QUANTUM TELEPORTATION USING ENTANGLED COHERENT STATES

Invited Talk by

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Classical information

Classical bit: ‘0’ or ‘1’

Usually implemented by voltage of either ‘0’ or ‘V’ across a capacitor.

Classical information is encoded as strings of bits

‘0100110100111000111101001011111’

Quantum information

Unit of Quantum information is ‘Quantum bit’ or ‘Qubit’

Qubit: Vector in 2-dim Hilbert space spanned by basis vector

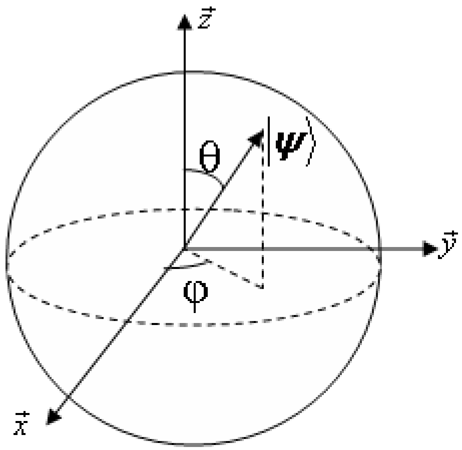
Qubit is physically represented by two-level quantum system state as a superposition of basis vectors in the form,

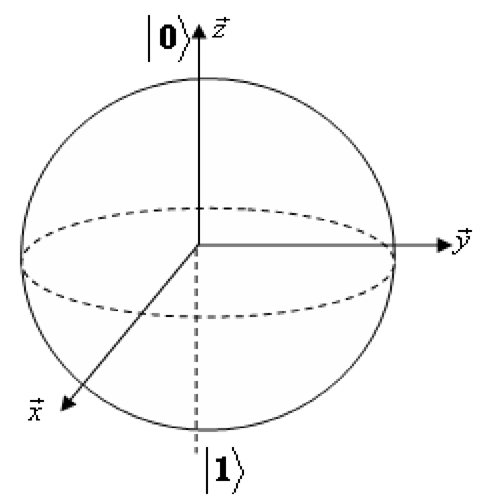
BLOCH SPHERE

Bloch sphere representation of qubit, (Left): an arbitrary qubit $|\psi\rangle$ can be parameterized with two real numbers $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ corresponding to polar and azimuthal angles in polar spherical coordinate system as

$$|\psi\rangle = \mathbf{cos}\,\frac{\theta}{2}|\mathbf{0}\rangle + e^{i\varphi}\,\mathbf{sin}\frac{\theta}{2}|\mathbf{0}\rangle$$

(right): Bloch representations of two frequently used qubit states $|0\rangle, |1\rangle$ corresponding to spin-up and spin-down eigenvectors along z-axis



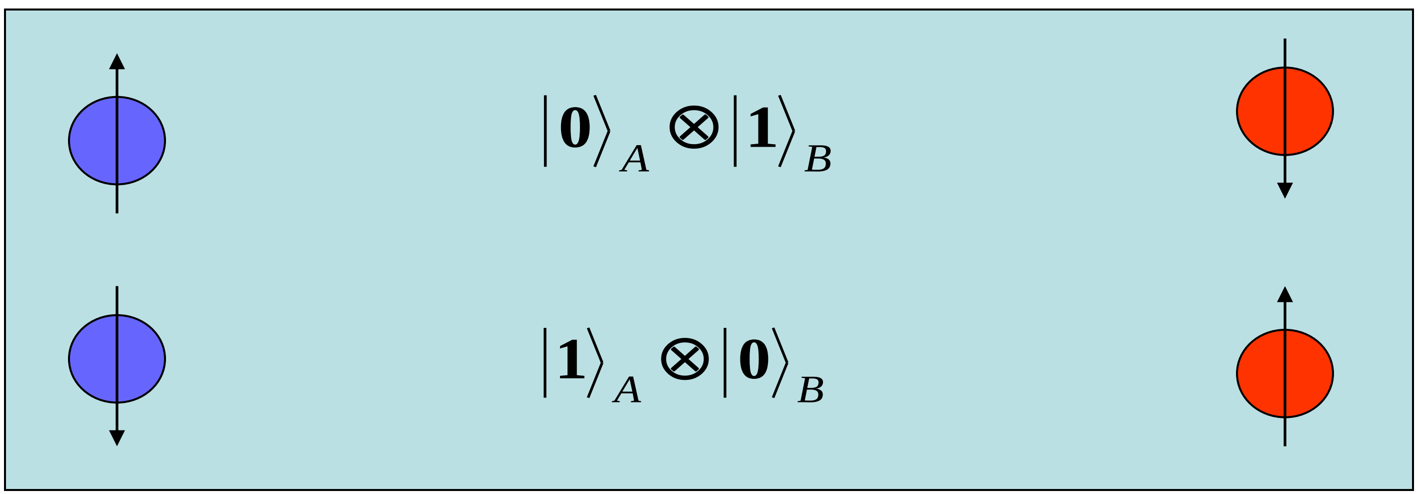


Quantum Entanglement

$$|\psi\rangle_{AB} \in H_1^{(2)} \otimes H_2^{(2)}$$

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right]$$

ALICE BOB



$$|0\rangle_A \otimes |1\rangle_B$$

$$|1\rangle_A \otimes |0\rangle_B$$

Quantum Entanglement: Concurrence

For pure states :

$$C(\psi)_{AB} = \left|\langle\psi|\sigma_y \otimes \sigma_y|\psi^*\rangle\right| = \left|\langle\psi|\tilde{\psi}\rangle\right|$$

For Mixed states :

$$\rho_{AB} = \sum_i p_i |\phi_i\rangle\langle\phi_i| ; \tilde{\rho}_{AB} = \sigma_y \otimes \sigma_y \rho^*_{AB} \sigma_y \otimes \sigma_y ,$$

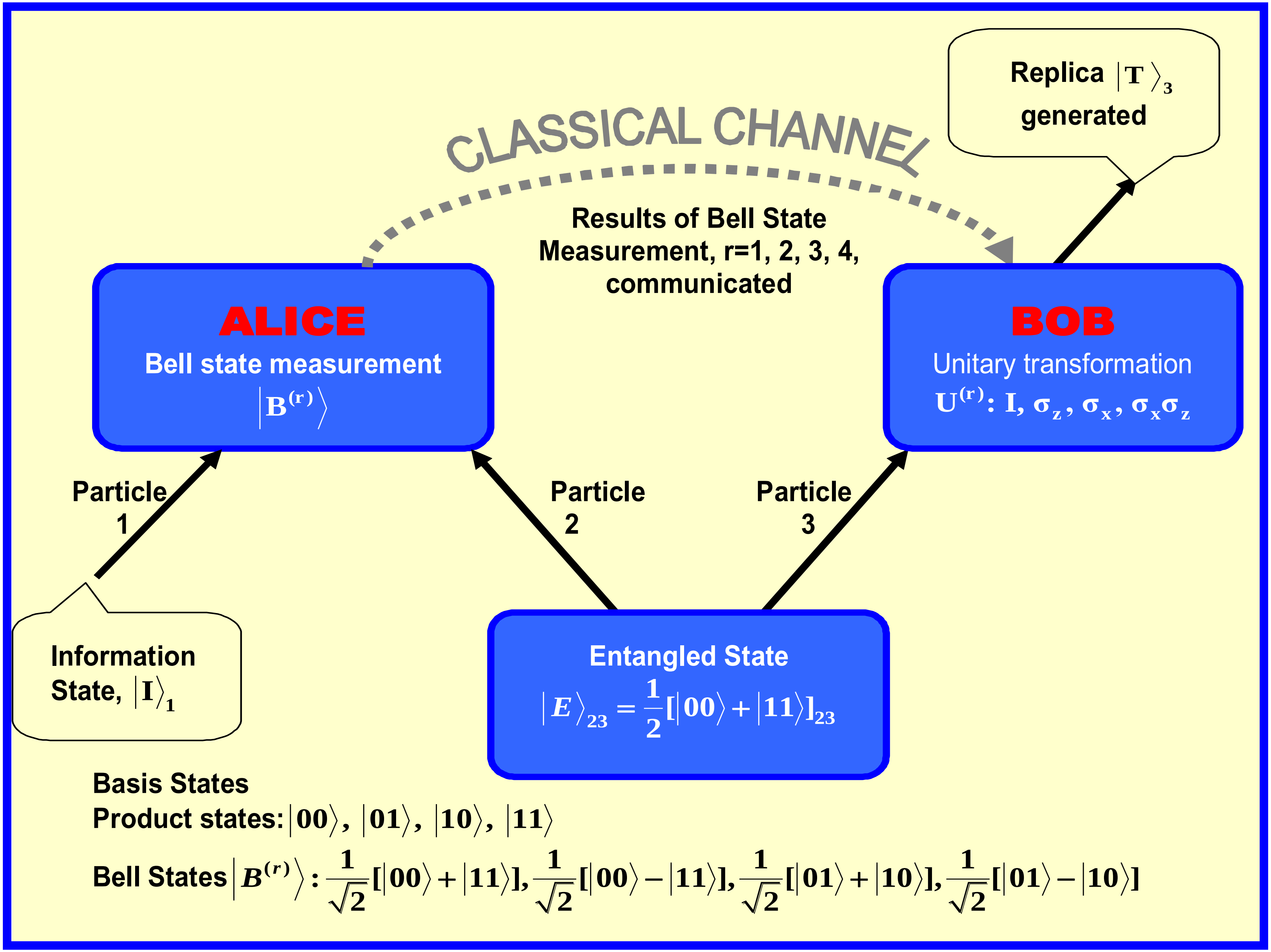
$$C(\rho_{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

where $\{\lambda_i\}$ are the square roots of eigen values of non harmitian matrix $\rho\tilde{\rho}$ with,

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0$$

[W.K Wooters, Phys. Rev. Lett. 80, 2245 (1998)]

$$|\psi\rangle_{AB} = \cos\frac{\theta}{2}|0\rangle_A|1\rangle_B + \sin\frac{\theta}{2}|1\rangle_A|0\rangle_B , C = \cos\theta$$



CLASSICAL CHANNEL
Replica $|\mathbf{T}\rangle_3$ generated
Results of Bell State Measurement, r=1, 2, 3, 4, communicated
ALICE
Bell state measurement
$|\mathbf{B}^{(r)}\rangle$
BOB
Unitary transformation
$\mathbf{U}^{(r)}$: $\mathbf{I}, \sigma_z, \sigma_x, \sigma_x\sigma_z$
Particle 1
Particle 2
Particle 3
Information State, $|\mathbf{I}\rangle_1$
Entangled State
$|E\rangle_{23} = \frac{1}{2}[|00\rangle + |11\rangle]_{23}$
Basis States
Product states: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$
Bell States $|B^{(r)}\rangle : \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle], \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle], \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle], \frac{1}{\sqrt{2}}[|01\rangle - |10\rangle]$

AN EXAMPLE

- Information state $\left|\mathbf{I}\right\rangle_1 = \mathbf{a}\left|0\right\rangle_1 + \mathbf{b}\left|1\right\rangle_1$
- Entangled state $\left|\mathbf{E}\right\rangle_{2,3} = \frac{1}{\sqrt{2}}[\left|0,0\right\rangle_{2,3} + \left|1,1\right\rangle_{2,3}]$
- Bell States $\left|\mathbf{B}^{(\mathbf{r})}\right\rangle$: $\left\langle\mathbf{B}^{(\mathbf{r})}\middle|\mathbf{B}^{(\mathbf{s})}\right\rangle = \boldsymbol{\delta}_{\mathbf{rs}}, \sum_{\mathbf{r}=1}^{4}\left|\mathbf{B}^{(\mathbf{r})}\right\rangle\left\langle\mathbf{B}^{(\mathbf{r})}\right| = 1$
- Combined State

$$\left|\boldsymbol{\psi}\right\rangle_{1,2,3} = \left|\mathbf{I}\right\rangle_1\left|\mathbf{B}^{(\mathbf{r})}\right\rangle_{2,3} = \sum_{\mathbf{r}=1}^{4}\left|\mathbf{B}^{(\mathbf{r})}\right\rangle_{2,3}\left\langle\mathbf{B}^{(\mathbf{r})}\middle|\boldsymbol{\psi}\right\rangle_{1,2,3}$$

- If result of BSM is r=3, for example

$$\left|\mathbf{B}^{(3)}\right\rangle_{2,3} = \frac{1}{\sqrt{2}}[\left|0,1\right\rangle_{2,3} + \left|1,0\right\rangle_{2,3}]$$

state of Bob is

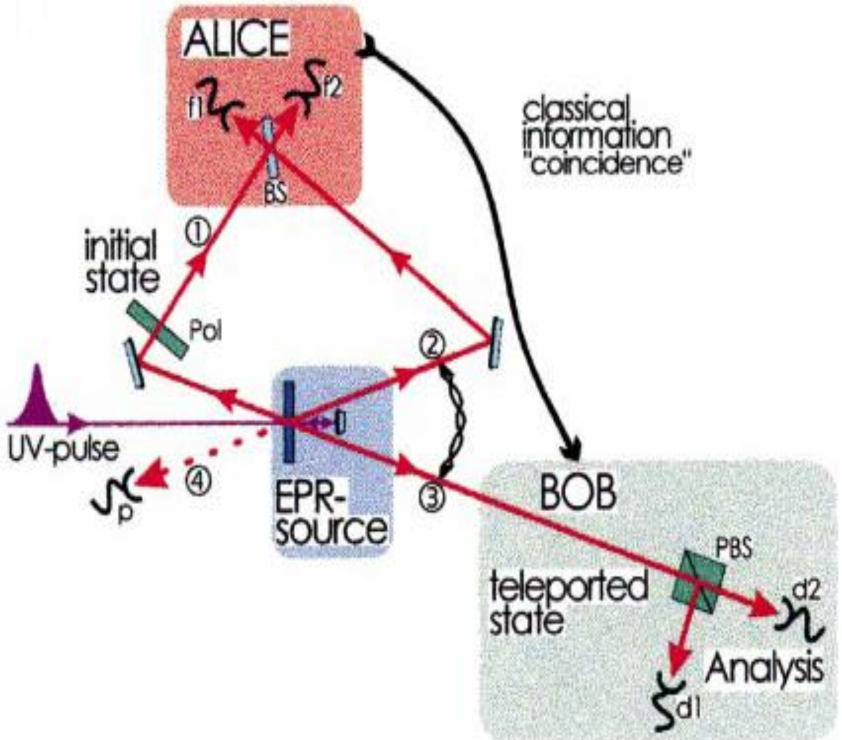
$$\left|\mathbf{Bob}^{(3)}\right\rangle_3 \square {}_{1,2}\left\langle\mathbf{B}^{(3)}\middle|\boldsymbol{\psi}\right\rangle_{1,2,3} \square\ \mathbf{a}\left|1\right\rangle_3 + \mathbf{b}\left|0\right\rangle_3$$

- Unitary transformation $\mathbf{U}^{(3)} = \boldsymbol{\sigma}_{\mathbf{x}}$ then gives

$$\left|\mathbf{T}^{(3)}\right\rangle = \mathbf{U}^{(3)}\left|\mathbf{Bob}^{(3)}\right\rangle = \left|\mathbf{I}\right\rangle$$

- Quality of Teleportation: Fidelity, F= $\left|\left\langle\mathbf{I}\middle|\mathbf{T}\right\rangle\right|^2$

Experimental Quantum Teleportation
(*Institut für Experimentalphysik, Universität Innsbruck*)

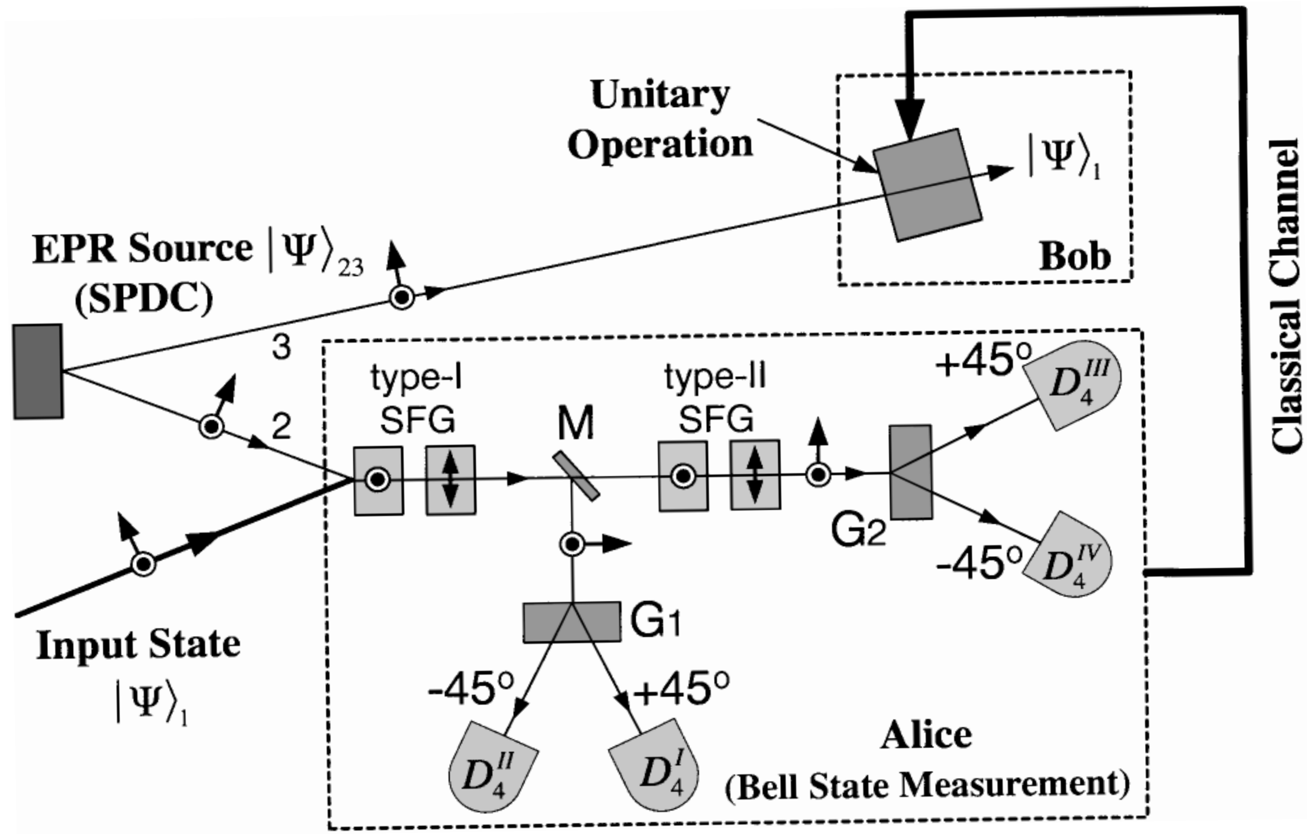


The experimental set-up: A pulse of ultra-violet radiation passing through a nonlinear crystal creates the ancillary pair of photons 2 and 3. After retroflection during its second passage through the crystal the ultraviolet pulse creates another pair of photons, one of which will be prepared in the initial state of photon 1 to be teleported, the other one serving as a trigger indicating that a photon to be teleported is under way. Alice then looks for coincidences after a beam splitter BS where the initial photon and one of the ancillaries are superposed. Bob, after receiving classical Information that Alice obtained coincidence count in detectors f1 and f2 identifying $|\psi^-\rangle_{1,2}$ Bell state, knows that his photon 3 is in the initial state of photon 1 which he then can check using polarization analysis with polarizing beam splitter PBS and detectors d1 and d2. The detector p provides Information that photon 1is under way.

$$|\psi^-\rangle_{23} = \tfrac{1}{\sqrt{2}}[|H\rangle_2|V\rangle_3 - |V\rangle_2|H\rangle_3]$$

[Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger, Nature 390,575 (1997)]

Experimental Quantum Teleportation with a Complete Bell State Measurement using Nonlinear Interaction



Principle schematic of quantum teleportation with a complete BSM. Nonlinear interactions (SFG) are used to perform the BSM. ⦿ and ↕ represent the respective horizontal and vertical orientations of the optic axes of the crystals.

[Yoon-Ho Kim, Sergei P. Kulik, and Yanhua Shih, Phys. Rev. Lett. 86, 1370 (2001)]

BSM is based on nonlinear interactions: optical sum frequency generation (SFG) (or “up-conversion”).

First type-I SFG crystal: $|V\rangle_1 |V\rangle_2 \to |H\rangle_4$

Second type-I SFG crystal: $|H\rangle_1 |H\rangle_2 \to |V\rangle_4$

First type-II SFG crystal: $|V\rangle_1 |H\rangle_2 \to |H\rangle_4$

Second type-II SFG crystal: $|H\rangle_1 |V\rangle_2 \to |V\rangle_4$

$$|\phi_\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}[|H\rangle_1 |H\rangle_2 \pm |V\rangle_1 |V\rangle_2] \xrightarrow{\text{Type}-\text{I SFG crystals}} |V\rangle_4 \pm |H\rangle_4$$

$$\xrightarrow{45\pm\ \text{polarization projector G1.}} |\pm 45^0\rangle_{DI,DII}$$

$$|\psi_\pm\rangle_{1,2} = \frac{1}{\sqrt{2}}[|H\rangle_1 |V\rangle_2 \pm |V\rangle_1 |H\rangle_2] \xrightarrow{\text{Type}-\text{II SFG crystals}} |V\rangle_4 \pm |H\rangle_4$$

$$\xrightarrow{45\pm\ \text{polarization projector G2.}} |\pm 45^0\rangle_{DIII,DIV}$$

• Van Enk & Hirota Scheme of QT

Entangled coherent states are stronger than standard bi-photonic states against possible photon transfer to reservoir modes

QT of one qubit : superposed coherent state,

Glauber's coherent states $|\alpha\rangle$ are defined as eigenstate of annihilation operator with eigenvalue α.

$$|\mathrm{I}\rangle = \varepsilon_+ |\alpha\rangle_o + \varepsilon_- |\alpha\rangle_o$$

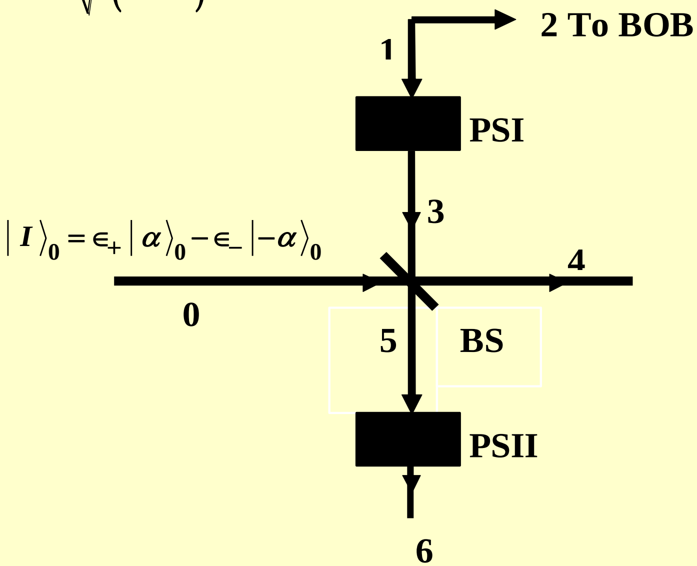
$$= A_+ |\mathrm{EVEN},\alpha\rangle + A_- |\mathrm{ODD},\alpha\rangle$$

$$|\mathrm{EVEN},\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2(1+x^2)}} \quad , \quad |\mathrm{ODD},\alpha\rangle = \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2(1-x^2)}} \quad , \quad x \equiv e^{-|\alpha|^2}$$

Entangled coherent state,

$$|\mathrm{E}\rangle_{1,2} = \frac{1}{\sqrt{2(1-x^4)}}[\ \ |\alpha,\alpha\rangle_{1,2} - |\alpha,\alpha\rangle_{1,2}\ \ ]$$

$$|E\rangle_{12} = \frac{1}{\sqrt{2\left(1-x^{4}\right)}}\left[\,|\alpha,\alpha\rangle_{12} - |-\alpha,-\alpha\rangle_{12}\right]$$



$$P.S.I: |\alpha\rangle_1 \rightarrow |-i\alpha\rangle_3$$

$$BS: |\beta,\gamma\rangle_{0,3} \rightarrow \left|\frac{(\beta+i\gamma)}{\sqrt{2}}, \frac{(i\beta+\gamma)}{\sqrt{2}}\right\rangle_{4,5}$$

$$P.S.II: |\eta\rangle_6 \rightarrow |i\eta\rangle_7$$

Figure 1 shows our scheme of quantum teleportation. Entangled state contains two modes 1 and 2, one of which (mode 2) goes to Bob directly We let Alice pass her part of the entangled state (mode 1) to pass through a phase shifter P.S.I which converts state in mode 1 to state in mode 3. Now Alice mixes state 3 with the state to be teleported (mode 0) by using a 50:50 beam splitter, modifies one of the two outputs (mode 5) by passing it through a phase-shifter P.S.II which changes the state 5 to 6, and then performs photon counting in the two final outputs 4 and 6. This result is then passed to Bob, which helps him to retrieve the information state by performing unitary transformation on the state 2.

- **Van Enk & Hirota noted that**
  - **One of the counts is always 0.**
  - **If the other counts are odd, teleportation is successful.**
  - **If the other counts are even, teleportation fails.**
  - **Success of teleportation is ½.**

- **Our Scheme differs from the above**

We divide counts in to:

Zero, nonzero even and odd

Zero counts gives failure

Odd counts gives perfect QT

Nonzero even counts gives almost perfect QT

For $|\alpha|^2 = 1, 2$ or 5 minimum average fidelity = 0.73, 0.9987, 0.9999 respectively

For photon counting modes 4 & 6 and Bob’s mode 2

$$|\Psi\rangle_{4,6,2} = \left[2\,(1-x^4)\right]^{-1/2}\Big[\in_+ \Big(\big|\sqrt{2}\alpha\big\rangle_4\big|0\big\rangle_6\big|\alpha\big\rangle_2 - \big|0\big\rangle_4\big|-\sqrt{2}\alpha\big\rangle_6\big|-\alpha\big\rangle_2\Big)$$
$$+ \in_- \Big(\big|0\big\rangle_4\big|\sqrt{2}\alpha\big\rangle_6\big|\alpha\big\rangle_2 - \big|-\sqrt{2}\alpha\big\rangle_4\big|0\big\rangle_6\big|-\alpha\big\rangle_2\Big)\Big]$$

One of the modes 4 and 6 is $|0\rangle$ and **Hence one of the count is always zero**

For count in the other mode Van Enk & Hirota wrote

$$|\pm\sqrt{2}\alpha\rangle = \sqrt{\frac{1}{2}\left(1+x^4\right)}\,|\mathrm{EVEN},\sqrt{2}\alpha\rangle \pm \sqrt{\frac{1}{2}\left(1-x^4\right)}\,|\mathrm{ODD},\sqrt{2}\alpha\rangle$$

We write

$$|\pm\sqrt{2}\alpha\rangle = x\,|0\rangle + \frac{1}{\sqrt{2}}(1-x^2)\,|\mathrm{NZE},\sqrt{2}\alpha\rangle \pm \sqrt{\frac{1}{2}(1-x^4)}\,|\mathrm{ODD},\sqrt{2}\alpha\rangle,$$

where

$$|\mathrm{NZE},\alpha\rangle = \Big(|\alpha\rangle + |-\alpha\rangle - 2\sqrt{x}\,|0\rangle\Big)/\sqrt{2}\,(1-x)$$

In the callouts: $|+\rangle \equiv |\mathrm{EVEN},\alpha\rangle$

$|-\rangle \equiv |\mathrm{ODD},\alpha\rangle$

$$U_{I} = 1$$

$$|\Psi\rangle_{4,6,2} = \frac{\sqrt{2x}}{1+x^2}|0\rangle_4|0\rangle_6\, A_+\,|\mathrm{ODD},\alpha\rangle_2$$

$$U_{II} = |+\rangle_{22}\langle -| + |-\rangle_{22}\langle +|$$

$$+\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\left\{|\mathrm{NZE},\sqrt{2}\,\alpha\rangle_4|0\rangle_6\left[A_+\sqrt{\frac{1-x^2}{1+x^2}}|\mathrm{ODD},\alpha\rangle_2 + A_-\sqrt{\frac{1+x^2}{1-x^2}}|\mathrm{EVEN},\alpha\rangle_2\right]\right.$$

$$U_{III} = |+\rangle_{2\,2}\langle -| - |-\rangle_{2\,2}\langle +|$$

$$\left.+|0\rangle_4|\mathrm{NZE},\sqrt{2}\,\alpha\rangle_6\left[A_+\sqrt{\frac{1-x^2}{1+x^2}}|\mathrm{ODD},\alpha\rangle_2 - A_-\sqrt{\frac{1+x^2}{1-x^2}}|\mathrm{EVEN},\alpha\rangle_2\right]\right\}$$

$$U_{IV} = 1$$

$$+\frac{1}{2}\left\{|\mathrm{ODD},\sqrt{2}\,\alpha\rangle_4|0\rangle_6\left[A_+|\mathrm{EVEN},\alpha\rangle_2 + A_-|\mathrm{ODD},\alpha\rangle_2\right]\right.$$

$$U_{V} = |+\rangle_{2\,2}\langle +| - |-\rangle_{2\,2}\langle -|$$

$$\left.+|0\rangle_4|\mathrm{ODD},\sqrt{2}\,\alpha\rangle_6\left[A_+|\mathrm{EVEN},\alpha\rangle_2 - A_-|\mathrm{ODD},\alpha\rangle_2\right]\right\}$$

These lead to

$$|T\rangle_{I} = |ODD,\alpha\rangle_2$$

$$|T\rangle_{II} = |T\rangle_{III} \sim A_+\sqrt{\frac{1-x^2}{1+x^2}}|EVEN,\alpha\rangle_2 + A_-\sqrt{\frac{1+x^2}{1-x^2}}|ODD,\alpha\rangle_2$$

$$|T\rangle_{IV} = |T\rangle_{V} = A_+|EVEN,\alpha\rangle_2 + A_-|ODD,\alpha\rangle_2$$

~ denotes unnormalized state

Fidelity defined by $F = |\langle I|T\rangle|^2$ or $\mathrm{Tr}\,[\rho_I\,\rho_T]$ where $\rho_I = |I\rangle\langle I|$ and $\rho_T = |T\rangle\langle T|$

$$F_I = |A_-|^2$$

$$F_{II} = F_{III} = \frac{\left[1 - x^2\left(|A_+|^2 - |A_-|^2\right)\right]^2}{1 + x^4 - 2x^2\left(|A_+|^2 - |A_-|^2\right)}$$

$$F_{IV} = F_V = 1$$

Information (i.e., A_+ and A_-) - arbitrary

Consider MASFI (Minimum Assured Fidelity), the minimum of F for variation of $A_\pm$

$$(\mathrm{MASFI})_{\mathrm{I}} = 0 \quad (\mathrm{MASFI})_{\mathrm{II,\,III}} = \left(1 - x^4\right) \quad (\mathrm{MASFI})_{\mathrm{IV,\,V}} = 1$$

If P_i is probability for occurrence of case I, we will define average fidelity as

$$\mathbf{F_{av.}} = \sum_{i=I}^{V} \mathbf{P_i\, F_i}$$

$$= 1 - 2\ (1 + x^2)^{-2} x^2 |A_+|^2 \left[x^2 |A_-|^2 + |A_+|^2 \right]$$

This has minimum value, which we call minimum average fidelity (MAVFI),

$$\mathbf{MAVFI = 1\text{-}2(1+x^2)^{-2}\, x^2 \approx 1 \quad for \quad x << 1}$$

For $|\alpha|^2 = 1$, 2 or 5, MAVFI = 0.73, 0.9987, 0.9999, respectively

Teleportation with non-maximally entangled coherent state

Recently H. Prakash and Manoj K. Mishra noted a very interesting thing that MAVFI increases on decreasing entanglement.

Use of non-maximally entangled coherent state,

$$|\mathbf{E}\rangle = \frac{1}{\sqrt{2(1+x^4)}}[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle],$$

in place of maximally entangled coherent state,

$$|\mathbf{E}\rangle = \frac{1}{\sqrt{2(1-x^4)}}[|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle],$$

gives, MASFI =0 for both zero count, F=1 for one nonzero even count and F≈1 for one odd count.

Minimum average fidelity in this case is greater then that when maximally entangled state used by an amount,

$$\mathbf{D} = \frac{x^2(3+x^4)(1-x^2)}{2(1+x^4)(1+x^2)^2}$$

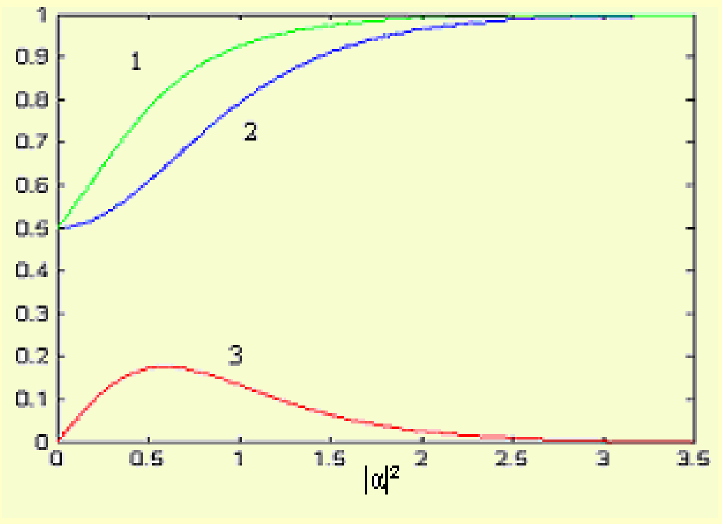


Fig. Curve 1 and 2 shows minimum average fidelity when non-maximally and maximally entangled coherent states respectively, are used as quantum channel. Curve 3 shows variation difference D with $|\alpha|^2$

It is seen that this is important as there is a marked increase in MAVFI at low |α|, and it is difficult to generate superposed coherent states with large |α|.

- **Wang's Teleportation of Bipartite State**

Phys. Rev. A <u>64</u>(2001)022302

Information: $|\Phi\rangle = \varepsilon_+ |\alpha,\alpha\rangle + \varepsilon_- |-\alpha,-\alpha\rangle$

Entangled State

$$\frac{1}{\sqrt{2(1-x^8)}}[|\sqrt{2}\alpha,\alpha,\alpha\rangle - |-\sqrt{2}\alpha,-\alpha,-\alpha\rangle]$$

Wang followed the van Enk Hirota scheme and reported success probability 1/2.

We modified the Wang's scheme and got near perfect teleportation.

H. Prakash, N. Chandra, R. Prakash & Shivani, Phys. Rev. A 75 (2007); also published in Virtual J. Q. Inf.

Our Scheme for QT of Bipartite State

$$|\, \mathrm{I}\,\rangle_{1,2} = \varepsilon_{+}\,|\alpha,\alpha\rangle_{1,2} + \varepsilon_{-}\,|{-\alpha},{-\alpha}\rangle_{1,2}$$

$$= A_{+}\,|\mathrm{EVEN},\alpha,\alpha\rangle_{1,2} + A_{-}\,|\mathrm{ODD},\alpha,\alpha\rangle_{1,2}$$

$$|\mathrm{EVEN},\alpha,\alpha\rangle_{1,2} = \frac{(|\alpha,\alpha\rangle + |{-\alpha}{-\alpha}\rangle)}{\sqrt{2(1+x^4)}}$$

$$|\mathrm{ODD},\alpha,\alpha\rangle_{1,2} = \frac{(|\alpha,\alpha\rangle - |{-\alpha}{-\alpha}\rangle)}{\sqrt{2(1-x^4)}}$$

$$A_{\pm} = (\varepsilon_{+} \pm \varepsilon_{-})\sqrt{\frac{(1 \pm x^2)}{2}}$$

$$\varepsilon_{\pm} = \frac{A_{+}}{\sqrt{2(1+x^2)}} \pm \frac{A_{-}}{\sqrt{2(1-x^2)}}$$

$$|\,E\rangle_{3,4,5} = \frac{1}{\sqrt{2(1-x^8)}}\,[|\sqrt{2}\alpha,\alpha,\alpha\rangle_{3,4,5} - |{-\sqrt{2}\alpha},{-\alpha},{-\alpha}\rangle_{3,4,5}\,]$$

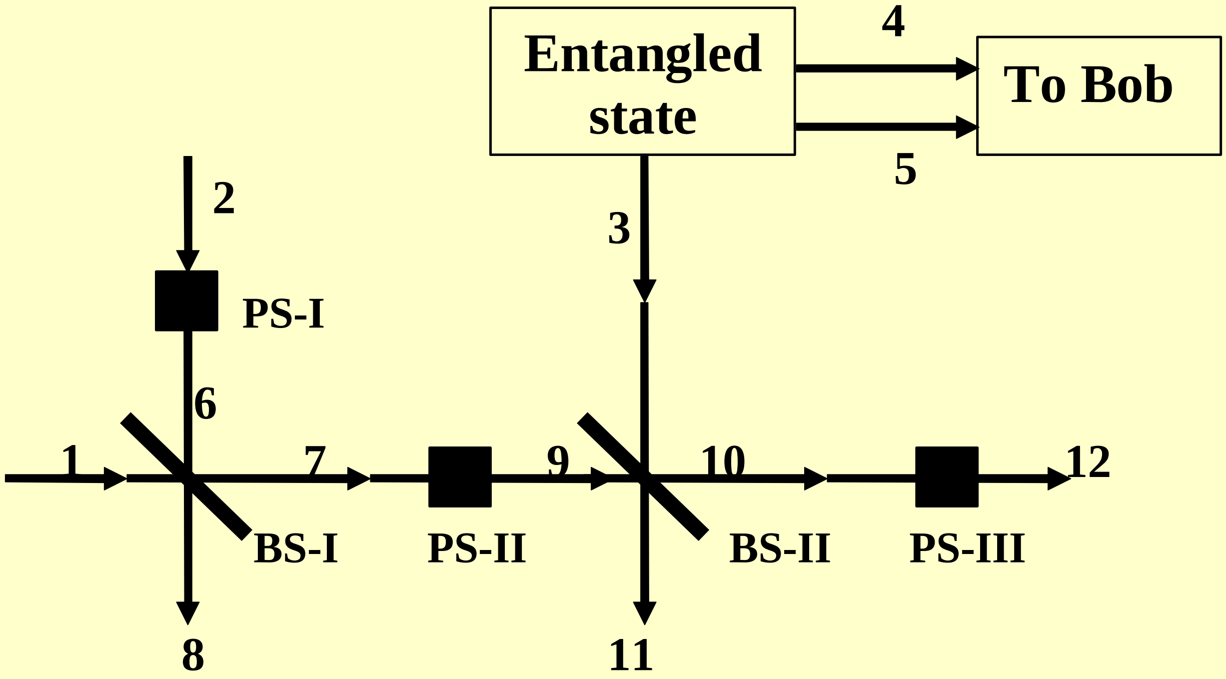


Figure 1. Numerals 1, 2, ..., 6 refers to modes. Entangled states of modes 1 and 2 are to be teleported to bob. Out of $|\mathrm{E}\rangle_{3,4,5}$, state in mode 3 goes to Alice while states in modes 4 and 5 go to Bob. Alice (*i*) converts state 2 to state 6 by using phase shifter PS-I, (*ii*) mixes state 6 with state 1 using a beam splitter BS-I, (*iii*) modifies output in 7 to state 9 using phase shifter PS-II, (*iv*) mixes state 9 with state 3 using beam splitter BS-II (*v*) modifies output in 10 to state 12 using phase shifter PS-III, and (*vi*) performs photon counting in mode 11 and 12. The results of photon counting, conveyed to Bob by a classical channel helps him construct the entangled state by making unitary transformation on state of mode 4 and 5

$$\boldsymbol{PSI}: |\alpha\rangle_2 \to |-i\alpha\rangle_6, \quad \boldsymbol{PSII}: |\eta\rangle_7 \to |-i\eta\rangle_9, \boldsymbol{PSIII}: |\xi\rangle_{10} \to |-i\xi\rangle_1$$

$$\boldsymbol{BSI}: |\beta,\gamma\rangle_{1,6} \to \left|\frac{1}{\sqrt{2}}(\beta + i\gamma), \frac{1}{\sqrt{2}}(\gamma + i\beta)\right\rangle_{7,8}, \boldsymbol{BSII}: |\lambda,\delta\rangle_{9,3} \to \left|\frac{1}{\sqrt{2}}(\lambda + i\delta), \frac{1}{\sqrt{2}}(\delta + i\lambda)\right\rangle_{10,11}$$

Possible cases of counts in modes 11 & 12 are

I – zero, zero   II – nonzero even, zero

III – zero, nonzero even,   IV – odd, zero,

V – zero, odd

Unitary Transformations are $U_I = U_{IV} = 1$

$$U_{II,III} = |EVEN,\alpha,\alpha\rangle\langle ODD,\alpha,\alpha| \pm |ODD,\alpha,\alpha\rangle\langle EVEN,\alpha,\alpha|$$

$$U_v = |EVEN,\alpha,\alpha\rangle\langle EVEN,\alpha,\alpha| - |ODD,\alpha,\alpha\rangle\langle ODD,\alpha,\alpha|$$

Teleported States are

$$|T\rangle_I = |ODD,\alpha,\alpha\rangle,$$

$$|T\rangle_{II} = |T\rangle_{III} = A_+\sqrt{\frac{1-X^4}{1+X^4}}\,|EVEN,\alpha,\alpha\rangle + A_-\sqrt{\frac{1+X^4}{1-X^4}}\,|ODD,\alpha,\alpha\rangle$$

$$|T\rangle_{IV} = |T\rangle_V = A_+\,|EVEN,\alpha,\alpha\rangle + A_-\,|ODD,\alpha,\alpha\rangle$$

This gives $F_I = |A_-|^2, F_{IV} = F_V = 1,$

$$F_{IV} = F_V = 1 - \frac{1 - x^4(|A_+|^2 - |A_-|^2)}{1 + x^8 - 2x^4(|A_+|^2 - |A_-|^2)}$$

$$F_{av} = 1 - \frac{2x^4 |A_+|^2 (x^4 |A_-|^2 - |A_+|^2)}{(1+x^4)^2}$$

$$F_{av,min} = 1 - \frac{2x^4}{(1+x^4)^2}$$

For $|\alpha|^2 = 2$, $F_{av,min} = 1 - 0.98 \times 10^{-4}$

TELEPORTATION OF 2-QUBIT INFORMATION ENCRYPTED IN SINGLE MODE SUPERPOSED COHERENT STATE

- **Information**: $$|I\rangle = \varepsilon_0|\alpha\rangle + \varepsilon_1|i\alpha\rangle + \varepsilon_2|-\alpha\rangle + \varepsilon_3|-i\alpha\rangle$$ $$= a_0|\alpha_0\rangle + a_1|\alpha_1\rangle + a_2|\alpha_2\rangle + a_3|\alpha_3\rangle$$
- $|\alpha_{0,1,2,3}\rangle$ **are superposed coherent states with 4n, 4n+1, 4n+2, 4n+3 photons respectively.**
- **Entangled state:** $$|E\rangle = |\alpha,\alpha\rangle + |i\alpha,i\alpha\rangle + |-\alpha,-\alpha\rangle + |-i\alpha,-i\alpha\rangle$$
- **Photon counting is done in four modes.**
- **Almost perfect fidelity is obtained if three counts are nonzero and one count zero**
- **Minimum average fidelity is ≥ 0.99 for $|\alpha| \geq 3.2$**

SCHEME FOR TELEPORTATION OF 2-QUBIT INFORMATION

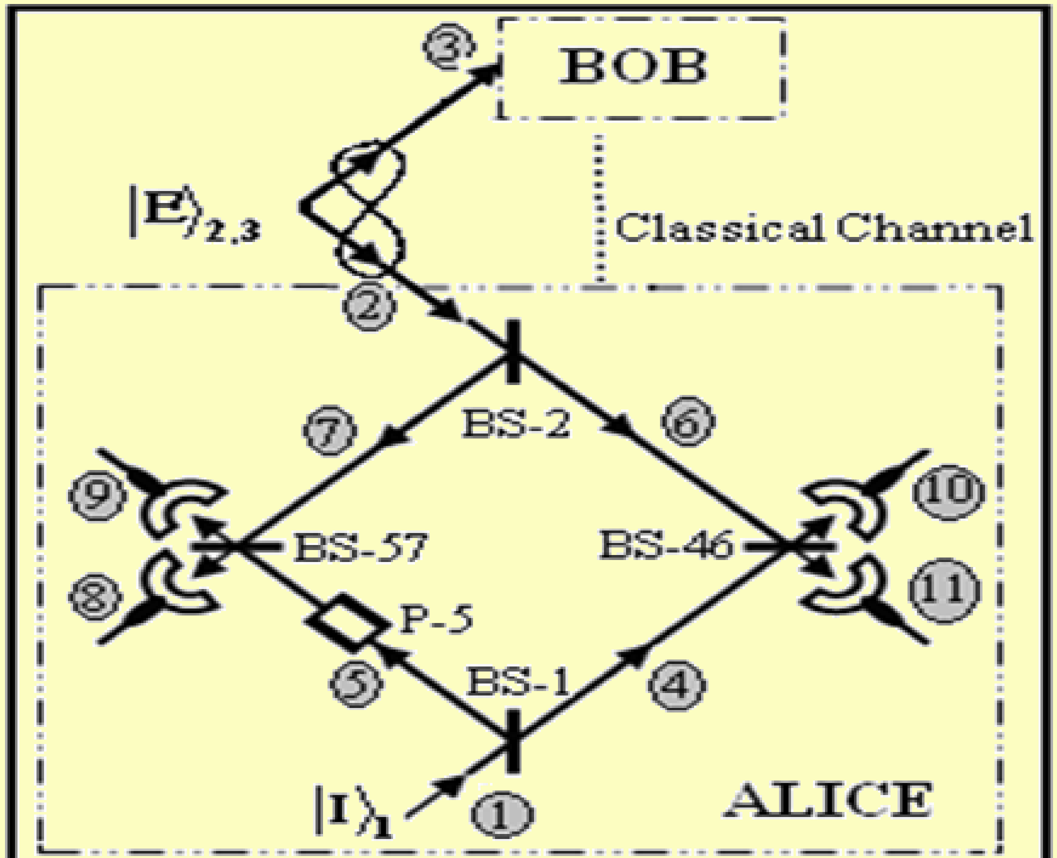


Fig.1: Encircled numbers denotes modes.
BS is 50/50 symmetric beam splitter.
P is $-\pi/2$ phase shifter.

• Taking Decoherence into Account

Following van Enk & Hirota, we also assume that

$$|\alpha\rangle|0\rangle_R \to |\sqrt{\eta}\alpha\rangle|\sqrt{1-\eta}\alpha\rangle_R, \quad \text{R is reservoir}$$

Information

$$|\alpha\rangle_0|0\rangle_R \to \quad |\tilde{1}\rangle_{0,R0} = \varepsilon_+|\tilde{\alpha}\rangle_0|k\rangle_{R0} + \varepsilon_-|-\alpha\rangle_0|-k\rangle_{R0}$$

$$\tilde{\alpha} = \sqrt{\eta}\alpha, \quad k = \sqrt{1-\eta}\alpha,$$

R0 is reservoir coupled to the mode 0.

If R0 contains more than one modes

$$|k\rangle_{R0} \to \Pi_i|k_i\rangle_{R0i}, \quad |k|^2 = \Sigma_i\ |k_i|^2 = (1-\eta)|\alpha|^2$$

Entangled State

$$|E\rangle_{1,2,R12} = \frac{1}{2(1-x^4)}[|\alpha\rangle_1|\alpha\rangle_2|K\rangle_{R12} - |-\alpha\rangle_1|-\alpha\rangle_2|-K\rangle_{R12}]$$

$$|K\rangle_{R12} \equiv \Pi_i|k_{1i}\rangle_{R1i}|k_{2i}\rangle_{R2i}, \quad \Sigma_i\ |k_{1i}|^2 = \Sigma_i\ |k_{2i}|^2 = (1-\eta)|\alpha|^2$$

The calculations are long and difficult but somewhat straight forward and give complicated results which we show graphically. It is interesting that even in the case of zero counts in both modes the minimum assured fidelity is nozero. The fidelity is zero only if the noise is zero and the information is an even state.

For nonzero counts, Minimum Assured Fidelity decreases with increase in $|\alpha|^2$ for low noise. For high noise however the Fidelity increases attains a maximum value and then decreases. The average Fidelity depends appreciably on Information for low values of $|\alpha|^2$ only.

Let us put

$$A_+ = \cos(\theta/2), \quad A_- = \sin(\theta/2)$$

Case with Nonzero Even Counts

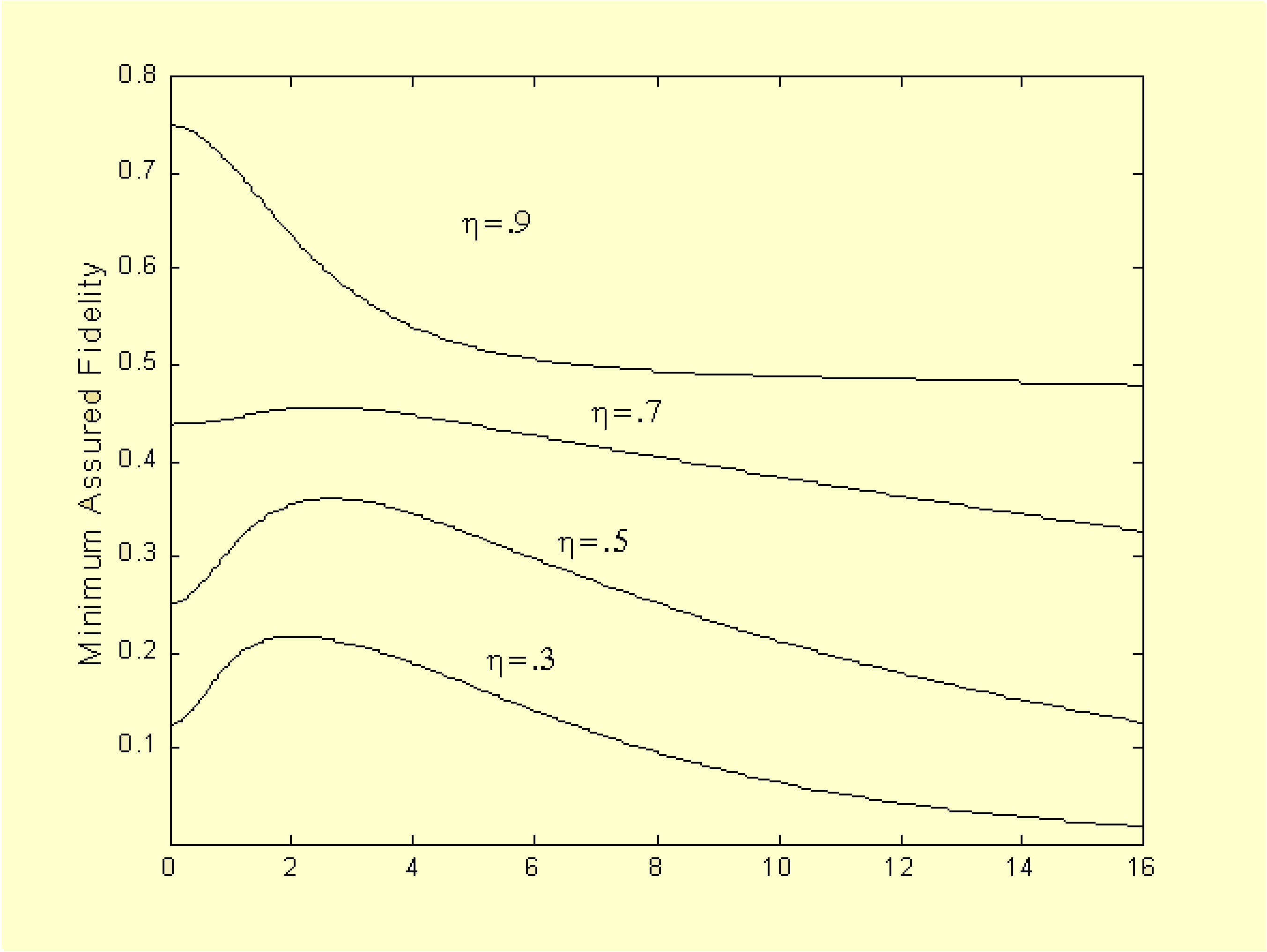


Figure 4: Variation of MAF with $|\alpha|^2$ for different values of η

$\eta > 0.721$: F decreases uniformly

$\eta < 0.721$: F has a Maximum, which decreases as noise increases

Case with Odd Counts

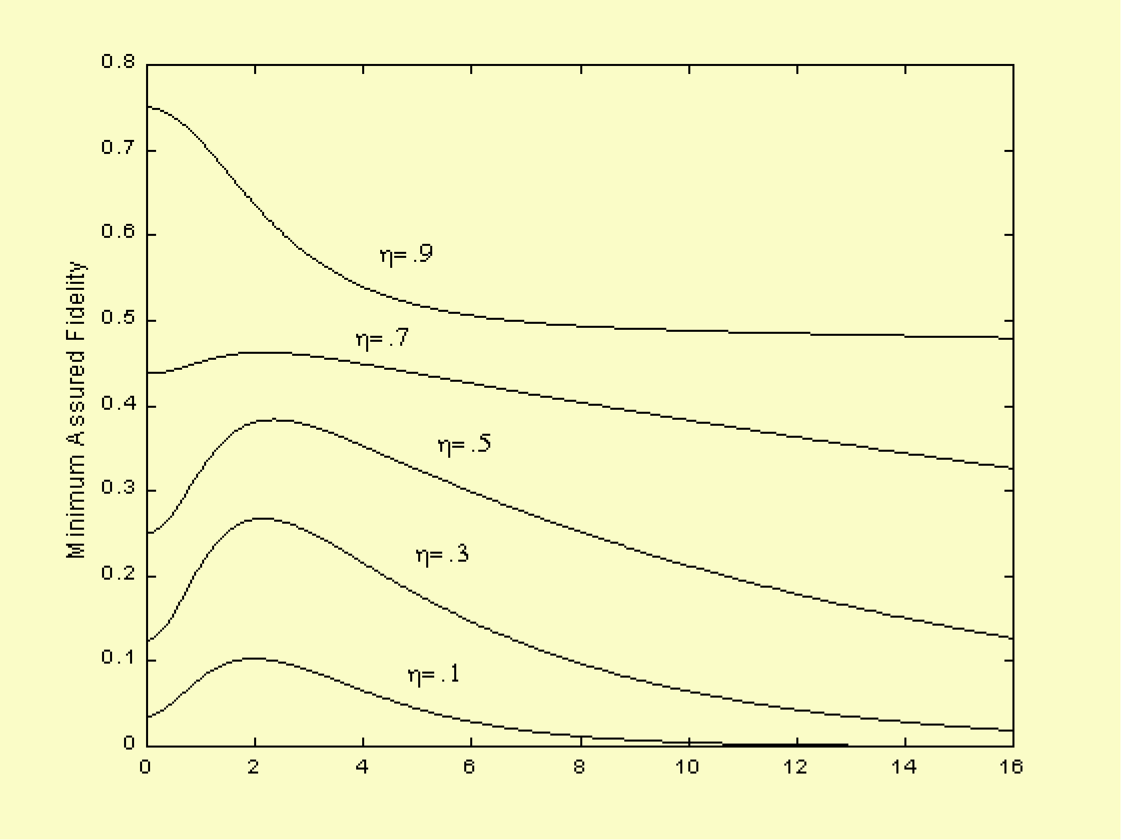


Figure 6: Variation of MAF with $|\alpha|^2$ for different values of η

Critical value of η = 0.738

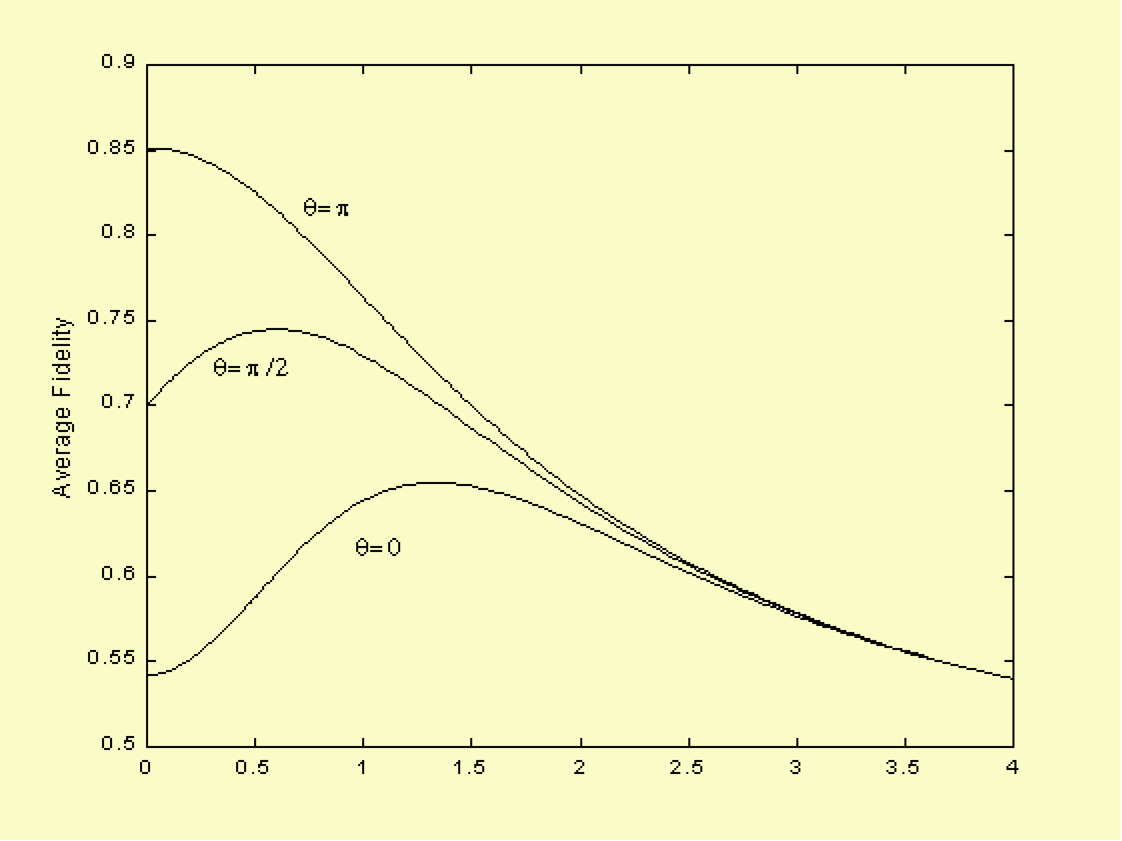


Figure7: Variation of average fidelity with $|\alpha|^2$ for different values of θ at $\eta = 0.9$.

Entanglement Diversion

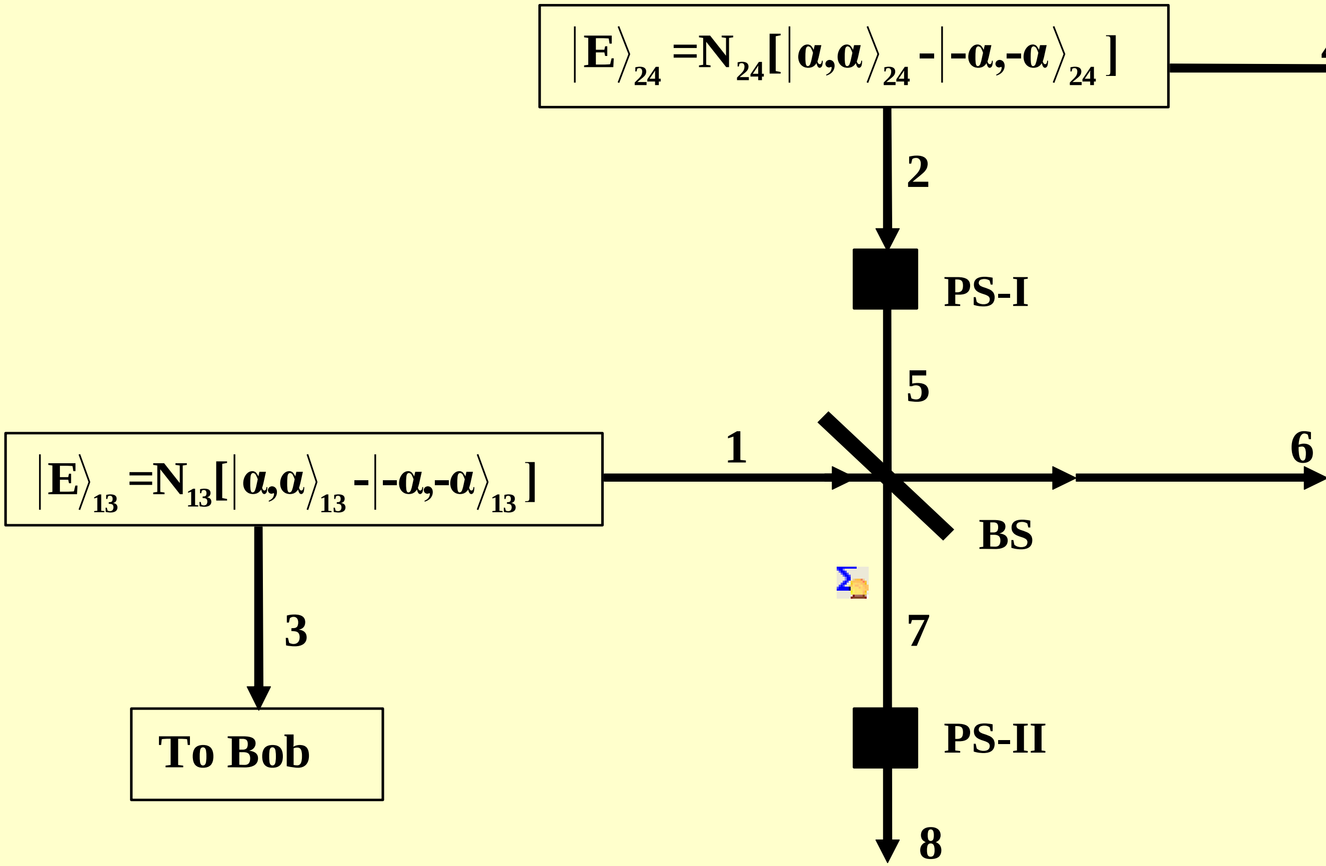


Figure 1. Numerals 1, 2, ..., 8 refers to modes. States in modes 1 and 2 are with Alice while state 3 and 4 are with Bob and Clair respectively. Alice (*i*) converts state 2 to state 5 using phase shifter PS-I, (*ii*) mixes state 1 with state 5 using a beam splitter BS, (*iii*) modifies output in 7 to state 8 using phase shifter PS-II, and (*iv*) performs photon counting in modes 6 and 8. The result, conveyed to Bob helps him to perform a unitary transformation on state 3 and 4 and generate an exact replica of entangled state.

LONG DISTANCE QT WITH PERFECT FIDELITY AND AS GOOD SUCCESS AS DESIREED USING REPEATED ATTEMPTS

- For QT using ECS
  Photon-counting in two modes – one count always zero
- Fidelity F = 0 when the other count is zero
  F= 1 when other counts are odd
  MASFI ≈ 1 when other counts is nonzero-even and $|\alpha|^2$ is appreciable
- In photon-counting information is destroyed and measurements cannot be repeated to increase success
- Repeated attempts can be possible only if the information is not destroyed in measurements
- We present another scheme of QT of state of a qubit with Alice on to a qubit with Bob using
  (i) a light pulse in even coherent state, $N_+[|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle], N_+ \equiv [2(1+e^{-4|\alpha|^2})]^{-1/2}$
  (ii) two light pulses in coherent state $|\alpha\rangle$
  (iii) Beam splitter assemblies and light detectors.
- This suits long distance Quantum Teleportation serving as a link between two quantum processors at long distances the photons fly and the quantum state of qubit is teleported.

- Measurements done in two steps

  (i) preliminary detection of light (dark/lighted states) in four modes, two with Alice and two with Bob, Bob's one c-bit result conveyed to Alice

  (ii) measurement by Alice on her qubit.

- Unique feature of this scheme – success/failure indicated by the first measurement.

- In the first step of measurement, one of the two modes with Alice and with Bob are always dark.

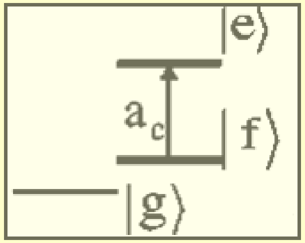
  If the other two are lighted, success is indicated

  If one more or all modes are dark, failure is indicated

- If success is indicated, Alice can perform second step of measurement and convey to Bob 2 c-bit information about the Unitary transformation to be done by Bob.

- If failure is indicated, the qubit states are not affected in spite of initial entanglement with light beams, and a second attempt may be started with a fresh even coherent state.

- Idea is based on results of Wang and Duan (005, PRA) for reflection of light in a coherent state from a single atom cavity.
- Atom-ground state $|g\rangle$, very low lying excited state $|f\rangle$, and an excited state $|e\rangle$.

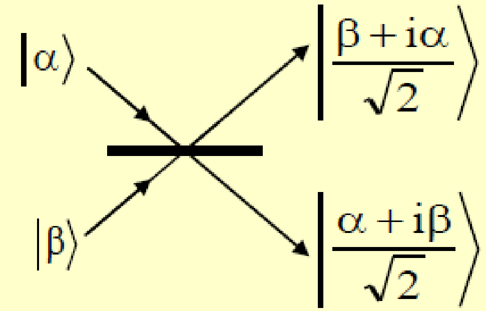


- States ($|f\rangle$, $|e\rangle$) are resonantly coupled to the cavity mode, which is driven by a resonant optical pulse in a coherent state $|\alpha\rangle$.
- States ($|g\rangle$, $|e\rangle$) are largely detuned.
- Cavity is one sided and if atom is in state $|g\rangle$ pulse in state is $|\alpha\rangle$ reflected without any appreciable interaction with atom but with a phase change giving output state $|-\alpha\rangle$.
- For atom in state $|f\rangle$, however, no phase change occurs and reflected light is in state $|\alpha\rangle$.
- Thus, $|g,\pm\alpha\rangle_{c,in} \rightarrow |g,\mp\alpha\rangle_{c,out}$; $|f,\pm\alpha\rangle_{c,in} \rightarrow |f,\pm\alpha\rangle_{c,out}$
- If atom is prepared in state $a|g\rangle + b|f\rangle$, $|a|^2 + |b|^2 = 1$

$$(a|g\rangle + b|f\rangle)|\alpha\rangle \rightarrow a|g\rangle|-\alpha\rangle + b|f\rangle|\alpha\rangle$$

and the atom and light becomes entangled.

- Other requirements are three Beam splitter assemblies (BSA) and four light detector which discriminates dark state (no photons) with light state (which have some photons).

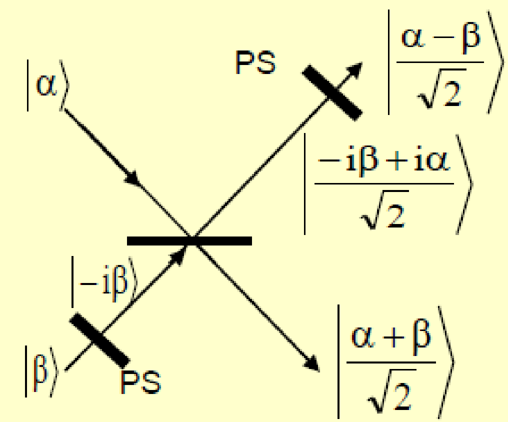


- A symmetric beam splitter converts coherent states,

$$|\alpha,\beta\rangle \xrightarrow{\text{BS}} \left|\frac{\alpha+i\beta}{\sqrt{2}}, \frac{\beta+i\alpha}{\sqrt{2}}\right\rangle$$

- If we make use of phase shifters (PS) which changes, $|\xi\rangle \xrightarrow{-\pi/2\ \text{PS}} |-i\xi\rangle$, we can construct a beam splitter assembly which converts states,

$$|\alpha,\beta\rangle \xrightarrow{\text{BSA}} \left|\frac{\alpha+\beta}{\sqrt{2}}, \frac{\alpha-\beta}{\sqrt{2}}\right\rangle$$



- When even coherent state, $N_+[|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle]$ is mixed with the vacuum using such a BSA, we get entangled coherent state,

$$N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle],\ N_+ \equiv [2(1+e^{-4|\alpha|^2})]^{-1/2}$$

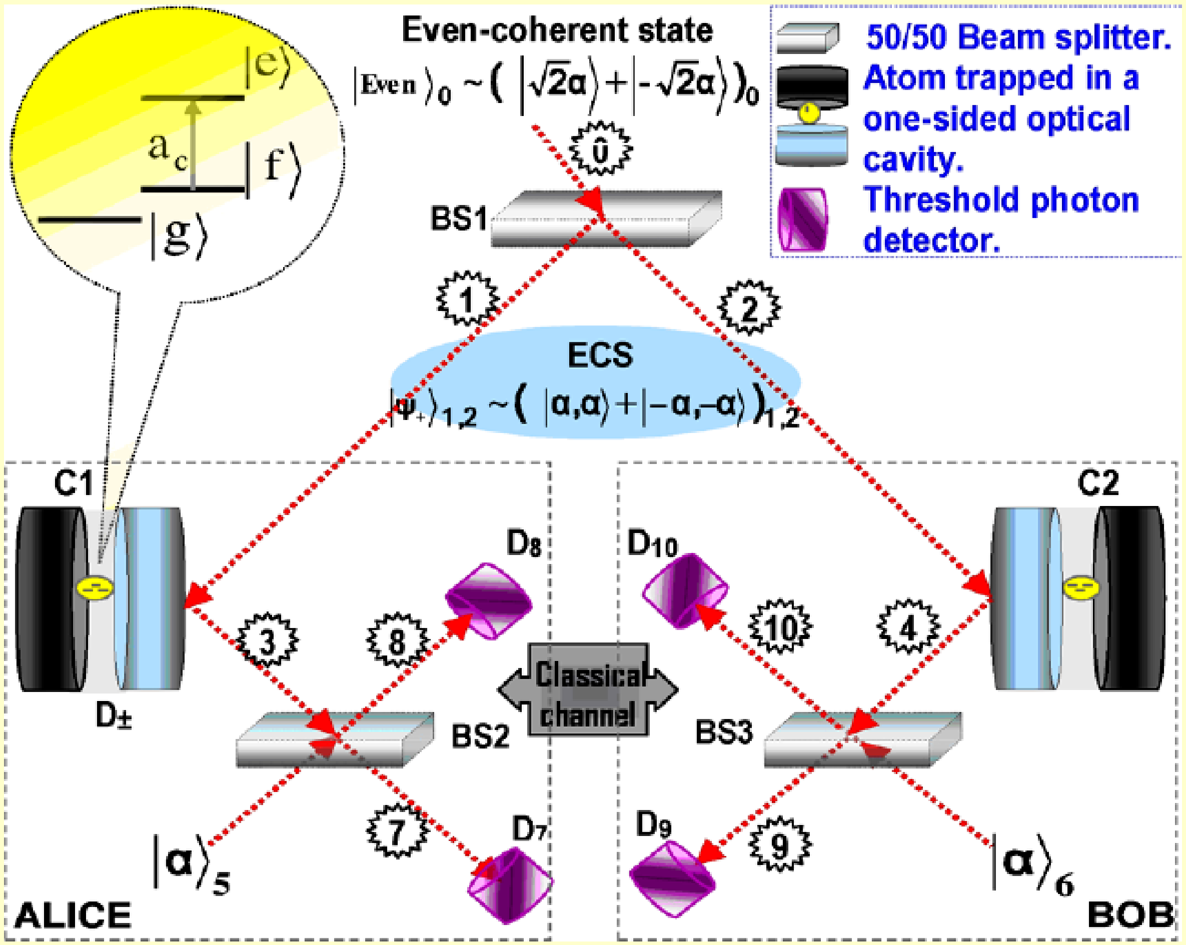


Fig.1: Scheme for teleportation of atomic-state trapped in cavity C1 to second atom in a distant cavity C2. Entangled coherent state (ECS) in modes 1 and 2 is produced by illuminating beam splitter BS1 with an Even-coherent state in mode 0. Inset shows level structure of atom. $D_{1,2,3,4}$ are photon detectors, atom in cavity C1 is measured in diagonal basis ±. Encircled numbers represent the quantum mode.

- The initial state is

$$|\psi\rangle_{c1,c2,1,2,5,6} = [a|g\rangle + b|f\rangle]_{c1} \frac{1}{\sqrt{2}}[|g\rangle + |f\rangle]_{c2} N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}|\alpha\rangle_5|\alpha\rangle_6$$

$$= \frac{1}{\sqrt{2}} N_+[a|g,g\rangle + a|g,f\rangle + b|f,g\rangle + b|f,f\rangle]_{c1,c2}[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}|\alpha\rangle_5|\alpha\rangle_6$$

- If atomic state is $|g,g\rangle$ or $|f,f\rangle$,

$$|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle \to |\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle$$ remains unaffected

If atomic state is $|g,f\rangle$ or $|f,g\rangle$

$$|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle \to |-\alpha,\alpha\rangle + |\alpha,-\alpha\rangle$$

$$\to |\alpha,-\alpha\rangle + |-\alpha,\alpha\rangle$$

- The state is then,

$$|\psi\rangle_{c1,c2,1,2,5,6} = \frac{1}{\sqrt{2}} N_+[(a|g,g\rangle + b|f,f\rangle)_{c1,c2}(|-\alpha,-\alpha\rangle + |\alpha,\alpha\rangle)_{3,4}|\alpha\rangle_5|\alpha\rangle_6$$

$$+ (a|g,f\rangle + b|f,g\rangle)_{c1,c2}(|-\alpha,\alpha\rangle + |\alpha,-\alpha\rangle)_{3,4}|\alpha\rangle_5|\alpha\rangle_6]$$

- If modes 3 and 5 are mixed to generate modes 7 and 8, and modes 4 and 6 are mixed to generate modes 9 and 10,

$$|\psi\rangle_{c1,c2,1,2,5,6} = \frac{1}{\sqrt{2}} N_+[(a|g,g\rangle + b|f,f\rangle)_{c1,c2}\left(\left|0,-\sqrt{2}\alpha,0,-\sqrt{2}\alpha\right\rangle + \left|\sqrt{2}\alpha,0,\sqrt{2}\alpha,0\right\rangle\right)_{7,8,9,10}$$

$$+ (a|g,f\rangle + b|f,g\rangle)_{c1,c2}\left(\left|0,-\sqrt{2}\alpha,\sqrt{2}\alpha,0\right\rangle + \left|\sqrt{2}\alpha,0,0,-\sqrt{2}\alpha\right\rangle\right)_{7,8,9,10}]$$

- Each coherent state $|\beta\rangle$ can be written as superposition of dark state $|D\rangle = |0\rangle$ and as lighted state $|L,\beta\rangle$ whose expansion has no terms in $|0\rangle$ writing,

$$|\beta\rangle = e^{-\frac{1}{2}|\beta|^2}|D\rangle + (1-e^{-|\beta|^2})^{1/2}|L,\beta\rangle$$

- Out of 4 modes 7,8,9 and 10, two are always dark and the other two may be dark or lighted.

- For two dark and two lighted states, the qubit states are given below,

7	8	9	10
×	✓	×	✓
✓	×	✓	×
×	✓	✓	×
✓	×	×	✓

$$|\psi\rangle_{c1,c2} = a|g,g\rangle + b|f,f\rangle = \tfrac{1}{\sqrt{2}}[|+\rangle_{c1}|I\rangle_{c2} + |-\rangle_{c1}\sigma_z|I\rangle_{c2}]$$

$$|\psi\rangle_{c1,c2} = a|g,f\rangle + b|f,g\rangle = \tfrac{1}{\sqrt{2}}[|+\rangle_{c1}\sigma_x|I\rangle_{c2} + |-\rangle_{c1}\sigma_x\sigma_z|I\rangle_{c2}]$$

$$|\pm\rangle = \tfrac{1}{\sqrt{2}}[|g\rangle + |f\rangle]$$

- The result is success with fidelity F=1 after unitary transformation by Bob which depends on results of second measurement by Alice $|\pm\rangle$ of on her qubit.

- For results- one lighted state or no lighted state the states of the two qubits is,

$$\sim (a|g,g\rangle + b|f,f\rangle) + (a|g,f\rangle + b|f,g\rangle) = (a|g\rangle + b|f\rangle)(|g\rangle + |f\rangle)$$

i.e.., the information state with Alice is not destroyed and a similar attempt can be repeated again.

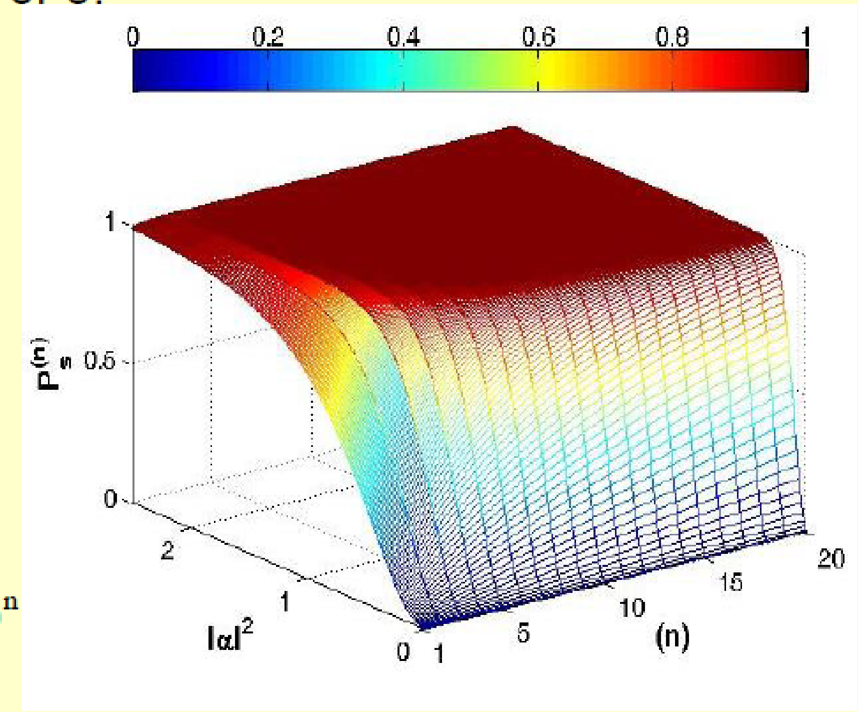
- The success of QT in one attempt is $P_s = (1-x^2)^2/(1+x^4)$ with $x \equiv e^{-|\alpha|^2}$ and this is equal to 0.734, 0.963 or 0.998 for $|\alpha|^2 = 1$, 2 or 3.

- We used non-maximally entangled coherent state . Use of maximally entangled coherent state (MES) changes information for one lighted state case and requires operation σ_z on the two qubits. Also, it is interesting to note that for MES, four dark states can does not arise.

$$\text{Success} = (1-x^2)^2/(1-x^4) = (1-x^2)/(1+x^2)$$

- Success for QT in 'n' attempts is, $P_S^{(n)} = 1-(P_f)^n$

- For $|\alpha|^2 = 1$ success becomes almost equal to unity in three attempts .



Variation of success probability ($P_s^{(n)}$) for different numbers of attempts 'n' with $|\alpha|^2$.

Thank You