Quantum information tools with single ions and photons

- Introduction
- Fundamentals: ion traps, quantum bits, quantum gates
- Implementations: 2-qubit gates, teleportation, …
- Quantum networks: atom-photon interfaces

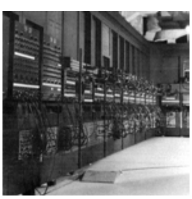
Jürgen Eschner

quanten photonik

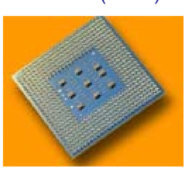
UNIVERSITÄT DES SAARLANDES

QIPA-2011 School, HRI, Allahabad, 20 February 2011

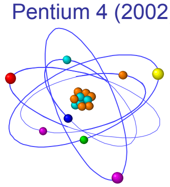
Towards quantum technology (Moore's Law)



ENIAC (1947)

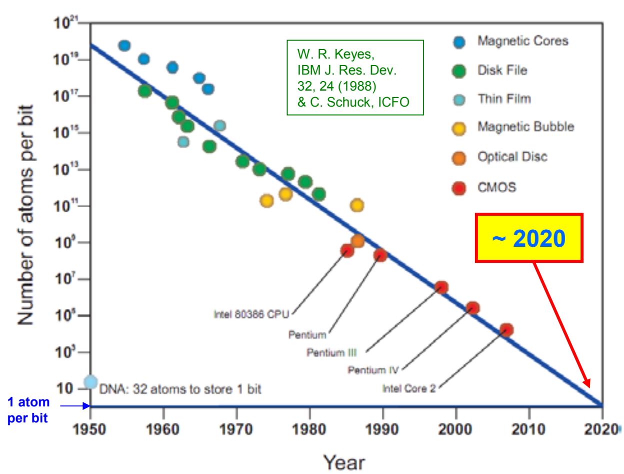


Pentium 4 (2002)



1 atom

How many atoms per bit of information?



- **Quantum effects will play a role – and open up new avenues**

Quantum information with single quantum systems

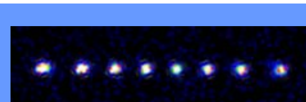
Applications in informatics and physics

- P. Shor, 1994: **factorization of large numbers is polynomial on a quantum computer, exponential on a classical computer**
- L. Grover, 1997: **data base search $N^{1/2}$ quantum queries, N classical**
- **simulation of Schrödinger equations or any unitary evolution**
- **quantum cryptography / repeaters / quantum links**
- **improved atomic clocks** (P. Schmidt et al., 2005)
- **(Gedanken-)experiments**
  fundamentals of QM, decoherence, entanglement

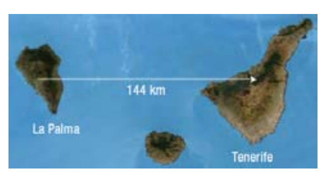
Fundamental phenomena ⇔ Quantum information ⇔ Technological applications

Quantum Information Technology

- **Quantum Computing & Simulation**
  - Factorization, data base search



- **Quantum Cryptography (QKD)**
- **Quantum Random Number Generation**
- **Quantum Communication**
  - Entanglement over large distance

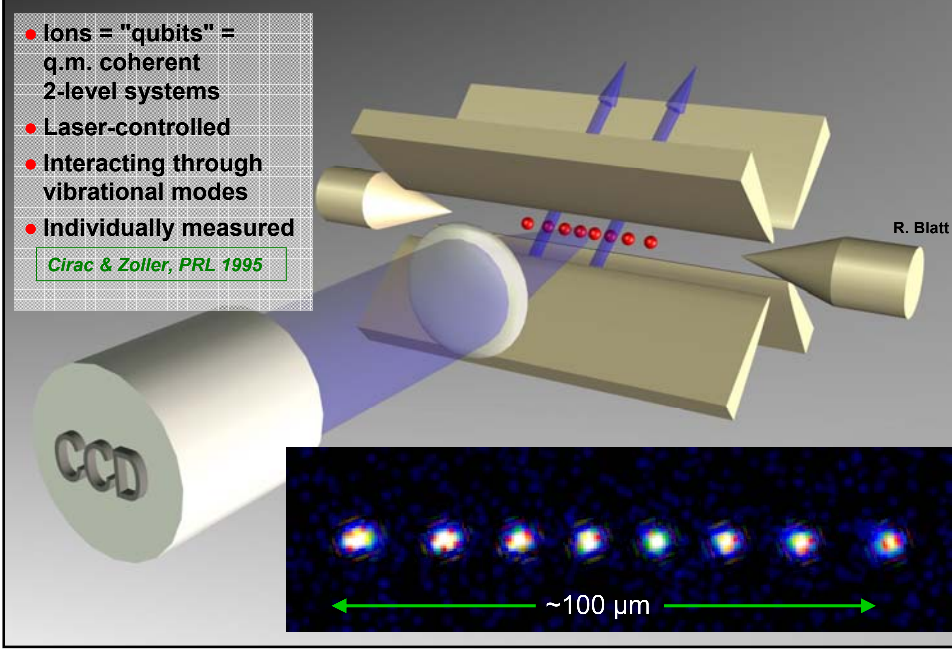


- **Quantum Metrology (clocks, sensors)**

QIP workhorse : linear Paul trap

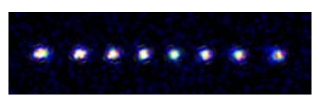
- Ions = "qubits" = q.m. coherent 2-level systems
- Laser-controlled
- Interacting through vibrational modes
- Individually measured

Cirac & Zoller, PRL 1995

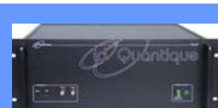


Quantum Optical Information Technology

- **Quantum Computing & Simulation**
  - Factorization, data base search



- **Quantum Cryptography (QKD)**
- **Quantum Random Number Generation**

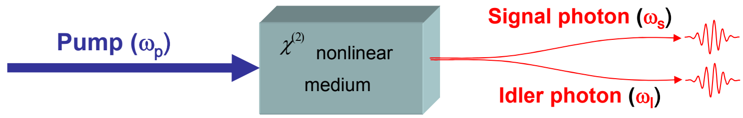


- **Quantum Communication**
  - Entanglement over large distance

- **Quantum Metrology (clocks, sensors)**

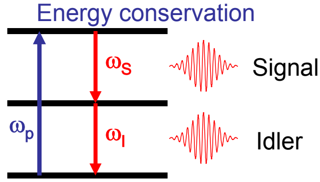
Quantum communication workhorse : SPDC

Spontaneous Parametric Down-Conversion



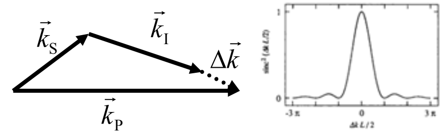
Phase matching conditions

Energy conservation



$\omega_P = \omega_S + \omega_I$

Momentum conservation

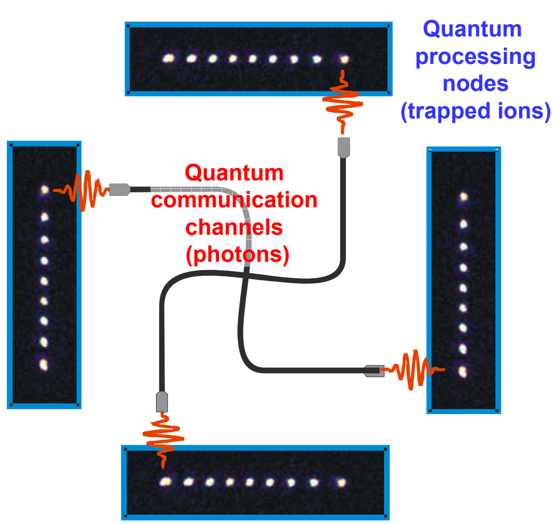


$\vec{k}_P = \vec{k}_S + \vec{k}_I + \Delta\vec{k}$

Type II phase matching: $H \rightarrow H + V$

Polarization entanglement: $|H_S\rangle|V_I\rangle - |V_S\rangle|H_I\rangle$

Integration : Quantum Network



Need:

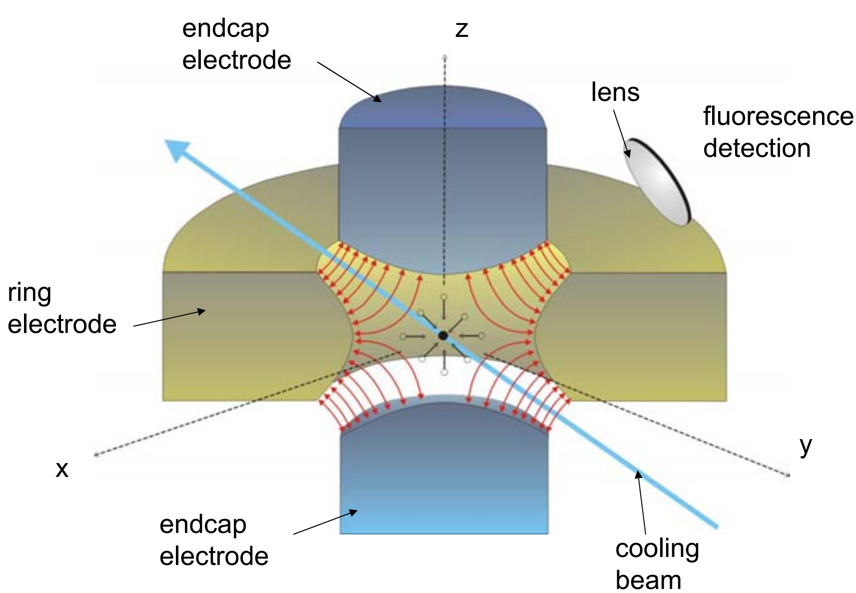
Atom - photon interfaces:
Coherently map states from one to the other

Distribute entanglement between distant nodes:
allows to scale up computing power of the system

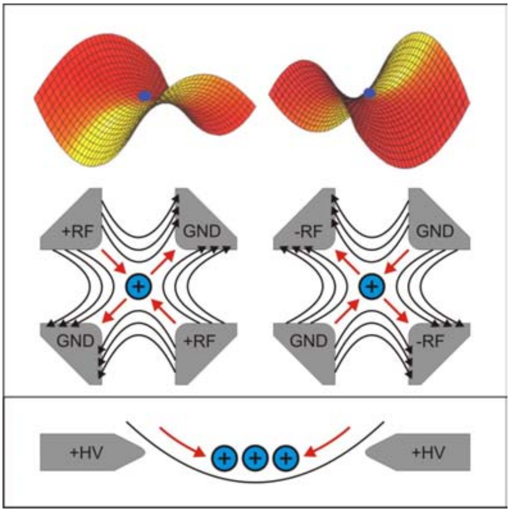
Cirac et al., PRL 1997
Kimble, Nature 2008

Ion traps

Classic Paul trap



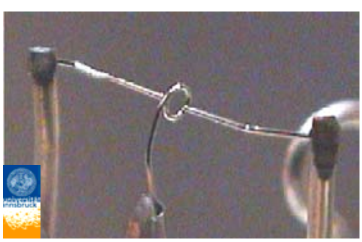
Trapping ions



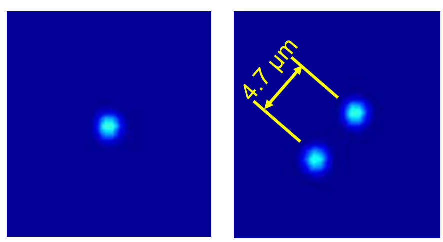
- Paul trap mechanism
  → Trapped charged particles
- Linear trap ~1 mm size
  → Ion strings
- Laser cooling
  → Localization << λ_{Laser}
- Coulomb repulsion
  → Interaction by common modes of motion

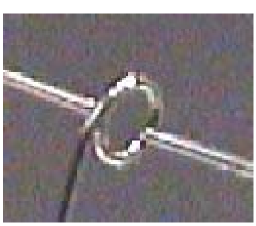
Legacy : circular trap for single ions

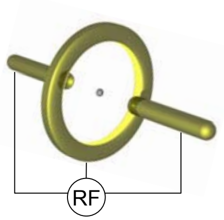
Miniature Paul trap



Single atoms (Ba^+) in in trap





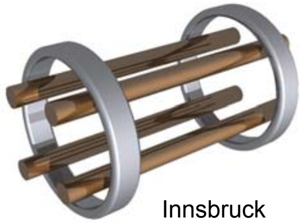


Ring Ø ~ 1 mm
RF ~25 MHz, ~ 500V

secular frequency ~ 1 MHz

Linear ion trap evolution

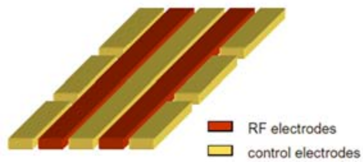
Paul mass filter



Innsbruck
Los Alamos

München

Boulder, Mainz, Aarhus

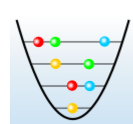


Boulder, Mainz, Innsbruck, …

Stationary / single-atom qubits

Examples
taken from:

ICFO
Institut
de Ciències
Fotòniques



Universität Innsbruck

Qubits in trapped ions (two-level systems, TLS)

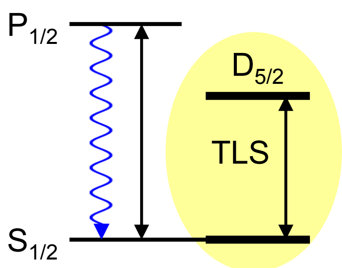
Encoding of quantum information requires longlived atomic states:

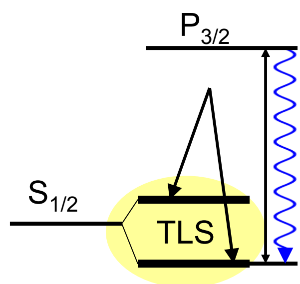
- optical transitions (forbidden transitions, intercombination lines) S – D transitions in earth alkalis: Ca^+, Sr^+, Ba^+, Ra^+, (Yb^+, Hg^+) etc.

  Innsbruck

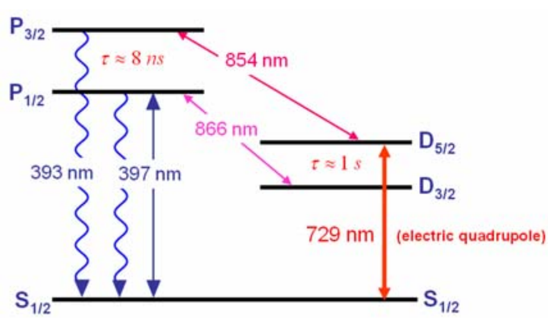
- microwave transitions (Hyperfine transitions, Zeeman transitions) earth alkalis: $^9Be^+$, $^{25}Mg^+$, $^{43}Ca^+$, $^{87}Sr^+$, $^{137}Ba^+$, $^{111}Cd^+$, $^{171}Yb^+$

  NIST

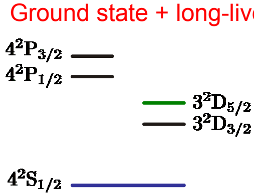




Qubits in $^{40}Ca^+$ ion



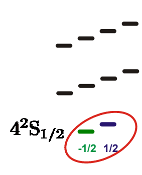
Ground state + long-lived excited state



$|1\rangle \rightarrow |D_{5/2}, m'\rangle$

$|0\rangle \rightarrow |S_{1/2}, m\rangle$

Zeeman substates of $4^2S_{1/2}$ level

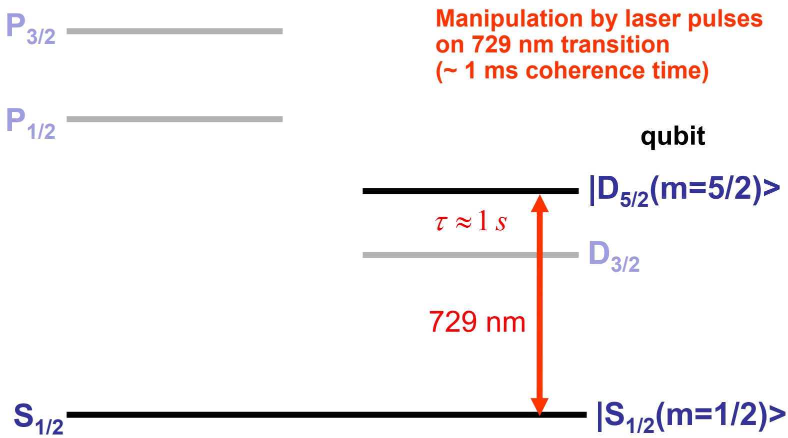


$|1\rangle \rightarrow |S_{1/2}, m = +\frac{1}{2}\rangle$

$|0\rangle \rightarrow |S_{1/2}, m = -\frac{1}{2}\rangle$

Optical qubit transition

Superpositions of $S_{1/2}(m=1/2)$ and $D_{5/2}(m=5/2)$ forms qubit



Qubit dynamics

Resonantly excited 2-level system

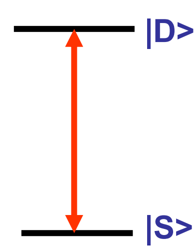
States $|S\rangle$ and $|D\rangle$ are atomic eigenstates: $|\psi\rangle = c_S|S\rangle + c_D|D\rangle$.

$$H = H_{\text{Atom}} + H_{\text{Atom-Laser}}$$
$$= \hbar\omega_S|S\rangle\langle S| + \hbar\omega_D|D\rangle\langle D| + (|S\rangle\langle D| + |D\rangle\langle S|)d\cdot\mathcal{E}\cos(\omega_L t)$$

After rotating wave approximation

$$H = \hbar\Delta|D\rangle\langle D| + \hbar\frac{\Omega}{2}(|S\rangle\langle D| + |D\rangle\langle S|)$$

where $\Delta = (\omega_D - \omega_S) - \omega_L$ and $\Omega = d\mathcal{E}/\hbar$.



Rabi oscillations

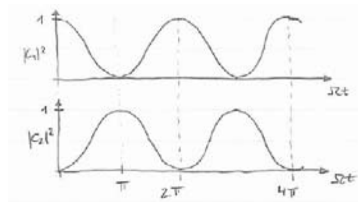
Schrödinger Equation $\begin{pmatrix} \dot{c}_S \\ \dot{c}_D \end{pmatrix} = -i \begin{pmatrix} 0 & \Omega/2 \\ \Omega/2 & \Delta \end{pmatrix} \begin{pmatrix} c_S \\ c_D \end{pmatrix}$

Simplest case $\Delta = 0$, "on resonance", $c_S(t=0) = 1$, $c_D(t=0) = 0$:

Solution $c_S(t) = \cos(\frac{\Omega}{2}t)$, $c_D(t) = -i\sin(\frac{\Omega}{2}t)$

Level populations $|c_{S,D}(t)|^2$ = probability of finding atom in state $|S\rangle, |D\rangle$ after excitation for time t show **Rabi oscillations**.

$|c_S(t)|^2 = \cos^2(t)$, $|c_D(t)|^2 = \sin^2(t)$



Note: in a single-atom experiment, the outcome of an individual experiment will be either $|S\rangle$ or $|D\rangle$. The populations are found by repeating the measurement (initial preparation, excitation, state determination) many times and doing statistics over the outcomes.

Rabi oscillations cont'd.

Special cases of coherent time evolution with excitation pulse of duration T:

$\Omega T = 2\pi$, "2π-pulse", populations are unchanged
$\Omega T = \pi$, "π-pulse", populations are inverted
$\Omega T = \pi/2$, "$\pi/2$-pulse", populations converted to superpositions and vice versa

Examples for $\pi/2$-pulse: $|S\rangle \to \frac{1}{\sqrt{2}}(|S\rangle - i|D\rangle)$, $|D\rangle \to \frac{1}{\sqrt{2}}(-i|S\rangle + |D\rangle)$

Note: the periodicity of the populations is with Ω, but the periodicity of the wave function is with $\Omega/2$, i.e. a 2π-pulse leaves the populations unchanged but it transforms $|\psi\rangle \to -|\psi\rangle$; only a 4π-pulse reproduces the original state. This is an important ingredient in quantum logical operations.

Note: a measurement of the state of the qubit in the basis of superpositions $|\pm\rangle = \frac{1}{\sqrt{2}}(|S\rangle \pm |D\rangle)$ (phase factors omitted) corresponds to a $\pi/2$-pulse followed by a state determination in the $|S, D\rangle$ basis.

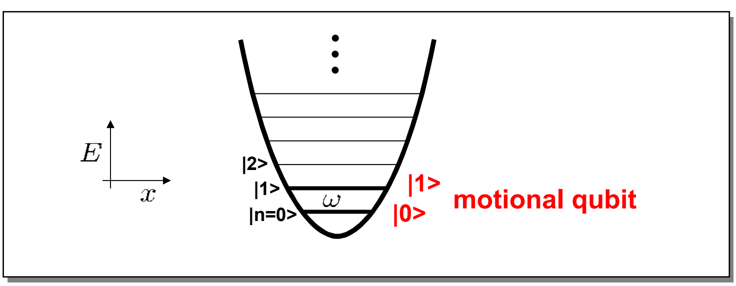
Motion and motional qubit

Quantized motion

$$E_{harm.osc.} = \frac{m}{2}\omega^2 x^2 + \frac{1}{2m}p^2$$

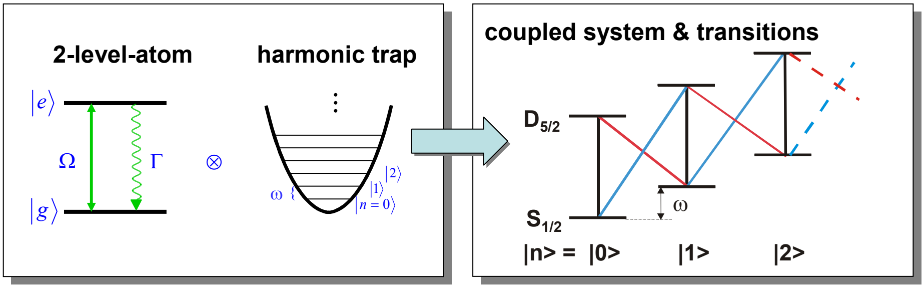
$$\rightarrow H_{mec} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

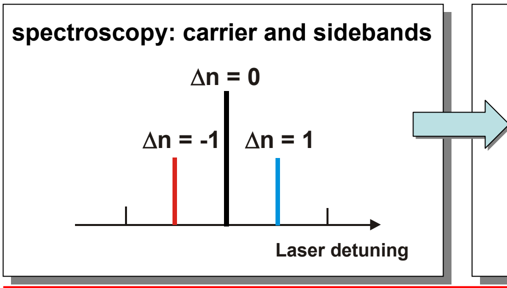
$$H_{mec}|n\rangle = E_n|n\rangle = \left(n + \frac{1}{2}\right)\hbar\omega|n\rangle$$



N ions $\rightarrow$ 3 N oscillators

Motional sidebands





Rabi frequencies

Carrier: Ω

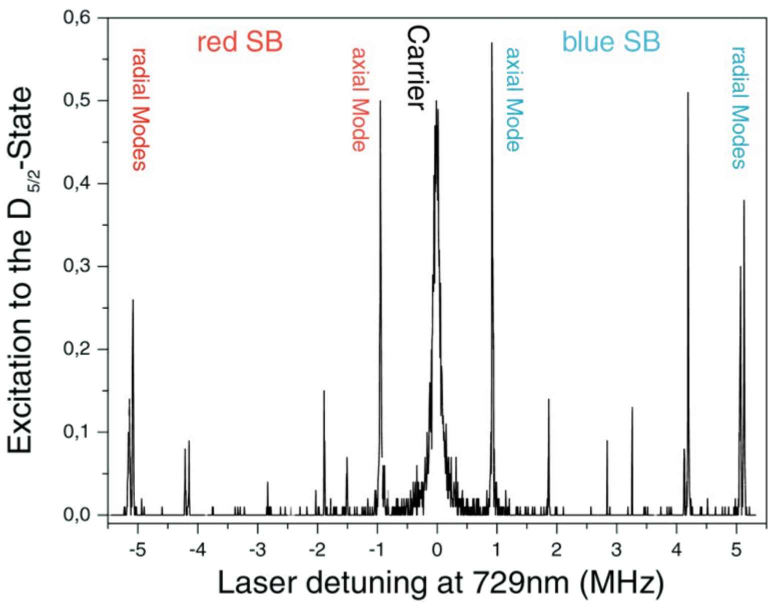
Red SB: $\Omega \cdot \eta \cdot \sqrt{n}$

Blue SB: $\Omega \cdot \eta \cdot \sqrt{n+1}$

$$\eta\sqrt{n} = k_L \langle n|x^2|n\rangle^{1/2} \ll 1 \ (*)$$

(*) Lamb-Dicke regime : ion localised to << laser wavelength : $\sqrt{\langle x^2 \rangle} \ll \lambda/2\pi$

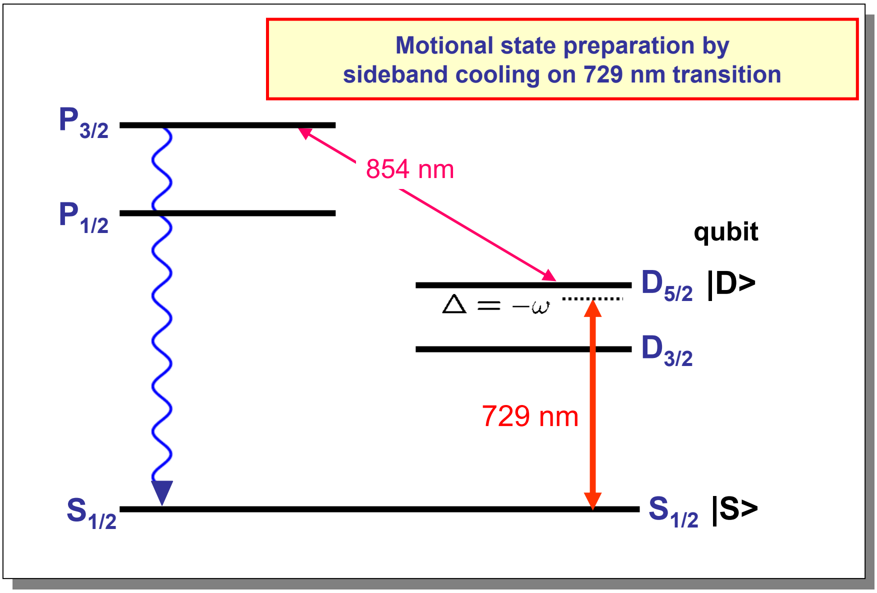
Excitation spectrum of the $S_{1/2}$ – $D_{5/2}$ transition



ω_{ax} = 1.0 MHz
ω_{rad} = 5.0 MHz

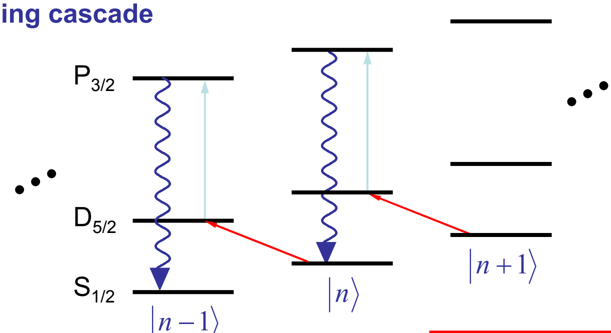
(only one Zeeman component)

Sideband cooling



Sideband cooling on $S_{1/2} – D_{5/2}$ transition

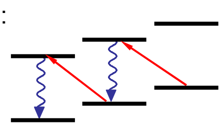
Cooling cascade



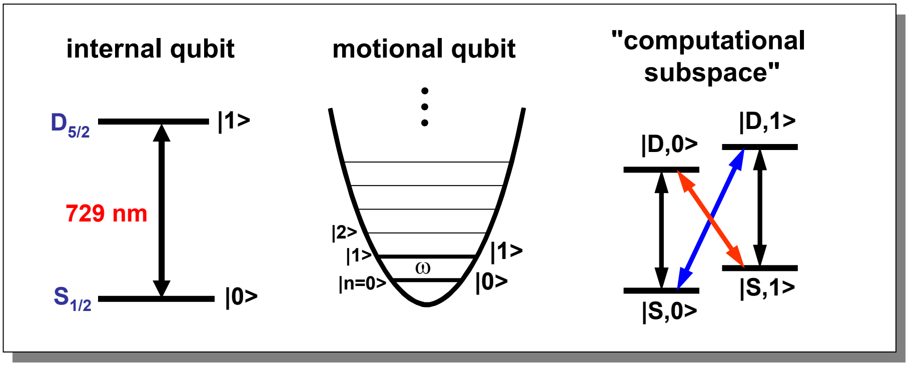
> 99.9% in |n=0>

- τ_D = 1 s : no spontaneous decay
- Quenching on $D_{5/2}$ - $P_{3/2}$ transition :

  Effective two-level system



Qubits in a single $^{40}Ca^+$ ion



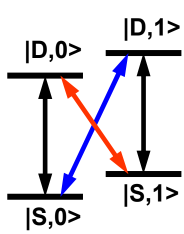
COHERENT LASER MANIPULATION (Rabi oscillations)

↔ **|S,n> ⇔ |D,n> : carrier transition** $(\Delta = 0)$

⇆ **|S,n> ⇔ |D,n±1> : sideband transition** $(\Delta = \pm\omega)$

First single-ion quantum gate: Monroe et al. (Wineland), PRL 75, 4714 (1995).

Qubit rotations in computational subspace



computational subspace

(levels with n>1 ignored)

Laser-driven transitions are described by unitary operators (if $\Omega >> \Gamma_D, \Gamma_{Laser}$) :

carrier:
$\theta = \Omega_C t$
$$R(\theta,\phi) = \exp\left[i\tfrac{\theta}{2}\left(e^{i\phi}\sigma^+ + e^{-i\phi}\sigma^-\right)\right]$$

red sideband:
$\theta = \Omega_{SB} t$
$$R_1^-(\theta,\phi) = \exp\left[i\tfrac{\theta}{2}\left(e^{i\phi}\sigma^+ a + e^{-i\phi}\sigma^- a^\dagger\right)\right]$$

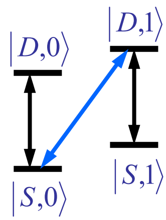
blue sideband:
$\theta = \Omega_{SB} t$
$$R_1^+(\theta,\phi) = \exp\left[i\tfrac{\theta}{2}\left(e^{i\phi}\sigma^+ a^\dagger + e^{-i\phi}\sigma^- a\right)\right]$$

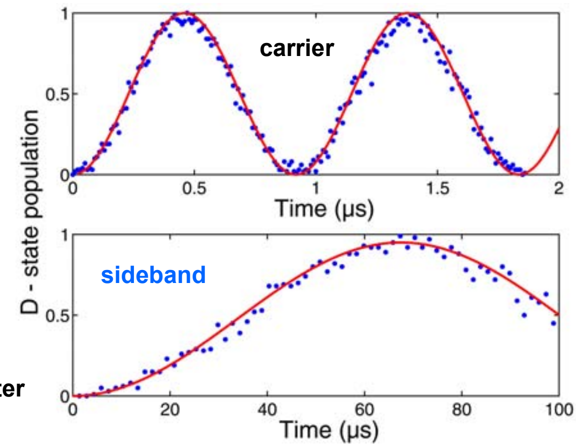
where $\sigma^+ = |D\rangle\langle S|,\ \sigma^- = |S\rangle\langle D$
$a = |0\rangle\langle 1|,\ a^\dagger = |1\rangle\langle 0|$

Example: excitation on blue sideband with $R^+(\frac{\pi}{2},0)$

$$|S,0\rangle \xrightarrow{R^+(\frac{\pi}{2},0)} \tfrac{1}{\sqrt{2}}(|S,0\rangle + i|D,1\rangle) \xrightarrow{R^+(\frac{\pi}{2},0)} i|D,1\rangle \xrightarrow{R^+(\frac{\pi}{2},0)} \tfrac{1}{\sqrt{2}}(-|S,0\rangle + i|D,1\rangle) \xrightarrow{R^+(\frac{\pi}{2},0)} -|S,0\rangle$$

Coherent state manipulation





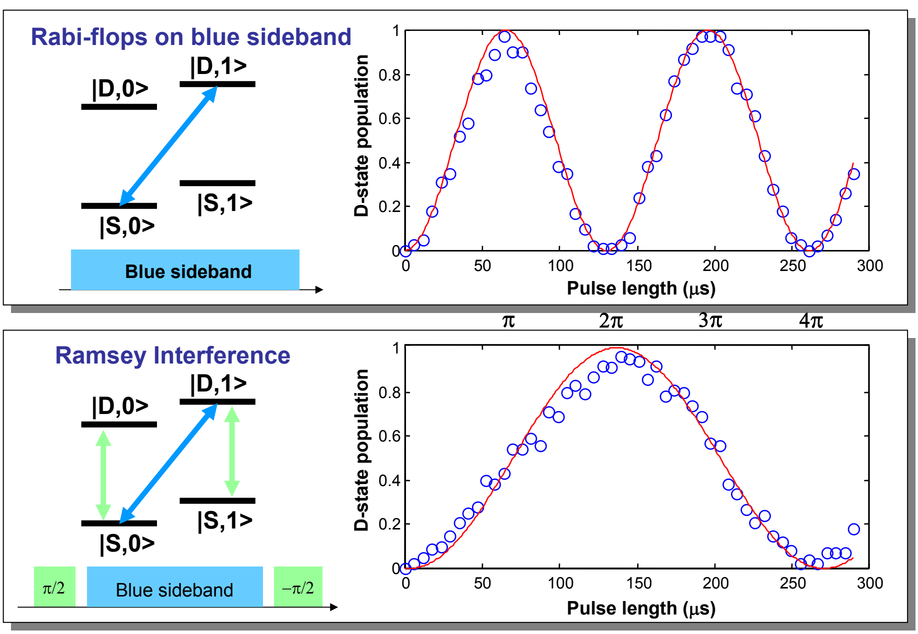
carrier and sideband Rabi oscillations with Rabi frequencies

$\Omega, \quad \eta\Omega\sqrt{n+1}$

$\eta = kx_0$ **Lamb-Dicke parameter**

Each point : average of 100-200 individual measurements, preparation – coherent rotation – state detection

Qubit rotations : phase of the wavefunction

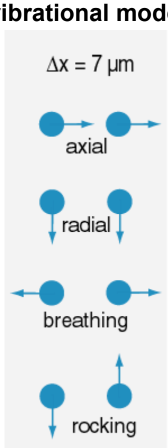


Quantum gates

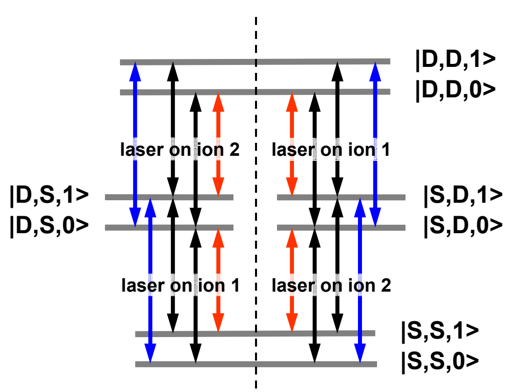
2 ions + motion = 3 qubits

With several ions, the motional qubits are shared
motional qubit acts as the "bus" between the ions

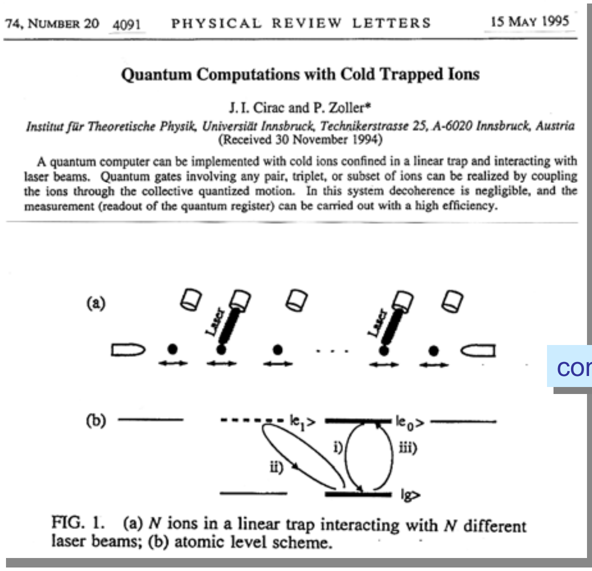
vibrational modes



computational subspace: 2 ions, 1 mode



Quantum gate proposal(s)


74, NUMBER 20 4091 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

phase gate

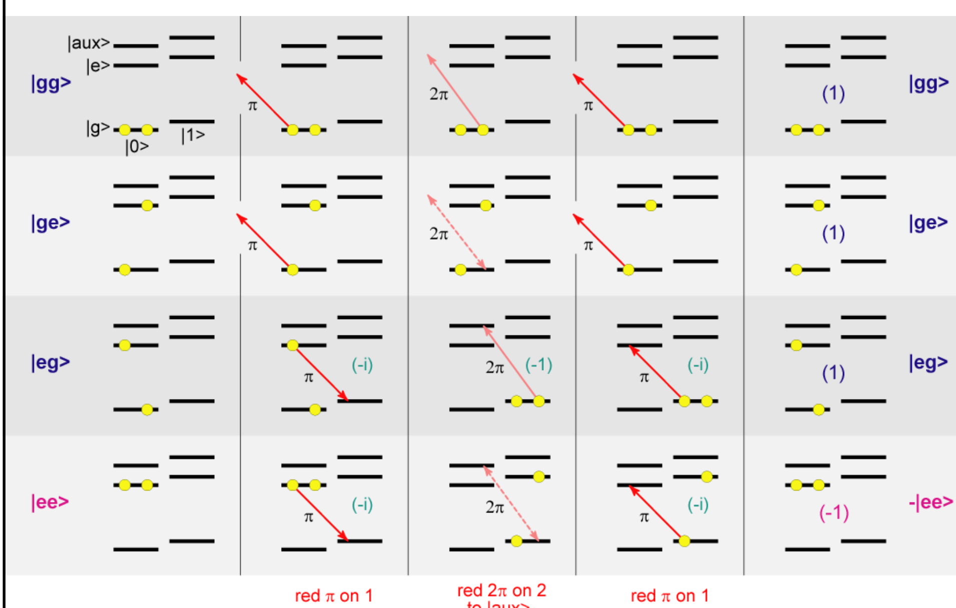
$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle$$
$$|1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle$$

control bit — target bit

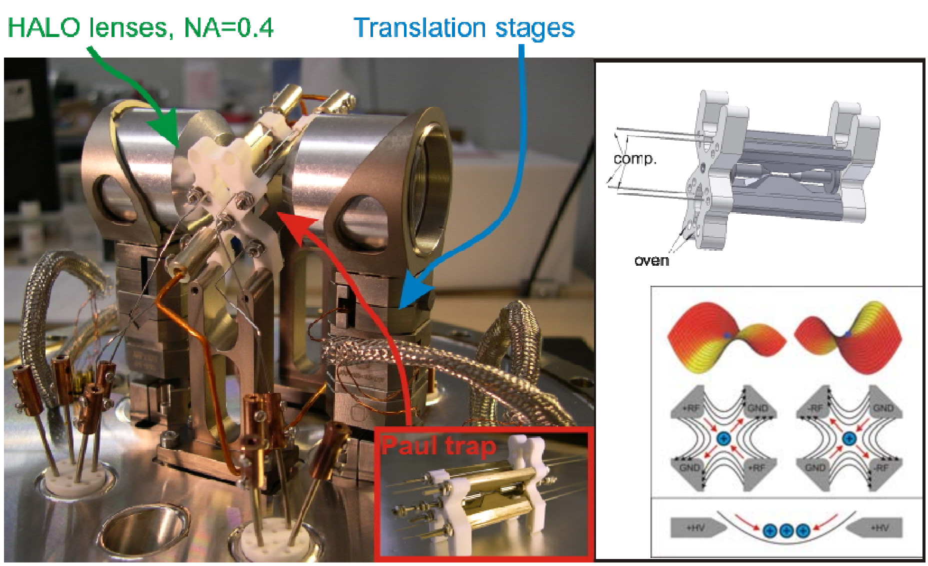
Further gate proposals:
- **Cirac & Zoller**
- **Mølmer & Sørensen, Milburn**
- **Jonathan & Plenio & Knight**
- **Geometric phases**

Details of C-Z gate operation (Phase gate)

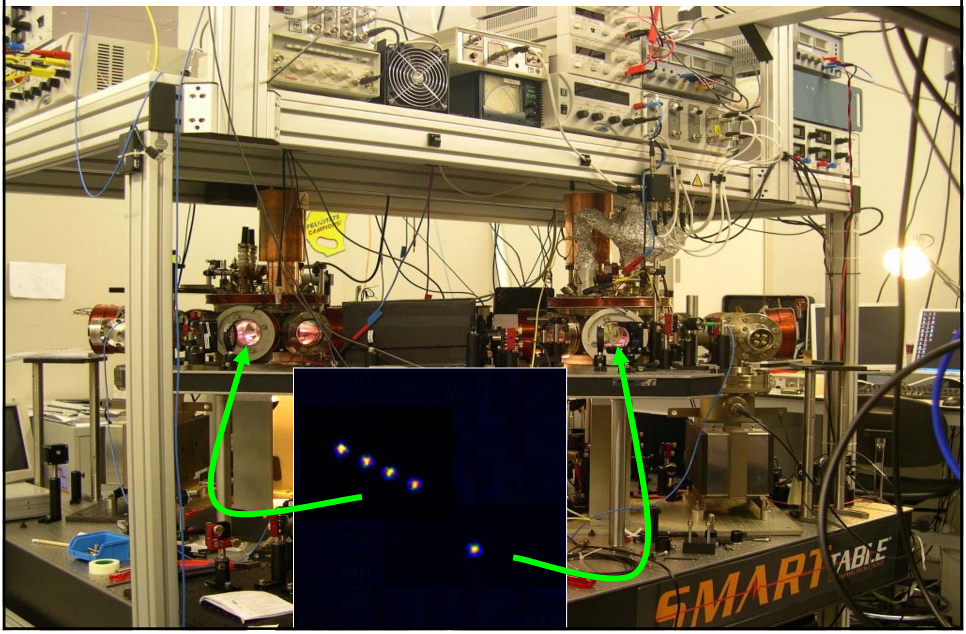


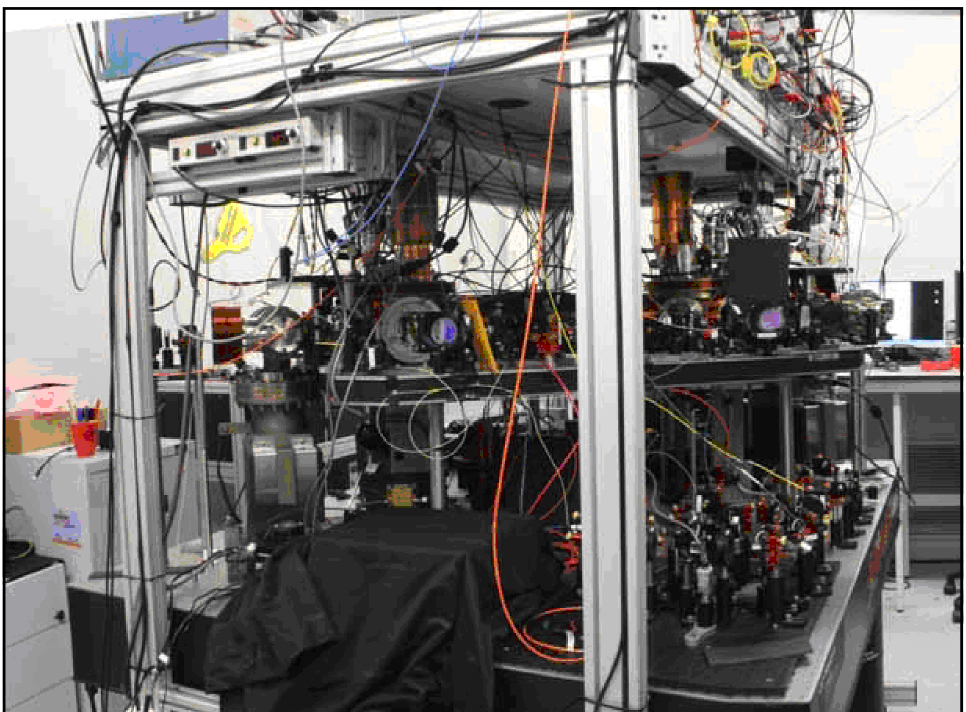
Experimental techniques

Ion trap setup

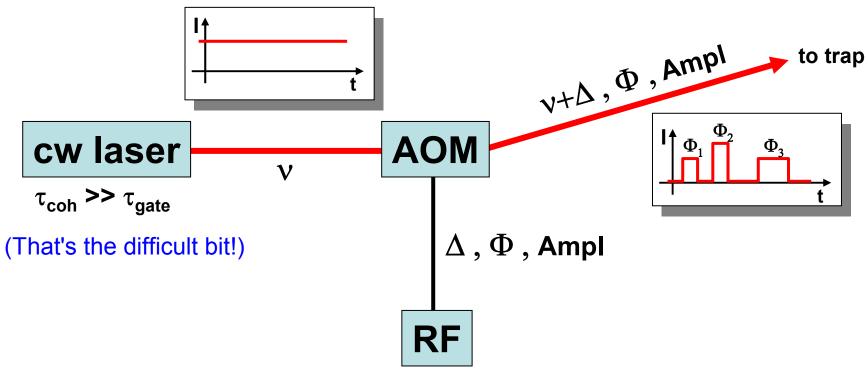


Double trap apparatus





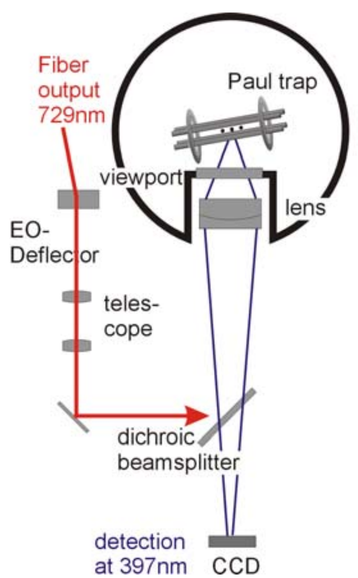
Laser pulses for coherent manipulation etc



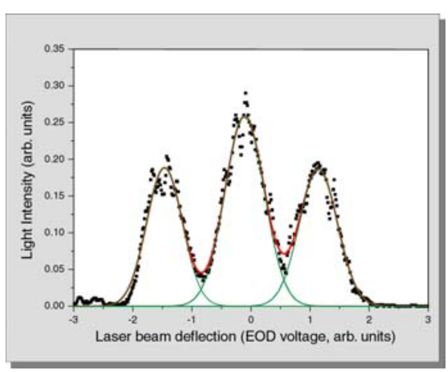
AOM = acousto-optical modulator, based on Bragg diffraction

"Ampl" = Amplitude, includes switching on/off

Addressing of ions in a string



Well-focussed laser beam

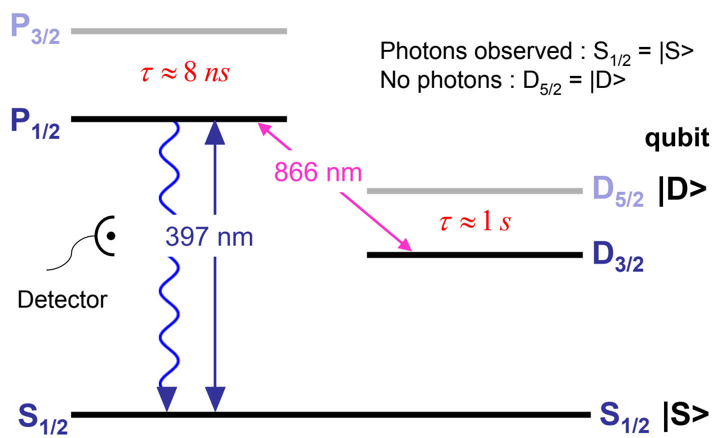


- **beam steering with electro-optical deflector**
- **addressing waist ~ 2.5 - 3.0 mm**
- **< 1/400 intensity on neighbouring ion**

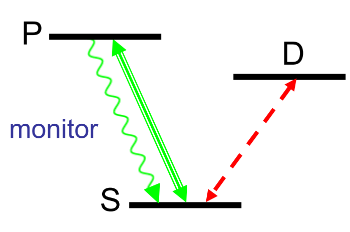
first demonstration: H.C. Nägerl et al., Phys. Rev. A 60, 145 (1999)

Discrimination of qubit states

State detection by photon scattering on $S_{1/2}$ to $P_{1/2}$ transition at 397 nm

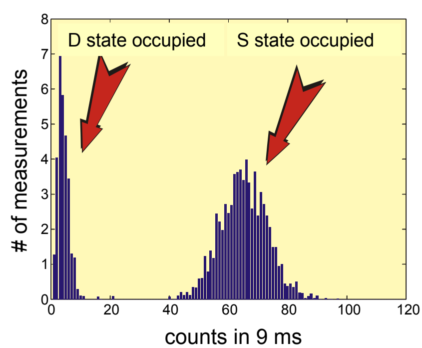


State detection: shelving



(Shelving level can, but need not be the same as qubit state)

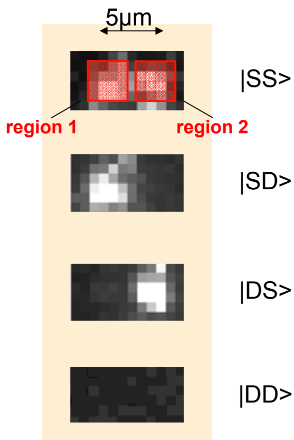
Histogram of counts in 9 ms
Poisson distribution N ± N½
discrimination efficiency 99.85%



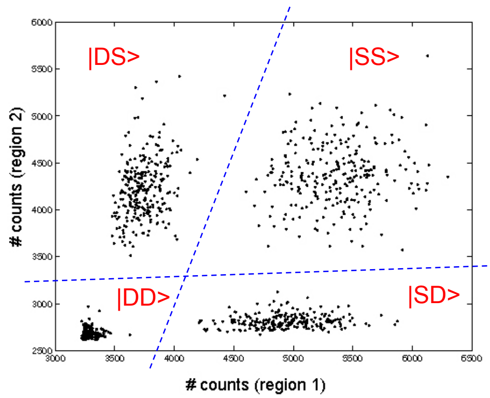
H. Dehmelt 1975

Quantum state discrimination with 2 ions

Individual ion detection on CCD camera



Two-ion histogram (1000 experiments)



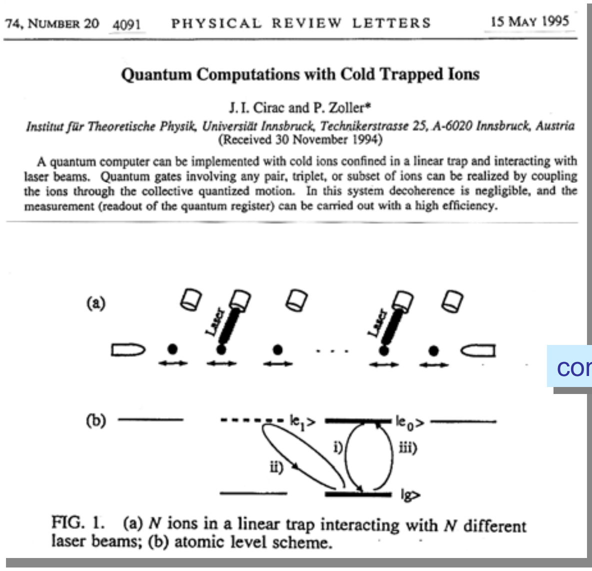
→ quantum state populations $p_{SS}, p_{SD}, p_{DS}, p_{DD}$

Cirac-Zoller quantum CNOT Gate with two trapped ions

"Realization of the Cirac–Zoller controlled-NOT quantum gate", F. Schmidt-Kaler et al., Nature 422, 408-411 (2003).

"Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate", D. Leibfried et al., Nature 422, 412 (2003).

Quantum gate proposal



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Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*
Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

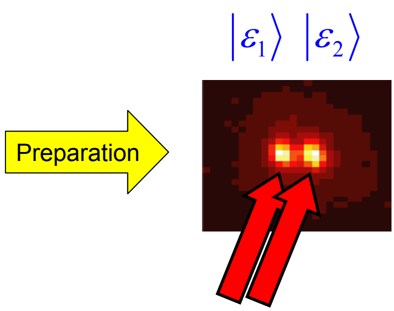
FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

phase gate

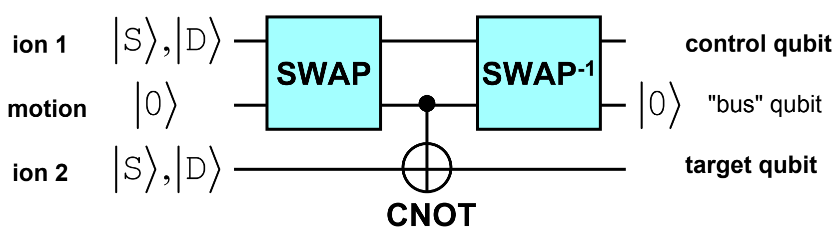
$$|0\rangle|0\rangle \to |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|0\rangle$$
$$|1\rangle|1\rangle \to -|1\rangle|1\rangle$$

control bit target bit

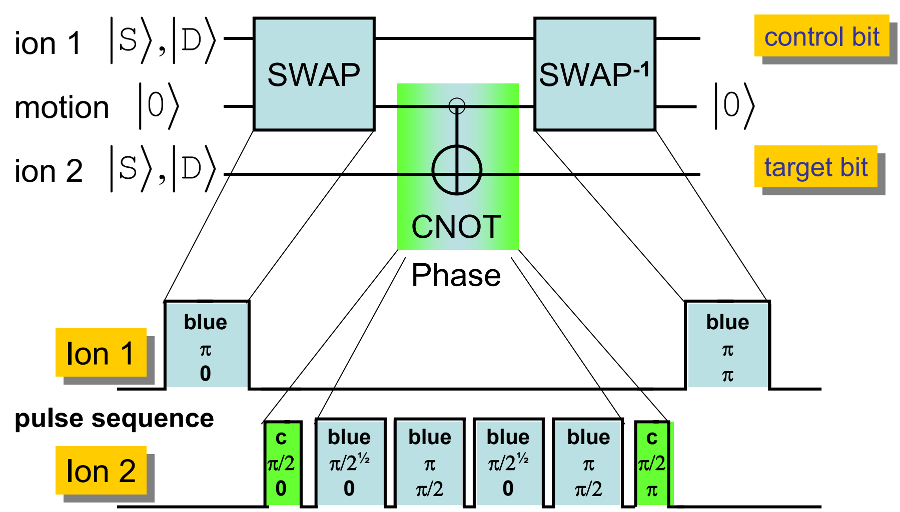
Cirac-Zoller two-ion controlled-NOT gate



|S> = bright
|D> = dark

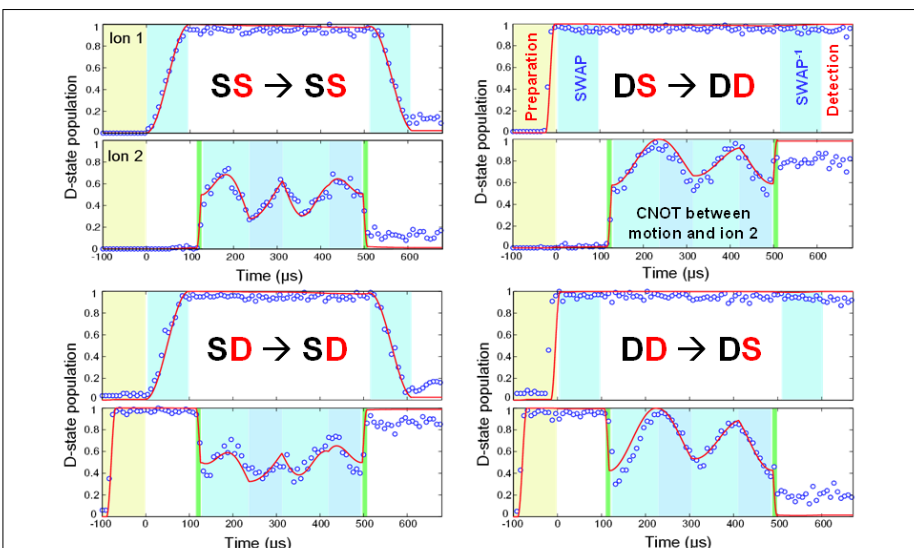


Cirac-Zoller two-ion controlled-NOT operation



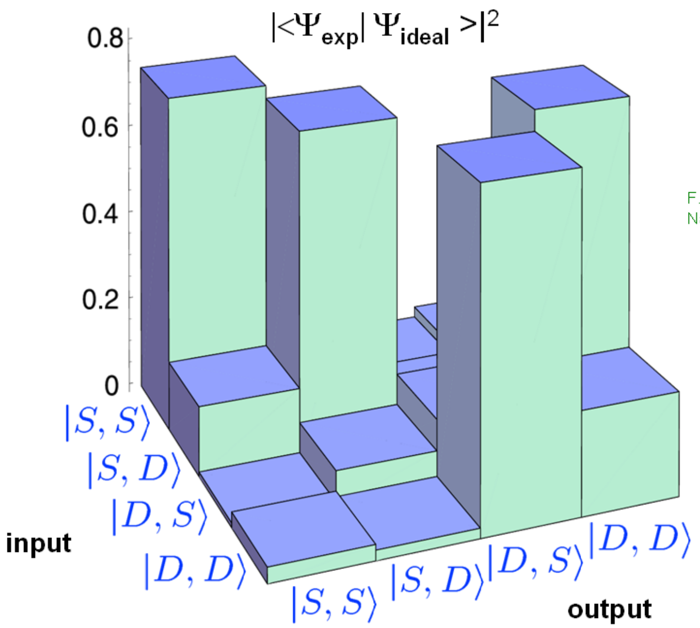
- **CNOT gate = Phase gate enclosed by π/2- pulses**
- **Phase gate implemented by composite pulses**

Result : full time evolution



every point = 100 single measurements, line = calculation (no fit)

Measured fidelity (truth table)



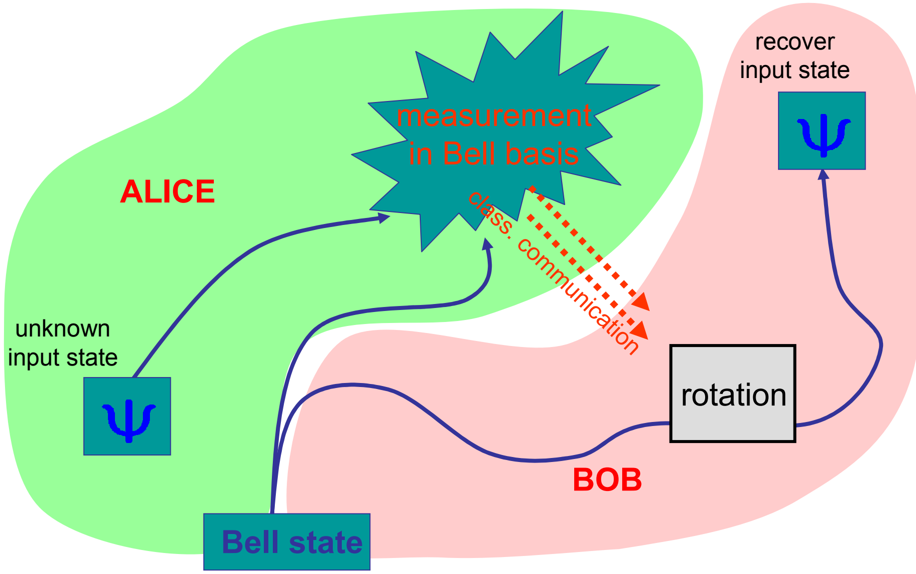
F. Schmidt-Kaler et al., Nature 422, 408 (2003)

Quantum teleportation

"Deterministic quantum teleportation with atoms", M. Riebe et al., Nature 429, 734 (2004).

"Deterministic quantum teleportation of atomic qubits", M. D. Barrett et al., Nature 429, 737 (2004).

Teleportation idea (Bennett 1993)



Full formal procedure

To teleport $|\psi\rangle_2 = \alpha|\uparrow\rangle_2 + \beta|\downarrow\rangle_2$

prepare $|\psi\rangle_{1,3} = |\Psi^-\rangle_{1,3} = |S\rangle_{1,3} = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$

The total state $|\Phi\rangle = |S\rangle_{1,3} \otimes |\psi\rangle_2$

can be rewritten as $\sum_{k=1}^{4} |\Psi_k\rangle_{1,2}(U_k|\psi\rangle_3)$

with Bell states $|\Psi_k\rangle \quad (k = 1 \ldots 4)$, and $|\psi\rangle_3 = \alpha|\uparrow\rangle_3 + \beta|\downarrow\rangle_3$

After Bell measurement on (1,2) shows Bell state $|\Psi_k\rangle$

apply U_k^{-1} to 3 to obtain $|\psi\rangle_3$.

Very simple description (see Bouwmeester et al.)

Particle 2 and 3 are prepared in an entangled state

$$|\Psi^-\rangle_{2,3} = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle)$$

where one particle is always in the "opposite" state of the other.

When the result of the Bell measurement between 1 and 2 is

$$|\Psi^-\rangle_{1,2} = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle)$$

then 2 is in the "opposite" state of the unknown state of 1.

Thus particle 3, being in the opposite state to 2, will be in the same state as 1.

If the result of the Bell measurement is a different state, the appropriate rotation has to be applied to particle 3.

Teleportation of atomic qubit states

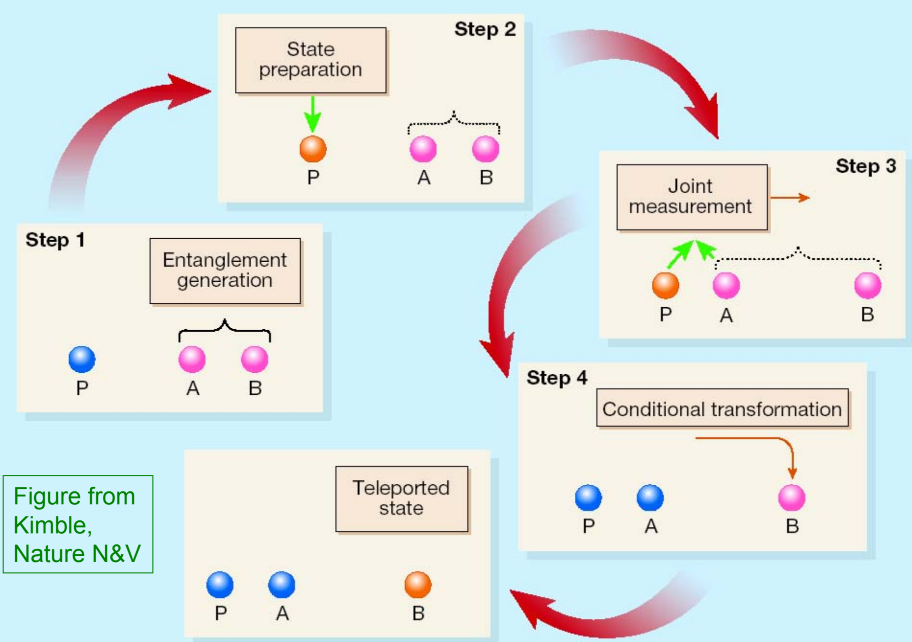
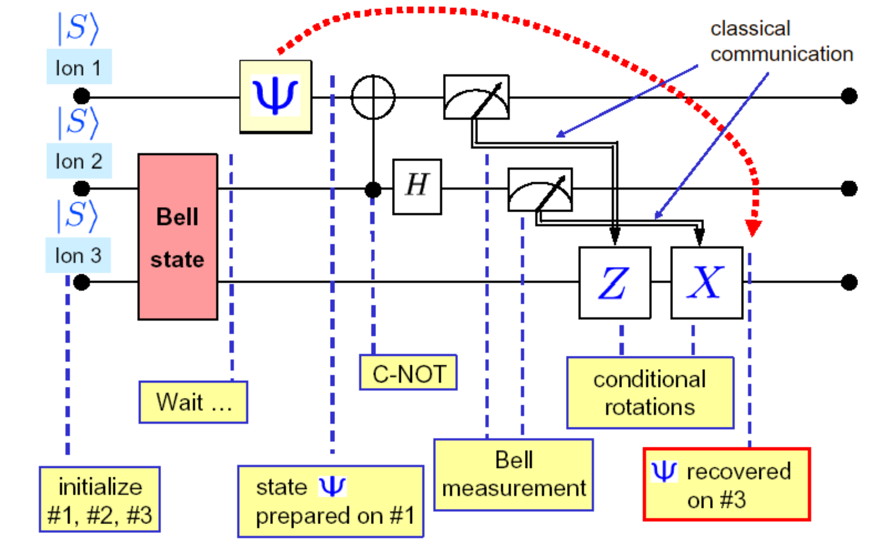
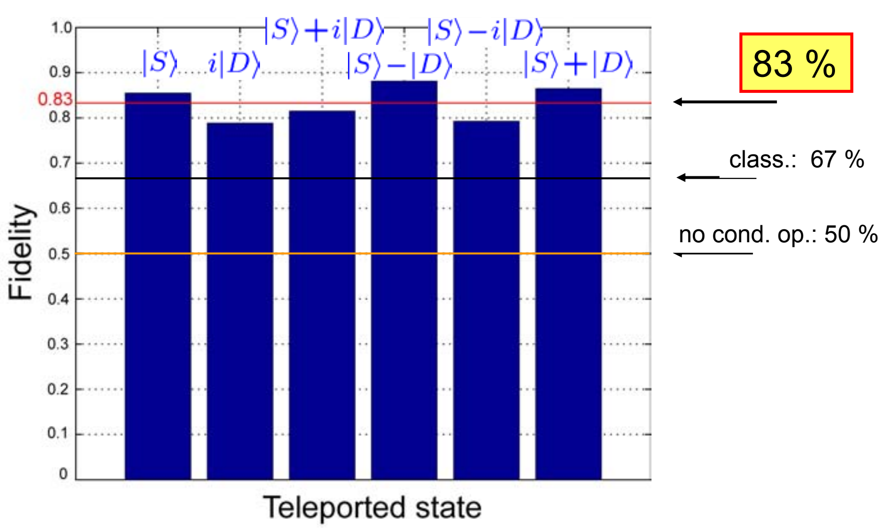


Figure from Kimble, Nature N&V

Teleportation protocol



Quantum teleportation with atoms: result

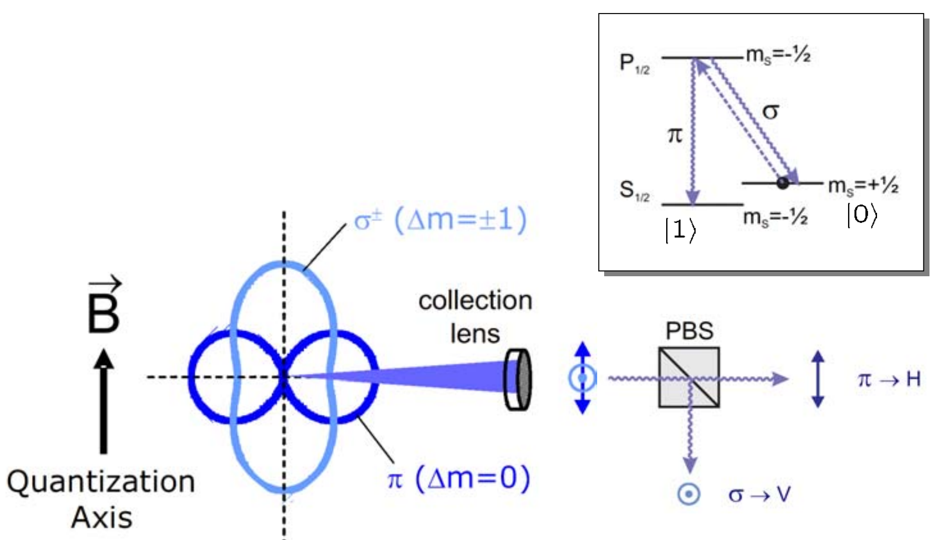


M. Riebe et al., Nature 429, 734 (2004)

Qubit interfacing

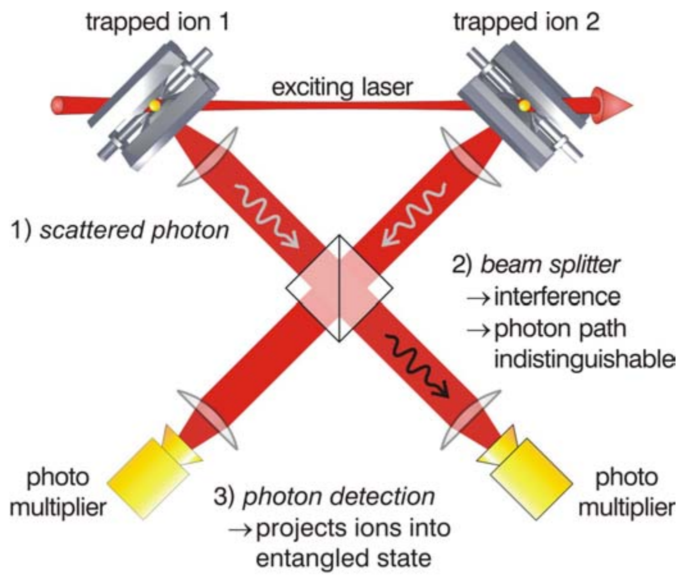
Quantum networks

Atom-photon entanglement



Blinov et al., Nature 2004

Probabilistic atom-atom entanglement



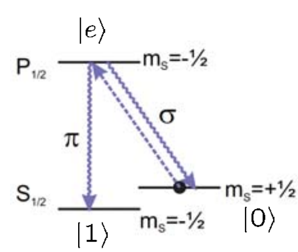
Entanglement scheme I

- Prepare initial 2-atom state $|0\rangle_1|0\rangle_2$
- Excite with weak σ pulse (ϵ)

$$|\psi_x\rangle = |0\rangle_1|0\rangle_2 + \epsilon(|e\rangle_1|0\rangle_2 + |0\rangle_1|e\rangle_2) + \epsilon^2|e\rangle_1|e\rangle_2$$

- Repeat until π photon is detected behind beamsplitter

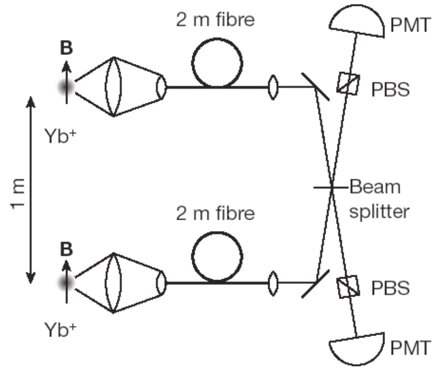
- Final state $|\psi_f\rangle = |1\rangle_1|0\rangle_2 + e^{i\Phi}|0\rangle_1|1\rangle_2$



Cabrillo et al., PRA 1999

Entanglement scheme II

- **Distant atomic entanglement**



- Atom-photon entanglement
- Ψ^- state produces coincidence detection
- Entanglement swapping
- Fidelity 63 %
- 1 pair / 39 s

Moehring et al., Nature 2007

References

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- Quantum Information Science and Technology Roadmap : http://qist.lanl.gov/