SUSY SU(5) with singlet plus adjoint matter and $A_4$ family symmetry
Cooper, King and Luhn arXiv:1004.3243[hep-ph]

Iain Cooper

School of Physics and Astronomy
University of Southampton

Nu Horizons IV, Feb 2011
Outline

1. Background
   - Introduction
   - The Seesaw Mechanism

2. Tribimaximal Mixing and A4
   - Mixing
   - Symmetries of the tribimaximal structure

3. The Model

4. Conclusions and Outlook
Ever since the first Super Kamiokande and SNO results were released between 1998 and 2001, it has been widely accepted that neutrinos change flavour and therefore possess a small but non-zero mass (since oscillation probability is $\propto \sin^2 \left( 1.27 \frac{\Delta m^2 L}{E} \right)$)
Furthermore, oscillation data are consistent with a highly symmetric pattern of mixing:

\[ U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \]
Introduction

- These two issues need explanation from a theoretical point of view.
- We present a model which attempts to do this using a family symmetry in conjunction with the Type III seesaw mechanism.
The Type I seesaw

- Introduce a heavy Majorana particle $\nu_R$

$$\mathcal{L}_N = - \left( L^T \tilde{H} Y_{\nu} \nu_R^* \right) - \frac{1}{2} \left( (\nu_R^c)^T M_{RR} \nu_R \right) + h.c.$$  

$$\tilde{H} = i\tau_2 H^*$$ with $H$ the Higgs $SU(2)_L$ doublet

- $\nu_R$ can be integrated out below $M_{RR}$ to give an effective theory
The Type I seesaw

\[ \nu_L \times m_{LL} \nu_L \]

- The Lagrangian now looks like a Majorana mass for the \( \nu_L \)

\[ \mathcal{L}_\nu = -\frac{1}{2} \nu_L^T m_{LL} \nu_L + h.c. \]

\[ m_{LL} \sim -v^2 Y_\nu M^{-1}_{RR} Y^T_\nu = -m_{LR} M^{-1}_{RR} m_{LR}^T \text{ with } v = \langle H \rangle \]

- This is equivalent to diagonalising the 6 \( \times \) 6 matrix

\[
\begin{pmatrix}
0 & (m_{LR}^D)^T \\
(m_{LR})^T & M_{RR}
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
m_{LL} & 0 \\
0 & M_{RR}
\end{pmatrix}
+ \text{ higher order terms}
\]

under the assumption that \( M_{RR} \gg m_{LR} \)
The Type III seesaw

- Take a closer look at one of the vertices
- Under $SU(2)_L$, $2 \otimes 2 = 1 + 3$ so the exchanged particle may also be a triplet with hypercharge 0 - this is the type III seesaw mechanism
- (There is also a Type II where the Higgs is a triplet - not discussed here)
Tribimaximal mixing

\[
U_{PMNS} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

- \( U_{PMNS} = V_{eL}V_{\nu_L}^\dagger \) describes the mismatch between neutrino and charged lepton bases
- Specializing to a basis where charged leptons are diagonal
  \[
  \begin{pmatrix}
  \nu_e \\
  \nu_\mu \\
  \nu_\tau
  \end{pmatrix} = \begin{pmatrix}
  U_{e1} & U_{e2} & U_{e3} \\
  U_{\mu1} & U_{\mu2} & U_{\mu3} \\
  U_{\tau1} & U_{\tau2} & U_{\tau3}
  \end{pmatrix}
  \begin{pmatrix}
  \nu_1 \\
  \nu_2 \\
  \nu_3
  \end{pmatrix}
  \]
- This is what gives rise to the observed neutrino oscillation effects
- Neutrino oscillation data is consistent with the choice
  \[
  \theta_{12} = \sin^{-1} \frac{1}{\sqrt{3}}, \quad \theta_{13} = 0, \quad \theta_{23} = \frac{\pi}{4} \Rightarrow U_{PMNS} = U_{TB} = \begin{pmatrix}
    \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
  \end{pmatrix}
  \]
Tribimaximal mixing

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
=
\begin{pmatrix}
\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & 0 \\
-rac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

- This is the tribimaximal mixing pattern
- A combination of:
  - Trimaximal mixing in the \( \nu_2 \) eigenstate
  - And bimaximal mixing in the \( \nu_3 \) eigenstate
- This is particularly simple - why should this be the case? Is there some hidden symmetry in play?
Symmetries of $U_{TB}$

- We want a clue as to which underlying symmetries could reproduce TBM.
- In our current basis $m_{LL}^{diag} = U_{TB}^\dagger m_{LL} U_{TB}^*$ and so

$$m_{LL} = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

- One can find the symmetries of this matrix (which is diagonalised by $U_{TB}$), i.e. transformations $W$ such that $W^* m_{LL} W^\dagger = m_{LL}$:

$$W = \begin{pmatrix} a & b & b \\ b & c & a + b - c \\ b & a + b - c & c \end{pmatrix}$$
Symmetries of $U_{TB}$

- Restricting attention to matrices with determinant = 1 we find

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{pmatrix}$$

denoting these $I$, $U$, $S$ and $US = SU$ respectively

- Since the charged lepton sector is diagonal and non-degenerate, its most general symmetry is

$$T = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}$$

with $\phi_i \neq \phi_j$
We thus expect TBM to arise from a Lagrangian initially symmetric under transformations generated by $S$, $T$, $U$ or subgroups of these.

In the current model we consider $A_4$, generated by (in the triplet representation)

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega = \exp\left(\frac{2\pi i}{3}\right)$

$A_4$ is the group of even permutations on four objects, or alternatively the rotations of a tetrahedron.

It has three inequivalent one dimensional representations $1$, $1'$ and $1''$, along with the triplet $3$. 
Multiplying triplets in A4

The approach we take in this model is to form a triplet of $A_4$ from the three families of lepton doublets and then introduce triplets called flavons in order to construct $A_4$ singlet terms.

Thus we need to know how to extract singlets from products like $3 \otimes 3$.

From the group character table we can show that

$$3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3 \oplus 3$$

But we would like to know which components of the triplets are used to form each representation.

To make the computations easier we first change basis such that

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
Multiplying triplets in $A_4$

\[
3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3 \oplus 3
\]

- Consider the 1. This is a singlet and so is invariant under the action of both $S$ and $T$.
- If we have two triplets $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, the combination of components giving a singlet is then $(a_1 b_1 + a_2 b_2 + a_3 b_3)$
- Carrying out the full computation gives
  \[
  3 \otimes 3 = (a_1 b_1 + a_2 b_2 + a_3 b_3) + (a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3) \\
  + (a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3) + (a_2 b_3, a_3 b_1, a_1 b_2) + (a_3 b_2, a_1 b_3, a_2 b_1)
  \]
- Since this rule contains a triplet, any product of two or more triplets will contain a singlet
The Model

- We investigate whether it is possible to impose an $A_4$ family symmetry upon a Type III seesaw mechanism within a SUSY GUT framework.

- In other words, we would like to combine:
  - A triplet mediated seesaw mechanism
  - Supersymmetric $SU(5)$ (including the Georgi-Jarlskog mechanism)
  - $A_4$ family symmetry in order to explain the tribimaximal mixing pattern.

- See ArXiv 0705.3589 'Adjoint SUSY SU(5) for the model without a family symmetry.'
Where is the seesaw particle?

- The first question is: where can we find an $SU(2)_L$ triplet with 0 hypercharge?

- The smallest $SU(5)$ representation which contains such a multiplet is the adjoint:

  $$24 = (8, 1)_0 \oplus (1, 3)_0 \oplus (3, 2)_{-5/6} \oplus (3, 2)_{5/6} \oplus (1, 1)_0$$

- Notice that this also contains a standard model singlet, making a hybrid Type I + Type III seesaw mechanism.

- However since both particles come from the same representation, they are constrained to give the same contribution to neutrino masses (up to a constant).

- This leads to only one massive neutrino; since two non-zero mass differences are observed, this is not enough.

- Adding a single right handed neutrino $N$ allows one to obtain two massive neutrinos.
How do we apply $A_4$?

- Combine the multiplets containing left handed neutrinos into an $A_4$ triplet:

$$F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$

- Use extra $A_4$ triplets called flavons to make Lagrangian invariant

- Allow these flavons to obtain VEVs in order to break $A_4$

- Recall the neutrino mass matrix:

$$m_{LL} = \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

- Each matrix is composed from an outer product of one of the eigenvectors of $m_{LL}$

- If we ensure the flavon VEVs are aligned with these eigenvectors, we will obtain TBM in the neutrino sector
Charge Assignment

<table>
<thead>
<tr>
<th>Field</th>
<th>$\psi_{24}$</th>
<th>$N$</th>
<th>$F$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$H_5$</th>
<th>$H_5$</th>
<th>$H_{45}$</th>
<th>$\varphi_{123}$</th>
<th>$\varphi_{23}$</th>
<th>$\varphi_3$</th>
<th>$\xi$</th>
<th>$\xi'$</th>
<th>$\varphi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(5)$</td>
<td>24</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>45</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A_4$</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$U(1)_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U(1)$</td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>-1</td>
<td>-4</td>
<td>0</td>
<td>$q_1$</td>
</tr>
<tr>
<td>$Z_2^1$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$Z_2^2$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flavon VEV</th>
<th>VEV alignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \varphi_1 \rangle$</td>
<td>$(1, 0, 0)^T$</td>
</tr>
<tr>
<td>$\langle \varphi_3 \rangle$</td>
<td>$(0, 0, 1)^T$</td>
</tr>
<tr>
<td>$\langle \varphi_{23} \rangle$</td>
<td>$\frac{1}{\sqrt{2}}(0, 1, -1)^T$</td>
</tr>
<tr>
<td>$\langle \varphi_{123} \rangle$</td>
<td>$\frac{1}{\sqrt{3}}(1, 1, 1)^T$</td>
</tr>
</tbody>
</table>

The extra $U(1)$ and $Z_2$s constrain the Yukawa couplings in order that we obtain the correct masses/mixings in the quark and charged lepton sectors (this includes the Georgi-Jarlskog relations)
What is the scale of the flavon VEVs? We define

\[ \eta_i = \frac{\langle |\varphi_i| \rangle}{\Lambda} \]

\( \varphi_i = \varphi_{123}, \varphi_{23}, \varphi_3, \xi \) or \( \xi' \)

In order to get the hierarchical structure of the quark and charged lepton mass matrices we assume

\[ \eta_{123}, \eta_{23}, \eta_{\xi'} = \epsilon^2 \quad \text{and} \quad \eta_3, \eta_{\xi} = \epsilon \]

The numerical values for \( \epsilon \) depend on the messenger scale of the relevant sector.

We present the superpotential terms of the quark and charged lepton sectors up to and including \( \mathcal{O}(\epsilon^5) \).
Neutrino sector

$W_\nu = \frac{\varphi_{123}^{123}}{\Lambda} cF \psi_{24} H_5 + \frac{\varphi_{23}^{23}}{\Lambda} pFNH_5 + \frac{\varphi_{23}^{23}}{2\Lambda} y_N NN + \frac{\xi^4}{2\Lambda^3} y_N NN + \frac{\varphi_{123}^{23}}{2\Lambda} yTr \left( \psi_{24}^2 \right)$

- $\Lambda$ is a heavy mass scale; the $\rho$ fields are the components of $\psi_{24}$
- The first two terms are Dirac masses, whilst the final three are Majorana
Neutrino sector

\[ W_\nu = \frac{\varphi_{123}}{\Lambda} c F \psi_{24} H_5 + \frac{\varphi_{23}}{2\Lambda} p F N H_5 + \frac{\varphi_{23}^2}{2\Lambda^3} y_N N N + \frac{\xi^4}{2\Lambda^3} y'_N N N + \frac{\varphi_{123}^2}{2\Lambda} y \text{Tr} \left( \psi_{24}^2 \right) \]

After allowing the flavons to obtain their VEVs, one can apply the seesaw formula \(-m_{LR} M_{RR}^{-1} m_{LR}^T\) to find the effective left handed neutrino mass matrix

\[ m_{LL} = \frac{2c^2 v_u^2}{15y\Lambda} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{p^2 v_u^2}{2(y_N + y'_N \xi^4 / \eta_{23}^2)\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \]

This has eigenvalues \(m_1 = 0, m_2 = m_3 = \frac{2c^2 v_u^2}{5y\Lambda}\) and \(m_3 = \frac{p^2 v_u^2}{(y_N + y'_N \xi^4 / \eta_{23}^2)\Lambda}\)

It is manifestly of tribimaximal form since the two matrices are both part of the generic mass matrix presented previously.
Down quark/charged lepton sector

\[ W_d \sim \frac{\varphi_{23}\xi^2}{\Lambda_d^3} T_1 FH_5 + \frac{\varphi_{123}\xi^2}{\Lambda_d^3} T_2 FH_5 + \frac{\varphi_{23}\xi}{\Lambda_d^2} T_2 FH_{45} + \frac{\varphi_3}{\Lambda_d} T_3 FH_5 \]

- \( \Lambda_d \) here is a heavy mass scale
- The terms are such that the down quark mass matrix is

\[
\begin{pmatrix}
0 & \eta_{23}\eta_{\xi}^2 & -\eta_{23}\eta_{\xi}^2 \\
\eta_{123}\eta_{\xi}^2 & \eta_{123}\eta_{\xi}^2 + k_f\eta_{23}\eta_{\xi} & \eta_{123}\eta_{\xi}^2 - k_f\eta_{23}\eta_{\xi} \\
0 & 0 & \frac{\eta_3}{\eta_{23}}
\end{pmatrix} v_d
\]

where \( k_f \) is a Clebsch factor from the \( H_{45} \): 1 for down quarks and -3 for charged leptons
Down quark/charged lepton sector

\[
\begin{pmatrix}
0 & \eta_{23} \eta_{\xi}^2 & -\eta_{23} \eta_{\xi}^2 \\
\eta_{123} \eta_{\xi}^2 & \eta_{123} \eta_{\xi}^2 + k f \eta_{23} \eta_{\xi} & \eta_{123} \eta_{\xi}^2 - k f \eta_{23} \eta_{\xi} \\
0 & 0 & \eta_{\xi}^3
\end{pmatrix}
\begin{pmatrix}
v_d
\end{pmatrix}
\]

This gives rise to mass matrices

\[
M_d \sim \begin{pmatrix}
0 & \epsilon^3 & -\epsilon^3 \\
\epsilon^3 & \epsilon^2 & -\epsilon^2 \\
0 & 0 & 1
\end{pmatrix} \epsilon v_d \quad \text{and} \quad M_e \sim \begin{pmatrix}
0 & \epsilon^3 & 0 \\
\epsilon^3 & -3\epsilon^2 & 0 \\
-\epsilon^3 & 3\epsilon^2 & 1
\end{pmatrix} \epsilon v_d
\]

with $\epsilon \sim 0.15$

Thus we obtain mass ratios of $\epsilon^4 : \epsilon^2 : 1$ for the down quarks and $\frac{\epsilon^4}{3} : 3\epsilon^2 : 1$ for the charged leptons

Left-handed down quark mixing angles $\theta_{12}^d \sim \epsilon$, $\theta_{13}^d \sim \epsilon^3$ and $\theta_{23}^d \sim \epsilon^2$ are also obtained in agreement with data (anticipating a near diagonal up sector)

The corresponding charged lepton mixing angles are $\theta_{12}^e \sim \frac{\epsilon}{3}$, $\theta_{13}^e \sim 0$ and $\theta_{23}^e \sim 0$

This implies deviation from exact tribimaximal mixing ($U_{PMNS} = V_{eL} V_{\nu L}^\dagger$), in accordance with, e.g. Antusch, King and Malinsky [arXiv:0711.4727]
Up quark sector

\[ W_u \sim \frac{(\xi')^2}{\Lambda_u^2} T_1 T_1 H_5 + \left( \frac{\varphi_{23}^2 \xi}{\Lambda_u^3} + \frac{\xi^5}{\Lambda_u^5} \right) (T_1 T_2 + T_2 T_1) H_5 + \frac{\varphi_{23} \varphi_3 \xi^2}{\Lambda_u^4} (T_1 T_3 + T_3 T_1) H_5 \]

\[ + \frac{\xi^2}{\Lambda_u^2} T_2 T_2 H_5 + \frac{\varphi_{123} \varphi_3 \xi^2}{\Lambda_u^4} (T_2 T_3 + T_3 T_2) H_5 + T_3 T_3 H_5 \]

- Again, \( \Lambda_u \) is a heavy mass scale
- This gives the matrix

\[ M_u \sim \left( \begin{array}{ccc} -\epsilon^4 & -\epsilon^5 & -\epsilon^5 \\ -\epsilon^5 & -\epsilon^2 & -\epsilon^5 \\ -\epsilon^5 & -\epsilon^5 & 1 \end{array} \right) v_u \]

implying negligible mixing and mass ratios of \( \epsilon^4 : \epsilon^2 : 1 \). Here \( \epsilon \sim 0.05 \)

- The negligible mixing means that the CKM angles are given simply by the predicted down quark angles
We have presented a model which successfully combines the Type III seesaw mechanism with a family symmetry.

It explains neutrino masses and mixing as well as giving successful outputs for quark and charged lepton parameters.

An important benefit of the model is that it has a naturally diagonal Majorana sector. This is normally assumed without a fundamental explanation.

What next? Higher order corrections to the VEV alignment could give further small deviations from TBM.