A New Model Of Inflation[1] [2]

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[1] **JCAP1001 : 029, 2010 - B. K. Pal**, *S. Pal, B. Basu*

[2] **arXiv:1010.5924 - B. K. Pal**, *S. Pal, B. Basu*

Plan of My Talk :

1. Motivation For Inflation....
2. Mutated Hilltop Inflation Model....
3. Quantum Fluctuation And Observable Parameters....
4. Matter Power Spectrum....
5. Temperature Anisotropies In CMB....
6. Summary And Future Aspects....

- Standard Big Bang theory very successfully explains:
  1. **Expanding Universe**
  2. **Big Bang Nucleosynthesis**
  3. **CMB Formation**.
- But there are some problems with **SBB**:
  1. **Flatness Problem**
  2. **Horizon Problem**
  3. **Structure Formation Problem**.
- The basic idea to solve those *problems* is to decrease **Comoving Hubble Radius**($\frac{1}{aH}$) sufficiently in the very early universe, which is possible only when $\ddot{a} > 0 \implies$ violation of **SEC**.
  Inflation[*Starobinsky*(1980), *Guth*(1981), *Linde*(1982)] $\longmapsto$ **Accelerated Expansion**.
- Violation of **SEC** but **how**?
  1. Cosmological constant....
  2. Scalar field....

Some Inflationary Potentials....

- **Chaotic Inflation**[Linde(1983)]

$$\mathbf{V}(\phi) = \frac{1}{2}\mathbf{m}^2\phi^2 + \frac{\lambda}{4}\phi^4 \tag{1}$$

- **Natural Inflation**[Freese et.al.(1990)]

$$\mathbf{V}(\phi) = \mathbf{V_0}\left[1 + \cos(\frac{\phi}{\mathbf{f}})\right] \tag{2}$$

- **Hybrid Inflation**[Linde(1994)]

$$\mathbf{V}(\phi) = \frac{\lambda}{4}(\chi^2 - \mathbf{M}^2)^2 + \frac{1}{2}\mathbf{g}^2\phi^2\chi^2 + \frac{1}{2}\mathbf{m}^2\phi^2 \tag{3}$$

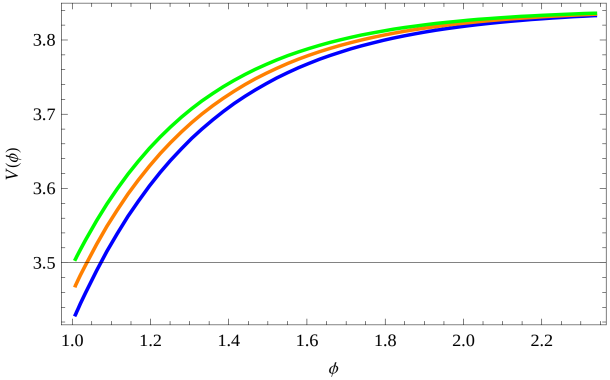
- **Hilltop Inflation**[Lyth(2005)]

$$\mathbf{V}(\phi) = \mathbf{V_0} - \frac{1}{2}\mathbf{m}^2\phi^2 + ... \tag{4}$$

  Many models can be converted to hilltop....
- Still no particular **Model of Inflation** satisfies all the **Observational Constraints**....

- Proposed form of the **Potential**:

$$\mathbf{V}(\phi) = \mathbf{V_0}\left(\mathbf{1} - \mathbf{sech}[\alpha\phi]\right) \tag{5}$$



- Different from original **Hilltop Inflation Model**:

Expressions For Different Quantities Involved....

- **Hubble Slow-Roll** Approximation [*Liddle & Lyth*(1992)]: $\epsilon_H << 1$ and $|\eta_H| << 1$.

$$\epsilon_{\mathbf{H}} \equiv 2\mathbf{M_P^2}\left[\mathbf{H}'(\phi)/\mathbf{H}(\phi)\right]^2$$
$$= \frac{\mathbf{M_P^2}}{2}\,\frac{\alpha^2\mathrm{sech}^2(\alpha\phi)\tanh^2(\alpha\phi)}{[1-\mathrm{sech}(\alpha\phi)]^2} \quad (6)$$

$$\eta_{\mathbf{H}} \equiv 2\mathbf{M_P^2}\left[\mathbf{H}''(\phi)/\mathbf{H}(\phi)\right]$$
$$= \mathbf{M_p^2}\frac{\alpha^2\mathrm{sech}(\alpha\phi)[\mathrm{sech}^2(\alpha\phi)-\tanh^2(\alpha\phi)]}{[1-\mathrm{sech}(\alpha\phi)]} - \epsilon_{\mathbf{H}} \quad (7)$$

Expressions For Different Quantities Involved....

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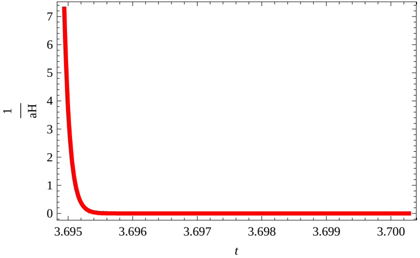
- Now applying *slow-roll* approximation and using **Friedmann** equations we obtain following expressions:
  1. The **scalar field** expression: $\sinh(\alpha\phi) = \alpha^2\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}\mathbf{M_P}(\mathbf{d}-\mathbf{t})$
  2. **Scale Factor**: $\mathbf{a}(\mathbf{t}) = \mathbf{a_1}\ \exp\left[-(\alpha\mathbf{M_p})^{-2}\sqrt{\mathbf{1}+\alpha^4\frac{\mathbf{V_0}}{\mathbf{3}}\mathbf{M_p^2}(\mathbf{d}-\mathbf{t})^2}\right]$

3. The **Comoving Hubble Radius**: $\frac{1}{aH} = \left[\frac{\sqrt{1+\alpha^4 \frac{V_0}{3} M_P^2 (d-t)^2}}{\frac{V_0}{3}\alpha^2 (d-t)}\right] a^{-1}(t)$



4. Amount Of **Inflation** Or no. of *e-foldings* :
$N \equiv \ln \frac{a(t_{end})}{a(t_{initial})} = (\alpha^2 M_p)^{-1}\left[\cosh(\alpha\phi) - \ln\cosh^2\left(\frac{\alpha\phi}{2}\right)\right]_{\phi_{end}}^{\phi_{in}}$
Observational Bound: $56 \leq N \leq 70$.

α M_P^{-1}	$\epsilon_H < 1$ $\phi \geq M_P$	$\lvert\eta_H\rvert < 1$ $\phi \geq M_P$	ϕ_{end} M_P	ϕ_{in} M_P	N
2.9	0.59886	1.02192	1.02192	2.44625	70
				2.39431	60
				2.37111	56
3.0	0.58759	1.00796	1.00796	2.38713	70
				2.33689	60
				2.31446	56
3.1	0.57681	0.99435	0.99435	2.33112	70
				2.28248	60
				2.26076	56

Quantum Fluctuations and Observable Parameters....

- The *Quantum Fluctuations* in the inflaton field were stretched outside the causal horizon producing macroscopic **Cosmological Perturbations**.
- **New Scalar Field** : $\mathbf{v} \equiv \mathbf{a}(\eta)\phi = -z\mathcal{R}$
  where $\mathcal{R}$ is the **Comoving Curvature Perturbation** and
  $\mathbf{z} \equiv \frac{\mathbf{a}\phi'}{\mathcal{H}} \approx \alpha^{-\mathbf{1}}\mathbf{M_P}\sqrt{\frac{\mathbf{3}}{\mathbf{V_0}}}|\eta|^{-\mathbf{1}}\left[\ln\left(\mathbf{a_1}\mathbf{M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}|\eta|\right)\right]^{-\mathbf{1}}$
- **Mukhanov-Sasaki** equation : $\mathbf{v_k''} + \left(\mathbf{k^2} - \frac{\mathbf{z''}}{\mathbf{z}}\right)\mathbf{v_k} = \mathbf{0}$

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- **Mukhanov-Sasaki** equation : $\mathbf{v_k''} + \left(\mathbf{k^2} - \frac{\mathbf{z''}}{\mathbf{z}}\right)\mathbf{v_k} = \mathbf{0}$
- And the complete solution is given by,

$$\mathbf{v_k} = \sqrt{\frac{1}{2\mathbf{k}}}\exp(-\mathbf{ik}\eta)\left(\mathbf{1} - \frac{\mathbf{i}}{\mathbf{k}\eta}\right) \tag{8}$$

- Therefore k^{th} Fourier mode of $\mathcal{R}_\mathbf{k}$ is

$$\mathcal{R}_\mathbf{k} = -\frac{\alpha\sqrt{\mathbf{V_0}}}{\mathbf{M_P}\sqrt{3}}\frac{|\eta|\mathbf{e}^{-\mathbf{ik}\eta}}{\sqrt{2\mathbf{k}}}(\mathbf{1} - \mathbf{i}/\mathbf{k}\eta)\left[\ln\left(\mathbf{a_1 M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{3}}|\eta|\right)\right] \tag{9}$$

- **Power Spectrum** of $\mathcal{R}$

$$\begin{aligned} \mathbf{P}_{\mathcal{R}}(\mathbf{k}) &\equiv \frac{\mathbf{k}^3}{2\pi^2}|\mathcal{R}_{\mathbf{k}}|^2 \\ &= \frac{\alpha^2 \mathbf{V_0}}{12\pi^2 \mathbf{M_P^2}}(1+\mathbf{k}^2\eta^2)\left[\ln\left(\mathbf{a_1 M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{3}}|\eta|\right)\right]^2 \\ \mathbf{P}_{\mathcal{R}}|_{\mathbf{k=aH}} &= \frac{\alpha^2 \mathbf{V_0}}{6\pi^2 \mathbf{M_P^2}}\left[\ln\left(\mathbf{a_1 M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{3}}|\eta|\right)\right]^2 \qquad (10) \end{aligned}$$

  where in the last equation we have evaluated power spectrum at the horizon crossing i.e., when $k = aH$.

- **Observational Bound** $[WMAP3, COBE, Spergel(2006)]$: $\mathbf{P}_{\mathcal{R}}^{1/2} \sim 5 \times 10^{-5}$.

- The *scale* dependence of *Power Spectrum* for **Comoving Curvature** perturbation is defined by the **Spectral Index** n_s

$$\mathbf{n_s} \equiv \mathbf{1} + \frac{\mathbf{d} \ln \mathbf{P}_{\mathcal{R}}(\mathbf{k})}{\mathbf{d} \ln \mathbf{k}}\Big|_{\mathbf{k}=\mathbf{aH}} = \mathbf{1} - \mathbf{2}\left[\ln\left(\mathbf{a_1 M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}|\eta|\right)\right]^{-\mathbf{1}} \tag{11}$$

Observational Bound [*WMAP*3, *COBE*, *Spergel et.al*.(2006)] : $\mathbf{0.948 < n_s < 1}$.

- The *scale* dependence of *Power Spectrum* for **Comoving Curvature** perturbation is defined by the **Spectral Index** n_s

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Observational Bound [*WMAP*3, *COBE*, *Spergel et.al.*(2006)] : $\mathbf{0.948 < n_s < 1}$.

- **Running** of the **Spectral Index** [*WMAP*3(2005)] :

$$\mathbf{n'_s} \equiv \frac{\mathbf{dn_s}}{\mathbf{d}\ln \mathbf{k}}\Big|_{\mathbf{k=aH}} = -\mathbf{2}\left[\ln\left(\mathbf{a_1 M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}|\eta|\right)\right]^{-2} \quad (12)$$

- WMAP3 data set indicates this *running*.

- The **tensor modes** satisfies

$$\mathbf{h}_\mathbf{k}'' + 2\mathcal{H}\mathbf{h}_\mathbf{k}' + \mathbf{k}^2\mathbf{h}_\mathbf{k} = \mathbf{0} \tag{13}$$

We define new tensor field: $\mathbf{h}_\mathbf{k} = \frac{\sqrt{2}}{\mathbf{M_P}}\frac{\mathbf{u}_\mathbf{k}}{\mathbf{a}}$
The above equation then becomes

$$\mathbf{u}_\mathbf{k}'' + \left(\mathbf{k}^2 - \frac{\mathbf{a}''}{\mathbf{a}}\right)\mathbf{u}_\mathbf{k} = 0 \tag{14}$$

- The exact solution is

$$\mathbf{u}_\mathbf{k} = \sqrt{\frac{\mathbf{1}}{\mathbf{2k}}}\exp(-\mathbf{i}\mathbf{k}\eta)\left(\mathbf{1} - \frac{\mathbf{i}}{\mathbf{k}\eta}\right) \tag{15}$$

The solutions are also known as **Gravitational Waves**.

- **Tensor Power Spectrum**

$$\mathbf{P_T} = \frac{\mathbf{V_0}}{3\pi^2\mathbf{M_P^4}}(\mathbf{1}+\mathbf{k}^2\eta^2) = \frac{\mathbf{2V_0}}{3\pi^2\mathbf{M_P^4}} \qquad (16)$$

- **Tensor Spectral Index**

$$n_T \equiv \frac{\mathbf{d}\ln\mathbf{P_T}}{\mathbf{d}\ln\mathbf{k}}\Big|_{\mathbf{k=aH}} = 0 \qquad (17)$$

So we get the tensor modes to be strictly **scale invariant**.

- **Tensor Power Spectrum**

$$\mathbf{P_T} = \frac{\mathbf{V_0}}{3\pi^2\mathbf{M_P^4}}(1+\mathbf{k}^2\eta^2) = \frac{2\mathbf{V_0}}{3\pi^2\mathbf{M_P^4}} \tag{16}$$

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- Tensor to scalar power spectrum ratio :

$$\mathbf{r} \equiv \frac{\mathbf{P_T}|_{\mathbf{k=aH}}}{\mathbf{P_{\mathcal{R}}}|_{\mathbf{k=aH}}} = \frac{\mathbf{4}}{\alpha^2\mathbf{M_P^2}}\left[\ln\left(\mathbf{a_1}\mathbf{M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}|\eta|\right)\right]^{-2}$$

Observational bound: $\mathbf{r} < \mathbf{0.1}$.

α M_P^{-1}	$P_{\mathcal{R}}^{1/2}$	n_s	r
2.9	3.7784×10^{-5}	0.9609	1.82×10^{-4}
3.0	3.9080×10^{-5}	0.9609	1.70×10^{-4}
3.1	4.0377×10^{-5}	0.9609	1.60×10^{-4}

- We see from the table, that **Power Spectrum, Scalar Spectral Index, Tensor To Scalar Power Spectrum Ratio** are well within the observational bound.

Matter Power Spectrum

- Using the super horizon limit of $\mathcal{R}_{\mathbf{k}}$; conservation law for $\zeta_{\mathbf{k}}$; equivalence of $\mathcal{R}_{\mathbf{k}}$ and $\zeta_{\mathbf{k}}$ in the super horizon limit for **adiabatic** perturbations we get, the expression for gravitational potential modes reentering the horizon during **matter dominated** era

$$\mathbf{\Phi}|_{\mathbf{k}=\mathbf{aH}} = -\frac{3}{5}\mathcal{R}|_{\mathbf{k}=\mathbf{aH}} = -\frac{3\alpha}{5\mathbf{M_P}}\sqrt{\frac{\mathbf{V_0}}{3}}\frac{\mathbf{i}}{\sqrt{2\mathbf{k}^3}}\ln\left(2\mathbf{a}_1\mathbf{M_P}^{-1}\sqrt{\frac{\mathbf{V_0}}{3}}\mathbf{k}^{-1}\right) \tag{18}$$

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$$\Phi|_{k=aH} = -\frac{3}{5}\mathcal{R}|_{k=aH} = -\frac{3\alpha}{5M_P}\sqrt{\frac{V_0}{3}}\frac{i}{\sqrt{2k^3}}\ln\left(2a_1M_P^{-1}\sqrt{\frac{V_0}{3}}k^{-1}\right) \tag{18}$$

- Poisson equation in GTR

$$3\mathcal{H}^2\Phi_k + 3\mathcal{H}\Phi_k' + k^2\Phi_k = -\frac{3}{2}\mathcal{H}^2\delta_k \tag{19}$$

- **Matter Power Spectrum**

$$P_\delta|_{k=aH} = \frac{16\alpha^2 V_0}{75\pi^2 M_P^2}\left[\ln\left(2a_1M_P^{-1}\sqrt{\frac{V_0}{3}}k^{-1}\right)\right]^2 \tag{20}$$

Matter Power Spectrum

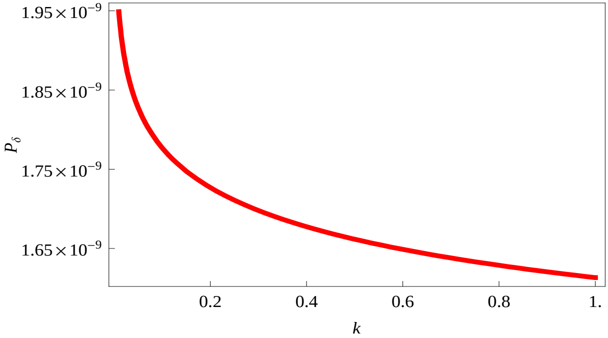


Figure: Variation of the matter power spectrum with comoving wave number

Matter Power Spectrum

- **Spectral index** for Matter Power Spectrum

$$\mathbf{n}_\delta \equiv 1 + \frac{d \ln \mathbf{P}_\delta(\mathbf{k})}{d \ln \mathbf{k}}\Big|_{\mathbf{k}=\mathbf{aH}} = 1 - 2\left[\ln\left(2\mathbf{a}_1 \mathbf{M}_\mathbf{P}^{-1}\sqrt{\frac{\mathbf{V}_0}{3}}\mathbf{k}^{-1}\right)\right]^{-1} \tag{21}$$

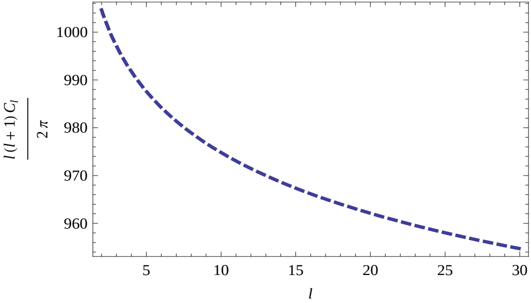
  So we have explicitly scale dependent *matter power spectrum* as it should be since our *primordial curvature perturbation* is scale dependent.

- Matter spectral index has negative **running** given by

$$\mathbf{n}_\delta^{'} \equiv \frac{d\mathbf{n}_\delta}{d \ln \mathbf{k}}\Big|_{\mathbf{k}=\mathbf{aH}} = -2\left[\ln\left(2\mathbf{a}_1 \mathbf{M}_\mathbf{P}^{-1}\sqrt{\frac{\mathbf{V}_0}{3}}\,\mathbf{k}^{-1}\right)\right]^{-2} \tag{22}$$

- Angular power spectrum for **Sachs-Wolfe**[*Sachs and Wolfe*(1967)] effect:

$$\mathbf{C_l^{SW}} = \frac{\alpha^2 \mathbf{V_0}}{\mathbf{75}\pi \mathbf{M_P^2}} \left(\ln \left[\mathbf{a_1 M_P^{-1}} \sqrt{\frac{\mathbf{V_0}}{\mathbf{3}} \frac{\mathbf{2}\eta_0}{\mathbf{l}}} \right] \right)^{\mathbf{2}} \frac{\mathbf{1}}{\mathbf{2l(l+1)}} \tag{23}$$



The Sachs-Wolfe plateau is slightly tilted towards larger l.

- **The acoustic oscillations of baryon-photon fluid is**

$$\frac{1}{4}\delta''_{\gamma\mathbf{k}} + \frac{1}{4}\frac{\mathbf{R}'}{1+\mathbf{R}}\delta'_{\gamma\mathbf{k}} + \frac{1}{4}\mathbf{k}^2\mathbf{c}_s^2\delta_{\gamma\mathbf{k}} = -\frac{\mathbf{k}^2}{3}\mathbf{\Phi}_\mathbf{k}(\eta) + \frac{\mathbf{R}'}{1+\mathbf{R}}\mathbf{\Phi}'_\mathbf{k}(\eta) + \mathbf{\Phi}''_\mathbf{k}(\eta)$$

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- **The complete solution in the tight coupling approximation reads**

$$\frac{1}{4}\delta_{\gamma\mathbf{k}} = \frac{3}{5}(\mathbf{1}+\mathbf{R})\mathcal{R}_\mathbf{k}\mathbf{T}_\mathbf{k} + \frac{1}{2}\mathbf{T}^0_\mathbf{k}\mathbf{\Phi}^0_\mathbf{k}\cos(\mathbf{k}\mathbf{r}_\mathbf{s}) \quad (24)$$

$$\mathbf{V}_{\gamma\mathbf{k}} = \frac{3\mathbf{c}_\mathbf{s}}{2}\mathbf{T}^0_\mathbf{k}\mathbf{\Phi}^0_\mathbf{k}\sin(\mathbf{k}\mathbf{r}_\mathbf{s}) \quad (25)$$

where $R = \frac{3\rho_b}{4\rho_\gamma}$ and $r_s \equiv \int_0^\eta c_s(\eta)d\eta$.

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where $R = \frac{3\rho_b}{4\rho_\gamma}$ and $r_s \equiv \int_0^\eta c_s(\eta)d\eta$.

- **The CMB anisotropy spectrum is therefore**

$$C_l = \frac{2}{\pi}\int_0^\infty |\theta_l|^2 k^2 dk \tag{26}$$

$$\Theta_l = \left([\frac{1}{4}\delta_\gamma(\eta,k) + \Phi(\eta,k)]j_l(k\eta_0) + V_\gamma(\eta,k)\frac{dj_l(k\eta_0)}{d(k\eta_0)}\right)e^{-(\sigma k\eta)^2}|_{\eta=\eta_{LS}} \tag{27}$$

- **After some simplifications we get** $[V.\ Mukhanov(2004)]$

$$
\begin{aligned}
C_l &= 4\pi P_{\Phi^0}\int_0^\infty\left[C - D\cos(\rho l x) + E\cos^2(\rho l x)\right] j_l^2(xl)\frac{dx}{x}\\
&+ 4\pi P_{\Phi^0}\int_0^\infty\left[F\left(1-\frac{l(l+1)}{l^2x^2}\right)\sin^2(\rho l x)\right] j_l^2(xl)\frac{dx}{x} \qquad (28)
\end{aligned}
$$

here $\mathbf{x} = \frac{\mathbf{k}\eta_0}{\mathbf{l}}$

$\mathbf{C} \equiv \frac{81}{100}\mathbf{R^2T^2(x)e^{-\frac{l^2x^2}{l_f^2}}}$

$\mathbf{D} \equiv \frac{9}{10}\mathbf{RT(x)T^0(x)e^{-1/2\frac{l^2x^2}{l_f^2+l_s^2}}}$

$\mathbf{E} \equiv \frac{1}{4}\mathbf{T^{0^2}(x)e^{-\frac{l^2x^2}{l_s^2}}}$

$\mathbf{F} \equiv \frac{9}{4}\mathbf{c_s^2T^{0^2}(x)e^{-\frac{l^2x^2}{l_s^2}}}$

$\mathbf{l_f^{-2}} \equiv \mathbf{2}\sigma^{\mathbf{2}}\left(\frac{\eta_{\mathbf{LS}}}{\eta_0}\right)^{\mathbf{2}}$

$\mathbf{l_s^{-2}} \equiv \mathbf{2}\left[\sigma^{\mathbf{2}} + (\mathbf{k_D}\eta_{\mathbf{Ls}})^{-\mathbf{2}}\right]\left(\frac{\eta_{\mathbf{LS}}}{\eta_0}\right)^{\mathbf{2}}$

$\rho \equiv \frac{\mathbf{1}}{\eta_0}\int_{\mathbf{0}}^{\eta_{\mathbf{LS}}}\mathbf{c_s d}\eta$ and

$$\mathbf{P_{\Phi^0}} \equiv \frac{\alpha^{\mathbf{2}}\mathbf{V_0}}{\mathbf{27}\pi^{\mathbf{2}}\mathbf{M_P^2}}\left[\ln\left(\mathbf{a_1M_P^{-1}}\sqrt{\frac{\mathbf{V_0}}{\mathbf{3}}}\ \frac{\eta_0}{\mathbf{l}}\right)\right]^{\mathbf{2}}$$

- In the large $l >> 1$ limit of *spherical bessel* function, the expression for C_l simplifies to

$$C_l = P_{\Phi^0} \int_1^{\infty} \frac{C - D\cos(\rho l x) + E\cos^2(\rho l x) + F(1 - \frac{l(l+1)}{l^2 x^2})\sin^2(\rho l x)}{l^2 x^2 \sqrt{x^2 - 1}/2\pi} dx \tag{29}$$

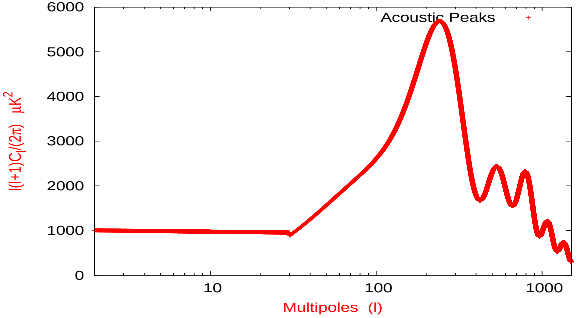

Figure: CMB angular power spectrum for adiabatic modes

Summary and Future Aspects....

1. **Mutated hilltop inflation** model provides analytical expressions for most of the *observable parameters*.
2. The *observable parameters* are in good agreement with the latest observations.
3. Scale dependent *matter power spectrum* has been obtained.
4. We have explicitly shown that *Sachs-Wolfe plateau* is tilted.
5. We have studied *acoustic oscillations* of the baryon-photon plasma and see the CMB power spectrum match well with observations.

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4. We have explicitly shown that *Sachs-Wolfe plateau* is tilted.
5. We have studied *acoustic oscillations* of the baryon-photon plasma and see the CMB power spectrum match well with observations.
6. We are now aiming to extend our work by incorporating **Integrated Sachs-Wolfe effect**, contribution from the **tensor modes** into our theory.

Thank You !