Quantum gravity and entanglement

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PLAN

1. Entanglement in quantum gravity

2. Entanglement from quantum gravity

I shall use natural units:

$$\hbar = \mathbf{1}, \qquad \mathbf{c} = \mathbf{1}, \qquad \mathbf{k}_{\mathbf{Boltzmann}} = \mathbf{1}$$

Entanglement in quantum gravity

In ordinary quantum mechanics entanglement arises when the system naturally has two or more parts.

We divide the system into two parts and if the wave-function is not a product of the wave-functions supported on the two parts, we call this an entangled state, e.g.

$$|\Psi\rangle = \mathbf{M}_{\mathbf{a}\beta} |\mathbf{a}\rangle_{\mathbf{1}} \otimes |\beta\rangle_{\mathbf{2}}$$

(Repeated indices a and β are summed over.)

$$|\Psi\rangle = \mathbf{M}_{\mathbf{a}\beta}|\mathbf{a}\rangle_{\mathbf{1}} \otimes |\beta\rangle_{\mathbf{2}}$$

If we perform measurement over system 1 only, we can trace over the states in system 2 and assign a density matrix to system 1

$$\rho_{entangle} = \mathbf{Tr}_{\mathbf{2}}|\Psi\rangle\langle\Psi| = \mathbf{M}_{\mathbf{a}\beta}\mathbf{M}^{*}_{\mathbf{b}\beta}|\mathbf{a}\rangle\langle\mathbf{b}|$$

The corresponding entanglement entropy

$$\mathbf{S}_{\mathbf{entangle}} = -\mathbf{Tr}(\rho \ln \rho)$$

measures the degree to which $|\Psi\rangle$ fails to behave as a pure state if we restrict our measurements to system 1 only.

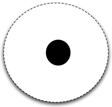
Thus the entanglement entropy reflects our unwillingness to perform measurement over the second system, just as the thermal entropy reflects our unwillingness to describe the precise quantum state of a many body system.

As in the case of thermal system, we have no problem in principle to describe the state as a pure quantum state by including both components 1 and 2.

This changes in a theory of gravity.

There are some systems, known as black holes, for which it is impossible to make measurement over the full system.

A black hole describes a compact massive object so heavy that not even light can escape a black hole.



Horizon

An imaginary surface surrounding the black hole, known as the event horizon, acts as a one way membrane, preventing an outside observer to determine the state of the system inside the black hole.

The presence of event horizon can be seen most easily in a two dimensional representation of the space-time known as the Penrose diagram.

Example: Minkowski space

r: radial distance from a fixed origin

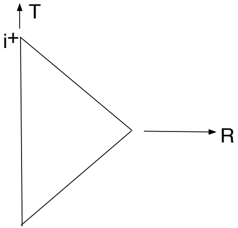
t: time coordinate

Introduce new coordinates

$$\mathbf{T} \pm \mathbf{R} = \mathbf{2}\tan^{-1}(\mathbf{t} \pm \mathbf{r})$$

The region $-\infty < t < \infty$, $0 \le r < \infty$ is mapped to a finite region in T-R space.

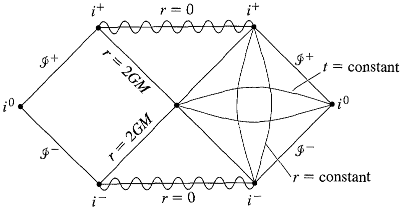
Penrose diagram of Minkowski space:



In this diagram signals travel at an angle $\leq$ 45 degrees with T-axis.

A static observer reaches the point i$^+$ at t $= \infty$ and can receive signal from every point of the Minkowski space.

Contrast this with the Penrose diagram of an uncharged black hole



A static observer outside the black hole reaches the point i^+ at infinite time and can receive signal only from the right of the event horizon.

Due to the presence of the event horizon, an observer sitting outside the blackhole cannot perform measurement over the part of the system inside the horizon.

As a result such an observer must describe the system by a density matrix ρ.

For a quantum field theory in such a background geometry one can calculate the density matrix for an observer staying outside the event horizon.

Result: Israel

$$\rho_{\text{entangle}} \propto \sum_n e^{-E_n/T_H} |n\rangle\langle n|$$

T_H: determined in terms of geometric parameters of the black hole.

$$\rho_{\mathbf{entangle}} \propto \sum_{\mathbf{n}} \mathbf{e}^{-\mathbf{E_n}/\mathbf{T_H}} |\mathbf{n}\rangle\langle\mathbf{n}|$$

– identical to the density matrix of a thermal system at temperature T_H.

Thus to an outside observer a quantum field theory in a black hole background appears as a thermal system.

A crude analogy:

The black hole is like a reservoir and the individual quantum field theories are smaller systems, and we try to guess the properties of the reservoir from the knowledge of how the smaller systems behave when in contact with the reservoir.

Since the quantum field theories acquire a temperature T_H in the presence of a black hole, we expect that the black hole itself has a temperature T_H.

Indeed black holes are known to ‘Hawking radiate’ at a temperature of T_H.

In an apparently independent study one finds that black holes obey a classical law which relates the change of the energy / mass of the black hole to the change of the area A_H of the event horizon.

$$\delta E = \frac{1}{4G_N} T_H \, \delta A_H$$

G_N: Newton's gravitational constant.

This looks like the first law of thermodynamics if we identify the entropy carried by the black hole as

$$S_{BH} = \frac{A_H}{4G_N}$$

Bekenstein, Hawking

Question: What is the interpretation of this entropy?

Can we regard this as $\ln \Omega$ where Ω is the number of quantum states of the black hole?

This is a much more difficult question since this time we are asking for a counting of states of the ‘reservoir’ produced by quantum gravity and for this we need a full fledged quantum theory of gravity.

For a spacial class of black holes, string theory can identify and count the quantum states describing a black hole and their number Ω is consistent with the relation Strominger, Vafa

$$S_{BH} = \ln \Omega$$

This analysis indicates that we should interpret black hole entropy as a thermal entropy.

– accounts for a large number of quantum states underlying a black hole.

However this analysis is somewhat indirect since the counting of states is done by ‘switching off gravity’.

With the help of supersymmetry one can show that the ‘switching off gravity’ does not change the number of quantum states.

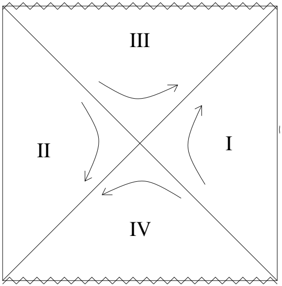
Can we also interpret the black hole entropy as an entanglement entropy? Srednicki

It turns that that this is possible in some cases, but the entanglement is in a quantum field theory rather than in quantum gravity.

– comes via ‘AdS/CFT’ correspondence: Maldacena

A theory of quantum gravity in d+2 dimensional anti de Sitter (AdS) space-time is equivalent to a d+1 dimensional conformal field theory (CFT) living at the boundary of AdS space-time.

Penrose diagram of a black hole in $\mathbf{AdS_{d+2}}$:



Note: This has two boundaries represented by the two vertical edges.

– must be described by a state living on two copies of the dual CFT.

Following the rules of AdS/CFT correspondence one can determine which state of CFT the black hole represents.

Result:

$$\sum_n e^{-E_n/2T_H} |n\rangle_1 \otimes |n\rangle_2$$

– an entangled state in the CFT.

For an observer staying at the boundary 1, the desnity matrix is

$$\rho_{\mathrm{entangle}} \propto \sum_n e^{-E_n/T_H} |n\rangle \otimes \langle n|$$

– a thermal density matrix with temperature T_H!

Maldacena

So far we have tried to describe the states of the black hole either by switching off gravity, or using the dual description as a CFT where we do not see gravity explicitly.

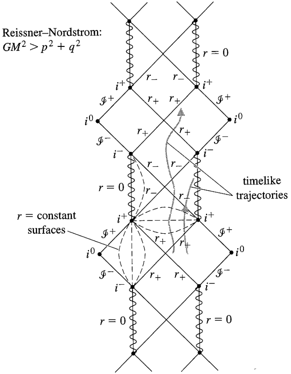
Can we identify the quantum states of the black hole directly as states in quantum gravity?

–requires constructing the wave-functions of these states in quantum gravity.

For uncharged black holes this seems difficult since any observer who could conceivably explore both sides of the horizon end up in the singularity.

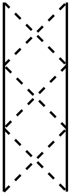
The situation changes when we add some charge.

Penrose diagram of a charged black hole:



An observer moving along the central vertical axis could live forever and collect data from both sides of the horizon.

The situation is even better in the zero temperature limit in which the singularity recedes away.



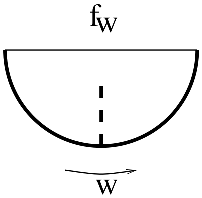
This in fact describes a two dimensional AdS space.

Now an observer moving up the central vertical axis can see the entire space-time.

– should be able to describe the quantum states of the black hole in the zero temperature limit.

Since the geometry has an AdS_2 factor we can use the rules of AdS/CFT correspondence to guess what these states will be. AS

– turns out to be states created by an appropriate euclidean path integral over a semi-infinite strip (or a half disk) with twisted boundary conditions.



Some states have been constructed this way, but much more work is needed to find all the symmetries which can be used to generate the twists.

Entanglement from quantum gravity

AdS/CFT correspondence relates quantum gravity in AdS space to a CFT living on its boundary.

In principle any question in quantum gravity in AdS can be translated to a question in CFT and vice versa.

So far we have been using the insights from CFT to address questions in quantum gravity.

But we can also turn it around and use quantum gravity in AdS to answer questions in CFT.

Given our poor understanding of quantum gravity, this seems a difficult problem.

But if we consider a limit of parameters in which the quantum corrections to gravity are small and classical gravity is a good approximation, then we can make use of this correspondence.

Usually such a limit corresponds to a strongly coupled CFT.

Thus classical gravity can be used to answer questions in strongly coupled CFT!

Take a CFT in d+1 dimensional flat space-time and divide the space into two regions A and B, separated by a (d-1) dimensional boundary K.

If we restrict our measurements only to region B, then we can take the trace over the states in region A and generate a density matrix ρ for B.

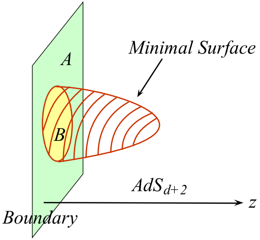
Associated entanglement entropy:

$$\mathbf{S}_{\mathbf{entangle}} = -\mathbf{Tr}(\rho \ln \rho)$$

– usually very difficult to calculate in a strongly coupled CFT.

Can we map this to a simple computation in the dual gravity theory?

Proposal: In the dual gravity theory in AdS_{d+2} we first find a d dimensional hypersurface of minimal area whose boundary coincides with the boundary K separating A and B.



If A_H is the area of this hypersurface, then

$$S_{entangle} = A_H / 4G_N$$

Ryu, Takayanagi

This converts a highly quantum problem in CFT to a simple classical geometry problem.

This proposal has passed many tests where the entanglement entropy in CFT can be computed independently.

Recently this has been ‘proved’ for the special case when the boundary is a sphere by relating this to a black hole entropy.

Casini, Huerta, Myers

A deeper understanding of this prescription is still awaited.

Conclusion

Entanglement is an essential feature of quantum gravity.

It seems to geometrize entanglement by dividing space-time into regions which cannot communicate.

A deeper understanding of quantum gravity is likely to go hand in hand with a deeper understanding of the role of entanglement in this theory.