symmetry of axion)

$$\delta V \sim \Lambda^4 \sum_n c_n \cos(n\phi/f_\phi)$$

(NB: slow roll is safe from *perturbative* loops of inflatons, gravitons)

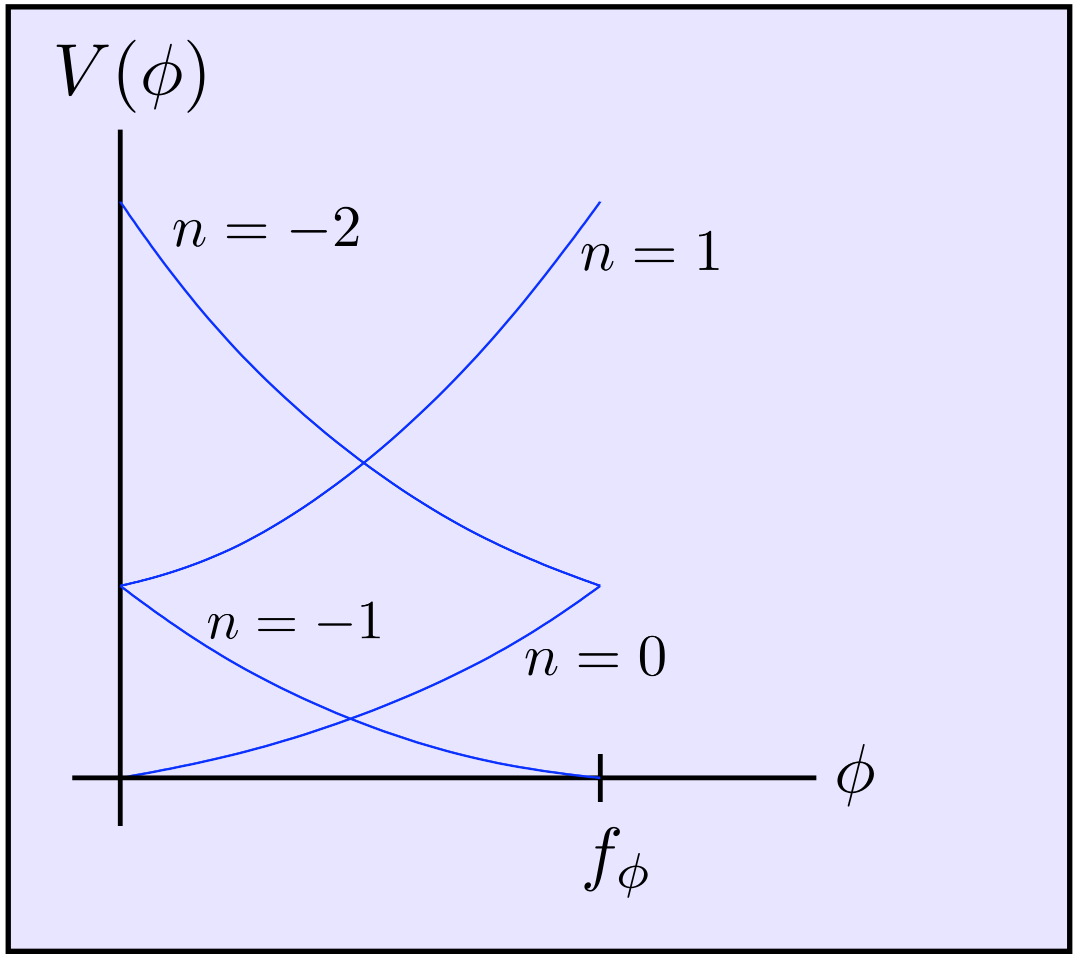
Smolin, Linde, KLS

Candidate solution: axion monodromy inflation

Silverstein and Westphal;
McAllister, Silverstein, and Westphal

Consider compact scalar field $\varphi \sim \varphi + f \ ; \ f \ll m_{pl}$

Theory invariant under shift $\varphi \to \varphi + f$ *physical state* need not be



Let axion wind N times such that $N f_\phi \gg m_{pl}$

Compactness of field space seems to control quantum corrections

Goal: 4d effective field theory analysis

Silverstein and Westphal;
McAllister, Silverstein, and Westphal

- Quantum corrections studied in specific string models and appear to be viable

- In all known string examples, effective potential for large fields has exotic power laws $V \sim M^{4-p}\varphi^{p<2}$

- Basic behavior, corrections have been studied model by model; models are of necessity complicated

Effective field theory approach

- Input basic fields, symmetries, topology of field space
- Expand action in powers of 1/M (M = UV scale), include all terms consisten with symmetries
- Pinpoints physics behind suppressing corrections to slow roll
- Isolates fine tuning required.
- Provides a framework for building new string models

String theory has a complicated landscape
Realistic models very hard to construct
Quantum corrections difficult to compute

$\Rightarrow$

4d effective field theory analysis is *always* important

II. 4d models of axion monodromy

Axion-four form model

Kaloper and Sorbo

$$S_{class} = \int d^4x\sqrt{g}\left(m_{pl}^2 R - \frac{1}{48}F^2 - \frac{1}{2}(\partial\varphi)^2 + \frac{\mu}{24}\varphi^* F\right)$$

$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]}$ U(1) gauge symmetry: $\delta A_{\mu\nu\lambda} = \partial_{[\mu} \Lambda_{\nu\lambda]}$

φ periodic: $\varphi \to \varphi + f_\varphi$

F does not propagate.
U(1) quantized $F_{\mu\nu\lambda\rho} = n e^2 \epsilon_{\mu\nu\lambda\rho}\ ;\ n \in \mathbb{Z}$

n can jump across domain walls/membranes

Dynamics

Single massive scalar degree of freedom Dvali; Kaloper and Sorbo

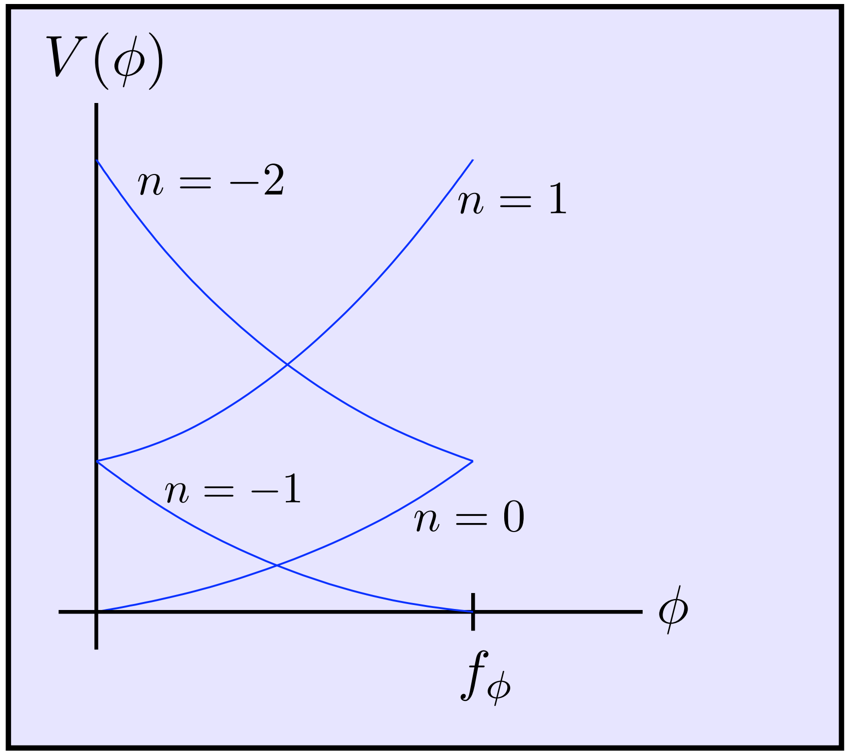
Hamiltonian: $H_{tree} = \frac{1}{2}p_\phi^2 + \frac{1}{2}\left(p_A + \mu\phi\right)^2 + \ grav.$

Compact U(1): $p_A = ne^2$

p_A conserved by H_{tree}

Jumps by membrane nucleation

Consistency condition: $\mu f_\varphi = e^2$



Realizes monodromy inflation: theory invariant if

$$\varphi \to \varphi + f_\varphi \ ; n \to n - 1$$

Good model for inflation: fits data well if $\mu \sim 10^{-6} m_{pl}$

+ observable GW

Large-N gauge dynamics

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{4g_{YM}^2} \mathrm{tr} G^2 - \frac{1}{2}(\partial\varphi)^2 + \frac{\varphi}{f_\varphi} \mathrm{tr} G \wedge G \right)$$

G: field strength for U(N) gauge theory with N large; strong coupling in IR

Instanton expansion breaks down — Witten; Giusti, Petrarca, and Taglienti

$$H_{tree} = H_{gauge} + \frac{1}{2} p_\varphi^2 + \frac{1}{2} \left(n\Lambda^2 + \mu\varphi \right)^2$$

Λ strong coupling scale of U(N) theory

$$\mu = \Lambda^2 / f_\varphi$$

Can be related to 4-form version: $F_{\mu\nu\lambda\rho} \sim \mathrm{tr}\ G_{[\mu\nu} G_{\lambda\rho]}$ Dvali

III. Quantum corrections

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

$\mu \sim 10^{-6} m_{pl}$ to match constraints on $\delta\rho/\rho, \ N_e$

What are the possible corrections?

Effective field theory:

- Allow all terms consistent with symmetries, topology of field space
- Dimenson-d operators suppressed by M_{uv}^{d-4}

Corrections controlled by:

- Compactness of scalar, U(1)
- Small coupling $\mu/M_{uv} \ll 1$

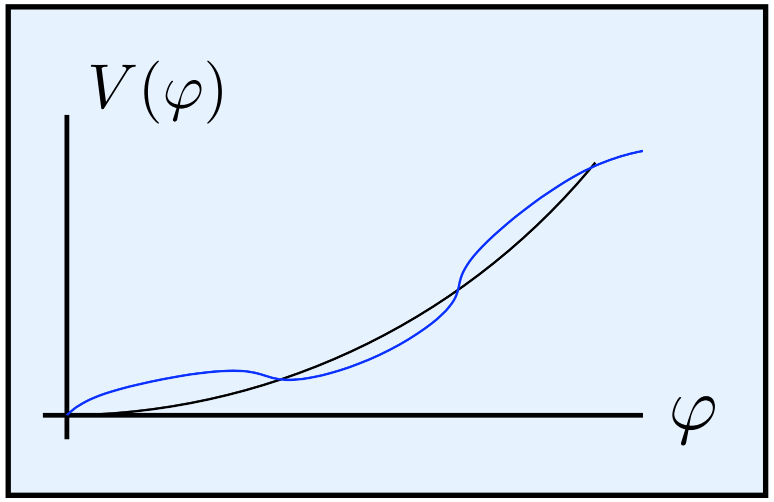
Direct corrections to $V(\varphi)$

Periodicity of φ $\quad\Rightarrow\quad$ quantum corrections to S must be

- Functions of $\partial^n \varphi$
- periodic functions of φ

$$\delta V \sim \Lambda^4 \sum_{n>1} c_n \cos(n\varphi/f_\varphi)$$

$$f_\phi \ll m_{pl}$$



Gauge dynamics: $\quad \Lambda = \Lambda_{QCD}$

from couplings $\frac{\varphi}{f_\varphi} \mathrm{tr}\, G \wedge G$

instanton corrections take above form (if dilute gas approx good)

Monodromy potential modulated by periodic effects

$$V_{corr} \ll \tfrac{1}{2}\mu^2\varphi^2 \Rightarrow \Lambda^4 \ll M_{gut}^4$$
$$\eta = m_{pl}^2 \frac{V''}{V} \ll 1 \Rightarrow \frac{\Lambda^4}{f_\varphi^2} \ll \frac{V}{m_{pl}^2} = H^2$$

Example: feasible if $\quad \Lambda \sim .1\ M_{gut},\ \ f > .01\ m_{pl}$

Resonant non-Gaussianity

Chen, Easther, Lim; Flauger, McAllister, Pajer, Westphal; Flauger and Estphal

Consider oscillations rapid on a Hubble time.

$$f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2}$$

ζ : gauge invariant scalar perturbation

$$\delta V \sim \cos\left(\frac{\varphi_0 + \dot{\varphi} t_{prop}}{f_\varphi}\right)$$

$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle$ enhanced when $\sum_i |k_i|_{phys} = \frac{\dot{\varphi}}{f_\varphi}$

$$f_{NL,res} \sim \Lambda^4 \sqrt{\frac{\dot{\varphi}}{H^5 f_\varphi^5}}$$

$\Lambda \sim .3 M_{gut},\, f_\varphi \sim 0.1 m_{pl}$ (slow roll starting to break down)

$$\Rightarrow f_{NL,res} \sim 34$$

Caveat: moduli stabilization

In any string theory: couplings in V will depend on moduli ψ

$$V = V_0(\psi) + \frac{1}{2}\mu^2\left(\frac{\psi}{m_{pl}}\right)\varphi^2 + \Lambda^4 \sum_n c_n \left(\frac{\psi}{m_{pl}}\right)\cos\left(\frac{n\varphi}{f_\varphi}\right)$$

Periodic corrections change sign many times since $f_\phi \ll m_{pl}$

Moduli must be stabilized by different effects than instantons coupling to inflaton

$$M_\psi^2 \equiv V_0''(\psi) \gg \frac{\Lambda^4}{m_{pl}^2}$$

Large $\varphi \gg m_{pl}$ sources potential for ψ

Stability requires $M_\psi^2 \gg \mu^2\varphi^2/m_{pl}^2 \sim \mu^2/\epsilon \sim H^2$

Indirect corrections to $V(\varphi)$

Additional corrections must respect periodicity of φ

$\Rightarrow$ corrections to dynamics of four-form F

$$S_{class} = \int d^4x\sqrt{g}\left(m_{pl}^2 R - \tfrac{1}{48}F^2 - \tfrac{1}{2}(\partial\varphi)^2 + \tfrac{\mu}{24}\varphi^* F\right)$$

Consider $\delta\mathcal{L} = \sum_n d_n \frac{F^{2n}}{M^{4n-4}}$

Integrate out F: $F \sim \mu\varphi + \ldots$

$$\delta V_{eff} = V_{class} \times \left(\sum_{n=1} d_{n+1} \frac{V_{class}^n}{M^{4n}}\right)$$

Safe if: $M^4 \gg V_{class} \sim M_{gut}^4$

Corrections of the form $\delta\mathcal{L} = \left(\sum_{n=1} d_{n+1}\frac{F^{2n}}{M^{4n}}\right)(\partial\varphi)^2$

Gives same effect after redefining φ to be canonically normalized

Small M not always fatal

Many string theory scenarios:

$$V(\varphi) = M_1^4 \sqrt{1 + \frac{\varphi^2}{M_2^2}} \qquad M_2 \ll m_{pl}$$

Silverstein and Westphal;
McAllister, Silverstein, and Westphal

- For small φ $\quad V \sim \frac{1}{2}\mu^2\varphi^2 \;; \mu = \frac{M_1^4}{M_2^2}$
- For $\varphi \gg m_{pl}$ $\quad V \sim m^3\varphi;\; m^3 = \frac{M_1^4}{M_2}$

Out of range of 4d effective field theory; requires understanding of UV completion (eg 10d SUGRA) to compute

Example: Coleman-Weinberg corrections

Consider scalar fields ψ_n (e.g. moduli, KK states, etc.)

$$\delta\mathcal{L} \sim \frac{1}{2}(\partial\psi_n)^2 - \frac{1}{2}M_n^2\psi_n^2 - \sum_k d_{n,k}\frac{F^{2n}}{M^{4n-2}}\psi_n^2$$

Integrate out F: $F^2 \sim V_{class} = \frac{1}{2}\mu^2\varphi^2$

Effective mass for ψ : $M_{eff}^2 = M_\psi^2 + M^2\sum_k d'_{n,k}\frac{V^2}{M^{4n}}$

Integrate out ψ_n : $\delta V_{CW}(\varphi) \sim M_{eff}(\phi)^4 \ln\frac{M_{eff}}{M}$

Must include all such states with $M_n^2 < M^2$

Corrections safe if $n_{eff}M_\psi^2 \ll M^2$; $V \ll M^4$

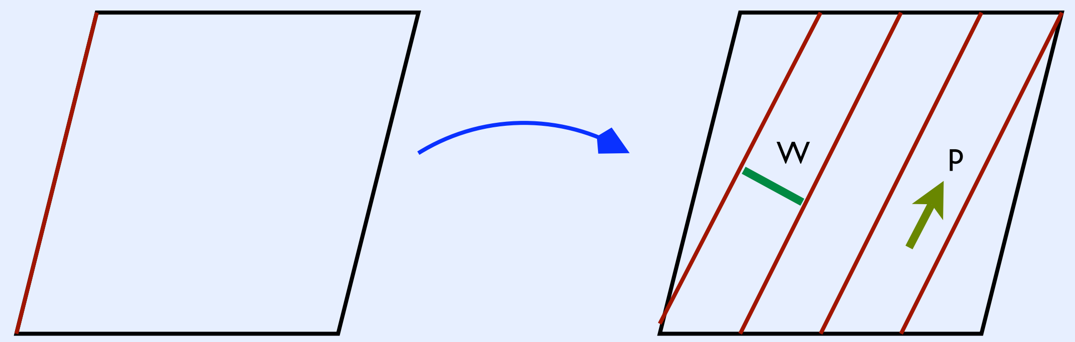
Kaluza-Klein corrections

Roughly $n_{eff} = \frac{m_{pl}^2}{m_*^2}$; $m_* = (m_s, m_{pl,10}) \gtrsim M_{gut}$

$$
\begin{aligned}
V_{CW} &= \sum_{KK} \int d^4q \, \ln\left(q^2 + M_{n,eff}^2\right) \\
&\sim \mathcal{V}_D \int d^{D+4}q \, \ln\left(q^2 + \sum_k d_k \frac{V_{tree}^k}{M^{4k-4} m_{pl}^2}\right) \\
&\sim m_*^{D+4} \mathcal{V}_D(\psi) + m_*^2 \mathcal{V}_{\mathcal{D}} \sum_k d_k \frac{V_{tree}^k}{M^{4k-4} m_{pl}^2} \\
&\sim \delta V(\psi) + V_{tree} \, F\left(\frac{V_{tree}}{M^4}\right)
\end{aligned}
$$

Corrections safe if $V_{class} \ll M^4$

Additional "stringy" light states



Shift τ n times; D-brane becomes n times as long.

Consider square torus with sides of length L; D4 wrapped n times

$$m_W^2 = \frac{m_s^4 L^2}{1+n^2};\ m_p^2 = \frac{1}{L(1+n^2)}\ ; n = \frac{\varphi}{f_\varphi} = \frac{F}{\mu f_\varphi}$$

n >> 1: strings have spectrum of asymmetric torus with sides of length

$$L_W = \frac{n}{m_s^2 L}\ ; L_p \sim \frac{n}{L}$$

and volume $V_{eff} \sim \dfrac{n^2}{m_s^2} \sim \dfrac{F^2}{m_s^2 e^4}$

where $e^2 = \mu f_\varphi$ is unit of quantization of F flux

Leading quantum correction

$$V_{CW} = \sum_{k,l} \int d^4q \, \ln \, \left(q^2 + m_{W,k}^2 + m_{p,k}^2\right) + \ldots$$
$$\sim \frac{F^2}{m_s^2 e^4} \int d^6 q \, \ln q^2 + \ldots$$
$$\sim \frac{m_s^4}{e^4} F^2 + \ldots$$

Effect is to renormalize $e^2 \to m_s^2 \sim M_{gut}^2 \sim 10^{-4} m_{pl}^2$

Dangerous: $\mu = 10^{-6} m_{pl}$ to match observation

$$\Rightarrow \quad f_\varphi \sim 10^2 \; m_{pl}$$

Must ensure renormalization of e is suppressed:

$$f_\varphi \sim .1 \; m_{pl} \Rightarrow \quad e^2 \sim 10^{-7} m_{pl}^2 \sim \left(3 \times 10^{-4} m_{pl}\right)^2 \sim H^2$$

- NB model above is crude (and known not to work for other reasons) so this is a caveat and not a fatal flaw

- Even if μ^2 pushed above $10^{-6} m_{pl}$

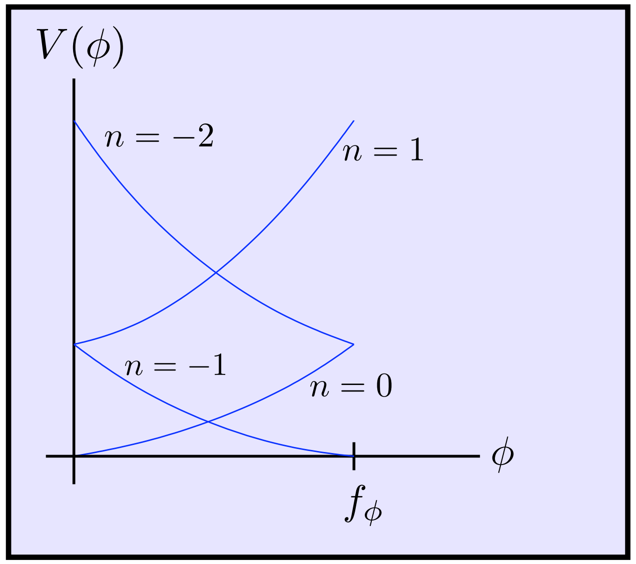
  we may still get successful large field inflation of the form, e.g.

$$V(\varphi) = M_1^4 \sqrt{1 + \frac{\varphi^2}{M_2^2}}$$

  but this requires more than our 4d EFT can do at present

IV. Stability

Sergei Dubovsky, AL, Matthew Roberts;
SD, AL, Raphael Flauger, in progress



Success of monodromy inflation requires that transition between branches is slow compared to time scale of inflation (must complete 60 efolds before such transitions)

Transitions occur by bubble nucleation. Let:

- T = tension of bubble wall
- E = energy difference between branches

Decay probability: $\Gamma \sim \exp\left(-\frac{27\pi^2}{2}\frac{T^4}{E^3}\right)$ (thin wall) Coleman

Phenomenological bound on T

$$\varphi = N f_\varphi \; ; \Delta\varphi = f_\varphi$$

$$f_\phi \sim .1\ m_{pl};\ N \sim 100; V \sim M_{gut}^4$$

$$\Rightarrow T^{1/3} \gg .2\ M_{gut}$$

V. Conclusions

- Check stability in explicit string, field theory models

Dubovsky, AL, Roberts; SD, AL, Flauger, in progress

- General issue: monodromy inflation does not seem *parametrically* safe. Should we worry?

Perhaps this is interesting:

- Implies number of e-foldings could be close to lower bound
- Implications for measurements of curvature, pre-inflation transients

- Gauge dynamics: $\Lambda = \Lambda_{QCD}$

  from couplings $\frac{\varphi}{f_\varphi}\mathrm{tr}\ G \wedge G$

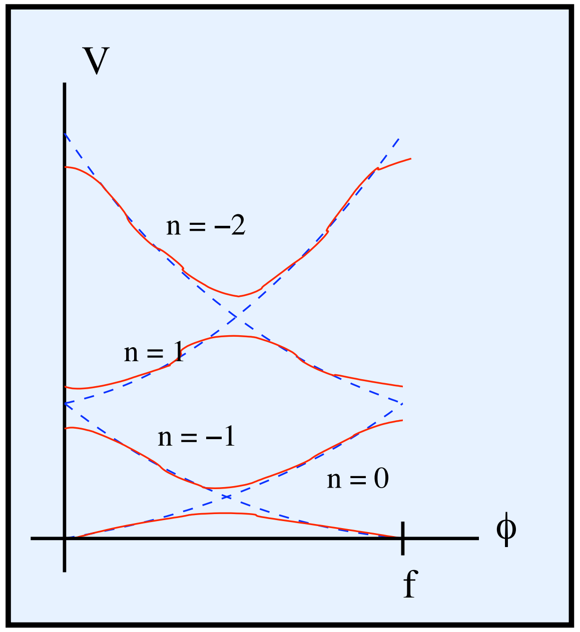
  instanton corrections take above form (if dilute gas approx good)

  strong coupling effects (when dilute gas aprox fails)

$$\delta V \sim \Lambda^4 \ \mathrm{min}_k \ F\left(\frac{\varphi}{f_\varphi} + k\right)$$

Witten; Giusti, Petrarca, and Taglienti

multibranched function of φ



When using this effect to generate monodromy potential: mixing between branches must be weak

When this generates corrections: mixing must be strong (else trapped in a fixed branch)

- Gravitational dynamics: $\Lambda^4 \sim \frac{f_\varphi^{n+4}}{m_{pl}^n}$

  gravitational instantons, wormholes, etc.