Quantum Information Processing with Ultracold Atoms and Molecules in Optical Lattices

Mirta Rodríguez
Instituto de Estructura de la Materia
CSIC, Madrid SPAIN
Overview

I. Introduction
   I. QIP requires the preparation, manipulation and detection of quantum states
   II. Neutral particles trapped in optical lattices

II. Manipulation of neutral atoms
   I. Trapping with magnetic and laser fields
   II. Optical lattices
   III. Long-range interactions

III. Quantum state engineering with time-dependent potentials
   I. Twin-Fock states for Heisenberg limited interferometry
   II. Fast oscillatory potentials for scanning the phase diagram
   III. Trains of pulses to align molecules

IV. Detection of strongly-correlated states of ultracold atoms
   I. Quantum Polarization Spectroscopy
   II. Atom counting

V. Quantum gate operations in optical lattices
   I. Experimental realization of gate operations with atoms
   II. QI with polar molecules: a theoretical proposal
I. Introduction

I. Requirements for QIP
II. Hubbard-Hamiltonians with optical lattices
I. Requirements for QIP (DiVincenzo)

- Quantum register: set of qubits
  - Superpositions & entanglement
- Isolated from environment → prevent decoherence
- Universal set of gates: two-qubit gate & single qubit operations
- Initial state preparation
- Detection of the quantum states
- Scalability of the system
- Networking ability
I. Requirements for QIP

⇒ Quantum State Engineering
Candidates for QIP

I. Introduction
II. Neutral particles trapped in Optical Lattices

- Lasers
- Trapped Atoms

D. Jaksch, Contemporary Physics 2004

- Zoller et al PRA 76,043604 (2007)

- I. Bloch’s group arxiv:1101.2076
Key elements

1) Qubits \( \{ |0\rangle_j, |1\rangle_j \} \)

\[
|\Psi\rangle = \sum_{j_1,j_2,...,j_N=0}^{1} c_{j_1,j_2,...,j_N} |j_1\rangle_1 \otimes |j_2\rangle_2 \otimes ... \otimes |j_N\rangle_N
\]

2) Low decoherence rates: neutral particles with small dipole moments

3) Large systems

4) High controllability: engineering of Hamiltonians and strongly correlated quantum states
Engineering Hamiltonians

\[ H = \sum_{\sigma j j'} T_{\sigma j} b_{\sigma j} b_{\sigma j'} + \sum_{\sigma j k} U_{\sigma \sigma' j k} b_{\sigma j} b_{\sigma j} b_{\sigma' k} b_{\sigma' k} \]

i) single and two-qubit interactions

ii) many-body quantum states are strongly correlated such that interactions between qubits of the same order as the single qubit energies

iii) many-body states are characterized by correlations \( \langle b_{\sigma j}^{(+)} b_{\sigma' k} \rangle \) which break the symmetries and define complicated phase diagrams characterized by \( U_{\sigma \sigma'}/T_{\sigma} \) quantum phase transitions

iv) strong correlations \( \rightarrow \) entanglement
5) Entanglement

Strongly correlated phases in optical lattices

Challenges:

- On-site resolution is difficult as interparticle distance in the order of the diffraction limit of light.
  On-site manipulation & detection is now possible ✔

- Neutral particles with small dipole moments result only in on-site interactions in the lattice.
  Long-range interactions appear in ultracold molecules ✔
II. Manipulation of neutral atoms

I. Ultracold samples are manipulated with magnetic and laser fields
II. Optical Lattice potentials
III. Long-range interactions
I. Ultracold atomic samples

- **Quantum degenerate regime**
  
  deBroglie wavelength $\lambda_T \sim (mT)^{-1/2} \sim \text{interparticle distance } n^{-1/3}$
  
  gas samples are dilute $\sim 10^{13-15} \text{ cm}^{-3}$ and ultracold $\sim 100 \text{nK}$

- **Alkali atoms**
  
  electronic spin $J=V+S, L=0 \rightarrow J=1/2$
  
  I nuclear spin
  
  total spin $F=I+J \rightarrow F=I \pm 1/2$
  
  $I=3/2$ for $^{87}\text{Rb}$

  Total spin $\rightarrow$ Fermi/Bose statistics

- **Dilute** $\rightarrow$ 2-body collisions whose sign and strength can be tuned

- **Trapping with magnetic fields and optical fields**

- **Imaging with CCD camera**

---

R. Hulet’s group
I. Magnetic and optical trapping

- Zeeman splitting

- Optical dipole force

Interaction of atom with laser field

\[ H_{\text{int}} = -d \mathbf{E} \cdot \mathbf{E}(r) \cos(\omega t + \phi) \]

If the laser is far detuned from the transition it remains in the ground state and experiences an effective dipole potential

\[ V(r) = \frac{\hbar \delta \Omega(r)^2}{\delta^2} \]

\[ \Omega(r) = \langle e|d \cdot \hat{\mathbf{E}}(r)e^{i\varphi(r)}|g\rangle / 2 \]

II. Manipulation of neutral atoms

Fig. 1  Energy Level Scheme for Rb\textsuperscript{87}
II. Optical lattice potentials

Counter-propagating laser beams result in a standing-wave configuration

$$V(r) \sim I = I_0 \cos^2 (k \cdot r)$$

I. Bloch’s group arxiv:1101.2076, Greiner’s group Nature 2009
II. Hubbard model

tight-binding regime: kinetic energy + two-body interactions

$$\hat{K} = \int dr \sum_{\sigma = \uparrow, \downarrow} \left[ \hat{\psi}_\sigma^\dagger (r) \left( -\frac{\hbar^2 \nabla^2}{2m} + V_\sigma (r) - \mu_\sigma \right) \hat{\psi}_\sigma (r) \right]$$

$$+ \frac{1}{2} \sum_{\sigma, \beta = \uparrow, \downarrow} \int dr \int dr' u_{\sigma\beta} (r - r') \hat{\psi}_\sigma^\dagger (r) \hat{\psi}_\beta^\dagger (r') \hat{\psi}_\beta (r') \hat{\psi}_\sigma (r),$$

$$\hat{\psi}_\sigma (r) = \sum_j \hat{c}_{j\sigma} w (r - r_j)$$

$$H_0 = J \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_j c_{j\uparrow}^\dagger c_{j\downarrow}^\dagger \hat{c}_{j\downarrow} \hat{c}_{j\uparrow}$$

$$J = - \int dr w^\dagger (r - r_i) \left[ -\frac{\hbar^2 \nabla^2}{2m} \right] w (r - r_j)$$

tunneling rate

$$U = \frac{4\pi \alpha_s \hbar^2}{m} \int dr |w (r)|^4$$
on-site two-body interaction
II. Superfluid to Mott-insulator transition: initialization of a qubit register

\[ H = -J \sum_j \left( a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j \right) + \frac{U}{2} \sum_j n_j(n_j - 1) \]

[Images of wavefunctions and energy levels]

\[ n \propto \sum_{i,j} e^{ik(r_i-r_j)} \langle a_i^\dagger a_j \rangle \]

Experiment:
III. Long-range interactions in neutral atoms

- Strong magnetic moments: Chromium atoms

- Cavity induced long-range interactions

- Coupling to Rydberg states of the atoms

T. Pfau’s group Stuttgart

T. Esslinger’s group Nature 464 (2010)
III. Dipolar molecules

- In the presence of external fields, molecules present high effective dipolar moments depending on the rotational states of the molecule.
- Strong electric dipole moments result in long-range dipole-dipole interactions between the molecules.
- Hubbard-Hamiltonians with long-range interactions could be realized.

$$H_{dd} = \frac{d_1 \cdot d_2 - 3(d_1 \cdot e_R)(e_R \cdot d_2)}{R^3}$$

Experiments with dipolar dimers at JILA and Innsbruck are at the edge of reaching quantum degeneracy!!
III. Quantum state engineering with time dependent fields

I. Twin-Fock states for Heisenberg-limited interferometry
II. Fast oscillatory potentials to cross the phase diagram
III. Orientation of molecules with trains of laser pulses
III.1
Generation of twin-Fock states for Heisenberg-limited interferometry

Melting the state

$$|\Psi_{ab}\rangle = \prod_{i=1}^{N} |ab\rangle_i$$
Two-component Mott insulators

- Components: *hyperfine states* of the atom
- MI regime $\rightarrow$ 2-body physics

Control of the spin interactions in an optical lattice

Rabi oscillations in 2-level system

$$P_{if} = \frac{\Omega_{if}^2}{2\delta_{if}} (1 - \cos(\Omega_{if} t))$$

$$\Omega_{if} = \sqrt{\Omega_{if}^2 + \delta_{if}^2}$$
Bose-Hubbard Hamiltonian

\[ H = -J \sum_{\langle i,j \rangle} (a_i^\dagger a_j + b_i^\dagger b_j) + U \sum_i n_i^a n_i^b + \sum_i \frac{V}{2} n_i^a (n_i^a - 1) + \sum_i \frac{V}{2} n_i^b (n_i^b - 1) \]

- \( J \) tunneling rate
- \( V \) intraspecies on-site interactions
- \( U \) interspecies interactions

\[ \frac{U}{V} \text{ fixed and } \frac{V}{J} \to 0 \]

II. Quantum State Engineering
Adiabatic Evolution

- During adiabatic evolution the system follows the instantaneous eigenstates.
- **Non-crossing rule:** If the Hamiltonian depends only on one parameter $\lambda$, the energies of the states as a function of $\lambda$ do not cross for states of the same symmetry.
- The curves approach each other at avoided crossings (Landau-Zener)
Adiabatic evolution

- We map initial superposition states into final superpositions with the same coefficients with the dynamical phase.

- Adiabatic time scale in a one-component system:
  \[ J t_r \gg \frac{VN}{JM} \]

  It does not scale with \( N \!).

II. Quantum State Engineering
Twin-Fock state (I)

- **MI regime** \((J \to 0)\)
  \[ |\psi_{ab}\rangle = \prod_{i=1}^{N} |ab\rangle_i \]
  Non-degenerate ground state if \(U < V\)

- **SF regime** \((V \to 0)\)
  \[ |\psi_{tf}\rangle = |N/2\rangle_{A_0} |N/2\rangle_{B_0} \]
  The non-degenerate ground state for each mode is the delocalized symmetric state \(A_0, B_0\)
  \[ A_0^\dagger \sim \sum_i a_i^\dagger \quad B_0^\dagger \sim \sum_i b_i^\dagger \]
  The two non-degenerate ground states are connected by adiabatic evolution

Adiabatic melting \( |\psi_{ab}\rangle \) provides a direct means of obtaining a twin-Fock state with zero relative number.

This state useful in atom interferometric experiments!!!
Exact calculation for M=6 sites

Spectrum of the 2 BHM in the symmetric subspace with \( N_a = N_b = 6 \) and \( V/U = 0.1 \).

Overlap of the instantaneous ground state of the BHM and the twin-Fock state.

II. Quantum State Engineering
TEBD (t-DMRG) calculation

II. Quantum State Engineering

Energy difference per atom and overlap between the ramped state and the final SF ground state for different ramping times $t_r$. ($V_0/J=20$, $M=25=N_a=N_b$)

Adiabatic ramping time scale

$J t_r \sim 3 V_0/J$
Large systems and interferometry

Objective: measure phase differences $\phi$ with highest sensitivity

**Shot noise limit**

$\Delta \phi \sim N^{-1/2}$

**Heisenberg limit**

$\Delta \phi \sim N^{-1}$
Sensitivity across the Mott insulator transition

\[ \Delta \phi(0) = \frac{1}{2\sqrt{\langle J_x^2 \rangle}} \]

\[ \langle J_x^2 \rangle = [\sum_i (\rho_{ii}^a + \rho_{ii}^b) + \sum_{i,j} (\rho_{ij}^a \rho_{ji}^b + \text{h.c.})]/4 \]

\[ \rho_{ij}^a = \langle a_i^{\dagger} a_j \rangle \]

\[ \Delta \Phi \sim N^{-\alpha} \]

\[ \rho_{ij}^{a,b} = 1 \rightarrow \Delta \phi(0) = 1/\sqrt{N^2/2 + N} \]

\[ \Delta \phi \sim N^{-1} \text{ Heisenberg limit} \]

\[ \rho_{ij} = \delta_{ij} \rightarrow \Delta \phi(0) = 1/\sqrt{2N} \]

\[ \Delta \phi \sim N^{-1/2} \text{ Standard limit} \]
Particle loss at a rate $\kappa$

One can obtain Heisenberg limited sensitivity using as input states the twin–Fock states resulting from a slow melting of MI states with two different atoms per lattice site.

$\kappa \ll J^2/30V_0 \implies$ Heisenberg-limited sensitivity

M. Rodriguez, S R Clark, D. Jaksch, PRA (R) 75 011601 (2007).
II.11

Disordered bosons in a fast oscillatory potential
I. Disordered Bose-Hubbard Model

\[ H_0 = -J \sum_j \left( a_j^{\dagger} a_{j+1} + a_{j+1}^{\dagger} a_j \right) + \frac{U}{2} \sum_j n_j (n_j - 1) \]

\[ + \frac{W}{2} \sum_j \epsilon_j n_j + C \sum_j j^2 n_j \]
I. Disordered-Bose-Hubbard Model


\[ \begin{align*}
W/U = 0 & \quad U/J \to \infty \text{ Mott Insulator} \\
W/U > 0 & \quad U/J \to 0 \text{ Superfluid} \\
W/U > 1 & \quad \text{Bose Glass} \\
W/U < 1 & \quad \text{MI, BG} \\
W/U & \to \infty \quad \text{Anderson Glass}
\end{align*} \]

III. Quantum State Engineering
II. Driven DBH model

\[ H(t) = H_0 + V(t) \]

\[ V(t) = V \cos(\omega t + \delta) \sum_j j n_j \]

- Time-dependent periodic Hamiltonian \( T = 2\pi/\omega \)

\[ i \frac{\partial}{\partial t} | \Psi(t) \rangle = H(t) | \Psi(t) \rangle \]

- Floquet theory

\[ H^F \equiv H(t) - i \frac{\partial}{\partial t} \]

\[ H^F(t) | \phi^e(t) \rangle = e | \phi^e(t) \rangle \]

\[ | \phi^e(t) \rangle = | \phi^e(t + T) \rangle \]
II. Floquet theory

We solve the Schrödinger eq. using the cyclic eigenstates of \( H_F \)

\[
\left| \Psi(t) \right\rangle = U(t, t_0; V) \left| \Psi(t_0) \right\rangle \quad \left| \Psi^e(t) \right\rangle = e^{-i\epsilon t} \left| \phi^e(t) \right\rangle
\]

\[
U(t, t_0; V) = \sum_\alpha \left| \Psi^{\epsilon_\alpha}(t) \right\rangle \left\langle \Psi^{\epsilon_\alpha}(t_0) \right| \quad \epsilon_\alpha \equiv e_\alpha \in \left( -\frac{1}{2} \omega, \frac{1}{2} \omega \right]
\]
III. Fast oscillatory regime: dynamical suppression of tunneling

\[ H_{\text{eff}} = -J_{\text{eff}} \sum_j \left( a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j \right) + \frac{U}{2} \sum_j n_j(n_j - 1) \]

\[ J_{\text{eff}} = J e^{\pm iV/\omega \sin \delta} J_0(V/\omega) \]


Many-body:
A. Eckardt, C. Weiss and M. Holthaus
PRL 95, 260404 (2005)

**Driven Mott-Insulator transition:**
one can tune the value of \( J_{\text{eff}}/U \) to zero driving a fast oscillatory force from 0 to \( V/\omega \approx 2.4 \)
III. Driven SF-MI transition: experimental realization

Arimondo’s group
PRL 102, 100403 (2009)

II. Quantum State Engineering
III. Fast oscillatory force

Ramping $U/J_{\text{eff}}$ and $W/J_{\text{eff}}$
IV. Effect of disorder in quasienergy levels

Quasienergy levels of the Floquet Hamiltonian as a function of the strength of the fast oscillatory force $V$ for $N=L=5$, $U/J=0.5$ and $\omega/J=25$

a) $W/J=0$  
b) $W/J=0.1$  
c) $W/J=1$

II. Quantum State Engineering
IV. Weak disorder: SF-MI transition

\[ \rho_k(t) = \sum_{i,j} e^{-ik(i-j)} \langle \Psi | a_i^+ a_j | \Psi \rangle \]
IV. Weak disorder: SF-MI transition

\[ \rho_{ij} = \langle \Psi | a_i^{\dagger} a_j | \Psi \rangle \]

a) $W/U=0.2 \ t=0$

b) $W/U=0.2 \ t=t_{\text{ramp}}$

II. Quantum State Engineering
IV. Weak disorder helps to localise for noncommensurate fillings

- a) $W/J=0$, $J_{\text{eff}}=J$
- b) $W/J=0$, $J_{\text{eff}}=0$
- c) $W/J=0.1$, $J_{\text{eff}}=J$
- d) $W/J=0.1$, $J_{\text{eff}}=0$

SF MI

SF BG
IV. Intermediate disorder: SF-BG

$t_{\text{ramp}} J \sim 1500$

$\omega / J = 25 \quad W / J = 1 \quad U / J = 0.5$
IV. Intermediate disorder: SF-BG transition

c) $W/U=2 \ t=0$

d) $W/U=2 \ t=t_{\text{ramp}}$
IV Spectra of the final states after ramping

\[ a) \quad \frac{\varepsilon}{J} \]

- \( W/U = 0.2 \)
- \( W/U = 2 \)
IV Strong disorder: AG-BG

\[ \text{t}_{\text{ramp}} J \sim 50 \]
\[ N = L = 5 \quad \omega / J = 25 \quad W / J = 7 \quad U / J = 0.05 \]
IV Strong disorder

e) $W/U=140 \; t=0$

f) $W/U=140 \; t=t_{\text{ramp}}$

AG

BG
IV Initial state for strong disorder: Anderson Glass
III.II Disordered bosons in a fast oscillatory potential

- A fast oscillatory force results in an effective tunneling
- One can drive adiabatic **dynamical** transitions across the phase diagram in experimentally relevant times

J. Santos, R. Molina, J. Ortigoso, M. Rodríguez, PRA 80, 063602 (2009)
Orientation and alignment of molecules
Alignment of molecules with a train of ultrafast laser pulses

Floquet Hamiltonian

\[ K(t') = -\frac{\hbar}{B} \frac{\partial}{\partial t'} + J^2 - \left( \Delta \omega \cos^2 \theta + \omega_\perp \right) g(t') \]

- Choosing the shape of the pulse train, one obtains rotational wave-packets with different orientation.
- Orientation defines the effective dipole moment of the molecule and thus the long-range dipole-dipole interactions between them.
- \( \rightarrow \) different Hubbard Hamiltonians

IV. Detection of strongly correlated states

I. Quantum Spin Polarization Spectroscopy
II. Atom counting
IV.I
Quantum spin polarization spectroscopy
Atom-light interfaces

\[ \hat{s}_z = \frac{1}{2} \left( \hat{a}^\dagger \hat{a}^+ - \hat{a}^\dagger \hat{a}^- \right) \]
\[ \hat{s}_x = \frac{1}{2} \left( \hat{a}^\dagger \hat{a}^- - \hat{a}^\dagger \hat{a}^+ \right) \]
\[ \hat{s}_y = \frac{1}{2i} \left( \hat{a}^\dagger \hat{a}^- - \hat{a}^\dagger \hat{a}^+ \right) \]

\[ \hat{H}_{eff} = -\int d\mathbf{r} a_0 \hat{1} \hat{\mathbf{s}} \cdot \hat{\mathbf{l}} + a_0 [\hat{\mathbf{1}} \hat{\mathbf{j}}^2 - \hat{s}_- \hat{\mathbf{j}}^2_+ - \hat{s}_+ \hat{\mathbf{j}}^2_-] \]

\[ \hat{H}_{eff} \sim -\kappa \int d\mathbf{r} \hat{s}_z \hat{J}_z (\mathbf{r}) \]

\[ a_i \sim n_{\gamma \lambda c} (10\kappa A \Delta) \]

IV. Detection
Propagation of light macroscopically polarized in the x-direction: Polarization rotation

\[ \hat{H}_{\text{eff}} \sim -\kappa \int d\mathbf{r} \hat{s}_z \hat{j}_z(\mathbf{r}) \]

Atoms

\[ [\hat{H}_{\text{eff}}, \hat{j}_z] = 0 \]

The collective spin is rotated around the z-axis by \( \chi = -\kappa t/\hbar \)

Light

\[ -i\hbar c \partial_r \hat{S}_i = [\hat{S}_i, \hat{H}_{\text{eff}}] \]

Faraday rotation of the Stokes operators in the x-y plane

\[ \hat{S}^\text{out}_y = \cos \theta \hat{S}^\text{in}_y + \sin \theta \hat{S}^\text{in}_x \]

\[ \hat{S}^\text{out}_x = -\sin \theta \hat{S}^\text{in}_y + \cos \theta \hat{S}^\text{in}_x \]

Experiments at NIST with F=1 spinor condensates: Y. Liu et al, PRL 102, 125301(2009)

\[ \langle \hat{S}^\text{in}_x \rangle = \frac{N_P}{2}, \quad \langle \hat{S}^\text{in}_y \rangle = \langle \hat{S}^\text{in}_z \rangle = 0 \]

\[ \langle \hat{J}_z \rangle \text{ constant} \]

\[ \langle \hat{X}^\text{out}_S \rangle \sim \langle \hat{J}_z \rangle \]

\[ \langle (\Delta \hat{X}^\text{out}_S)^2 \rangle = \frac{1}{2} + \frac{\kappa^2}{F N_{\text{at}}} \langle (\hat{J}_z - \langle \hat{J}_z \rangle)^2 \rangle \]

IV. Detection
Probing strongly correlated states

Heisenberg Hamiltonian
Spin  $F=1$

$$H = \sum_{\langle n,n' \rangle} \cos \beta \hat{j}(z_n) \cdot \hat{j}(z_{n'}) + \sin \beta \left[ \hat{j}(z_n) \cdot \hat{j}(z_{n'}) \right]^2$$

Paramagnetic states

IV. Detection

$$\langle \hat{X}_s^{out} \rangle \sim \langle \hat{J}_z \rangle = 0$$

$$\langle (\delta \hat{X}_s^{out})^2 \rangle - \frac{1}{2} = \frac{\kappa^2}{2N_{at}} \langle (\hat{J}_z - \langle \hat{J}_z \rangle)^2 \rangle = \frac{5\kappa^2}{2}$$

Antiferromagnetic states

$$\langle \hat{X}_s^{out} \rangle \sim \langle \hat{J}_z \rangle = 0$$

$$\langle (\delta \hat{X}_s^{out})^2 \rangle - \frac{1}{2} = 0$$
Spatially resolved probing: access to the effective atomic spin

Averaging over phases $\phi$: access to the magnetic structure factor

\[ \langle \delta \hat{X}_S^{\text{out}}(k)^2 \rangle = \frac{1}{2} + \frac{\kappa^2 V}{4N_{\text{at}}} \left[ 4S_m(0) + S_m(2k) + S_m(-2k) \right] \]

\[ S_m(k) = \frac{1}{V} \int r \int r' e^{-i k \cdot (r - r')} \langle \delta \hat{J}_z(r) \delta \hat{J}_z(r') \rangle \]

\[ \langle \hat{X}_S^{\text{out}} \rangle \sim \langle \hat{J}_z^{\text{eff}} \rangle \]

\[ \langle (\delta \hat{X}_S^{\text{out}})^2 \rangle = \frac{1}{2} + \frac{\kappa^2}{N_{\text{at}}} \langle (\hat{J}_z^{\text{eff}} - \langle \hat{J}_z^{\text{eff}} \rangle)^2 \rangle \]

\[ \hat{J}_z^{\text{eff}} = \int dz \cos^2(k_P z + \phi) \hat{J}_z(z) \]

IV. Detection
Standing-wave probing of F=1 antiferromagnetic states

Dimer

Trimer
Magnetic structure factor for attractive fermions

\[ H = -t \sum_{i, \sigma} \left( \hat{c}_{i, \sigma}^\dagger \hat{c}_{i+1, \sigma} + \text{h.c.} \right) - U \sum_i \hat{n}_{i \uparrow} \hat{n}_{i \downarrow} \]

FFLO pairs with momentum

\[ Q = k_{F \uparrow} - k_{F \downarrow} \]

QMC calculation
IV.I Quantum Polarization Spectroscopy

- QPS can be used to distinguish strongly correlated antiferromagnetic phases of the spin-1 Heisenberg Hamiltonian

- Can distinguish superfluidity and the FFLO phase in a 1D chain of attractive fermions

Eckert et al, PRL 98, 100404 (2007)


IV.II-Counting statistics of atoms

- After expansion
- In the lattice

M. Schellekens et al Science 310, 648 (2005)

Greiner’s group Nature 462 2009
Bloch’s group Nature 467 2010
Chen’s group Nature 460 2009

IV. Detection
Counting formalism [Kelley & Kleiner 1964, Glauber 1965]

The probability distribution of counting $m$ fermionic/ bosonic atoms

$$p(m) = \frac{(-1)^m}{m!} \frac{d^m}{d\lambda^m} Q \bigg|_{\lambda=1}$$

can be derived from a generating function

$$Q(\lambda) = \text{Tr}(\rho : e^{-\lambda I :})$$

with the normal/appex order of the intensity of the atoms at the detector

$$\mathcal{I} = \int_0^t dt' \int dr' \Omega(r') \tilde{\Psi}^\dagger(r', t') \tilde{\Psi}(r', t')$$
\[ \hat{H} = -J \sum_{j=1}^{N} (\hat{c}_j^\dagger \hat{c}_{j+1} + \gamma \hat{c}_j^\dagger \hat{c}_j + h.c. - 2g \hat{c}_j^\dagger \hat{c}_j + g). \]

\[ H_{xy} = -J \sum_{j=1}^{N} \left[ (1 + \gamma) S_j^x S_{j+1}^x + (1 - \gamma) S_j^y S_{j+1}^y + g S_j^z \right] \]

Figure 5.1: a) Counting probability distribution \( p(m) \) of finding \( m \) particles as a function of \( m \) for the fermionic system eq. (5.1) with \( \gamma = \kappa = 1, \ g = 0 \) and \( N = 1000 \). b) Mean \( \bar{m}/N \) (blue squares) and variance \( \sigma^2/N \) (red circles) of the counting distribution as a function of the transverse field \( g \).
Detection of the QPT

\[
H_{xy} = -J \sum_{j=1}^{N} \left[ (1 + \gamma) S_j^x S_{j+1}^x + (1 - \gamma) S_j^y S_{j+1}^y + g S_j^z \right]
\]

Figure 5.4: (Color online.) a)-c) Mean \( \bar{m}/N \) (blue squares) and variance \( \text{var}/N \) (red circles) of the fermion counting distribution as a function of \( g \) for \( \kappa = 1 \), and indicated values of \( \gamma \). d)-f) Derivatives of the mean (blue squares) and variance (red circles) for the respective values of \( \gamma \).

Figure 6.2: Derivative of the mean \( m/N \) (blue squares) and the variance \( \sigma^2/N \) (red circles) of the counting distribution of the fermionic system eq. (5.1) with \( \gamma = 1 \) as a function of the transverse field \( g \). a) \( k_B T/J = 0 \), \( N_d = 0 \) for all \( g \); b) \( k_B T/J = 0.05 \) and \( N_d/N \approx 0 \) at \( g = 0 \); c) \( k_B T/J = 0.3 \) and \( N_d/N = 0.03 \) at \( g = 0 \); d) \( k_B T/J = 1 \) and \( N_d/N = 0.27 \) at \( g = 0 \).

Figure 5.3: (Color online.) Mean \( \bar{m}/N \) (blue squares) and variance \( \text{var}/N \) (red circles) of the fermion counting distribution as a function of \( g \) for \( \gamma = 1 \) (Ising model), and the indicated values of \( \kappa \).
Counting bosons after expansion

S. Braungardt, M. Rodriguez, A. Sen, U. Sen, M. Lewenstein (to be submitted)

Figure 7.3: MI vs. SF as a function of distance from detector. Probability distribution for MI (black bars) and superfluid (white bars) states in a 3x3x3 lattice. $\Delta_x = \Delta_y = 2$ mm, $\Delta_z = 2$ cm, $\kappa = 1$. a) $Z_0 = 1$ cm, b) $Z_0 = 3$ cm, c) $Z_0 = 5$ cm.

Figure 7.4: var($m$)/$N$ of the counting distribution for the MI (blue squares) and SF (green circles) state with respect to the distance from the detector $Z_0$. $\Delta_x = \Delta_y = 2$ mm, $\Delta_z = 2$ cm, $\kappa = 1$.

IV. Detection
V. QI processing with OL

I. Atoms
II. Molecules
I. Entangling quantum gates  


\[
\frac{1+e^{-i\varphi}}{2} |1_j\rangle |1_{j+1}\rangle + \frac{1-e^{-i\varphi}}{2} (|0_j\rangle(|0_{j+1}\rangle - |1_{j+1}\rangle) + |1_j\rangle(|0_{j+1}\rangle + 1_{j+1})) / 2
\]

V. Examples QIP
Signature: visibility of the Ramsey fringes

I. Swap gate via control of the exchange interaction


\[
\begin{align*}
|\psi_S\rangle &= \phi_S(x_1, x_2)|S\rangle = \left( |1_c, 0_g\rangle - |0_c, 1_g\rangle \right) / \sqrt{2} \\
|\psi_T^0\rangle &= \phi_T(x_1, x_2)|T^0\rangle = \left( |1_c, 0_g\rangle + |0_c, 1_g\rangle \right) / \sqrt{2} \\
|\psi_T^-\rangle &= \phi_T(x_1, x_2)|T^-\rangle = |0_c, 0_g\rangle \\
|\psi_T^+\rangle &= \phi_T(x_1, x_2)|T^+\rangle = |1_c, 1_g\rangle
\end{align*}
\]

Table 1 | Truth table for SWAP and $\sqrt{SWAP}$ gates

<table>
<thead>
<tr>
<th>Initial</th>
<th>State after time $t$</th>
<th>$\sqrt{SWAP}$ $t = \pi \hbar / 2U_{eg} = T_{SWAP} / 2$</th>
<th>SWAP $t = \pi \hbar / U_{eg} \equiv T_{SWAP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>0_c, 0_g\rangle$</td>
<td>$e^{-iU_{eg}t/2\hbar}</td>
<td>0_c, 0_g\rangle$</td>
</tr>
<tr>
<td>$</td>
<td>0_c, 1_g\rangle$</td>
<td>$\cos(U_{eg}t/2\hbar)</td>
<td>0_c, 1_g\rangle - i \sin(U_{eg}t/2\hbar)</td>
</tr>
<tr>
<td>$</td>
<td>1_c, 0_g\rangle$</td>
<td>$-i \sin(U_{eg}t/2\hbar)</td>
<td>0_c, 1_g\rangle + \cos(U_{eg}t/2\hbar)</td>
</tr>
<tr>
<td>$</td>
<td>1_c, 1_g\rangle$</td>
<td>$e^{-iU_{eg}t/2\hbar}</td>
<td>1_c, 1_g\rangle$</td>
</tr>
</tbody>
</table>

The table ignores a global phase factor $e^{-i\Omega t / \hbar}$. 
III. Quantum computation with trapped polar molecules
De Mille, PRL 88 (2002)

CNOT gate

Dipole dipole interactions

\[ E_{\text{int}} = -\sum_j d_{\text{eff}}/|x_i - x_j|^3 \]

\[ v_i = v_0 + d_{\text{eff}}E_i/h \]

\[ v_0 = hBJ(J+1) \]

\[ d_{\text{eff}} = d_g - d_e \]

\[ E_i = E_0 + V_i \]

\[ v_i \text{ in the GHz regime} \]

\[ d_{\text{eff}}^3/(\lambda/2)^3 \text{ in the 1/10 GHz regime} \]
Conclusions

- OL provide promising systems for the implementation of Quantum Information Processing
  - low decoherence rates
  - large systems
  - controllable, implement Hubbard and Spin Hamiltonians.

- Quantum state engineering with time-dependent potentials: adiabatic ramping, fast oscillatory potential, trains of pulses.

- Detection of strongly correlated phases is possible.

- Some quantum gates have already been implemented experimentally.

→ recent experimental breakthrough: single-site addressing
Thank you very much for your attention!!

1-year post-doctoral position in Madrid available

mirta.rodriguez@iem.cfmac.csic.es