Local Realism of Macroscopic Correlations

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Outline of the talk

PART I

- Local Realism and Bell inequalities
- Monogamy of Bell inequalities violations

PART II

- Classicality of macroscopic correlations

PART I

Local Realism

Extremely orthodox point of view: quantum theory is about correlations

Quantum correlations are "strange"

Father founders of QM were clearly bothered

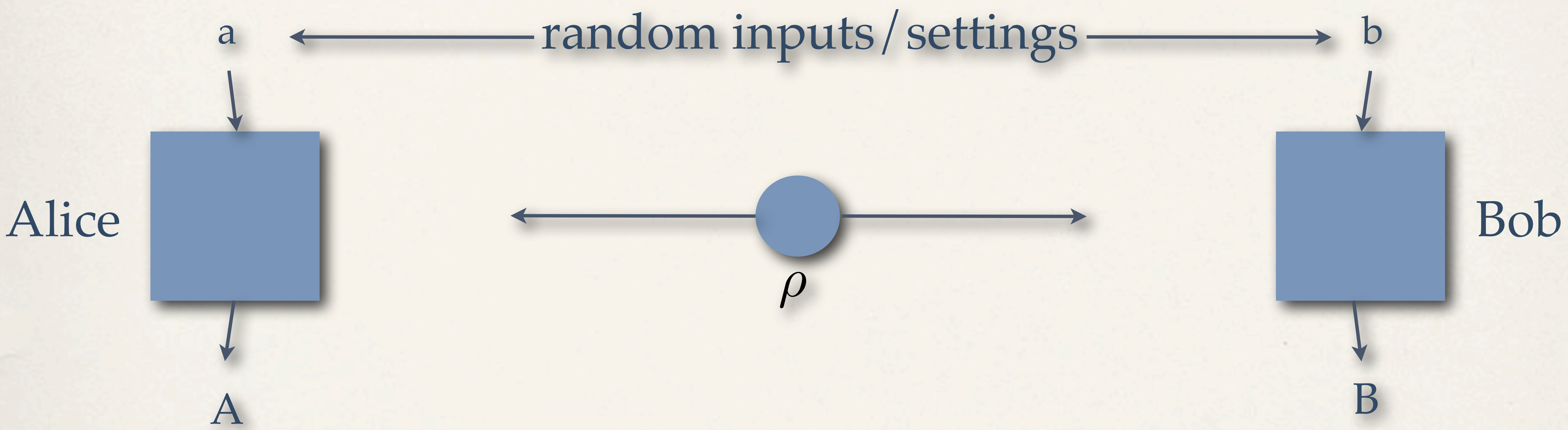
Some of them even wrote papers about it

Shroedinger 30's: concept of entanglement

Einstein, Podolsky, Rosen 30's: concept of local realism

Local Realism

Generic experiment testing quantum correlations



$$p(A, B|a, b)$$

Local Realism

Correlation function

$$E(ab) = \sum_{A,B} f(A) f(B) p(A, B|a, b)$$

local interpretation of outcomes

Remark: correlation function contains less information then probabilities

Local Realism

Quantum mechanics predicts

$$p_{QM}(A, B|a, b) = Tr(\rho P(A, a) \otimes P(B, b))$$

Projectors

$$E_{QM}(a, b) = Tr(\rho A(a) \otimes B(b))$$

$$A(a) = \sum_A f(A) P(A, a)$$

Local Realism

Local Realism (LR) assumes that

1. Outcomes of measurements exist before the act of measurement
2. Relativistic locality holds

LR is a very intuitive/common sense view of Nature

All reasonable classical physical theories are LR

Local Realism

LR implies the following

$$p_{LR}(A, B|a, b) = \int d\lambda \mu(\lambda) p(A|a, \lambda) p(B|b, \lambda)$$

$$E_{LR}(a, b) = \int d\lambda \mu(\lambda) I(a, \lambda) J(b, \lambda)$$

Local response functions

$$\min_A f(A) \leq I(a, \lambda) \leq \max_A f(A)$$

Bell inequalities

However intuitive LR is, it does not agree with QM (Bell 1964)

CHSH inequality for two qubits (a,b=0,1; f(A/B)=-1,+1)

$$|E_{LR}(0,0) + E_{LR}(0,1) + E_{LR}(1,0) - E_{LR}(1,1)| \leq 2$$

QM gives for a maximally entangled state

$$|E_{QM}(0,0) + E_{QM}(0,1) + E_{QM}(1,0) - E_{QM}(1,1)| = 2\sqrt{2}$$

Bell inequalities

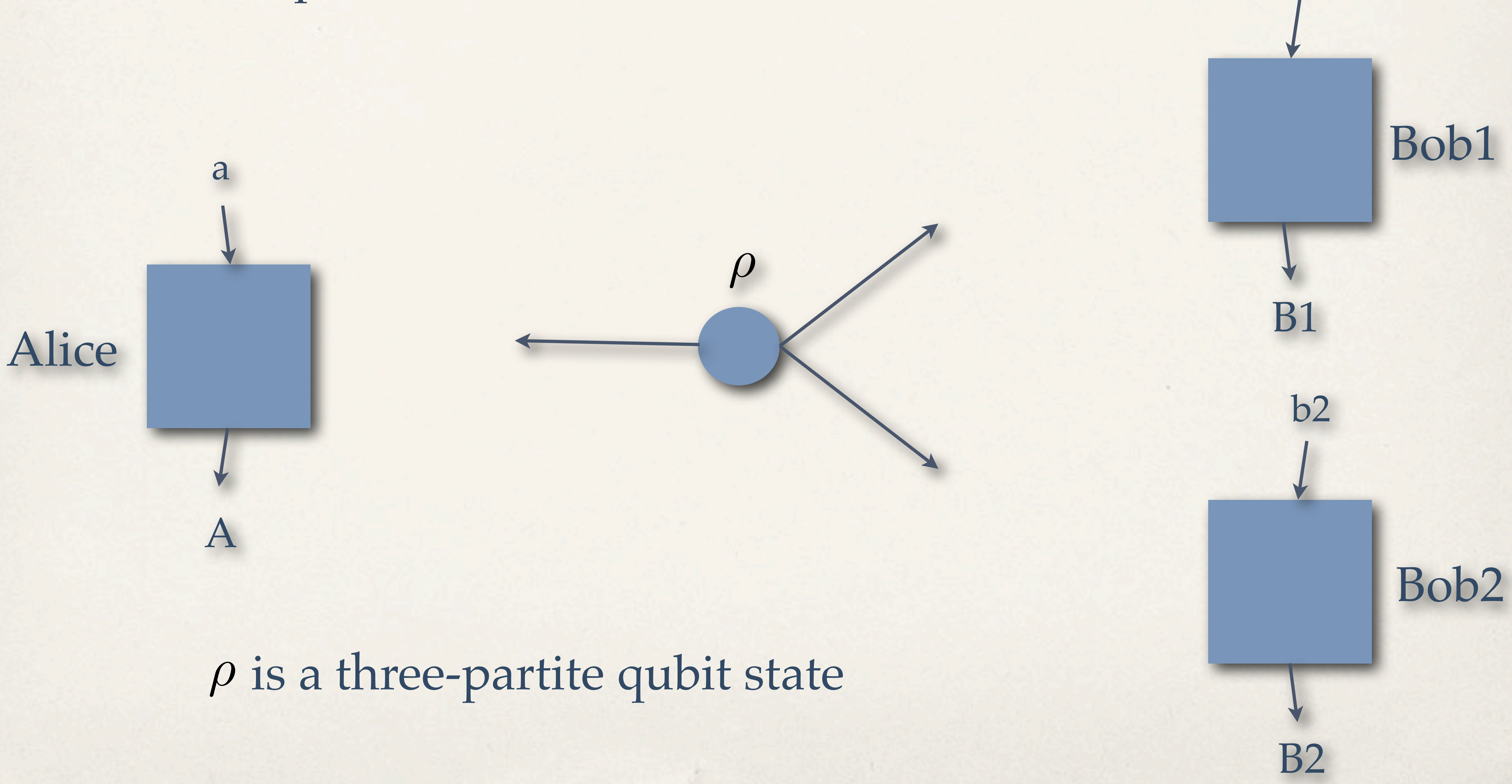
Multi-partite, higher dimensional versions of Bell inequalities exist

Such Bell inequalities are violated by QM and are more robust with the increasing number of particles and/or their dimension

$$\sum_{s=0}^{D} |s\rangle_A |s\rangle_B \quad \text{is “less” LR than} \quad |\uparrow\rangle_A |\downarrow\rangle_B + |\downarrow\rangle_A |\uparrow\rangle_B$$

Monogamy relations

Consider three parties: Alice, Bob1 and Bob2



ρ is a three-partite qubit state

Monogamy relations

Probabilities in this experiment

$$p_{QM}(AB_1B_2|ab_1b_2)$$
$$p_{QM}(AB_1|ab_1)$$
$$p_{QM}(AB_2|ab_2)$$

Here: A,B1,B2,a,b1,b2=0,1 and f(A/B)=-1,+1

$$E_{QM}(ab_k) = \sum_{A,B_k} f(A)f(B_k)p_{QM}(AB_k|ab_k)$$

Monogamy relations

Can Alice and Bob1 violate CHSH inequality together with Alice and Bob2?

The answer is NO!

The proof is very simple but we need a trivial observation first

Monogamy relations

Consider a set of mutually anti-commuting hermitian operators A_i

$$F = \sum_i \langle A_i \rangle A_i$$

$$var(F) = \langle F^2 \rangle - \langle F \rangle^2 \geq 0$$

$\langle F^2 \rangle = \sum_i \langle A_i \rangle^2 \langle A_i^2 \rangle$ because $\{A_i, A_j\} = 0$

$$\max_k \langle A_k^2 \rangle \geq \sum_i \langle A_i \rangle^2$$

Monogamy relations

WWZB 2001: CHSH is not violated iff a certain parameter L is not larger than 1

L*L is bounded by

$$\sum_{i,j=x,y} T_{ij}^2, \; T_{ij} = Tr(\sigma_i \otimes \sigma_j \rho)$$

We have

$$L_{AB_1}^2 + L_{AB_2}^2 \leq \sum_{i,j} (T_{ij}^{AB_1})^2 + \sum_{i,k} (T_{ik}^{AB_2})^2$$

Monogamy relations

Grouping

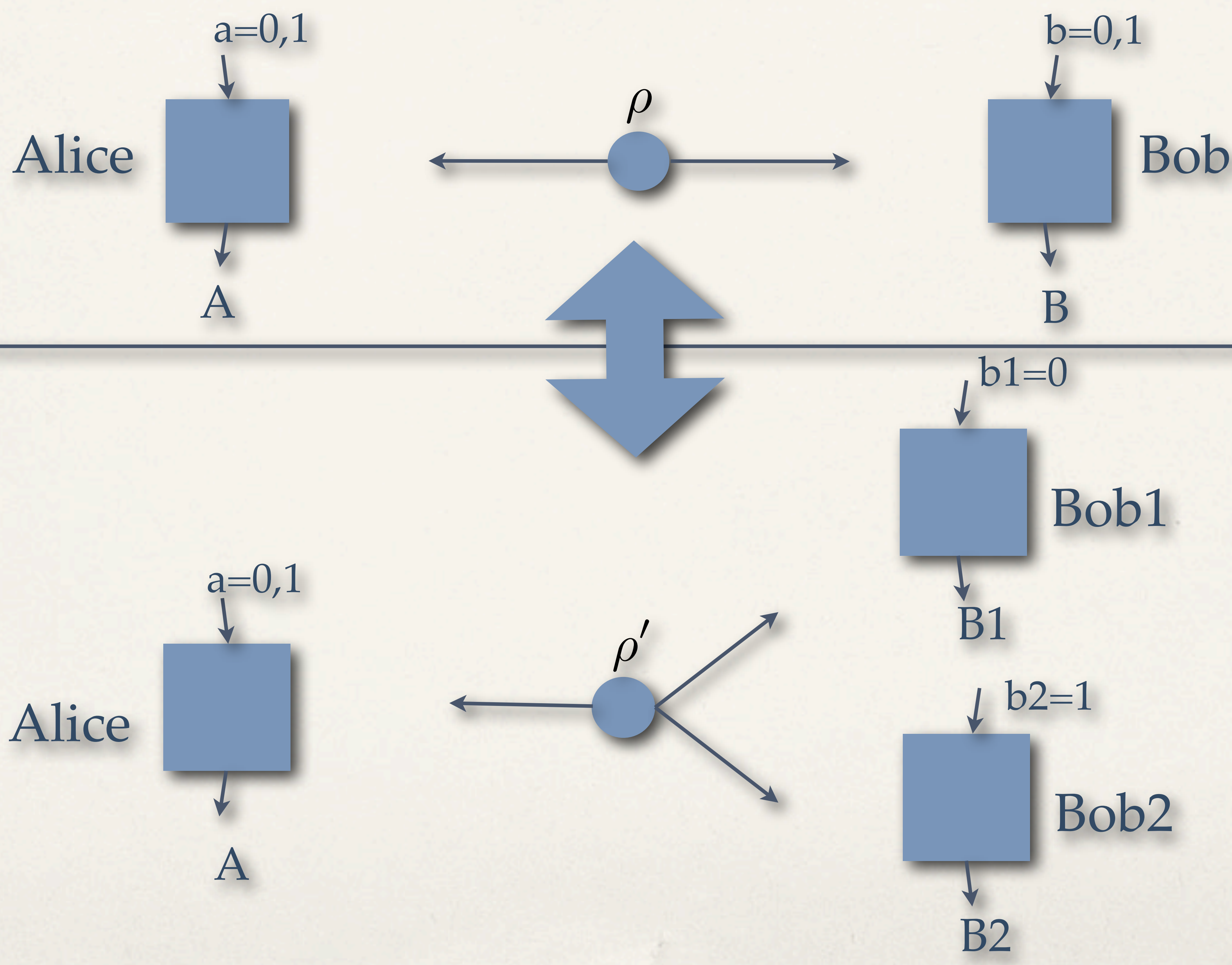
$$(XXI, XYI, YIX, YIY)$$
$$(YXI, YYI, XIX, XIY)$$

In each group sum of the squares of average values is bounded by 1

Thus

$$L^2_{AB_1} + L^2_{AB_2} \leq 2$$

Monogamy relations



Monogamy relations

$$Tr_{B_2}(\rho') = \rho = Tr_{B_1}(\rho')$$

symmetric extension

$$p(A_0, B_1, B_2) = Tr(\rho' P(A_0, a = 0) \otimes P(B_0, b_1 = 0) \otimes P(B_1, b_2 = 1))$$

$$p(A_1, B_1, B_2) = Tr(\rho' P(A_1, a = 0) \otimes P(B_0, b_1 = 0) \otimes P(B_1, b_2 = 1))$$

$$p(A_k, B_1) = \sum_{B_2} p(A_k, B_1, B_2) = Tr(\rho P(A_k, a = k) \otimes P(B_1, b_1 = 0))$$

Monogamy relations

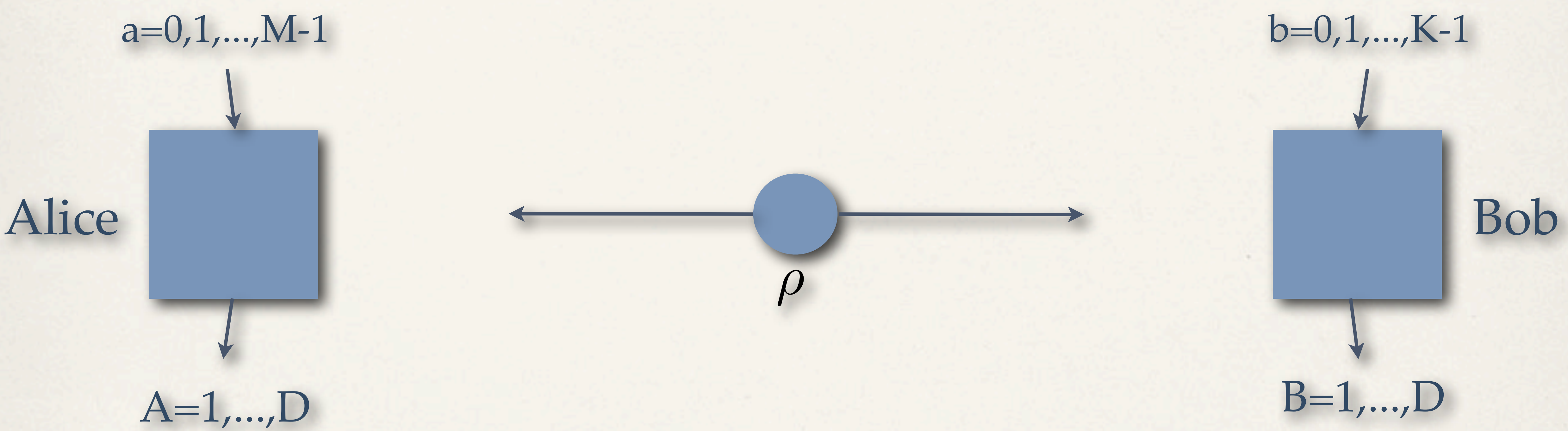
However, if symmetric extension exists

$$p(A_0, A_1, B_0, B_1) := p(A_0, B_0, B_1)p(A_1, B_0, B_1)/p(B_0, B_1)$$

This joint probability is equivalent to LR

Monogamy relations

LR exists if symmetric extension exists to K Bobs



Moreover, Alice and Bob can measure POVM's with D outcomes

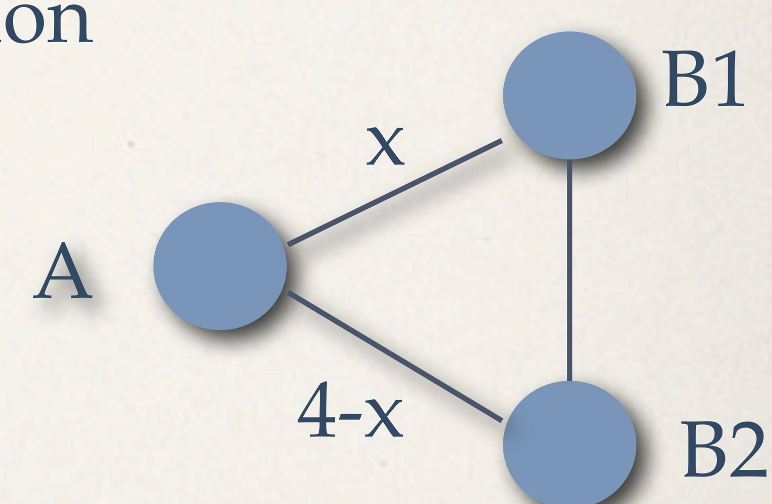
Monogamy relations

Symmetric extension vs monogamy

Consider two states $\rho_{AB_1B_2}$ and $\rho_{eff} = \frac{1}{2}(\rho_{AB_1} + \rho_{AB_2})$

symmetric extension

$$\rho_{perm} = \frac{1}{2}(\rho_{AB_1B_2} + \rho_{AB_2B_1})$$



If AB1 violates CHSH then AB2 can't violate it

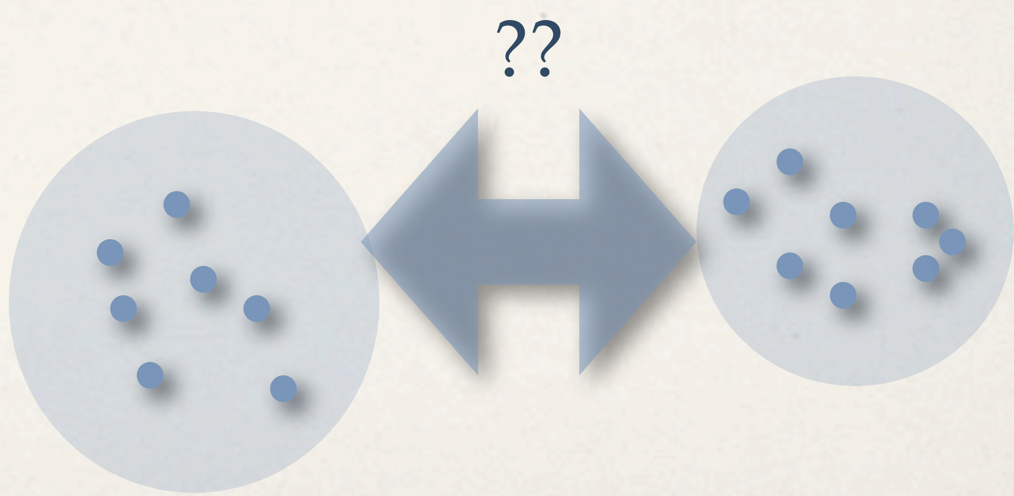
PART II

Macroscopic correlations

Macroscopic world appears LR regardless of the fact that the fabric of reality is quantum

Imagine two spatially separated regions containing many quantum particles

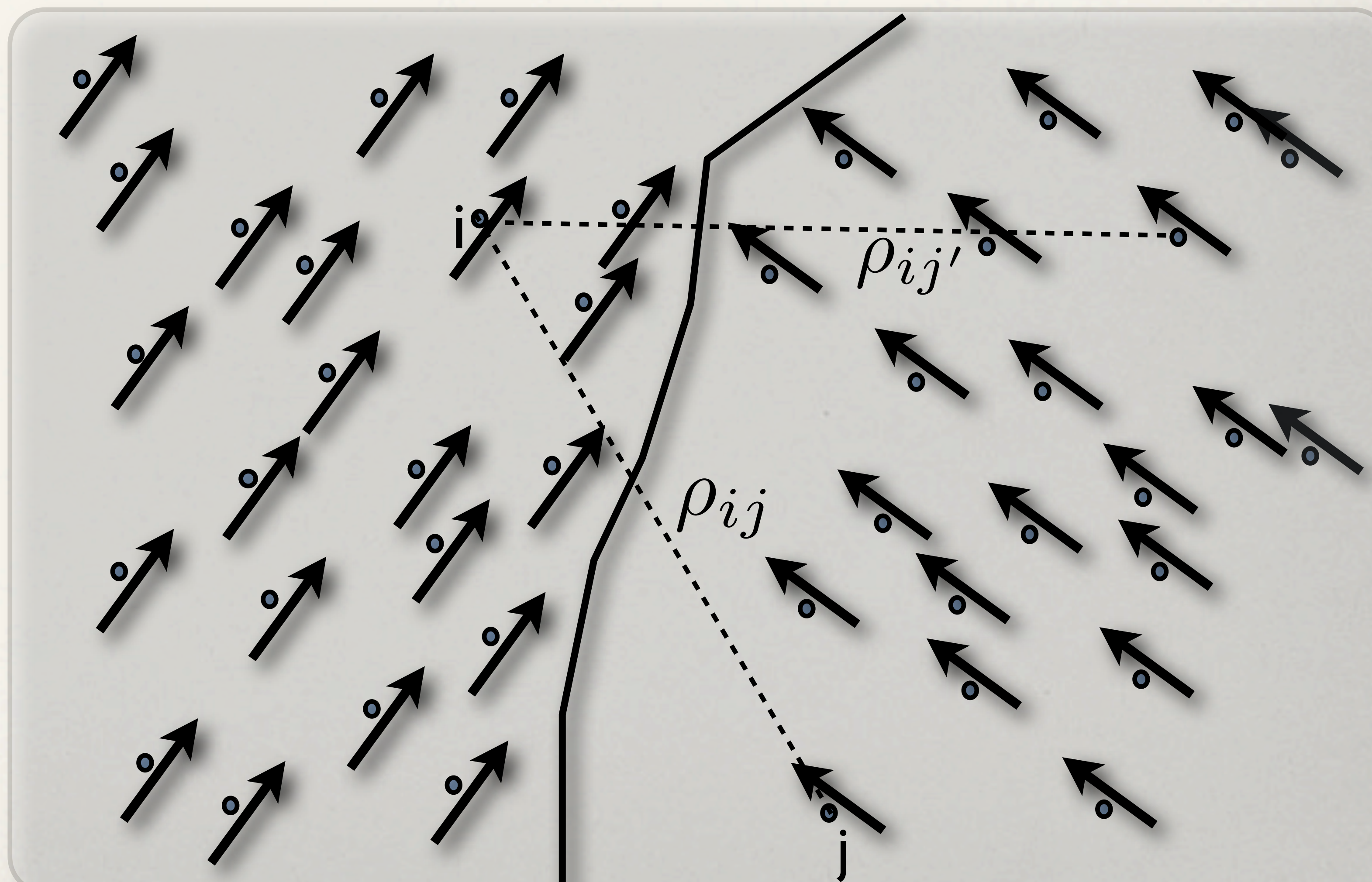
Can they be correlated in a non-classical way?



Macroscopic correlations

Lattice of qubits

Alice and Bob measure correlations between magnetizations



Macroscopic correlations

Magnetization operator in region K=A,B $M_K(\vec{n}) = \vec{n} \cdot \sum_{k \in K} \vec{\sigma}_k$

$$\langle M_A(\vec{n}) \otimes M_B(\vec{m}) \rangle = N_A N_B Tr(\vec{n} \cdot \vec{\sigma} \otimes \vec{m} \cdot \vec{\sigma} \rho_{eff})$$

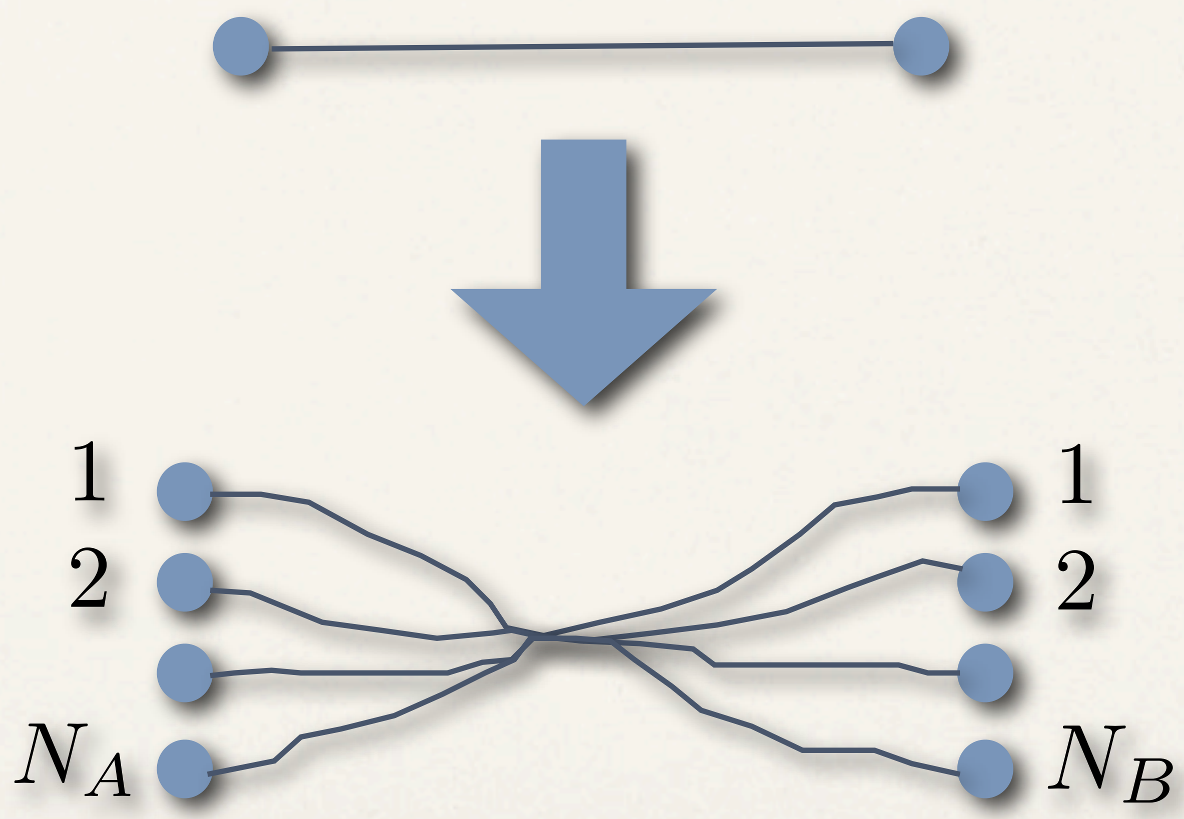
number of qubits in A and B

$$\rho_{eff} = \frac{1}{N_A N_B} \sum_{i \in A, j \in B} \rho_{ij}$$

reduced density operators

Macroscopic correlations

$$\rho_{eff} = \frac{1}{N_A N_B} \sum_{i \in A, j \in B} \rho_{ij}$$ is a two-qubit state that has the following symmetric extension



$$\rho' = \frac{1}{N_A! N_B!} \sum_{\Pi_A, \Pi_B} \Pi_A \otimes \Pi_B \rho \Pi_A^\dagger \otimes \Pi_B^\dagger$$

Macroscopic correlations

ρ_{eff} has LR model as long as the number of settings for Alice and Bob is not more than N_A, N_B ← LARGE numbers!

$$\langle M_A(\vec{n}) \otimes M_B(\vec{m}) \rangle = N_A N_B \int d\lambda \mu(\lambda) I(a,\lambda) J(b,\lambda)$$

The above result can be generalized as follows

Macroscopic measurements

$$M_s^{(j)} = \sum_{k \in j} \sum_{l} f(l,s) E_{l,s}^{(j)}$$

$$\sum_{l} E_{l,s}^{(j)} = I_j$$

POVM for qudit

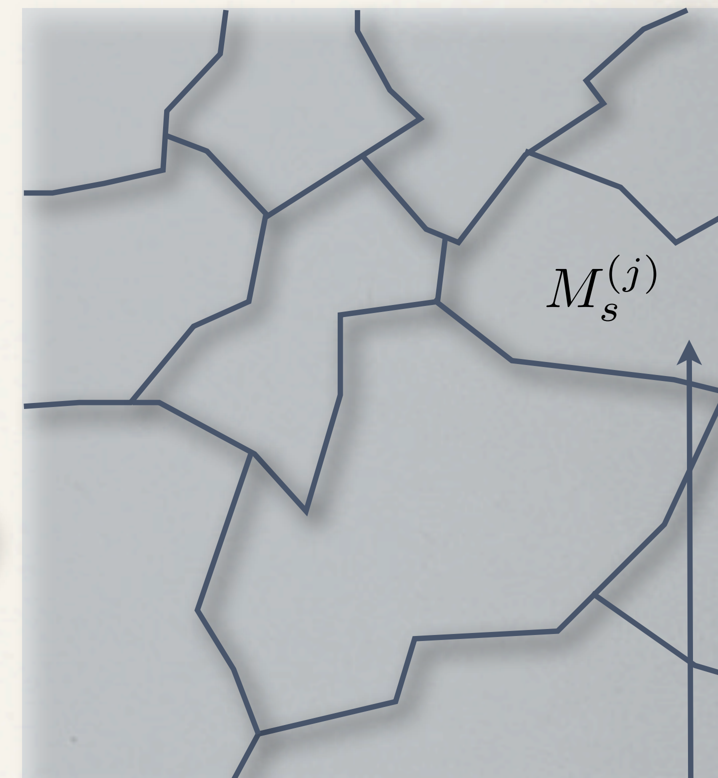
settings defining measurement;
s<= number of qudits in jth
region

generalized "magnetization"

jth region

Macroscopic measurements

$$\rho_{eff} = \frac{1}{N_A \ldots N_K} \sum_{a \in A, \ldots, k \in K} \rho_{a \ldots k}$$



$$\rho' = \frac{1}{N_A! \ldots N_K!} \sum_{\Pi_A \ldots \Pi_K} (\Pi_A \otimes \ldots \otimes \Pi_K) \rho (\Pi_A^\dagger \otimes \ldots \otimes \Pi_K^\dagger)$$

Permutations of particles

jth

Symmetric extension

Macroscopic correlations

Generalized multi-partite "magnetization" correlation measurements always have LR model (if number of settings does not exceed number of particles)

Important assumption: we considered only correlations between average values of "magnetizations" (one body operators)

Can we do better than that?

Macroscopic correlations

If each region contains Avogadro number of particles we are unable to measure anything but average values of "magnetization"

Single measurement is always extremely close to the average value of "magnetization"

Macroscopic correlations

Measurement of M-body observables is simply equivalent to reducing number of settings to N/M

still LARGE number!

$$O = O_{12} + O_{23} + O_{13}$$

- 1
- 2
- 3

Macroscopic measurements

Moreover for a class of rotationally invariant qubit systems, LR description exists for any number of settings

$$\rho_{eff} = V|singlet\rangle\langle singlet| + (1-V)\frac{I_A \otimes I_B}{4}$$

$$V \leq \frac{R+2}{3R}$$

$$R = \max(N_A, N_B)$$

For more than 7 particles in each region the effective state is LR for any number of POVM measurements

Conclusions

Monogamy of Bell ineq. violations

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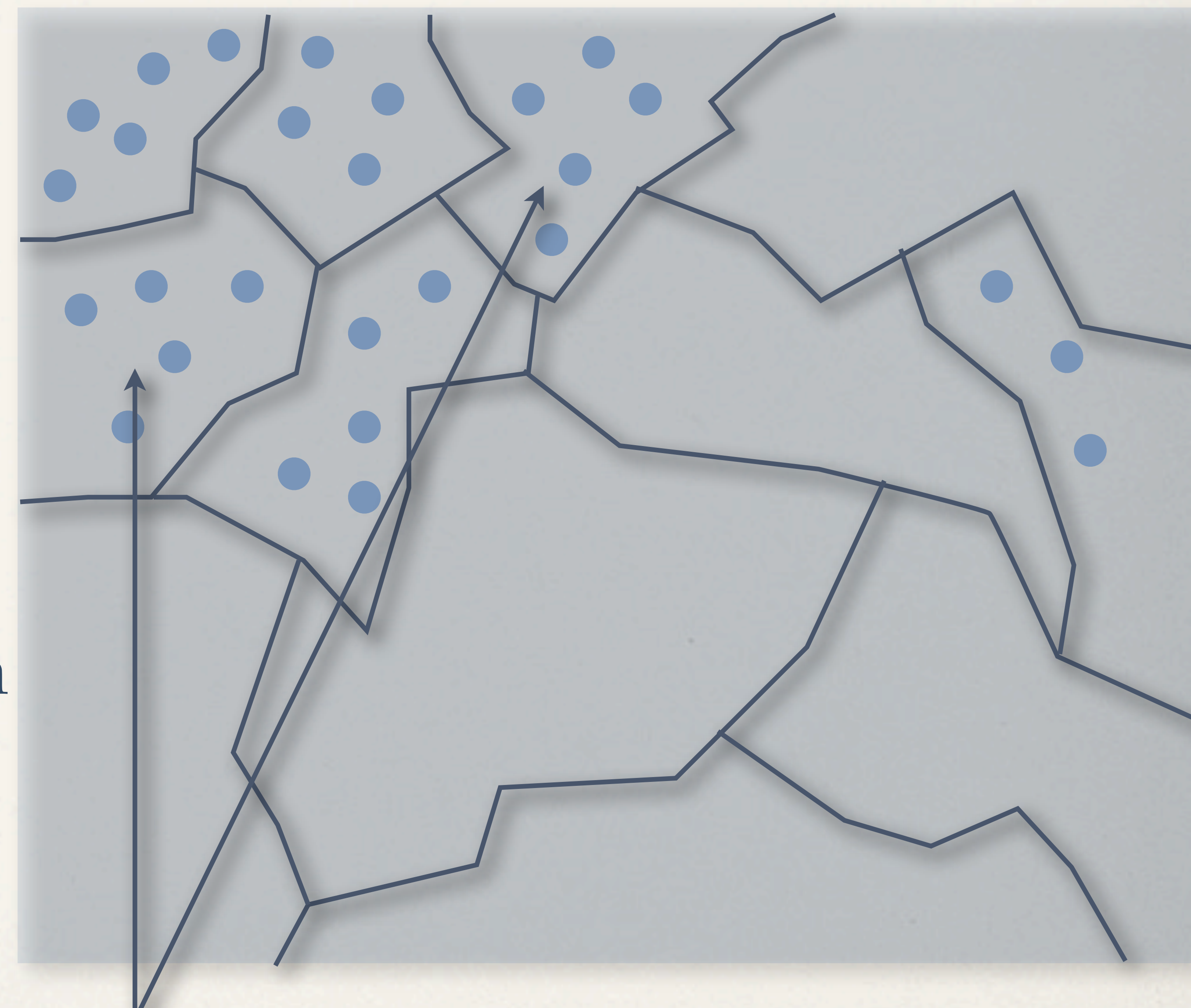
Realistic measurements

Classicality of correlations between large quantum systems

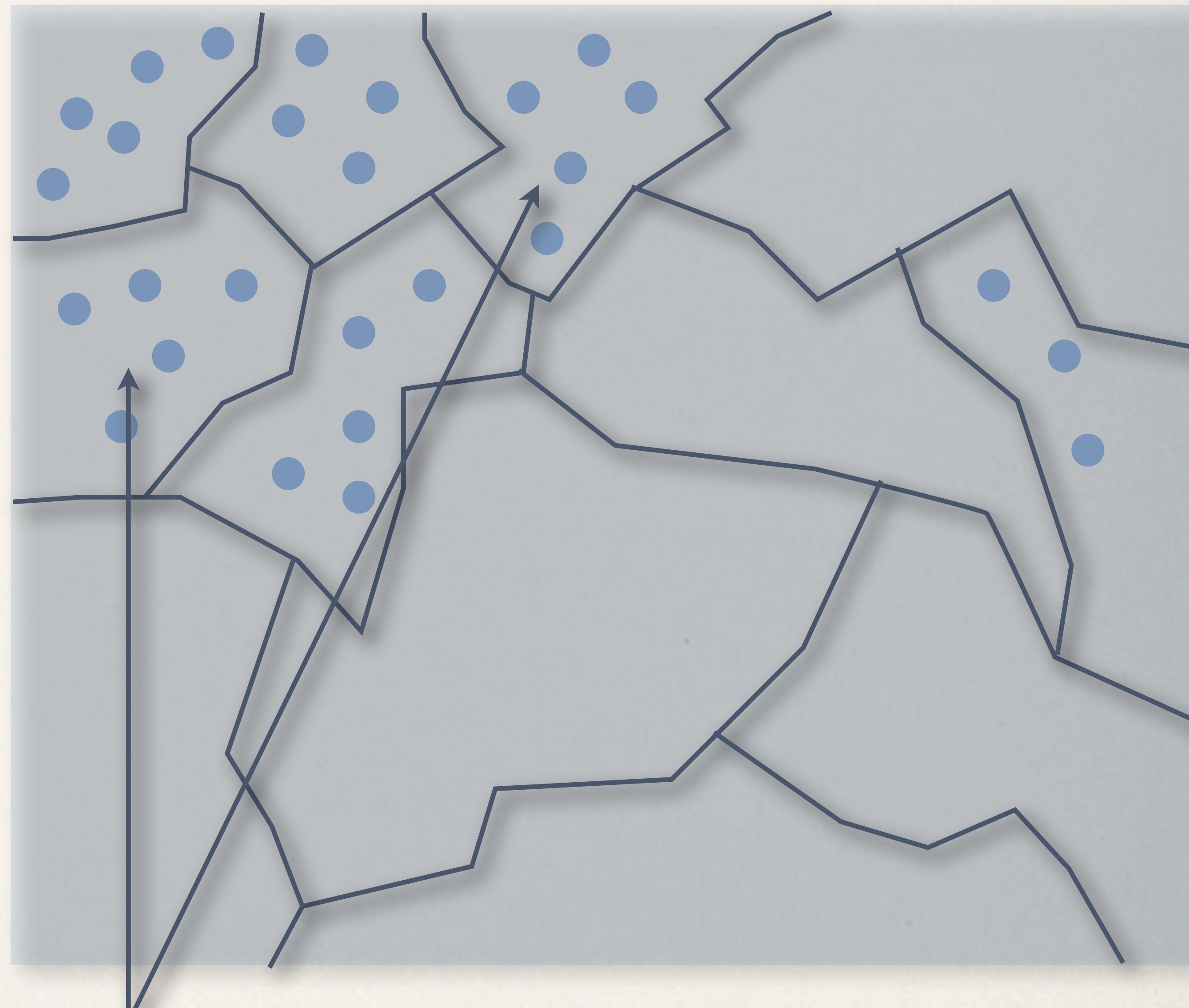


Large number of particles

Conclusions

Non-LR correlations have a chance to appear if the number of settings is larger than number of particles

Mezoscopic region



Large number of particles