Theoretical constraints on the rare tau decays

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Lepton flavour violation is a very powerful tool to probe physics beyond the Standard Model.

In the Standard Model

\[ U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_e \times U(1)_\mu \times U(1)_\tau \]

Charged lepton masses

Family lepton numbers and total lepton number are strictly conserved.

Consistent with experiments searching for neutrinoless double beta decay and rare lepton decays, but not with neutrino oscillation experiments.
In the Standard Model with massive neutrinos

\[ U(3)_{e_R} \times U(3)_L \longrightarrow U(1)_{\text{lep}} \quad \text{Dirac Mass} \]

\[ U(3)_{e_R} \times U(3)_L \longrightarrow \text{nothing} \quad \text{Majorana Mass} \]

The predictions for the rare lepton decays are

\[ \text{BR}(\mu \rightarrow e\gamma) \approx 10^{-57}, \text{BR}(\tau \rightarrow \mu\gamma) \approx 10^{-54}, \text{BR}(\tau \rightarrow e\gamma) \approx 10^{-57}, \]

Still consistent with experiments searching for rare lepton decays.
Searches for Lepton Number Violation

UL Branching Ratio (Conversion Probability) vs. Year

- $\mu \rightarrow e \gamma$
- $\mu^- N \rightarrow e^- N$
- $\mu^+ e^- \rightarrow \mu^- e^+$
- $\mu \rightarrow eee$
- $K_L \rightarrow \pi^+ \mu e$
- $K_L \rightarrow \mu e$
- $K_L \rightarrow \pi^0 \mu e$

AJSTÖ et al.

J-PARC, MEG, PSI

Projected NUFACHT
Bounds on new physics from $\mu \to e\gamma$

Lowest dimension operator which induces $\mu \to e\gamma$

$$-\mathcal{L} = m_\mu \bar{\mu}(f_{M1}^{\mu e} + \gamma_5 f_{E1}^{\mu e})\sigma^{\mu\nu} e F_{\mu\nu} + \text{h.c.}$$

The rate for the rare muon decay is:

$$\text{BR}(\mu \to e\gamma) = \frac{96\pi^3 \alpha}{G_F^2} (|f_{E1}^{\mu e}|^2 + |f_{M1}^{\mu e}|^2)$$

The present experimental bound $\text{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ gives:

$$|f_{E1}^{\mu e}|, |f_{M1}^{\mu e}| \lesssim 10^{-12} \text{GeV}^{-2}$$

Naively,

$$f^{\mu e} \sim \frac{1}{\Lambda^2} \quad \longrightarrow \quad \Lambda \gtrsim 300 \text{TeV}$$
In most models the contact interaction arises as a result of quantum effects (new particles interacting with the muon and the electron circulating in loops).

\[ f_{\mu e} \sim \frac{\theta_{\mu e}^2 \alpha}{\Lambda^2} \]

Then, the present bound on \( BR(\mu \rightarrow e\gamma) \) requires

\[ \Lambda \gtrsim 20\text{TeV} \quad \text{if} \quad \theta_{\mu e} \sim \frac{1}{\sqrt{2}} \]
\[ \theta_{\mu e} \lesssim 0.01 \quad \text{if} \quad \Lambda \sim 300\text{GeV} \]

A large mass scale for the new particles and/or small coupling between the electron/muon and the new particles.
Rare tau decays

Complementary probe of lepton flavour violation.

Until very recently, not as interesting as $\mu \rightarrow e\gamma$ for constraining models.

**PDG 2004**

$$\text{BR}(\tau \rightarrow e\gamma) \leq 2.7 \times 10^{-6} \quad \text{CL} = 90\%$$
$$\text{BR}(\tau \rightarrow \mu\gamma) \leq 1.1 \times 10^{-6} \quad \text{CL} = 90\%$$

The experimental bound $\text{BR}(\tau \rightarrow \mu\gamma) < 1.1 \times 10^{-6}$ yields:

$$\Lambda \gtrsim 800\text{GeV} \quad \text{if} \quad \theta_{\tau\mu} \sim \frac{1}{\sqrt{2}} \quad \text{(compare to 20TeV!)}$$
$$\theta_{\tau\mu} \lesssim 0.3 \quad \text{if} \quad \Lambda \sim 300\text{GeV}$$
Impressive experimental progress in the last years!!
**PDG 2004**
\[
\text{BR}(\tau \rightarrow e\gamma) \leq 2.7 \times 10^{-6} \quad \text{CL} = 90\%
\]
\[
\text{BR}(\tau \rightarrow \mu\gamma) \leq 1.1 \times 10^{-6} \quad \text{CL} = 90\%
\]

**PDG 2005**
\[
\text{BR}(\tau \rightarrow e\gamma) \leq 3.9 \times 10^{-7} \quad \text{CL} = 90\%
\]
\[
\text{BR}(\tau \rightarrow \mu\gamma) \leq 3.1 \times 10^{-7} \quad \text{CL} = 90\%
\]

**PDG 2006**
\[
\text{BR}(\tau \rightarrow e\gamma) \leq 1.1 \times 10^{-7} \quad \text{CL} = 90\%
\]
\[
\text{BR}(\tau \rightarrow \mu\gamma) \leq 6.8 \times 10^{-8} \quad \text{CL} = 90\%
\]

**January 2009**
\[
\text{BR}(\tau \rightarrow e\gamma) \leq 1.1 \times 10^{-7} \quad \text{CL} = 90\%
\]
\[
\text{BR}(\tau \rightarrow \mu\gamma) \leq 4.5 \times 10^{-8} \quad \text{CL} = 90\%
\]

**Projected sensitivity of present B-factories**
\[
\text{BR}(\tau \rightarrow e\gamma) \sim 10^{-8}
\]
\[
\text{BR}(\tau \rightarrow \mu\gamma) \sim 10^{-8}
\]
The present experimental bounds on the rare tau decays yield:

From $\tau \rightarrow e\gamma$

$\Lambda \gtrsim 1300\text{GeV}$ \quad \text{if} \quad \theta_{\tau e} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau e} \lesssim 0.2$ \quad \text{if} \quad \Lambda \sim 300\text{GeV}$

From $\tau \rightarrow \mu\gamma$

$\Lambda \gtrsim 1700\text{GeV}$ \quad \text{if} \quad \theta_{\tau \mu} \sim \frac{1}{\sqrt{2}}$

$\theta_{\tau \mu} \lesssim 0.1$ \quad \text{if} \quad \Lambda \sim 300\text{GeV}$

fairly stringent constraints!!
What can we learn from these experimental results?

Can we make any model independent prediction on the rates of the rare decays?
If $\tau \rightarrow \mu \gamma$ is observed, the tau and muon family numbers are necessarily broken. However, the electron family symmetry might still be preserved. Then, the processes $\tau \rightarrow e \gamma$ and $\mu \rightarrow e \gamma$ will have vanishing rates.
If $\tau \to \mu \gamma$ is observed, the tau and muon family numbers are necessarily broken. However, the electron family symmetry might still be preserved. Then, the processes $\tau \to e \gamma$ and $\mu \to e \gamma$ might have vanishing rates.

If $\tau \to \mu \gamma$ and $\tau \to e \gamma$ are both observed, all the family lepton numbers are broken. The rate for $\mu \to e \gamma$ is necessarily non-vanishing.

\[ \text{BR}(\mu \to e \gamma) \gtrsim C \times \text{BR}(\tau \to \mu \gamma) \text{BR}(\tau \to e \gamma) \]
A simple example

The Feynman diagrams for the dipole transitions $\tau \rightarrow l\gamma$ are

Even if the dipole transition $\mu \rightarrow e\gamma$ does not exist at tree level, there is no symmetry which forbids this transition. It will arise at the quantum level.

$$BR(\mu \rightarrow e\gamma) \simeq C \times BR(\tau \rightarrow \mu\gamma)BR(\tau \rightarrow e\gamma)$$
The result of the calculation gives

\[
\text{BR}(\mu \to e\gamma) \gtrsim 4 \times 10^{-23} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

Not very useful in practice, but model independent

Alternatively, using \(\text{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}\) we can derive a theoretical (and model independent) constraint on the rates of the rare tau decays

\[
\left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right) \lesssim 3 \times 10^{11} \left( \frac{\text{BR}(\mu \to e\gamma)}{1.2 \times 10^{-11}} \right)
\]
The effective theory approach is very powerful (model independent), but extremely conservative.

If the dipole transitions $\tau \rightarrow \mu \gamma, \tau \rightarrow e \gamma$ are generated at one loop, diagrams like

\[ \begin{array}{c}
\mu \\
\tau \\
\gamma \\
\tau \\
e
\end{array} \]

generate $\mu \rightarrow e \gamma$ at three loops! (hence the poor result)

More interesting constraints are derived in more specific models:

- Minimal Supersymmetric Standard Model
- Supersymmetric see-saw model
The scalar sector of the MSSM contains additional sources of flavour violation in the soft SUSY breaking Lagrangian:

\[-L_{\text{soft}}^{\text{lep}} = (m_{L}^2)_{ij} \tilde{L}_i^* \tilde{L}_j + (m_{e}^2)_{ij} \tilde{e}_{Ri}^* \tilde{e}_{Rj} + \left( A_{eij} \tilde{e}_{Ri}^* H_d \tilde{L}_j + \text{h.c.} \right).\]

Many possibilities for the origin of the lepton flavour violation:
LL, RR, RL, LR
In general, \((m^2_L)_{12}, (m^2_e)_{12}, A_{e12}\) are different from zero, thus inducing the process \(\mu \rightarrow e\gamma\).

Assume the worst case for the detection of \(\mu \rightarrow e\gamma\), namely all \((m^2_L)_{12}, (m^2_e)_{12}, A_{e12}\) are equal from zero.

Still, if the rates for \(\tau \rightarrow \mu \gamma\) and \(\tau \rightarrow e\gamma\) do not vanish, a non-vanishing rate for \(\mu \rightarrow e\gamma\) will be generated at one loop via a double mass insertion.

Again, we find \(\text{BR}(\mu \rightarrow e\gamma) \gtrsim C \times \text{BR}(\tau \rightarrow \mu\gamma)\text{BR}(\tau \rightarrow e\gamma)\)
The process $\mu \rightarrow e\gamma$ can be generated by 16 combinations of mass insertions:

But also LL-LR, RR-RL, RL-LR, etc

We find that the 16 combinations can be classified in four classes (same dependence with $\tan\beta$, the fermion masses and the overall size of the scalar masses)
• **Class I**: LL-LL, RR-RR

\[
\text{BR}(\mu \to e\gamma) \gtrsim 9 \times 10^{-10} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class II**: LL-RR, RR-LL, LR-RR, RR-LR, RL-LL, LL-RL

\[
\text{BR}(\mu \to e\gamma) \gtrsim 3 \times 10^{-7} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class III**: LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL

\[
\text{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-14} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class IV**: LR-RL, RL-LR

\[
\text{BR}(\mu \to e\gamma) \gtrsim 2 \times 10^{-11} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]
• **Class I**: LL-LL, RR-RR

\[
\text{BR}(\mu \to e\gamma) \gtrsim 9 \times 10^{-10} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class II**: LL-RR, RR-LL, LR-RR, RR-LR, RL-LL, LL-RL

\[
\text{BR}(\mu \to e\gamma) \gtrsim 3 \times 10^{-7} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class III**: LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL

\[
\text{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-14} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class IV**: LR-RL, RL-LR

\[
\text{BR}(\mu \to e\gamma) \gtrsim 2 \times 10^{-11} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]
• **Class I:** $\text{LL-LL, RR-RR}$

\[
\text{BR}(\mu \to e\gamma) \gtrsim 9 \times 10^{-10} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class II:** $\text{LL-RR, RR-LL, LR-RR, RR-LR, RL-LL, LL-RL}$

\[
\text{BR}(\mu \to e\gamma) \gtrsim 3 \times 10^{-7} \left( \frac{\tilde{m}}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class III:** $\text{LL-LR, LR-LL, RR-RL, RL-RR, LR-LR, RL-RL}$

\[
\text{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-14} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]

• **Class IV:** $\text{LR-RL, RL-LR}$

\[
\text{BR}(\mu \to e\gamma) \gtrsim 2 \times 10^{-11} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right) \left( \frac{\text{BR}(\tau \to e\gamma)}{1.1 \times 10^{-7}} \right)
\]
Or alternatively, one could derive theoretical constraints on the rare tau decays in the MSSM
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Region of the parameter space allowed by searches of rare tau decays
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Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \to e\gamma$. 

AI, Shindou, Simonetto

SPS1a
Or alternatively, one could derive theoretical constraints on the rare tau decays in the MSSM.
Or alternatively, one could derive theoretical constraints on the rare tau decays in the MSSM

Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu \rightarrow e\gamma$ (BR<10$^{-13}$)

Region where both $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ could be observed at present B-factories

SPS1a

AI, Shindou, Simonetto
Or alternatively, one could derive theoretical constraints on the rare tau decays in the MSSM.

**Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu \rightarrow e\gamma$ ($BR<10^{-13}$)**

If MEG reaches the sensitivity $BR(\mu \rightarrow e\gamma) \sim 10^{-13}$ without finding a signal, present B-factories will not discover both $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$.

**Region where both $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ could be observed at present B-factories**
Class I

Class II

Class III

Class IV
If both $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e \gamma$ are observed, our SM vacuum is metastable. Problems with vacuum stability?
Supersymmetric see-saw model

1) Extend the particle content of the MSSM with two or three singlet superfields (right-handed neutrinos)

\[ W_{\text{lep}} = e^c_{Ri} Y_{eij} L_j H_d + \nu^c_{Ri} Y_{\nu ij} L_j H_u - \frac{1}{2} \nu^c_{Ri} M_{ij} \nu^c_{Rj} \]

2) Assume that the right-handed neutrinos are very heavy

\[ W_{\text{lep}}^{\text{eff}} = e^c_{Ri} Y_{eij} L_j H_d + \frac{1}{2} \left( Y_T^T M^{-1} Y_{\nu} \right)_{ij} (L_i H_u)(L_j H_u) \]

\[ M = (Y_T^T M^{-1} Y_{\nu}) \langle H_u^0 \rangle^2 \]
Even if the mechanism of mediation of SUSY breaking is flavour blind, so that the off diagonal elements of \((m^2_L), (m^2_e), A_e\) vanish at high energies, quantum effects induced by right-handed neutrinos will generate radiatively non-vanishing off-diagonal elements.

\[
\begin{align*}
(m^2_L)_{ij} &\approx -\frac{1}{8\pi^2}(3m^2_0 + |A_0|^2)(Y^\dagger \nu Y^\nu)_{ij} \log \left(\frac{M_X}{M_{maj}}\right) \\
(m^2_e)_{ij} &\approx 0 , \\
(A_e)_{ij} &\approx -\frac{3}{8\pi^2} A_0 Y_e(Y^\dagger \nu Y^\nu)_{ij} \log \left(\frac{M_X}{M_{maj}}\right) ,
\end{align*}
\]

Borzumati, Masiero
Assume the worst case for the detection of $\mu \rightarrow e \gamma$, namely all $(m^2_{L})_{12}$, $(m^2_{e})_{12}$, $A_{e12}$ are equal to zero at low energies

- $(m^2)_{12}$, $(m^2)_{12}$, $A_{e12}$ vanish at high energies
  (no LFV in the soft terms at tree level)

AND

- $(Y^\dagger_v Y_v)_{12} = 0$
  (no LFV in the soft terms at one loop level)
Crucial assumption in what follows: 
\((Y_v^+ Y_v)_{12} = 0\) without cancellations 

\((Y_v^+ Y_v)_{12} = y_{11}^* y_{12} + y_{21}^* y_{22} + y_{31}^* y_{32}\) vanishes naturally only when each term vanishes. There are only eight possibilities

\[
Y_v = \begin{pmatrix}
0 & \times & \times \\
0 & \times & \times \\
\times & 0 & \times
\end{pmatrix} \quad Y_v = \begin{pmatrix}
x & 0 & \times \\
0 & \times & \times \\
\times & 0 & \times
\end{pmatrix}
\]

\[
Y_v = \begin{pmatrix}
0 & \times & \times \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix} \quad Y_v = \begin{pmatrix}
x & 0 & \times \\
0 & \times & \times \\
\times & 0 & \times
\end{pmatrix}
\]

\[
Y_v = \begin{pmatrix}
0 & \times & \times \\
\times & 0 & \times \\
\times & 0 & \times
\end{pmatrix} \quad Y_v = \begin{pmatrix}
|\times & 0 & \times \\
\times & 0 & \times \\
\times & 0 & \times
\end{pmatrix}
\]
Crucial assumption in what follows: \( (Y^\dagger_Y)_v^{12} = 0 \) without cancellations

\[
(Y^\dagger_Y)_v^{12} = \gamma_{11}^* \gamma_{12} + \gamma_{21}^* \gamma_{22} + \gamma_{31}^* \gamma_{32}
\]

vanishes naturally only when each term vanishes. There are only eight possibilities.

\[
Y_v = \begin{pmatrix}
0 & \times & \times \\
0 & \times & \times \\
0 & \times & \times \\
\end{pmatrix}
\]

In conflict with experiments: the number conserved
Crucial assumption in what follows:

\[(Y_\nu^+ Y_\nu)\sub{12} = 0\] without cancellations

\[(Y_\nu^+ Y_\nu)\sub{12} = y_{11} y_{12} + y_{21} y_{22} + y_{31} y_{32}\] vanishes naturally only when each term vanishes. There are only eight possibilities

\[Y_\nu = \begin{pmatrix} 0 & \times & \times \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}\]

\[Y_\nu = \begin{pmatrix} 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}\]

\[\mathcal{M} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & 0 \\ \times & 0 & \times \end{pmatrix}\]

In conflict with experiments: \(\mu\) number conserved

\[Y_\nu = \begin{pmatrix} \times & 0 & \times \\ \times & 0 & \times \\ \times & 0 & \times \end{pmatrix}\]
Crucial assumption in what follows: 

\[ (Y^{\dagger}_\nu Y_{\nu})_{12} = 0 \text{ without cancellations} \]

\[ (Y^{\dagger}_\nu Y_{\nu})_{12} = y_{11}^{*} y_{12} + y_{21}^{*} y_{22} + y_{31}^{*} y_{32} \]

vanishes naturally only when each term vanishes. There are only eight possibilities.

\[ M \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix} \]

\[ Y_{\nu} = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix} \]

\[ Y_{\nu} = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix} \]

\[ Y_{\nu} = \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix} \]

\[ Y_{\nu} = \begin{pmatrix} |\times & 0 & \times \\ \times & 0 & \times \\ \times & 0 & \times \end{pmatrix} \]

Consistent by experiments: violates all lepton family numbers.
All the allowed models which lead to vanishing \((Y^\dagger \nu Y^\dagger \nu)_{12}\) lead to

\[
Y^\dagger \nu Y^\dagger \nu \sim \begin{pmatrix}
\times & 0 & \times \\
0 & \times & \times \\
\times & \times & \times 
\end{pmatrix}
\]

**Implications:**

- Off-diagonal soft terms are generated at one loop level in the 13 and 23 sectors. A non vanishing rate for \(\mu \rightarrow e\gamma\) will be induced through the double mass insertion

\[
BR(\mu \rightarrow e\gamma) \simeq C \times BR(\tau \rightarrow \mu\gamma)BR(\tau \rightarrow e\gamma)
\]
All the allowed models which lead to vanishing $(Y_\nu^+ Y_\nu)_{12}$ lead to

$$Y_\nu^+ Y_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$$

**Implications:**

- Off-diagonal soft terms are generated at one loop level in the 13 and 23 sectors. A non vanishing rate for $\mu \rightarrow e \gamma$ will be induced through the double mass insertion.

- That structure for $Y_\nu^+ Y_\nu$ likely to hold at the cut-off scale. Off-diagonal soft terms are generated at two loop level in the 12 sector. Another contribution to $\text{BR}(\mu \rightarrow e \gamma)$

\[
(m^2_L)_{21}(M_{maj}) \simeq \left(\frac{1}{16\pi^2}\right)^2 m^2_S(Y_\nu^+ Y_\nu)^*_{32}(Y_\nu^+ Y_\nu)_{31} \log\left(\frac{M_X}{M_{maj}}\right)^2
\]

Again,

$$\text{BR}(\mu \rightarrow e \gamma) \simeq C \times \text{BR}(\tau \rightarrow \mu \gamma) \text{BR}(\tau \rightarrow e \gamma)$$
In the worst case neutrino scenario, where $Y^\dagger Y \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$

$$BR(\mu \rightarrow e\gamma) \simeq C \times BR(\tau \rightarrow \mu\gamma)BR(\tau \rightarrow e\gamma)$$

In any other scenario, with $(Y^\dagger Y)_{12} \neq 0$,

$$BR(\mu \rightarrow e\gamma) \geq C \times BR(\tau \rightarrow \mu\gamma)BR(\tau \rightarrow e\gamma)$$

C depends just on SUSY parameters and is independent of see-saw parameters

This bound holds for all the see-saw models that reproduce the low energy data. The only assumption is the absence of cancellations.
There is though some dependence on SUSY parameters. Roughly,

$$BR(\mu \rightarrow e\gamma) \gtrsim 10^{-9} \left( \frac{m_S}{200 \text{ GeV}} \right)^4 \left( \frac{\tan \beta}{10} \right)^{-2} \left( \frac{BR(\tau \rightarrow \mu\gamma)}{6.8 \times 10^{-8}} \right) \left( \frac{BR(\tau \rightarrow e\gamma)}{1.1 \times 10^{-7}} \right)$$

Alternatively, one can set theoretical constraints on the rare tau decays in the SUSY see-saw model

$$BR(\tau \rightarrow e\gamma) \lesssim 10^{-9} \left( \frac{m_S}{200 \text{ GeV}} \right)^{-4} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{BR(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}} \right) \left( \frac{BR(\tau \rightarrow \mu\gamma)}{6.8 \times 10^{-8}} \right)^{-1},$$

$$BR(\tau \rightarrow \mu\gamma) \lesssim 7 \times 10^{-10} \left( \frac{m_S}{200 \text{ GeV}} \right)^{-4} \left( \frac{\tan \beta}{10} \right)^2 \left( \frac{BR(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}} \right) \left( \frac{BR(\tau \rightarrow e\gamma)}{1.1 \times 10^{-7}} \right)^{-1}.$$

Valid for every neutrino model compatible with the low energy data, with the only assumption of absence of cancellations.
Region of the parameter space allowed by searches of rare tau decays.
Region of the parameter space allowed by searches of rare tau decays + present bound on $\mu \rightarrow e \gamma$

Region where both $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow e \gamma$ could be observed at present B-factories

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Region where both $\tau \to \mu \gamma$ and $\tau \to e \gamma$ could be observed at present B-factories

Region of the parameter space allowed by searches of rare tau decays + projected bound on $\mu \to e \gamma$ (BR$<10^{-13}$)
Conclusions

- Lepton flavour violation is a very powerful tool to probe physics beyond the Standard Model.
- Huge experimental effort ongoing to constrain (hopefully discover) leptonic rare decays. On the theory side, there is also an intense activity computing predictions in very particular scenarios.
- We have presented a relation among the branching ratios of the rare decays, which applies in full generality:

\[ \text{BR}(\mu \rightarrow e\gamma) \gtrsim C \times \text{BR}(\tau \rightarrow \mu\gamma) \text{BR}(\tau \rightarrow e\gamma) \]

- Effective field theory. Not practical
- MSSM. Fairly stringent in most scenarios
- SUSY see-saw. Very stringent: present B-factories should not discover both $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$. 