Quantum Information Processing by NMR: Recent Experimental Developments

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Achievements of NMR - QIP

√ 1. **Preparation of Pseudo-Pure States**

√ **2. Quantum Logic Gates**

√ **3. Deutsch-Jozsa Algorithm**

√ **4. Grover's Algorithm**

√ **5. Hogg's algorithm**

√ **6. Berstein-Vazirani parity algorithm**

√ **7. Quantum Games**

√ **8. Creation of EPR and GHZ states**

√ **9. Entanglement transfer**

√ **10. Quantum State Tomography**

√ **11. Geometric Phase in QC**

√ **12. Adiabatic Algorithms**

√ **13. Bell-State discrimination**

14. Error correction

15. Teleportation

16. Quantum Simulation

17. Quantum Cloning

18. Shor's Algorithm

√ **19. No-Hiding Theorem**

√ **Also performed in our Lab.**

Maximum number of qubits achieved in our lab: 8

Liquid-State Room-Temperature NMR:
Using spins in molecules as qubits:

-- Pseudo-Pure States (PPS)
-- One qubit Gates
-- Multiqubit Gates
-- Implementation of DJ and Grover's Algorithms
-- How to increase the number of Qubits
    -- Quadrupolar Nuclei as multiqubits
        -- Spin 1 as qudit
    -- Dipolar Coupled spin ½ Nuclei- up to 8 qubits
-- Geometric Phase and its use in Quantum Algorithms
-- Quantum Games
-- Adiabatic Algorithms

Recent Developments in our Laboratory

1. Experimental Proof of No-Hiding theorem.
2. Non-Destructive discrimination of Bell States.
3. Non-destructive discrimination of arbitrary set of orthogonal quantum States by phase estimation.
4. Use of Nearest Neighbour Heisenberg XY interaction for creation of entanglement on end qubits in a linear chain of 3-qubit system.

Experimental Proof of Quantum No-Hiding Theorem#

NMR Experimental verification of No-Hiding Theorem is described here.

Jharana Rani Samal*, Arun K. Pati and Anil Kumar,
(PRL- Accepted).

Also available in arXiv:quant-ph.1004.5073v1, 28 April 2010

*** Deceased 12 November 2009**

#This paper is dedicated to the memory of Ms. Jharana Rani Samal

No-Hiding Theorem

S.L. Braunstein & A.K. Pati, PRL 98, 080502 (2007).

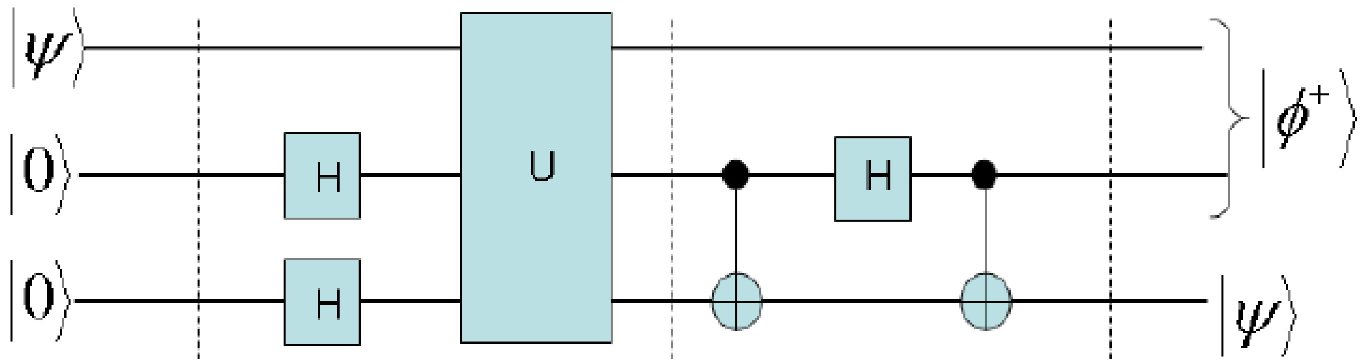
Any physical process that bleaches out the original information is called "Hiding". If we start with a pure state, this bleaching process will yield a "mixed state" and hence the bleaching process in Non-Unitary". However, in an enlarged Hilbert space, this process can be represented as a "unitary". The No-Hiding Theorem demonstrates that the initial pure state, after the bleaching process, resides in the ancilla qubits from which, under local unitary operations, is completely transformed to one of the ancilla qubits.

The above paper shows that for a 1-qubit pure state, "quantum state randomization" (QSR), which yields a completely mixed state, can be performed with an "ancilla" of 2-qubits. In such a case the "randomization process is a "Unitary" and the "missing information resides "completely in the ancilla qubits, from where it can be transformed to one of the qubits using only "local Unitary" operations.

In the end; the first two qubits are in Bell states and the initial pure state is transferred from 1^{st} to the 3^{rd} qubit.

Quantum Circuit for Test of No-Hiding Theorem using State Randomization (operator U).

H represents Hadamard Gate and dot and circle represent CNOT gates.



After randomization the state |ψ> is transferred to the second Ancilla qubit proving the No-Hiding Theorem.

(S.L. Braunstein, A.K. Pati, PRL 98, 080502 (2007).

Creation of ψ and Hadamard Gates after preparation of |000> PPS

$$|0\rangle_1 \xrightarrow{(\theta_\varphi)^1} \Psi = \cos(\theta/2)\,|0\rangle_1 + e^{[i(\varphi - \pi/2)]} \sin(\theta/2)\,|1\rangle_1$$

$$|00\rangle_{2,3} \xrightarrow{(\pi/2)^{2,3}} |A_{2,3}\rangle = [|(0 + 1)\rangle]_2 \otimes [(0 + 1)\rangle]_3$$

- The randomization operator is given by,

$$U = \sum_{k=0}^{3} \sigma_k \otimes |A_k\rangle\langle A_k| \qquad \text{Eq. (1)}$$

where $\sigma_0 = I,\quad \sigma_k\ (k = 1,2,3)$ Are Pauli Matrices

$$|A\rangle = \frac{1}{2}\sum_{k=0}^{3} |A_k\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

Eq. (2)

With this randomization operator it can be shown that any pure state is reduced to completely mixed state if the ancilla qubits are traced out.

Using Eqs (1) and (2), the U is given by,

The Randomization Operator is obtained as

U =

<table>
<tr><th></th><th>|000></th><th>|001></th><th>|010></th><th>|011></th><th>|100></th><th>|101></th><th>|110></th><th>111></th></tr>
<tr><td>|000></td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
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<tr><td>|110></td><td></td><td></td><td>-1</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>|111></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>-1</td></tr>
</table>

Blanks = 0

Local unitary for transforming information to one of the ancilla qubits

$$|\psi\rangle = \frac{1}{2}\sum_{k=0}^{3} \sigma_k |\psi\rangle |A_k\rangle \longrightarrow \frac{1}{2}\sum_{k=0}^{3} \sigma_k |\psi\rangle U_{23} |A_k\rangle = CNOT_{23}(I_2 \otimes H_3) CNOT_{23} |\psi\rangle$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|\psi\rangle = |\psi_{out}\rangle$$

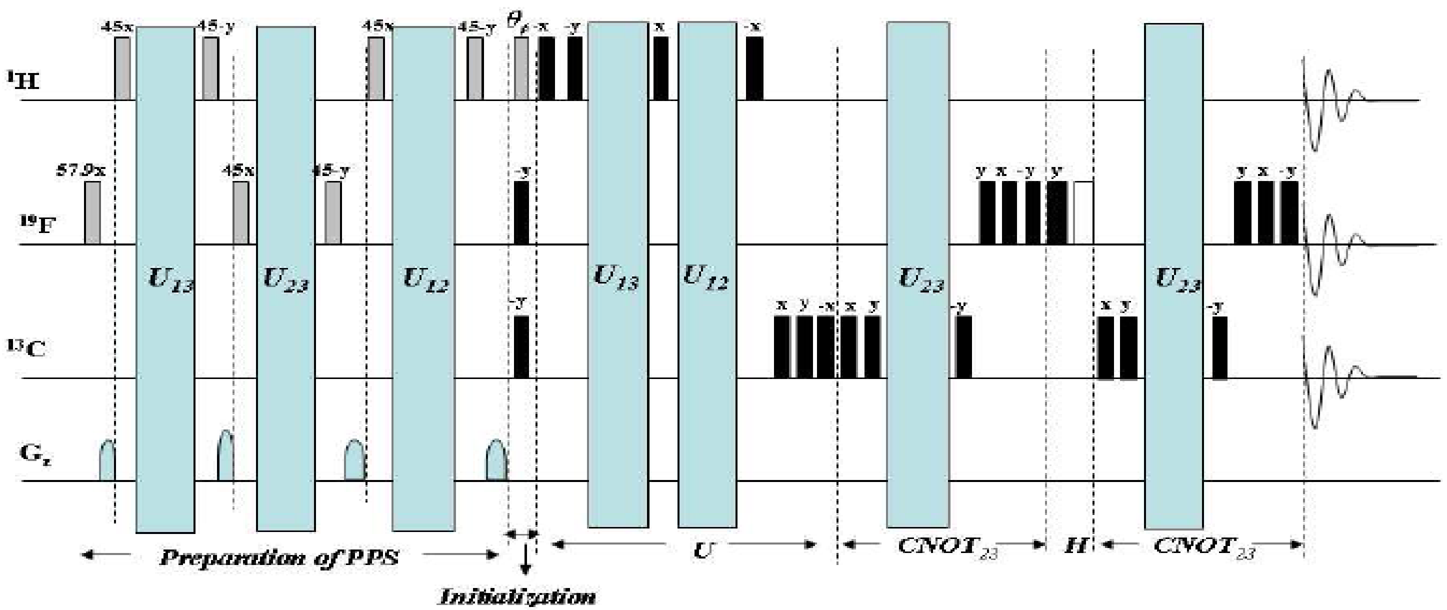
This shows that the first two qubits are in Bell State and the |ψ> has been transferred to the 3rd qubit.

Conversion of the U-matrix into an NMR Pulse sequence has been achieved here by a Novel Algorithmic Technique, developed in our laboratory by Ajoy et. al (to be published). This method uses Graphs of a complete set of Basis operators and develops an algorithmic technique for efficient decomposition of a given Unitary into Basis Operators and their equivalent Pulse sequences.

The equivalent pulse sequence for the U-Matrix is obtained as

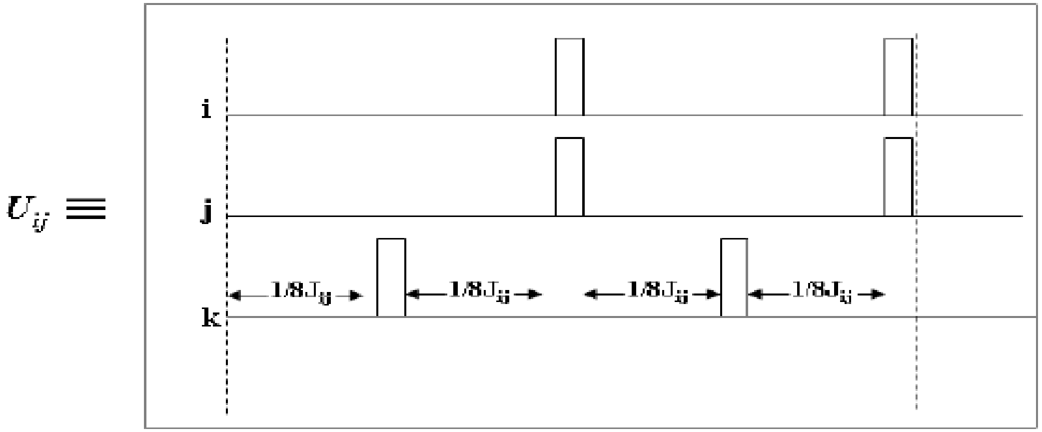
$$U = exp(-i\frac{\pi}{4}\mathbf{1})exp(i\frac{\pi}{2}I_{3z})exp(-i\pi I_{1y}I_{2z})exp(-i\pi I_{1z}I_{3z})exp(i\frac{\pi}{2}I_{1x})exp(i\frac{\pi}{2}I_{1z}).$$

NMR Pulse sequence for the Proof of No-Hiding Theorem



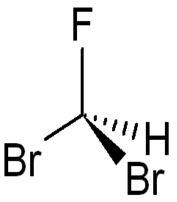
(a)

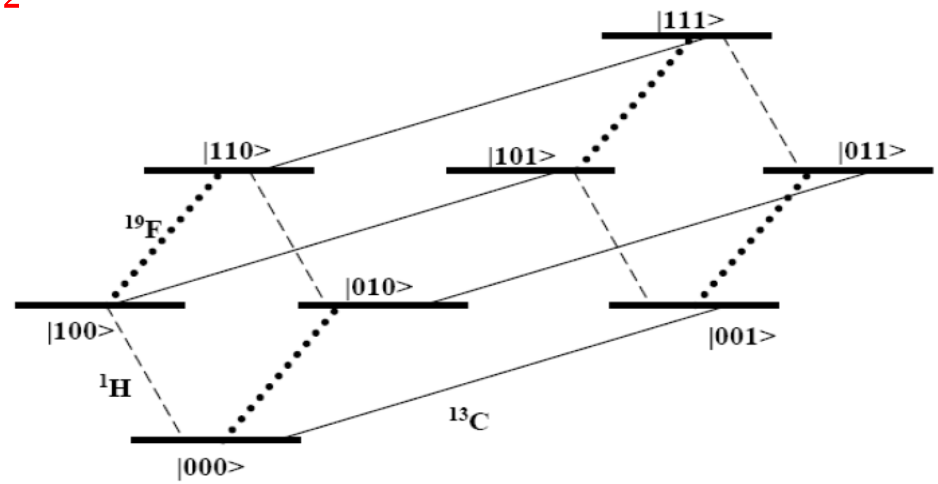
The initial State ψ is prepared for different values of θ and φ



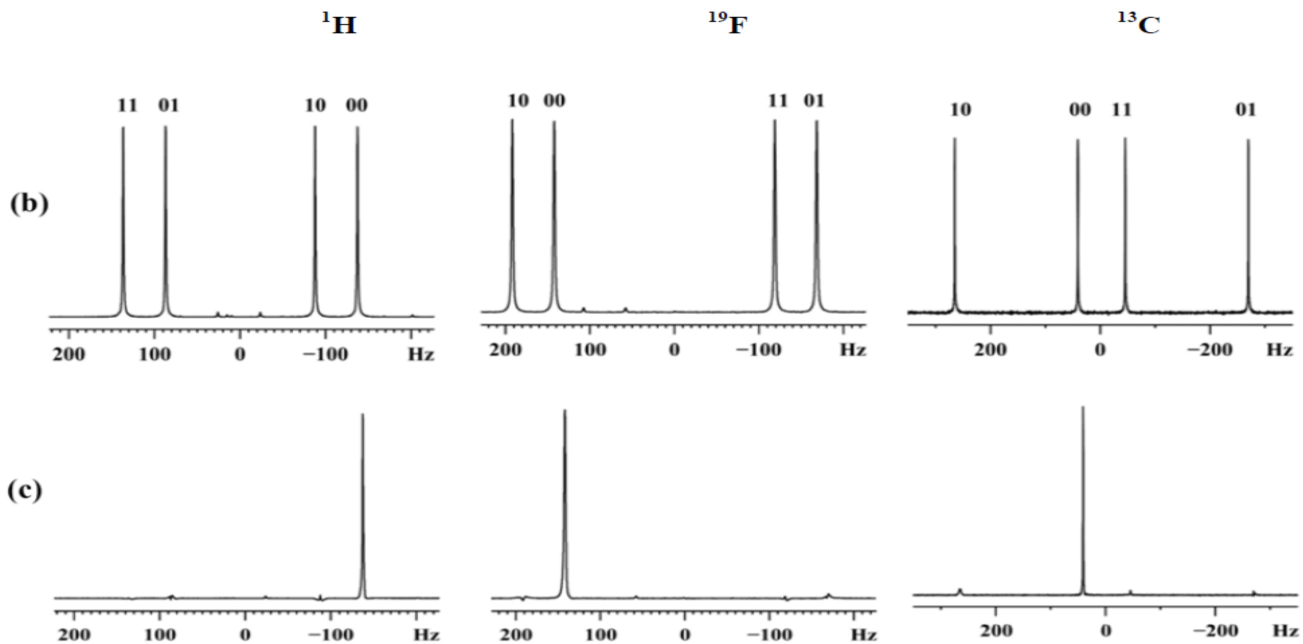
Jharana et al

$^{13}CHFBr_2$





Three qubit Energy Level Diagram



Equilibrium Spectra of three qubits

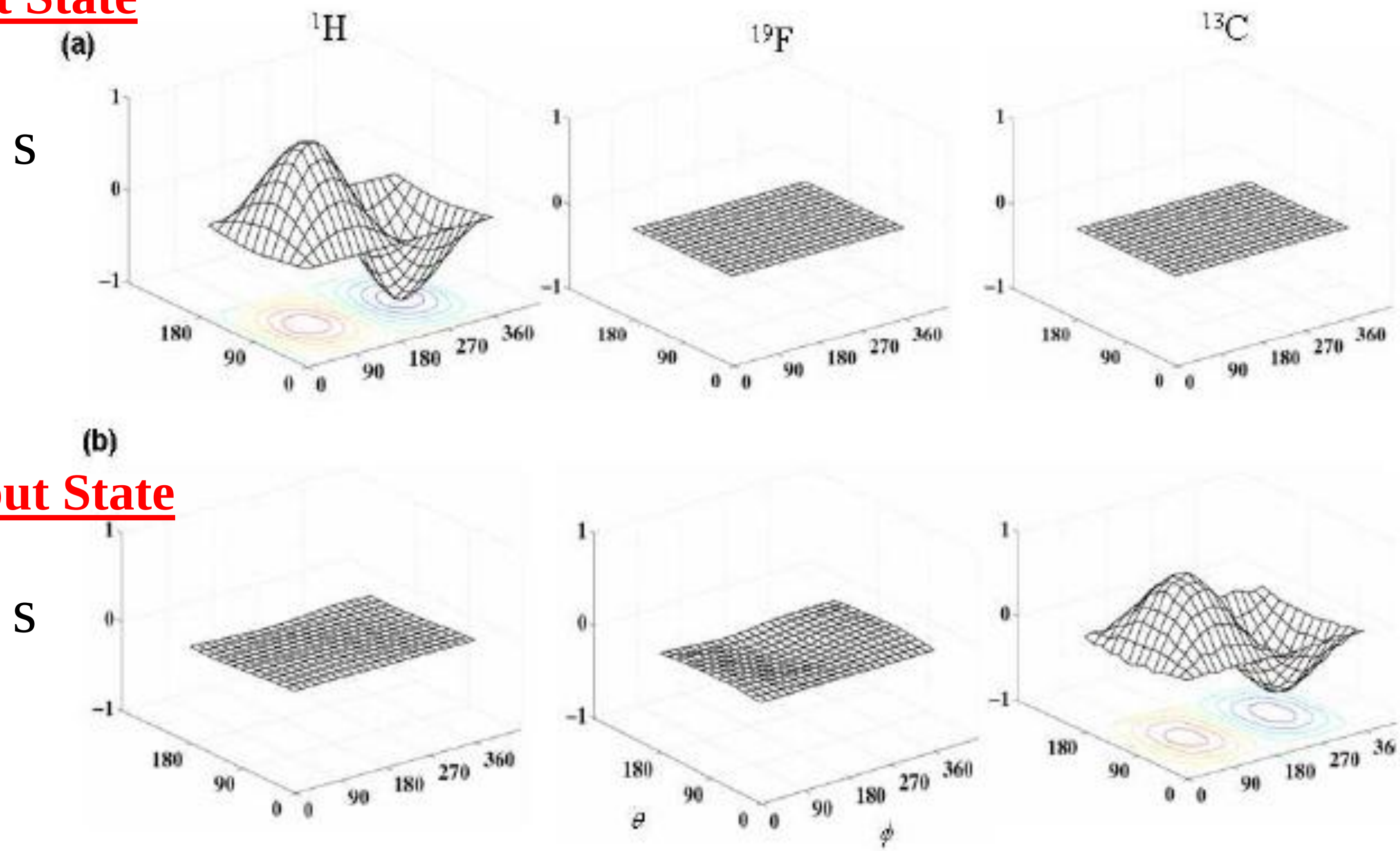
Spectra corresponding to |000> PPS

Experimental Result for the No-Hiding Theorem.

The state ψ is completely transferred from first qubit to the third qubit

Input State

Output State

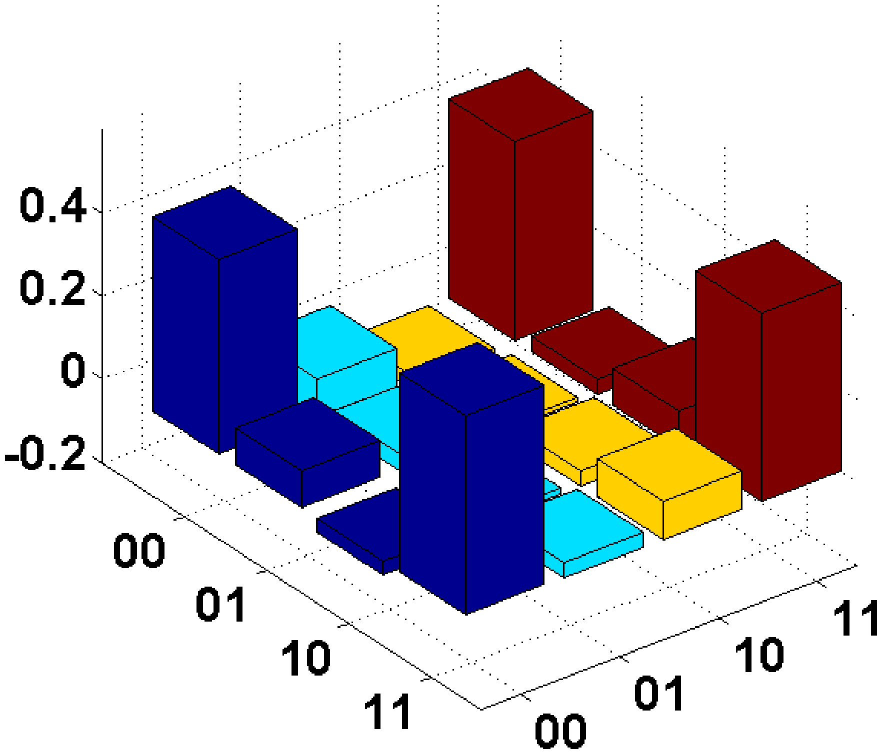


S = Integral of real part of the signal for each spin

325 experiments have been performed by varying θ and φ in steps of 15°

All Experiments were carried out by Jharana (Dedicated to her memory)

PRL-Accepted



Tomography of first two qubits showing that they are in Bell-States.

PRL-Accepted

Non-destructive discrimination of Bell States

Bell States are Maximally Entangled 2-qubit states.
There are 4 Bell States

$|\Phi^{+}\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ $|\Phi^{-}\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$

$|\psi^{+}\rangle = (|01\rangle + |10\rangle)\ \sqrt{2}$ $|\psi^{-}\rangle = (|01\rangle - |10\rangle)\sqrt{2}$

Bell states play an important role in teleportation protocols

Non-destructive Discrimination of Bell States

Manu Gupta and P. Panigrahi (quant-ph/0504183v)

Have given a Quantum circuit for non destructive discrimination of Bell States by using two ancilla qubits and making phase and parity measurements on each ancilla.

Jharana has experimentally implemented the above protocol, using one ancilla and two measurements.

Jharana Rani Samal*, Manu Gupta, P. Panigrahi and Anil Kumar, J. Phys. B, 43, 095508 (2010).

***Deceased 12 November 2009**

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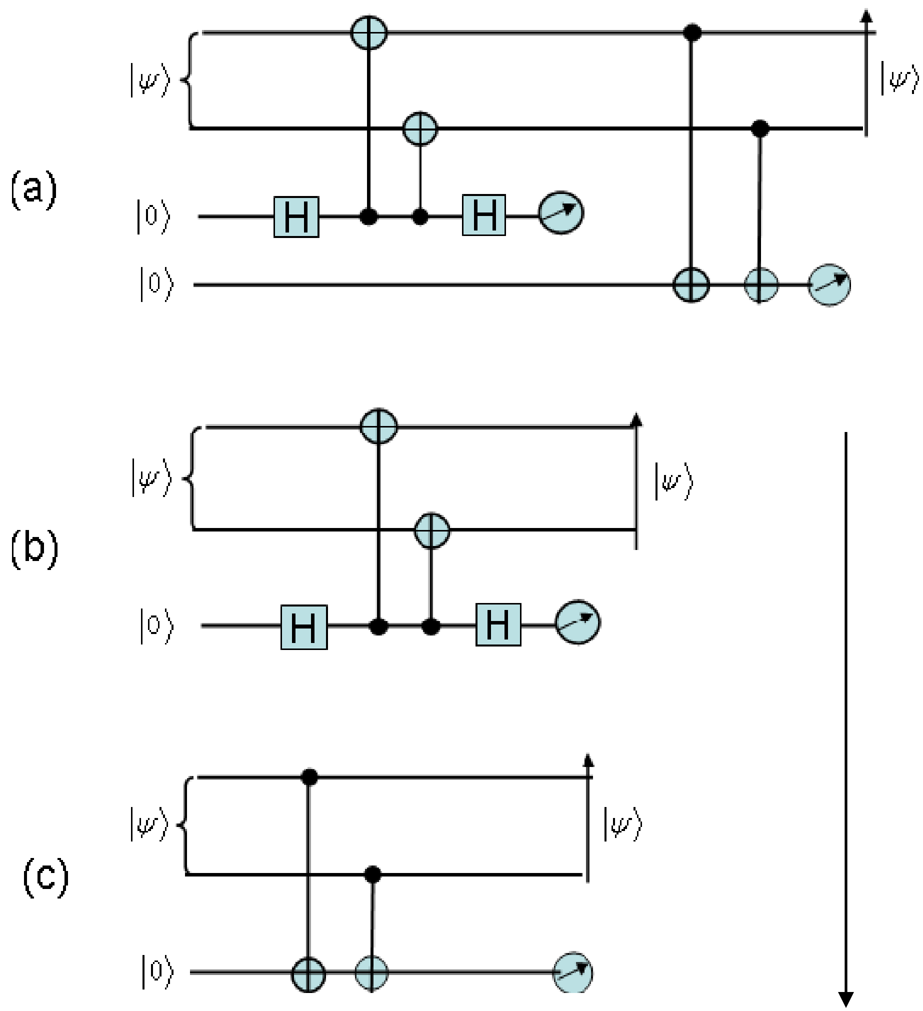


Panigrahi Circuit

Jharana Circuits

For Phase Measurement

For Parity Measurement

Bell State	1st Measurement	2nd Measurement
$\|\phi^+\rangle$	$\|0\rangle$	$\|0\rangle$
$\|\phi^-\rangle$	$\|1\rangle$	$\|0\rangle$
$\|\psi^+\rangle$	$\|0\rangle$	$\|1\rangle$
$\|\psi^-\rangle$	$\|1\rangle$	$\|1\rangle$

NMR Pulse Sequence for Discrimination of Bell States using one Ancilla Qubit

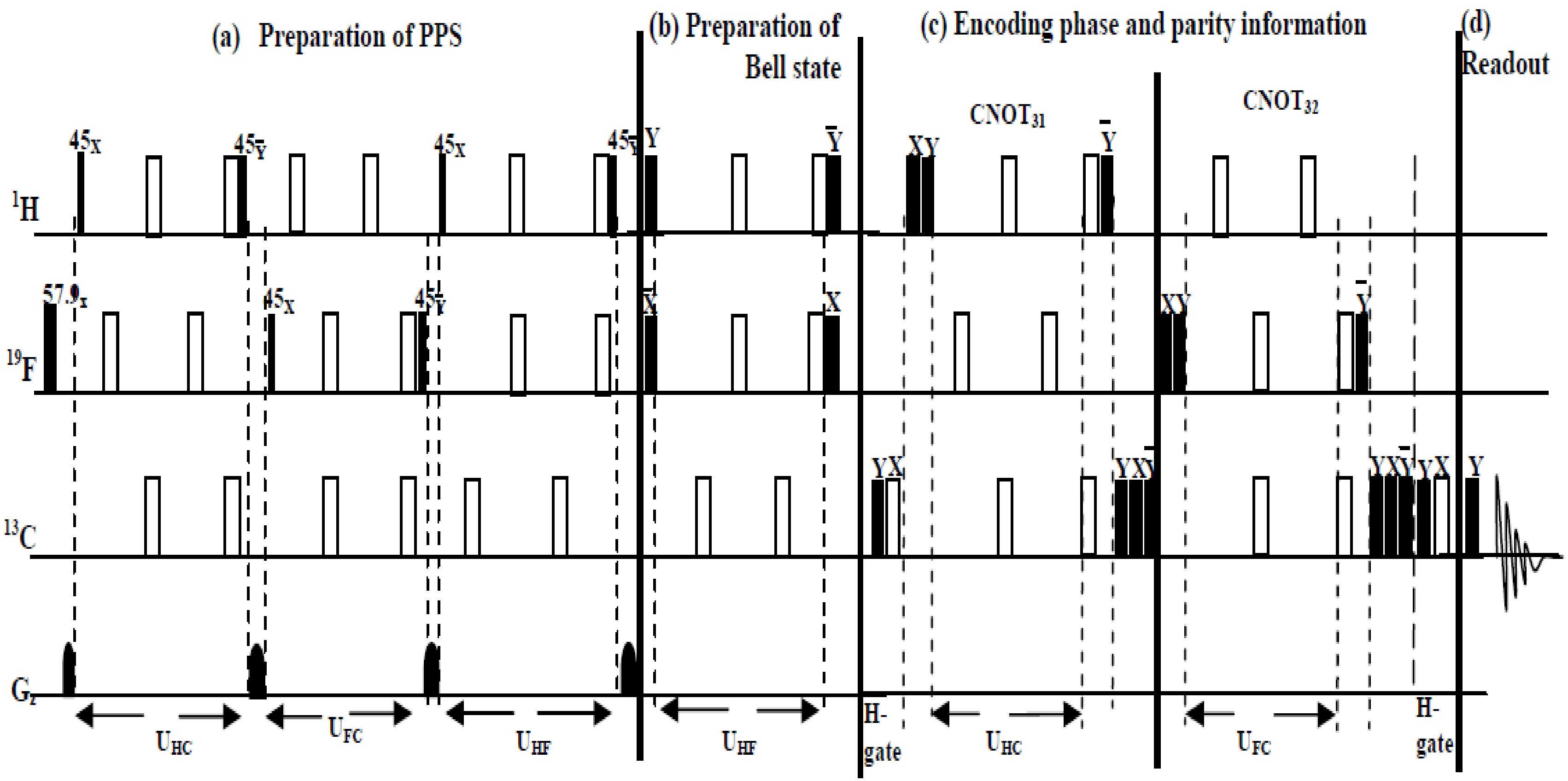


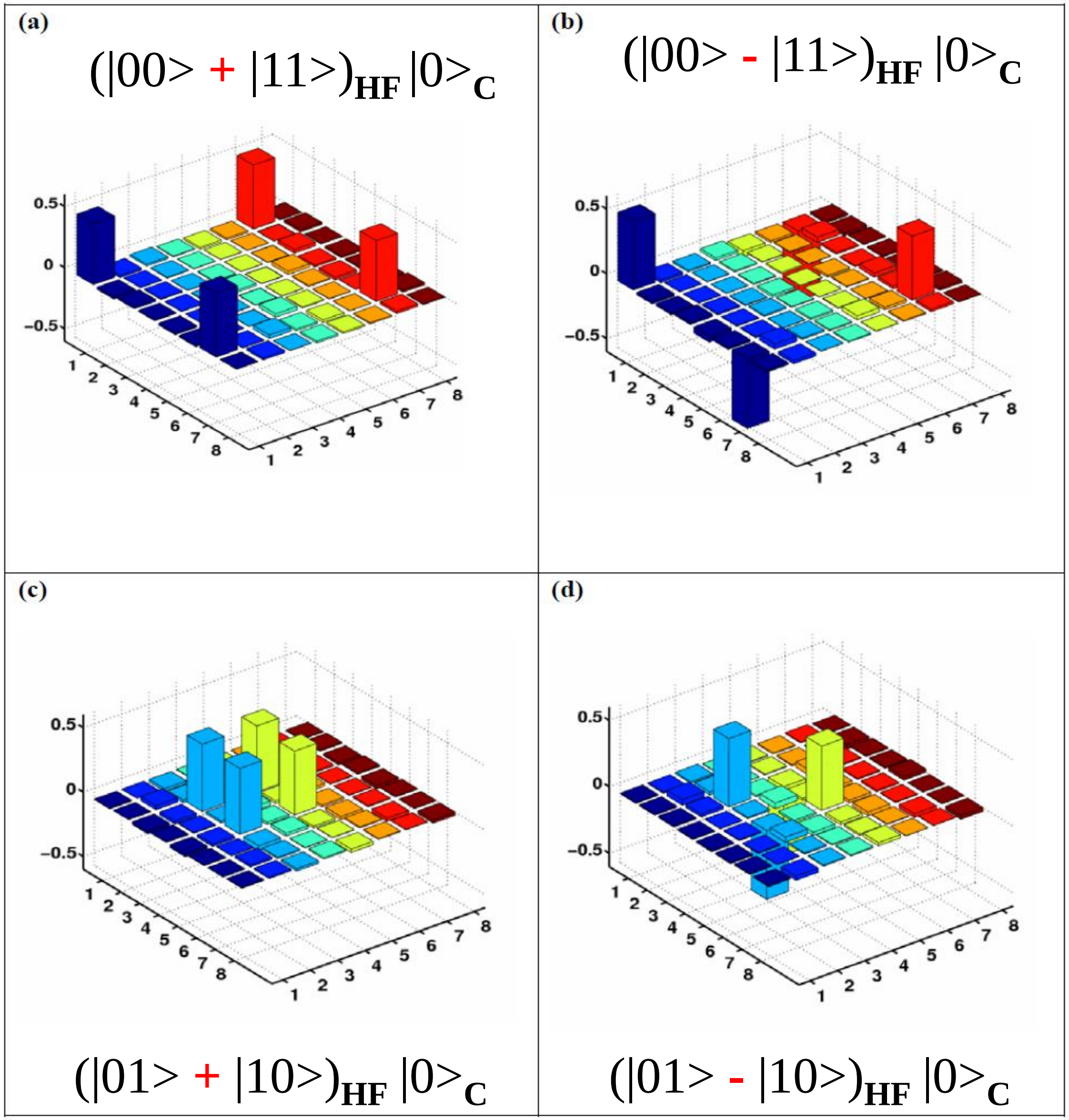
Fig. 2

For Parity measurement the Hadamard gates are removed and the CNOT Gates are reversed

Jharana et al, J.Phys. B., 43, 095508 (2010)

Created Bell States

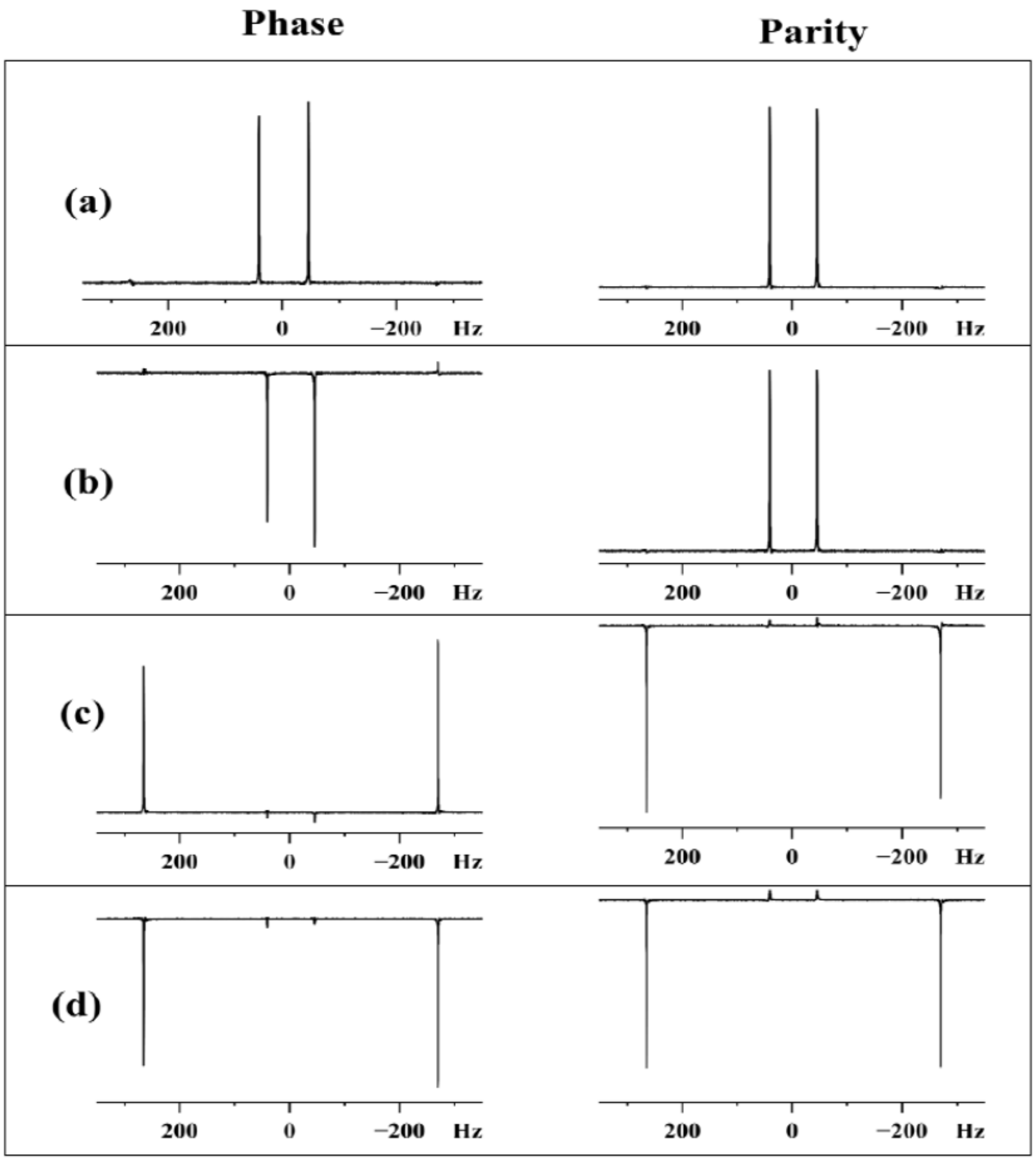
$|\Phi^{+}\rangle$ $|\Phi^{-}\rangle$



$|\psi^{+}\rangle$ $|\psi^{-}\rangle$

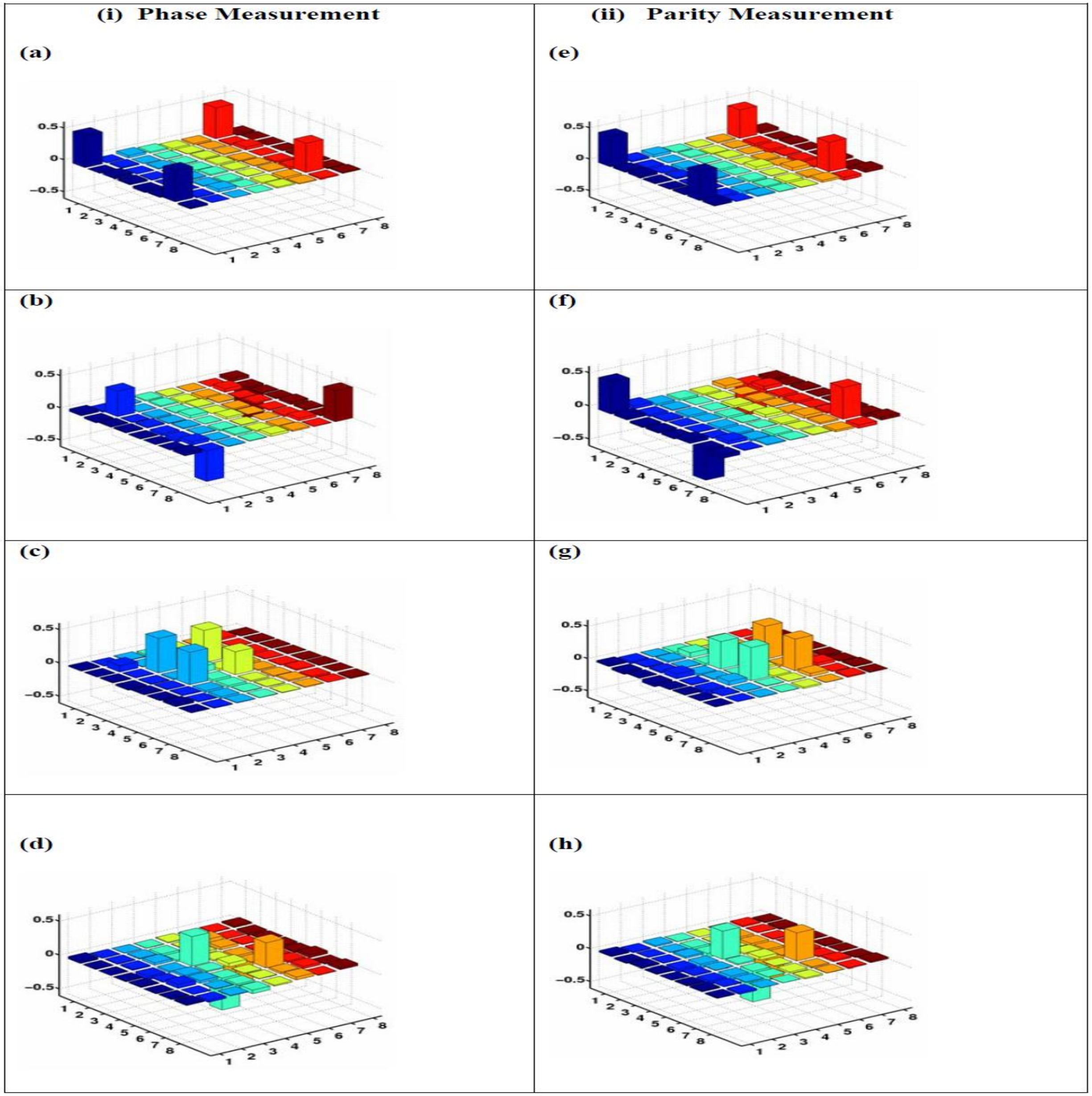
1 = |000>; 7 = |110>; 3 = |010>; 5 = |100>

Population Spectra of ^{13}C



	Phase	Parity
$\|\Phi^+\rangle$	0	0
$\|\Phi^-\rangle$	1	0
$\|\psi^+\rangle$	0	1
$\|\psi^-\rangle$	1	1

Tomograph of the real part of the Density matrix confirming the Phase and Parity measurement.



0 0

1 0

0 1

1 1

Jharna et al J.Phys.B 43, 095508 (2010)

Non-Destructive Discrimination of Arbitrary set of Orthogonal Quantum states by NMR using Quantum Phase Estimation.

V. S. Manu and Anil Kumar, PRA, Submitted

We present here an algorithm for Non-destructive discrimination of a set of Orthogonal Quantum States using ONLY Phase estimation.

For this algorithm, the states need not have definite PARITY (and can even be in a coherent superposition state).

This algorithm is thus more general than the just described Bell-State Discrimination.

For a given eigen-vector |φ> of a Unitary Operator U, Phase Estimation Circuit, can be used for finding the eigen-value of |φ>.

Conversely, with defined eigen-values, the Phase Estimation can be used for discriminating eigenvectors.

By logically defining the operators with preferred eigen-values, the discrimination, as shown here, can be done with certainty.

Quantum Phase Estimation

- **Suppose a unitary operation *U* has a eigen vector |*u*> with eigen value $e^{-i\varphi}$.**
- **The goal of the Phase Estimation Algorithm is to estimate *φ*.**

As the state is the eigen-state, the evolution under the Hamiltonian during phase estimation will preserve the state.

Finding the *n* Operators U_j

Let M be the diagonal matrix formed by eigen-value array $\{e^i\}_j$ of U_j.

And

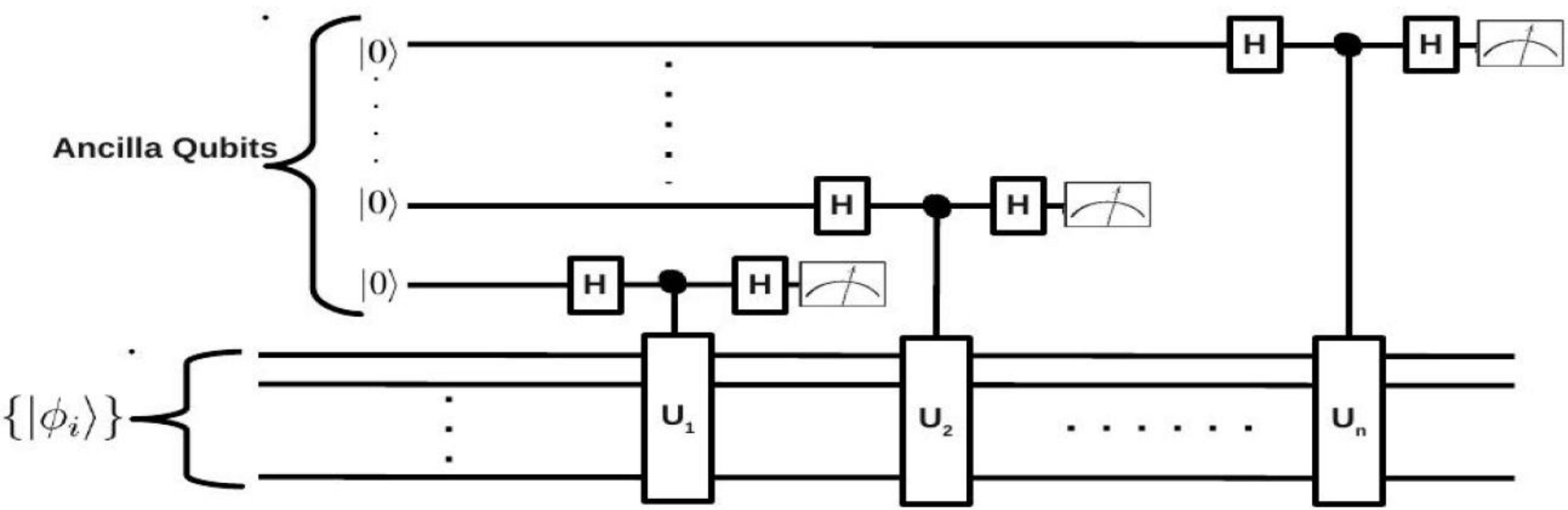
V is the matrix formed by the column vectors $\{|\varphi_k>\}$,

$$U_j = V^{-1} \times M_j \times V$$

Forming Eigen-value arrays

1. Eigen-value arrays $\{ e^i \}$ should contain equal number of +1 and -1
2. 1^{st} eigen value array can have any order of +1 and -1.
3. 2^{nd} onwards should also contain equal number of +1 and -1, but should not be equal to earlier arrays or their complements.

The General Procedure (n-qubit case)



The general circuit for Quantum State Discrimination. For discriminating ***n*** qubit states it uses ***n*** number of ancilla qubits with n controlled operations. ***n*** ancilla qubits are first prepared in the state **|00…0>**. Here **H** represents Hadamard transform and the meter represents measurement of the qubit state.

Two Qubit Case

Consider the following set of orthogonal 2-qubit states

$$\left\{S\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right\}=\left\{\begin{matrix}\frac{1}{\sqrt{2}}(|00\rangle+|01\rangle),\frac{1}{\sqrt{2}}(|10\rangle+|11\rangle),\\ \frac{1}{\sqrt{2}}(|10\rangle-|11\rangle),\frac{1}{\sqrt{2}}(|00\rangle-|01\rangle)\end{matrix}\right\}$$

States having no definite parity

A complete set of orthogonal States, which are not Bell states.

Here the 1st qubit in state |0> or |1> and the 2nd qubit in a superposed State (|0> ± |1>)

Eigen Value Arrays,

$$\{e_1\}=\{1,1,-1,-1\},\qquad \{e_1\}=\{1,-1,1,-1\},$$

U_1 and U_2 can be shown as,

$$U_1=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix},\quad U_2=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}\qquad \ldots\ldots\ldots (3)$$

Experimental implementation of this case is performed here by NMR

For the operators U_1 and U_2 described in Eqn. (3)

$$Controlled - U_1 = e^{-iH_1} \qquad Controlled - U_2 = e^{-iH_2}$$

In terms of NMR Product Operators The Hamiltonians are given by

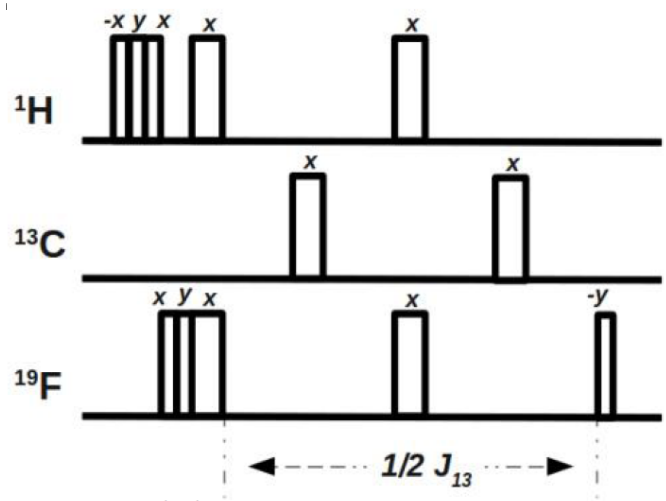
$$H_1 = \left(\frac{\pi}{4}I - \frac{\pi}{2}I_z^1 - \frac{\pi}{2}I_z^3 + \pi I_z^1 I_x^3\right)$$

$$H_2 = \left(\frac{\pi}{4}I - \frac{\pi}{2}I_z^1 - \pi I_z^2 I_x^3 + 2\pi I_z^1 I_z^2 I_x^3\right).$$

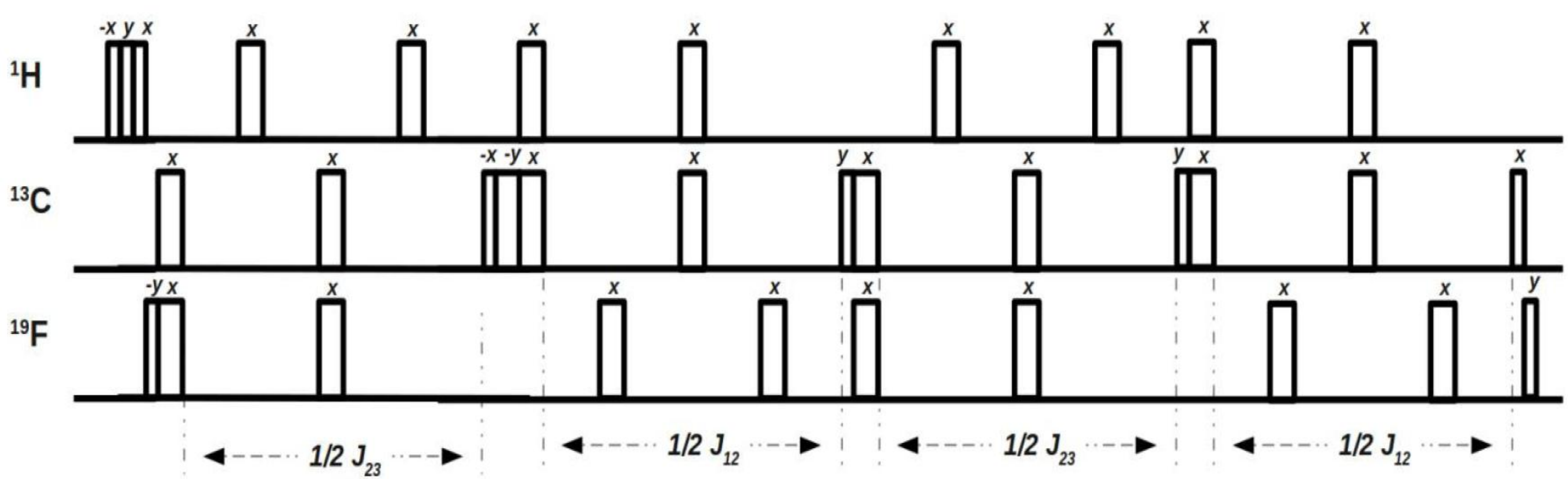
Since various terms in H_1 and H_2 commute each other, we can write,

$$Controlled - U_1 = e^{i\frac{\pi}{4}I} \times e^{-i\frac{\pi}{2}I_z^1} \times e^{-i\frac{\pi}{2}I_x^3} \times e^{i\pi I_z^1 I_x^3}.$$

$$Controlled - U_2 = e^{i\frac{\pi}{4}I} \times e^{-i\frac{\pi}{2}I_z^1} \times e^{i\pi I_z^1 I_x^3} \times e^{i2\pi I_z^1 I_z^2 I_x^3}$$



(a) *Controlled-U_1*

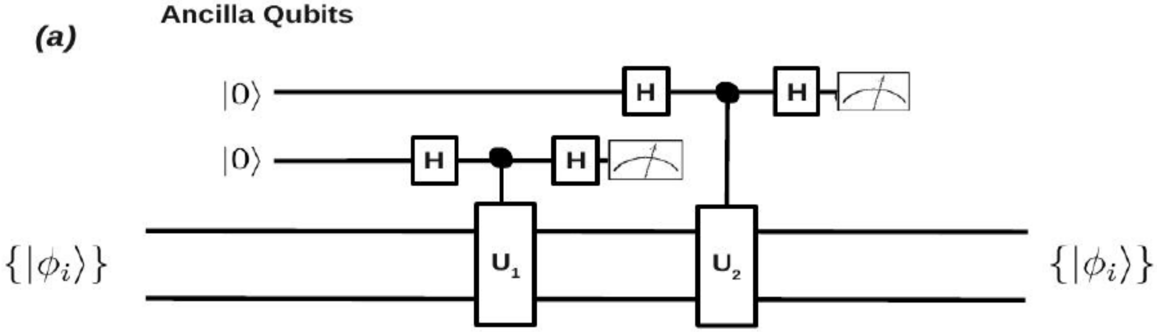


(b) *Controlled-U_2*

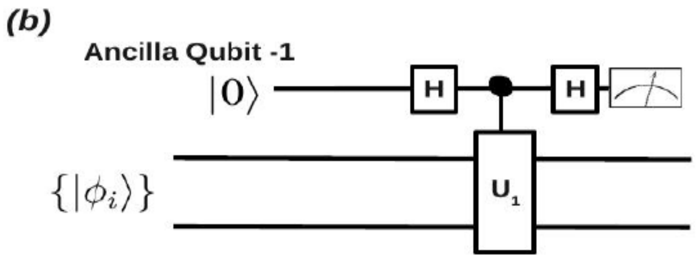
Thin pulses are $\pi/2$ and broad pulses are π pulses. Phase of pulses on top

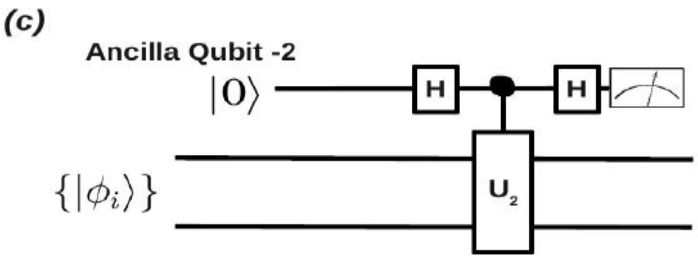
Non-destructive Discrimination of two-qubit orthonormal states.

Original Circuit Needing 2-ancilla qubits



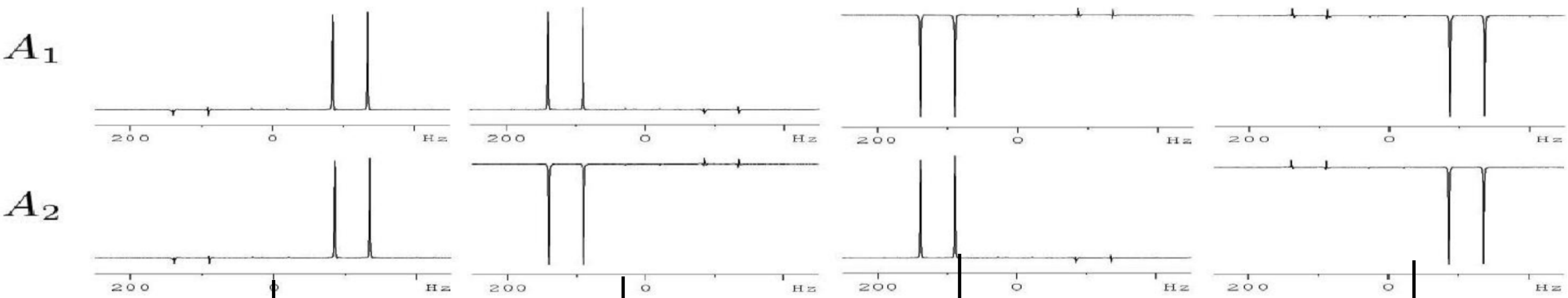
Split Circuit needing 1-ancilla qubit





Results for Ancilla measurements

(i) φ_1 (ii) φ_2 (iii) φ_3 (iv) φ_4



A_1 → +ve signal → |0> state.
A_2 → +ve signal → |0> state.

→ (1/√2) (|00> + |01>)

A_1 → +ve signal → |0> state.
A_2 → -ve signal → |1> state.

→ (1/√2) (|10> + |11>)

A_1 → -ve signal → |1> state.
A_2 → +ve signal → |0> state.

→ (1/√2) (|10> - |11>)

A_1 → -ve signal → |1> state.
A_2 → -ve signal → |1> state.

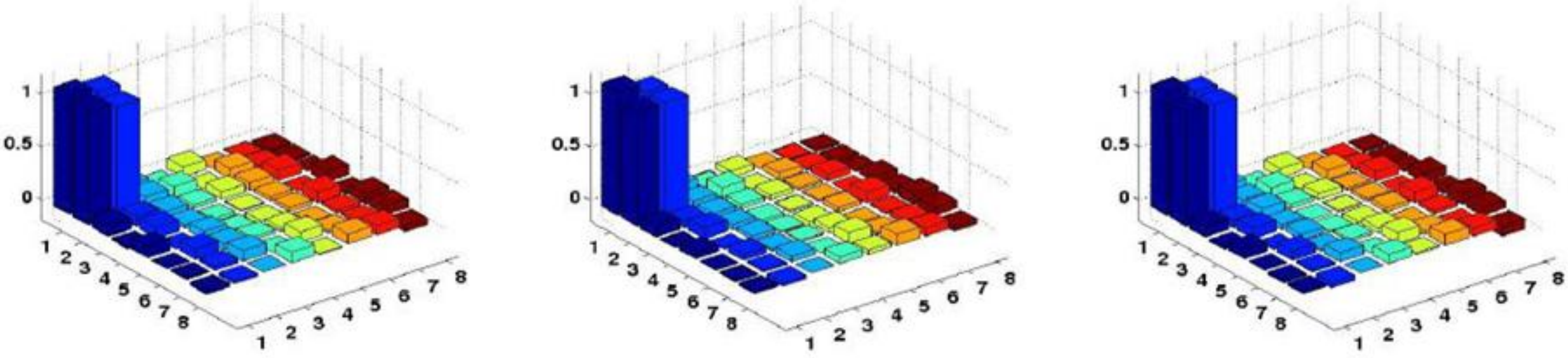
→ (1/√2) (|00> - |01>)

Complete density matrix tomography has done to

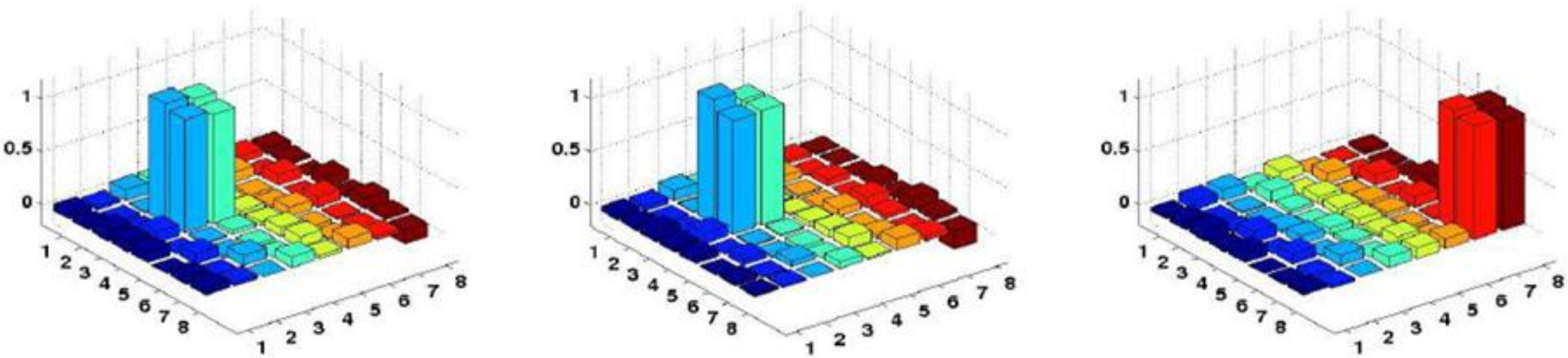
1. Show the state is preserved
2. Compute fidelity of the experiment.

Initial State | **After First Experiment** | **After Second Experiment**

$(i)\quad |0\rangle(\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)) \qquad |0\rangle(\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)) \qquad |0\rangle(\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle))$

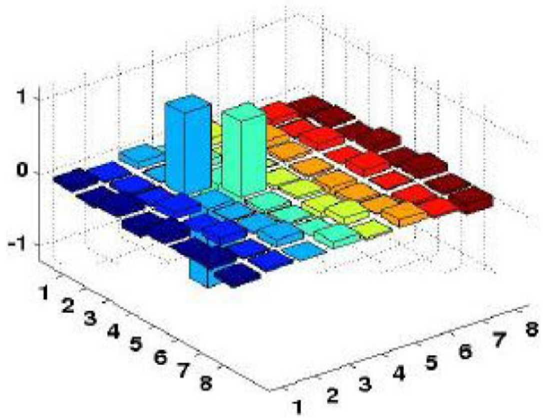


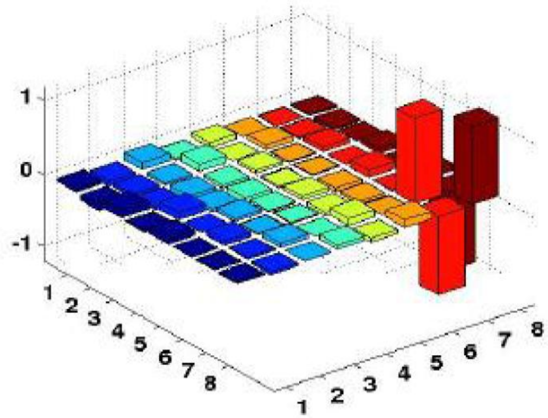
$(ii)\quad |0\rangle(\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)) \qquad |0\rangle(\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)) \qquad |1\rangle(\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle))$

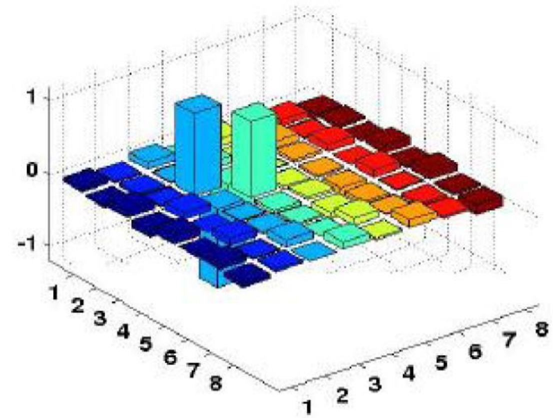


Initial State | **After First Experiment** | **After Second Experiment**

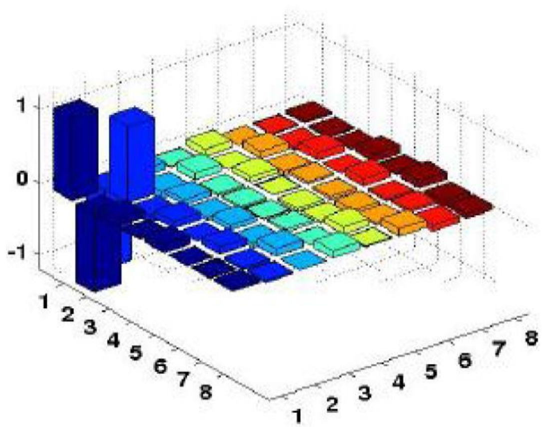
(iii) $|0\rangle(\frac{1}{\sqrt{2}}(|10\rangle - |11\rangle))$ $|1\rangle(\frac{1}{\sqrt{2}}(|10\rangle - |11\rangle))$ $|0\rangle(\frac{1}{\sqrt{2}}(|10\rangle - |11\rangle))$

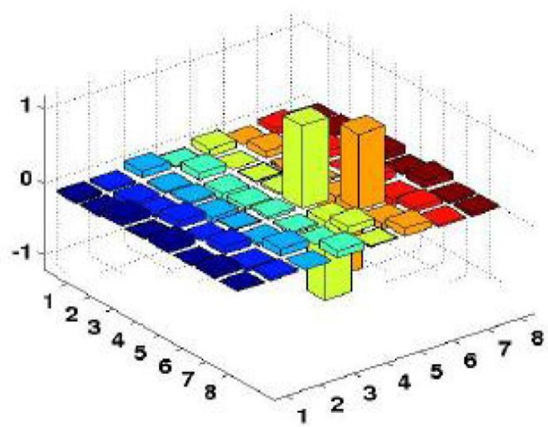


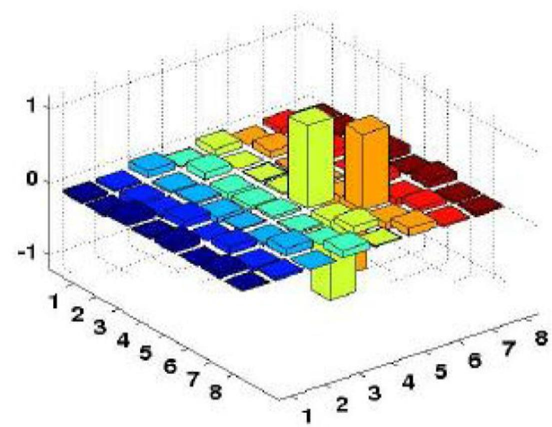




(vi) $|0\rangle(\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle))$ $|1\rangle(\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle))$ $|1\rangle(\frac{1}{\sqrt{2}}(|00\rangle - |01\rangle))$







Conclusions of the State Discrimination

- **A general scalable method for quantum state discrimination using quantum phase estimation algorithm is discussed, and experimentally implemented for a two qubit case by NMR.**

- **As the direct measurements are performed only on the ancilla, the discriminated states are preserved.**

Use of nearest neighbour Heisenberg-XY interaction.

Until recently we have been looking for qubit systems, in which all qubits are coupled to each other with unequal couplings, so that all transitions are resolved and we have a complete access to the full Hilbert space.

However it is clear that such systems are not scalable, since remote spins will not be coupled.

Solution

Use Nearest Neighbour Interactions

Creation of Bell states between end qubits and a W-state using nearest neighbour Heisenberg-XY interactions in a 3-spin NMR quantum computer

Heisenberg XY interaction is normally not present in liquid state NMR: We have only ZZ interaction available.

We create the XY interaction by transforming the ZZ interaction into XY interaction by the use of 90^0 RF pulses.

Rama K. Koteswara Rao and Anil Kumar, PRA, to be submitted

Heisenberg-interaction

$$H = \sum_{i<j} J_{ij} (\sigma_i \cdot \sigma_j)$$

Where σ are Pauli spin matrices

Linear Chain: Nearest Neighbour Interaction

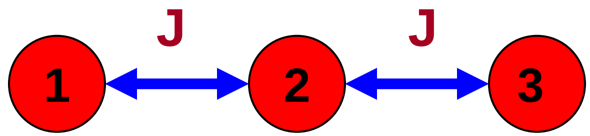
$$H = \sum_{i=1}^{N-1} J_i (\sigma_i \cdot \sigma_{i+1}) = \sum_{i=1}^{N-1} J_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$$

Nearest neighbour Heisenberg XY Interaction

$$H_{XY} = \tfrac{1}{2} \sum_{i=1}^{N-1} J_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y)$$

$$= \sum_{i=1}^{N-1} J_i \left(\sigma_i^+ \sigma_{i+1}^- + \sigma_i^- \sigma_{i+1}^+\right)$$

Consider a linear Chain of 3 spins with equal couplings



$$H_{XY} = \tfrac{1}{2} J\left(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2 + \sigma_x^2\sigma_x^3 + \sigma_y^2\sigma_y^3\right)$$

$\sigma^j_{x/y}$ are the Pauli spin matrices and J is the coupling constant between two spins.

Divide the H_{XY} into two commuting parts

$$U(t) = e^{-iH_{XY}t}$$

$$\left[\sigma_x^1\sigma_x^2 + \sigma_y^2\sigma_y^3,\ \sigma_y^1\sigma_y^2 + \sigma_x^2\sigma_x^3\right] = 0$$

$$U(t) = U_A(t)\,U_B(t) = e^{-\frac{i}{2}Jt\left(\sigma_x^1\sigma_x^2 + \sigma_y^2\sigma_y^3\right)} e^{-\frac{i}{2}Jt\left(\sigma_y^1\sigma_y^2 + \sigma_x^2\sigma_x^3\right)}$$

$$U_A(t) = e^{-\frac{i}{2}Jt\left(\sigma_x^1\sigma_x^2 + \sigma_y^2\sigma_y^3\right)}$$

$$U_B(t) = e^{-\frac{i}{2}Jt\left(\sigma_y^1\sigma_y^2 + \sigma_x^2\sigma_x^3\right)}$$

Jingfu Zhang et al., Physical Review A, 72, 012331(2005)

$$\left(\frac{\sigma_x^1\sigma_x^2+\sigma_y^2\sigma_y^3}{\sqrt{2}}\right)^2=I$$

$$\left[\begin{array}{l} A \textit{ is a matrix such that } A^2=I, \textit{then} \\ \qquad e^{-iA\theta}=\cos(\theta)\,I-i\sin(\theta)\,A \end{array}\right]$$

$$U_A(t)=e^{-\frac{i}{2}Jt\left(\sigma_x^1\sigma_x^2+\sigma_y^2\sigma_y^3\right)}$$

$$=e^{-i\frac{Jt}{\sqrt{2}}\frac{\left(\sigma_x^1\sigma_x^2+\sigma_y^2\sigma_y^3\right)}{\sqrt{2}}}$$

$$=\cos(\tfrac{Jt}{\sqrt{2}})\,I-\tfrac{i}{\sqrt{2}}\sin(\tfrac{Jt}{\sqrt{2}})\;(\sigma_x^1\sigma_x^2+\sigma_y^2\sigma_y^3)$$

$$U_B(t)=e^{-\frac{i}{2}Jt\left(\sigma_y^1\sigma_y^2+\sigma_x^2\sigma_x^3\right)}$$

$$=e^{-i\frac{Jt}{\sqrt{2}}\frac{\left(\sigma_y^1\sigma_y^2+\sigma_x^2\sigma_x^3\right)}{\sqrt{2}}}$$

$$=\cos(\tfrac{Jt}{\sqrt{2}})\,I-\tfrac{i}{\sqrt{2}}\sin(\tfrac{Jt}{\sqrt{2}})\;(\sigma_y^1\sigma_y^2+\sigma_x^2\sigma_x^3)$$

$$U(t)=U_A(t)\;U_B(t)$$

$$=\left(\cos\varphi\,I-\tfrac{i}{\sqrt{2}}\sin\varphi\;(\sigma_x^1\sigma_x^2+\sigma_y^2\sigma_y^3)\right)\left(\cos\varphi\,I-\tfrac{i}{\sqrt{2}}\sin\varphi\;(\sigma_y^1\sigma_y^2+\sigma_x^2\sigma_x^3)\right)$$

$$\left\{\varphi=Jt/\sqrt{2}\right\}$$

Jingfu Zhang et al., Physical Review A, 72, 012331(2005)

The operator U in Matrix form for the 3-spin system

$$U(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos^2\varphi & -\frac{i}{\sqrt{2}}\sin(2\varphi) & 0 & -\sin^2\varphi & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{2}}\sin(2\varphi) & \cos(2\varphi) & 0 & -\frac{i}{\sqrt{2}}\sin(2\varphi) & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos^2\varphi & 0 & -\frac{i}{\sqrt{2}}\sin(2\varphi) & -\sin^2\varphi & 0 \\ 0 & -\sin^2\varphi & -\frac{i}{\sqrt{2}}\sin(2\varphi) & 0 & \cos^2\varphi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{2}}\sin(2\varphi) & 0 & \cos(2\varphi) & -\frac{i}{\sqrt{2}}\sin(2\varphi) & 0 \\ 0 & 0 & 0 & -\sin^2\varphi & 0 & -\frac{i}{\sqrt{2}}\sin(2\varphi) & \cos^2\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where $\varphi = Jt/\sqrt{2}$

Quantum State Transfer

When $t = \pi / \sqrt{2}J, \quad \varphi = \pi/2$

$$U\left(\frac{\pi}{\sqrt{2}J}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U|000\rangle = |000\rangle$$

$$U|010\rangle = -|010\rangle$$

$$U|101\rangle = -|101\rangle$$

$$U|111\rangle = |111\rangle$$

$$U|001\rangle = -|100\rangle$$

$$U|100\rangle = -|001\rangle$$

$$U|011\rangle = -|110\rangle$$

$$U|110\rangle = -|011\rangle$$

Interchanging the states of 1st and 3rd qubit

[Ignore the phase (minus sign)]

Two qubit Entangling Operator

$$When \;\; t = \frac{\pi}{2\sqrt{2}J}, \qquad \varphi = \pi/4$$

$$U\left(\frac{\pi}{2\sqrt{2}J}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{-i}{\sqrt{2}} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{-i}{\sqrt{2}} & 0 & 0 & \frac{-i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{-i}{\sqrt{2}} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{-i}{\sqrt{2}} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-i}{\sqrt{2}} & 0 & 0 & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & -\frac{-i}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$U|101\rangle = \tfrac{-i}{\sqrt{2}}\left(|011\rangle + |110\rangle\right)$$

$$U|010\rangle = \tfrac{-i}{\sqrt{2}}\left(|100\rangle + |001\rangle\right)$$

Bell States

Three qubit Entangling Operator

$$When \quad t = \frac{\tan^{-1}(\sqrt{2})}{\sqrt{2}J}, \qquad \varphi = \frac{\tan^{-1}(\sqrt{2})}{2}$$

$$U(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.79 & \frac{-i}{\sqrt{3}} & 0 & -0.21 & 0 & 0 & 0 \\ 0 & \frac{-i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{-i}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.79 & 0 & \frac{-i}{\sqrt{3}} & -0.21 & 0 \\ 0 & -0.21 & \frac{-i}{\sqrt{3}} & 0 & 0.79 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-i}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & \frac{-i}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & -0.21 & 0 & \frac{-i}{\sqrt{3}} & 0.79 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U(t)|101\rangle = \tfrac{1}{\sqrt{3}}|101\rangle - \tfrac{i}{\sqrt{3}}|011\rangle - \tfrac{i}{\sqrt{3}}|110\rangle$$

Phase Gate on 2nd spin

$$|0\rangle\langle 0| + i\,|1\rangle\langle 1|$$

$$\tfrac{1}{\sqrt{3}}|101\rangle + \tfrac{1}{\sqrt{3}}|011\rangle + \tfrac{1}{\sqrt{3}}|110\rangle$$

W - State

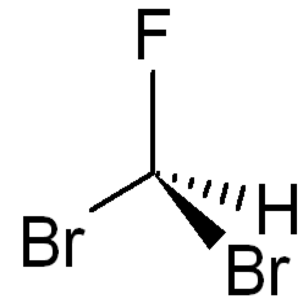
Experiments using nearest neighbour interactions in a 3-spin system

1. Pseudo-Pure States.

2. Bell states on end qubits.

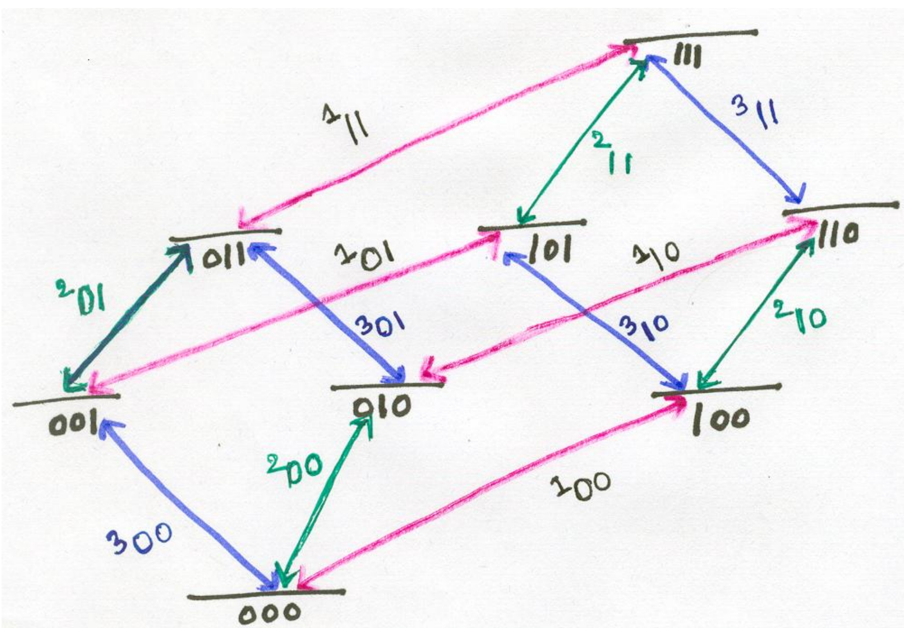
3. W-state.

$^{13}CHFBr2$

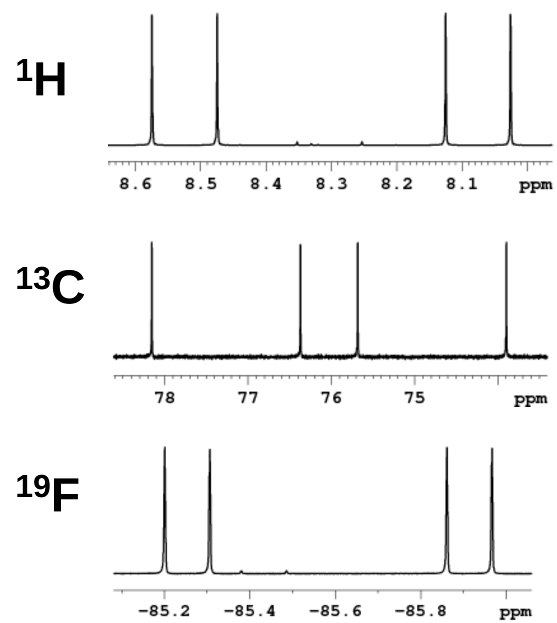


J_{HC} = 224.5 Hz, J_{CF} = -310.9 Hz and J_{HF} = 49.7 Hz.

Energy Level Diagram



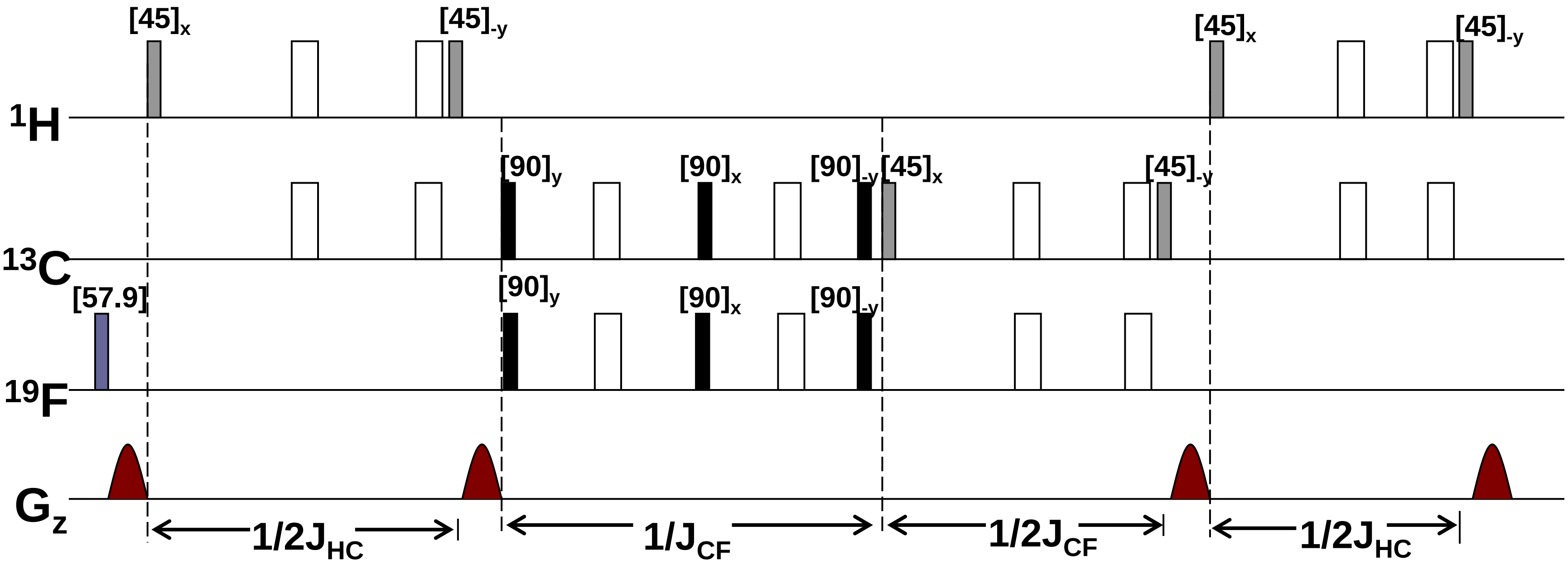
Equilibrium spectra



Pseudo-Pure States using only nearest neighbour interactions

$$\rho_{eq} = \gamma_H I_z^H + \gamma_F I_z^F + \gamma_C I_z^C = \gamma_H (I_z^H + 0.94\, I_z^F + 0.25\, I_z^C)$$

$$\rho_{000} = I_z^1 + I_z^2 + I_z^3 + 2I_z^1 I_z^2 + 2I_z^2 I_z^3 + 2I_z^1 I_z^3 + 4I_z^1 I_z^2 I_z^3$$



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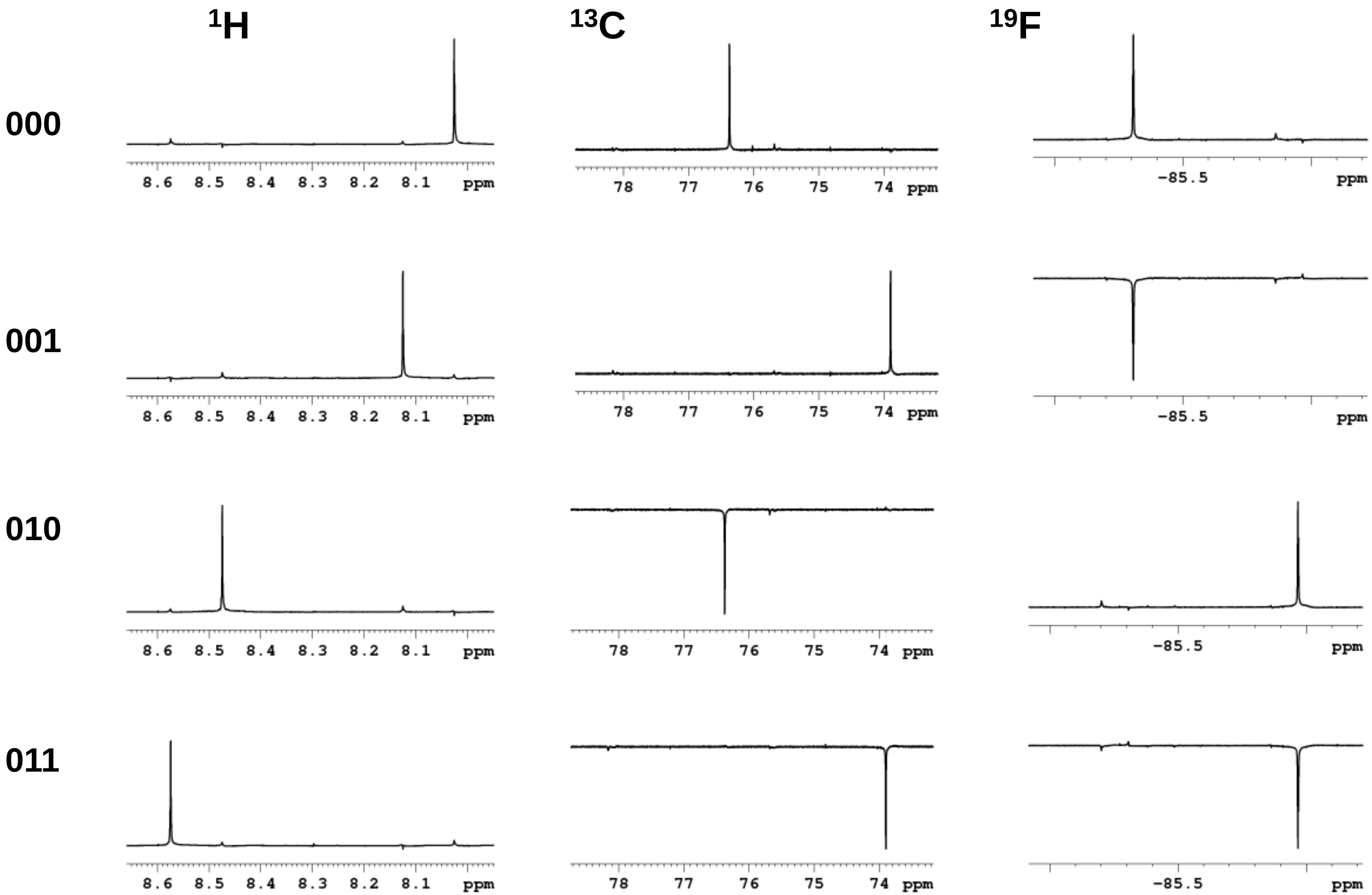
Pseudo-Pure States:



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Bell state on end qubits:

$$When\ \ t = \frac{\pi}{2\sqrt{2}J}, \quad \varphi = Jt2\sqrt{2} = \pi/4$$

$$U|010\rangle = \tfrac{-i}{\sqrt{2}}\left(|100\rangle + |001\rangle\right)$$

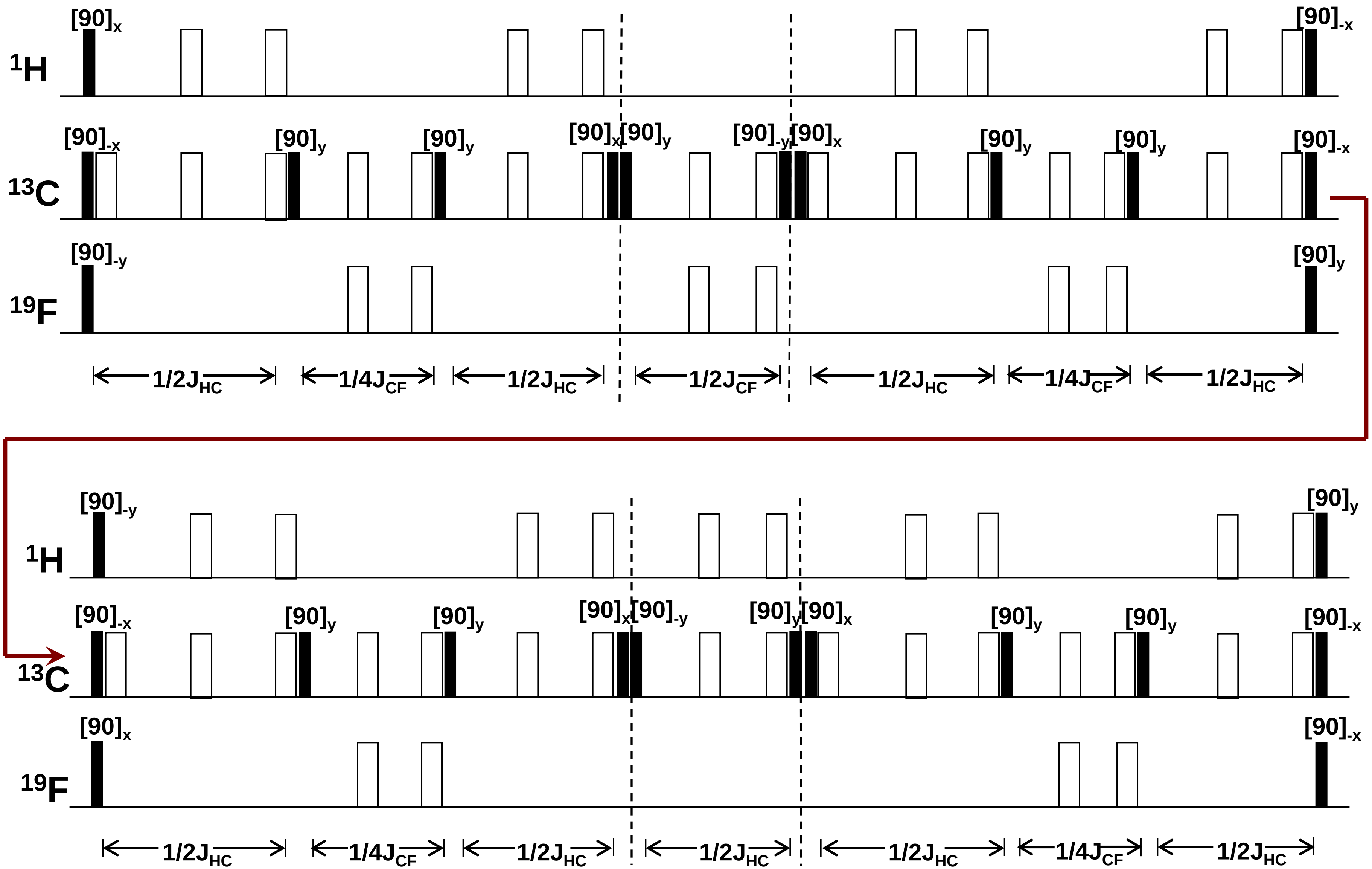
Starting from 010 pps the U yields Bell states in which the Middle qubit in state 0

$$U|101\rangle = \tfrac{-i}{\sqrt{2}}\left(|011\rangle + |110\rangle\right)$$

Starting from 101 pps the U yields Bell states in which the Middle qubit in state 1

Bell state on end qubits:

Pulse sequence for implementing the unitary operator U(t)

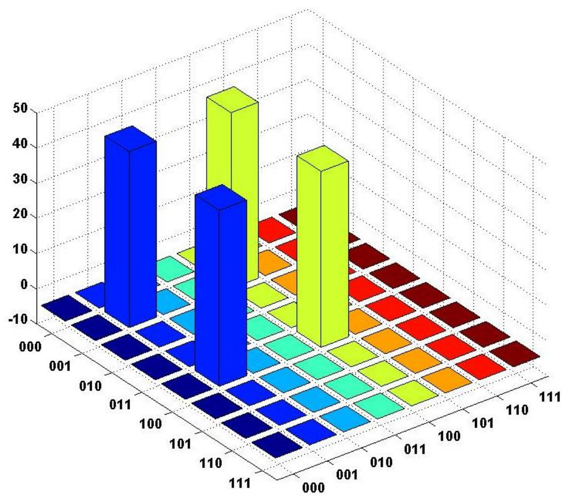
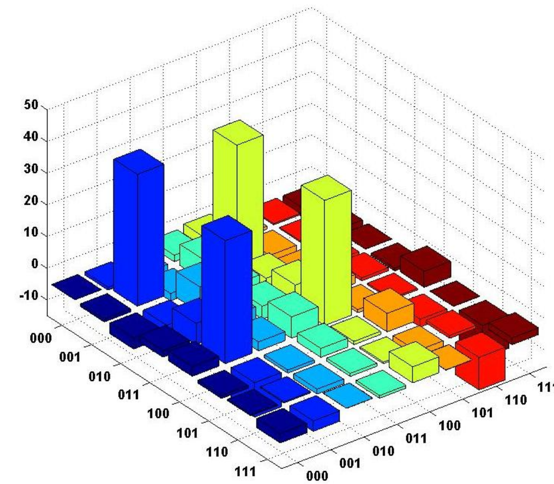
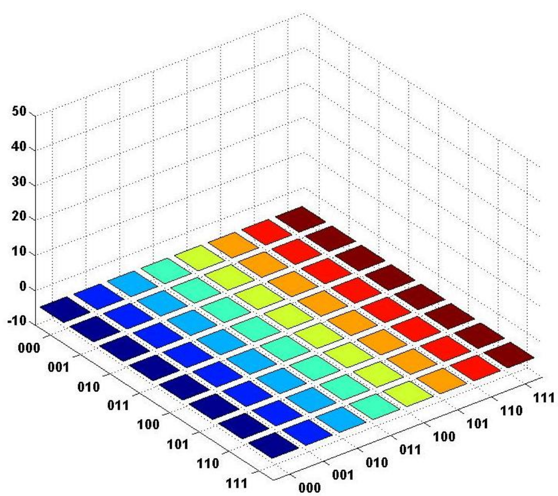
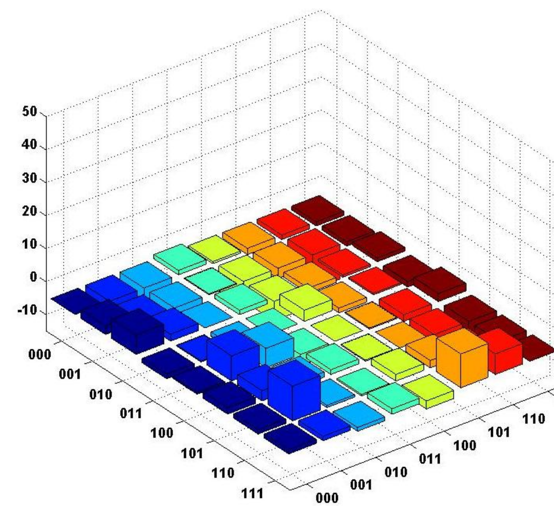


Bell state on end qubits:

$$U|010\rangle = \frac{-i}{\sqrt{2}}\left(|100\rangle + |001\rangle\right)$$

Middle qubit is in state 0

Starting from 010 pps

Simulated		Experiment
	**Real Part**	
	**Imaginary Part**	

Bell state on end qubits:

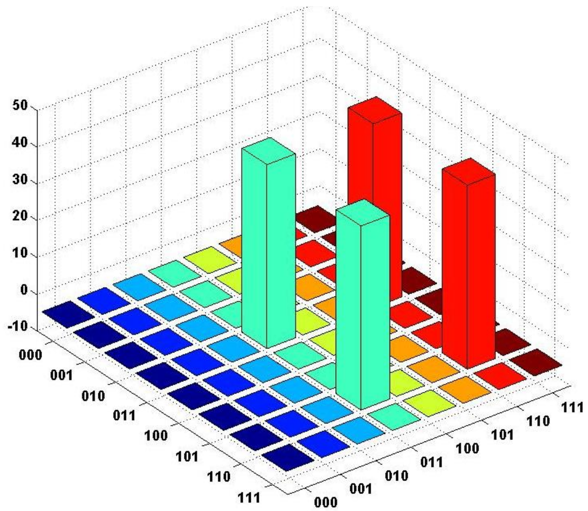
$$U\left|101\right\rangle = \frac{-i}{\sqrt{2}}\left(\left|011\right\rangle + \left|110\right\rangle\right)$$

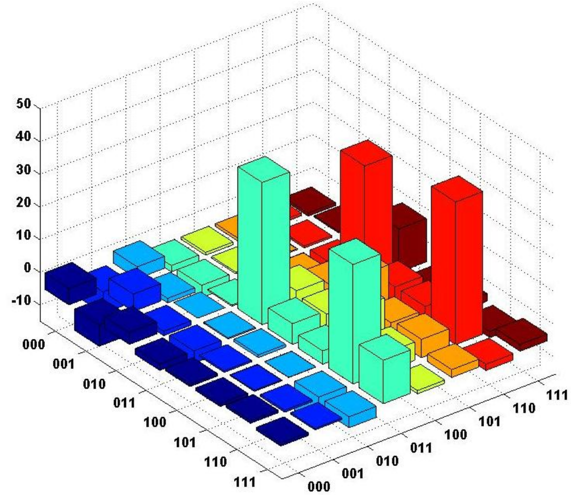
Middle bit in state 1

Starting from 101 pps

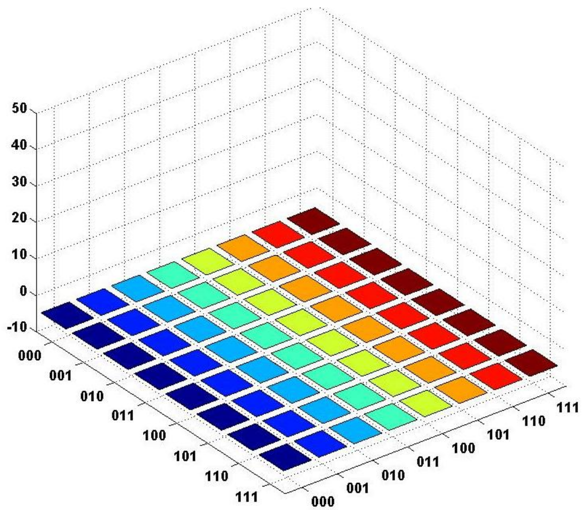
Simulated | **Experiment**

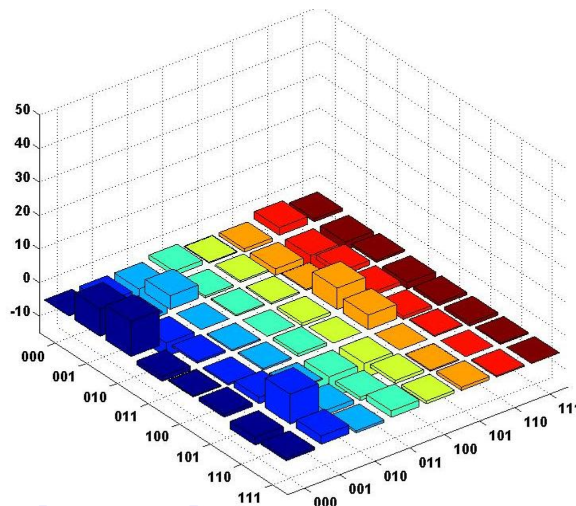
Real Part





Imaginary Part





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W-State

$$When \quad t = \frac{\tan^{-1}(\sqrt{2})}{\sqrt{2}J}, \qquad \varphi = \frac{\tan^{-1}(\sqrt{2})}{2}$$

$$U(t)|101\rangle = \tfrac{1}{\sqrt{3}}|101\rangle - \tfrac{i}{\sqrt{3}}|011\rangle - \tfrac{i}{\sqrt{3}}|110\rangle$$

$$\left[\frac{\pi}{2}\right]_z^2 \Rightarrow \left[\frac{\pi}{2}\right]_y^2 \left[\frac{\pi}{2}\right]_x^2 \left[\frac{\pi}{2}\right]_{-y}^2$$

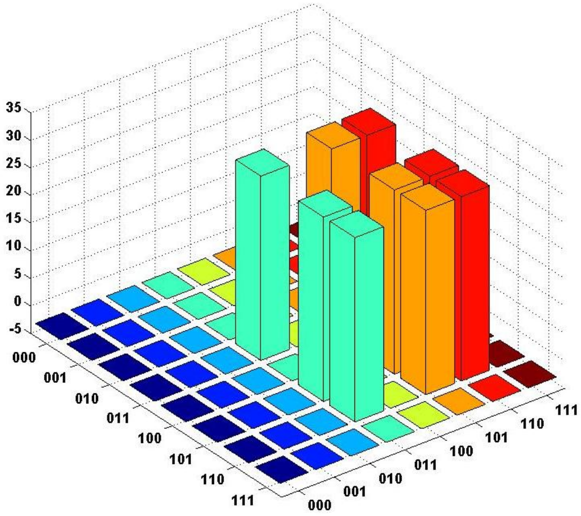
$$\tfrac{1}{\sqrt{3}}|101\rangle + \tfrac{1}{\sqrt{3}}|011\rangle + \tfrac{1}{\sqrt{3}}|110\rangle$$

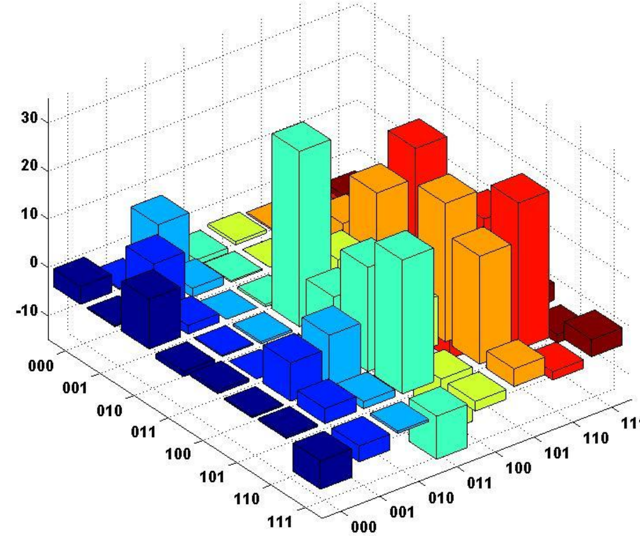
W-State:

$$\frac{1}{\sqrt{3}}\left|101\right\rangle+\frac{1}{\sqrt{3}}\left|011\right\rangle+\frac{1}{\sqrt{3}}\left|110\right\rangle$$

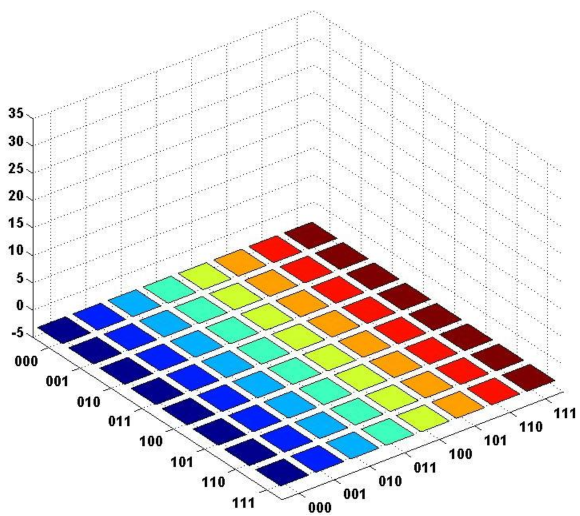
Simulated | **Experiment**

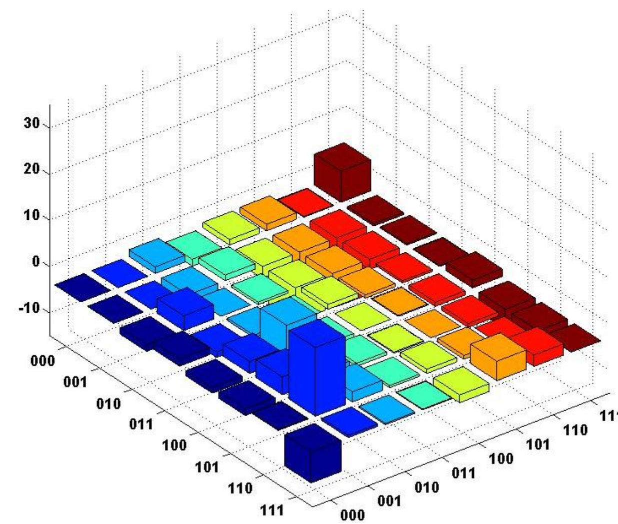
Real Part





Imaginary Part





Rama K. Koteswara Rao -(to be submitted)

Future Directions:

(i) Use Collective Modes of linear chains.

(ii) **Backbone of a C-13, N-15 labeled protein forming a linear chain:**

Lucio Frydman: Using nearest neighbour Heisenberg XY Interaction has performed State transfer using C-13 of the side-chain of Leucine forming a six qubit system:

Phys. Rev. A 81, 060302 (2010)

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This lecture is dedicated
to the memory of
Ms. Jharana Rani Samal*
(*Deceased, Nov., 12, 2009)

Current QC IISc - Students

Mr. Rama K. Koteswara Rao
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Other Collaborators

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Thank You

Single Qubit Case

$\{|\varphi_1> = (\alpha|0> +\beta|1>),\ |\varphi_2> = (\alpha|0> -\beta|1>)\}$

with $|\alpha|^2+|\beta|^2=1$

- operator U for eigenvalue array $\{1,-1\}$ can be shown as,

$$U = \begin{pmatrix} Cos(\theta) & Sin(\theta) \\ Sin(\theta) & -Cos(\theta) \end{pmatrix}$$

$$\theta = 2 \times Tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

- For the selected M_1 if the given state is $|\varphi_1\rangle$ then ancilla will be in the state $|0\rangle$ and if the given state is $|\varphi_2\rangle$ ancilla will be in the state $|1\rangle$.

Two Qubit Case

Consider a set of orthogonal states :

$$\{S(\alpha,\beta)\} = \left\{ \begin{matrix} (\alpha|00\rangle + \beta|01\rangle), (\alpha|10\rangle + \beta|11\rangle), \\ (\beta|10\rangle - \alpha|11\rangle), (\beta|00\rangle - \alpha|01\rangle) \end{matrix} \right\}.$$

Eigen Value Arrays,

$$\{e_1\} = \{1, 1, -1, -1\}, \qquad \{e_1\} = \{1, -1, 1, -1\},$$

$$U_1 = \begin{pmatrix} Cos(\theta) & Sin(\theta) & 0 & 0 \\ Sin(\theta) & -Cos(\theta) & 0 & 0 \\ 0 & 0 & Cos(\theta) & Sin(\theta) \\ 0 & 0 & Sin(\theta) & -Cos(\theta) \end{pmatrix}$$

$$U_2 = \begin{pmatrix} Cos(\theta) & Sin(\theta) & 0 & 0 \\ Sin(\theta) & -Cos(\theta) & 0 & 0 \\ 0 & 0 & -Cos(\theta) & -Sin(\theta) \\ 0 & 0 & -Sin(\theta) & Cos(\theta) \end{pmatrix}$$

Where $\theta = 2 \times Tan^{-1}\left(\frac{\beta}{\alpha}\right)$ .... **(2)**

The Ancilla Measurement Results can be Tabulated as,

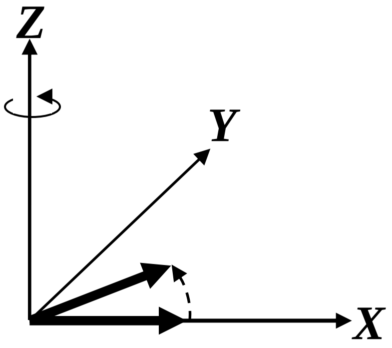
States	Ancilla -1	Ancilla-2
$\|\varphi_1> = (\alpha\|00> + \beta\|01>)$	$\|0>$	$\|0>$
$\|\varphi_2> = (\alpha\|10> + \beta\|11>)$	$\|0>$	$\|1>$
$\|\varphi_3> = (\beta\|10> - \alpha\|11>)$	$\|1>$	$\|0>$
$\|\varphi_4> = (\beta\|00> - \alpha\|01>)$	$\|1>$	$\|1>$

Simulating the 3-spin XY chain using liquid state NMR

Define $L_x^A = \sigma_x^1 \sigma_x^2 / 2$ $L_y^A = \sigma_y^2 \sigma_y^3 / 2$ $L_z^A = \sigma_x^1 \sigma_z^2 \sigma_y^3 / 2$ Such that

$$\left[L_x^A, L_y^A\right] = iL_z^A \qquad \left[L_y^A, L_z^A\right] = iL_x^A \qquad \left[L_z^A, L_x^A\right] = iL_y^A$$

$$\begin{aligned} U_A(t) &= e^{-\frac{i}{2} Jt\left(\sigma_x^1 \sigma_x^2 + \sigma_y^2 \sigma_y^3\right)} \\ &= e^{-iJt(L_x^A + L_y^A)} \\ &= e^{-i(\pi/4)L_z^A} \, e^{-i\sqrt{2}\, JtL_x^A} \, e^{i(\pi/4)L_z^A} \\ &= e^{-i(\pi/8)\sigma_x^1 \sigma_z^2 \sigma_y^3} \, e^{-i(Jt/\sqrt{2})\sigma_x^1 \sigma_x^2} \, e^{i(\pi/8)\sigma_x^1 \sigma_z^2 \sigma_y^3} \end{aligned}$$



Jingfu Zhang et al., Physical Review A, 72, 012331.

Converting 3-spin operators to 2-spin operators using J-evolution

$$e^{-i(\pi)I_z^1 I_z^2 I_z^3}$$

$$\downarrow$$

$$e^{-i(\pi/2)I_x^2} e^{-i(\pi/2)I_y^2} \left(e^{-i(\pi)I_z^1 I_y^2 I_z^3}\right) e^{i(\pi/2)I_y^2} e^{i(\pi/2)I_x^2}$$

$$I_x^2 \xrightarrow{e^{-i\pi I_z^1 I_z^2}} 2I_z^1 I_y^2$$

$$I_z^2 \xrightarrow{e^{-i\pi I_z^1 I_x^2}} -2I_z^1 I_y^2 \qquad 2I_z^2 I_z^3 \xrightarrow{e^{-i\pi I_z^1 I_x^2}} -4I_z^1 I_y^2 I_z^3$$

$$e^{-i(\pi)I_z^1 I_y^2 I_z^3}$$

$$\downarrow$$

$$e^{i(\pi)I_z^1 I_x^2} e^{-i(\pi/2)I_z^2 I_z^3} e^{-i(\pi)I_z^1 I_x^2}$$

$$\downarrow$$

$$e^{i(\pi/2)I_y^2} e^{-i(\pi)I_z^1 I_z^2} e^{-i(\pi/2)I_y^2} e^{-i(\pi/2)I_z^2 I_z^3} e^{-i(\pi/2)I_y^2} e^{-i(\pi)I_z^1 I_z^2} e^{i(\pi/2)I_y^2}$$

$$e^{-i(\pi)I_z^1 I_z^2 I_z^3} \Rightarrow e^{-i(\pi/2)I_x^2} e^{-i(\pi)I_z^1 I_z^2} e^{-i(\pi/2)I_y^2} e^{-i(\pi/2)I_z^2 I_z^3}$$

$$e^{-i(\pi/2)I_y^2} e^{-i(\pi)I_z^1 I_z^2} e^{i(\pi)I_y^2} e^{i(\pi/2)I_x^2}$$

D. G. Cory et al., Physical Review A, 61, 012302, (1999).

Generating NMR pulse sequence

$$e^{-i(\pi)I_z^1 I_z^2 I_z^3} \Rightarrow e^{-i(\pi/2)I_x^2} e^{-i(\pi)I_z^1 I_z^2} e^{-i(\pi/2)I_y^2} e^{-i(\pi/2)I_z^2 I_z^3} e^{-i(\pi/2)I_y^2} e^{-i(\pi)I_z^1 I_z^2} e^{i(\pi)I_y^2} e^{i(\pi/2)I_x^2}$$

$$e^{-i(\pi)I_z^1 I_z^2 I_z^3} \Rightarrow \left[\frac{\pi}{2}\right]_{-x}^2 - [\pi]_{-y}^2 - \left[\frac{1}{2J_{12}}\right] - \left[\frac{\pi}{2}\right]_{y}^2 - \left[\frac{1}{4J_{23}}\right] - \left[\frac{\pi}{2}\right]_{y}^2 - \left[\frac{1}{2J_{12}}\right] - \left[\frac{\pi}{2}\right]_{x}^2$$

$$e^{i(\pi)I_z^1 I_z^2 I_z^3} \Rightarrow \left[\frac{\pi}{2}\right]_{x}^2 - [\pi]_{-y}^2 - \left[\frac{1}{2J_{12}}\right] - \left[\frac{\pi}{2}\right]_{y}^2 - \left[\frac{1}{4J_{23}}\right] - \left[\frac{\pi}{2}\right]_{y}^2 - \left[\frac{1}{2J_{12}}\right] - \left[\frac{\pi}{2}\right]_{-x}^2$$

D. G. Cory et al., Physical Review A, 61, 012302, 1999.