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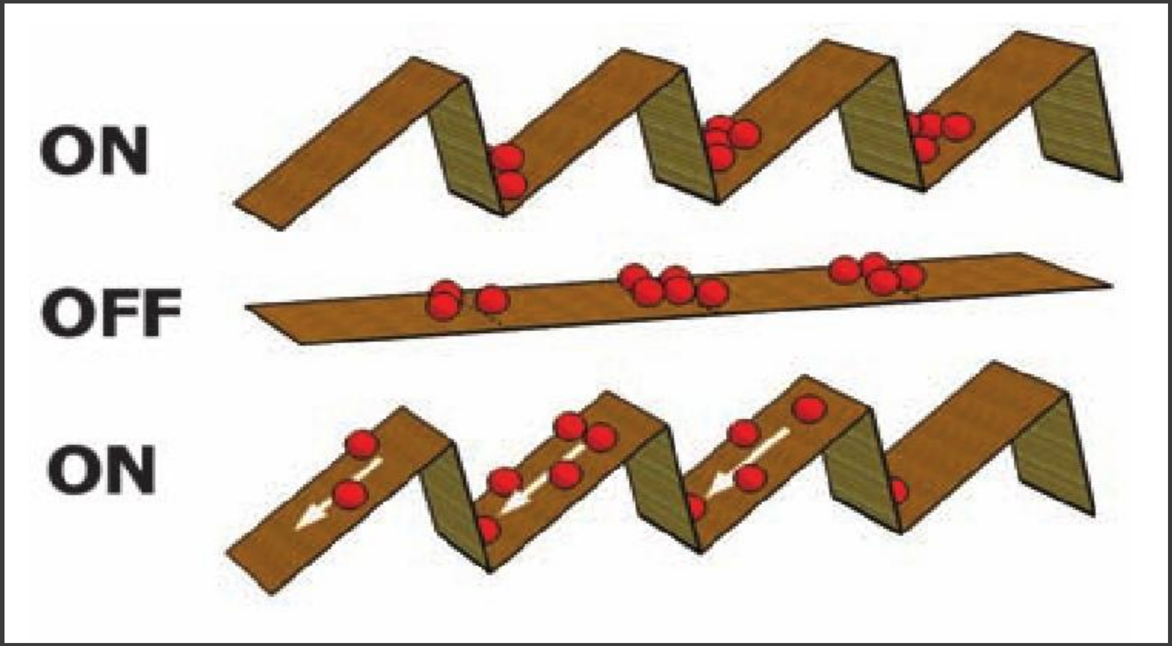
TUNABLE PARTICLE CURRENT WITH A FLASHING PERIODIC POTENTIAL

Motivation: ratchet phenomena

- In general, a time-dependent potential with non-zero average results in a directed current.
- It is normally expected that the opposite case i.e. a time-dependent field with zero mean may not lead to a directed current.
- This is however not the case: ratchet phenomena.
- In a ratchet system a directed current is generated along a periodic structure without using a biased field, i.e. using a time dependent field of zero mean.
- Unidirectional motion in some biological systems

Classical ratchet: flashing sawtooth potential



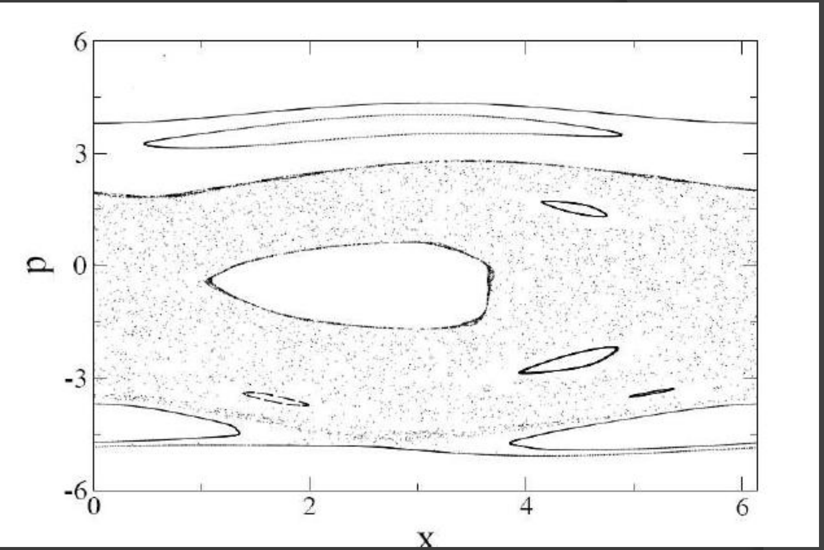
Picture from T. Salger et al
Science 326,2009

One obtains a directed current from a stochastic force with zero average

- On: particles role down a potential hill.
- Off: Stochastic Brownian motion redistributes particles.
  When the potential is off, particles are able to jump over the potential barrier.
- Thus particles move to the left even if on average the potential is zero.

Symmetries underlying classical ratchet dynamics

S. Flach, O. Yevtushenko, Y Zolotaryuk, PRL 84 (2000)



- Classical trajectory: $X(t,X_0,P_0)$ $P(t,X_0,P_0)$

  S_t $V(t+\varsigma)=V(-t+\varsigma)$ $t \rightarrow -t+2\varsigma$

  $X(-t+2\varsigma,X_0,P_0)$ $-P(-t+2\varsigma,X_0,P_0)$

  S_x $V(t)=-V(t+T/2)$, $V(x+\chi)=-V(-x+\chi)$

  $t \rightarrow t+T/2$ $x \rightarrow -x+2\chi$

  $-X(t+T/2,X_0,P_0+2\chi)$ $-P(t+T/2,X_0,P_0)$

  ergodic theorem $\rightarrow$ average momentum zero

  If either S_x or S_t symmetries are fulfilled, there is no ratchet current

- Quantum ratchets follow a similar principle and both **symmetries S_t and S_x need to be broken in order to induce a directed current.**

Hänggi et al, Flach et al

Evolution in a periodic time-dependent field

$$H(t) = H_0 + V(t) \qquad H_0 = -J\sum_l \left(b_l^\dagger b_{l+1} + b_{l+1}^\dagger b_l\right)$$

$$V(l,t) \;=\; Kf(t)V(l) \qquad \begin{array}{c} f(t) = f(t+T) \\ V(l) = V(l+L) \end{array}$$

$$\boxed{H(t)\,|\,\psi(t)\rangle = i\frac{\partial}{\partial t}\,|\,\psi(t)\rangle}$$

Initial state $|\,\psi(t_0)\rangle$ with **zero** momentum

Floquet theory for time-periodic potentials

$$H^F = H(t) - i\frac{\partial}{\partial t} \qquad H(t) = H(t+T)$$

$$H^F\,|\,\phi_\alpha(t)\rangle = \varepsilon_\alpha\,|\,\phi_\alpha(t)\rangle \qquad |\,\phi_\alpha(t)\rangle = |\,\phi_\alpha(t+T)\rangle$$

quasienergies

cyclic states

One can expand the wave function in the cyclic state basis with the corresponding quasienergies .
The initial state is the zero momentum eigenstate of H_0.

$$|\psi(t)\rangle = \sum_j e^{-i\varepsilon_j t} c_j\,|\,\phi_j(t)\rangle$$

$$c_j = \langle\phi_j(0)|\psi(0)\rangle,$$

Current

- The momentum of the particle at any time t is given by

$$\langle p(t)\rangle = \sum_{j,j'} c_j c^*_{j'} e^{-it(\varepsilon_j-\varepsilon'_j)} \langle\phi_{j'}(t)|\, p \,|\phi_j(t)\rangle$$

- The interesting quantity is the **average current** after n cycles which is the average of the average current during period m

$$\mathcal{I}(nT) = \tfrac{1}{n}\textstyle\sum_{m=1}^{n} \mathcal{I}_m$$

$$\mathcal{I}_m = \frac{1}{T}\int_{(m-1)T}^{mT} \langle p(t')\rangle dt'.$$

asymptotic current & symmetries

$$\mathcal{I}(nT) = \frac{1}{n}\sum_{j,j'} c_j c^*_{j'} \langle p\rangle_{jj'} \frac{1-e^{-inT(\varepsilon_j-\varepsilon'_j)}}{1-e^{iT(\varepsilon_j-\varepsilon'_j)}}$$

$$\langle p\rangle_{jj'} \equiv \frac{1}{T}\int_0^T \langle \phi_j(t)\,|\,p\,|\,\phi_{j'}(t)\rangle\, e^{-it(\varepsilon_j-\varepsilon'_j)}dt.$$

- If S_t **and** S_x are broken ,then $<p>_{jj}$ non zero → ratchet

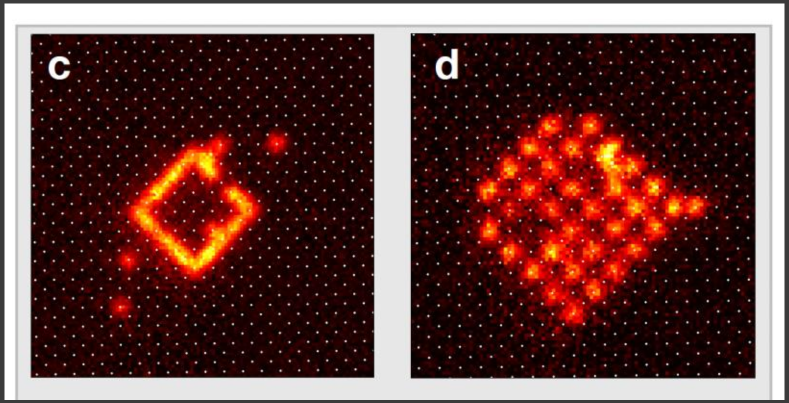
$$\mathcal{I}(\infty) = \sum_j |c_j|^2 \langle p\rangle_{jj}$$

- If either of the symmetries is preserved $<p>_{jj}$ =0→ no asymptotic current

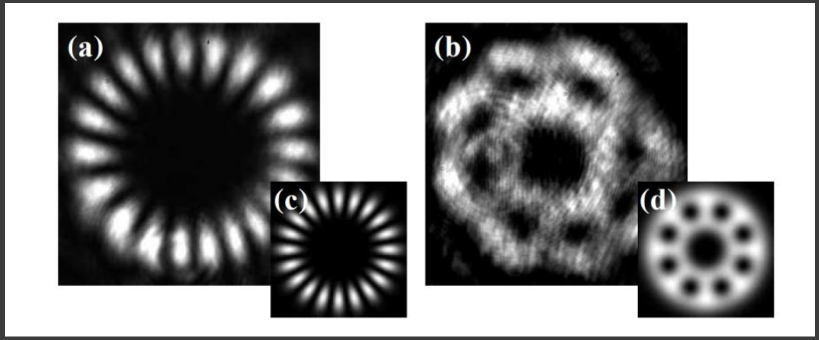
S.Denisov, L. Morales-Molina. S. Flach, P. Hanggi, PRA 75,063242 (2007)

  - Unless one exploits some resonance in the system

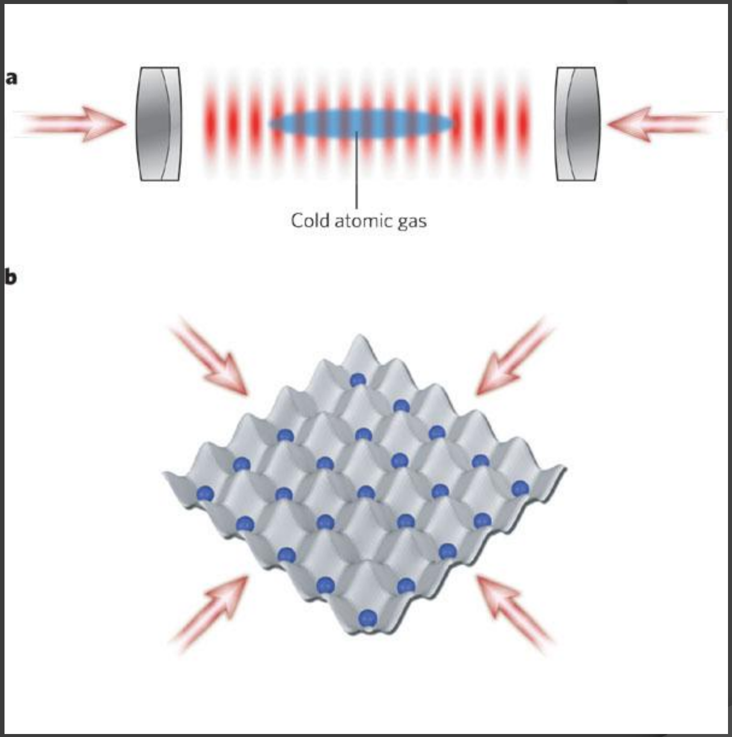
Ultracold atomic samples→ experimental realization of a quantum ratchet

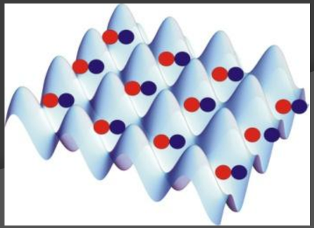


pics from I. Bloch's group

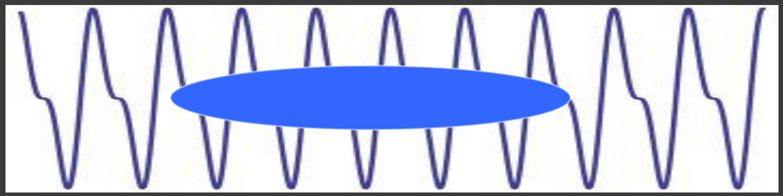


pics from S. Franke-Arnold et al OE 2007



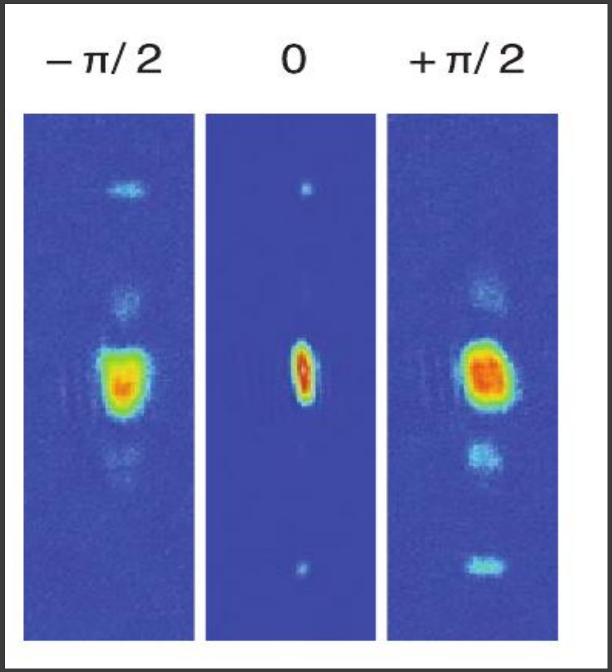


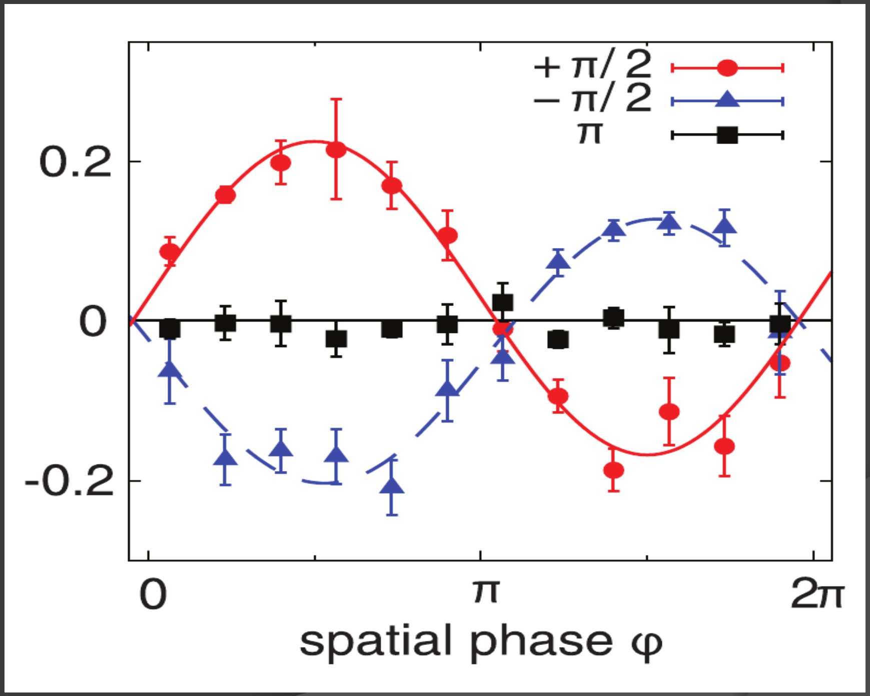
Experimental realization of a non-dissipative quantum ratchet



$$V(x,t) = A(t)[V_1 \cos(2kx) + V_2 \cos(4kx + \phi)]$$
$$A(t) = A_1 \sin^2(\omega t/2) + A_2 \sin^2(\omega t + \theta/2)$$

θ



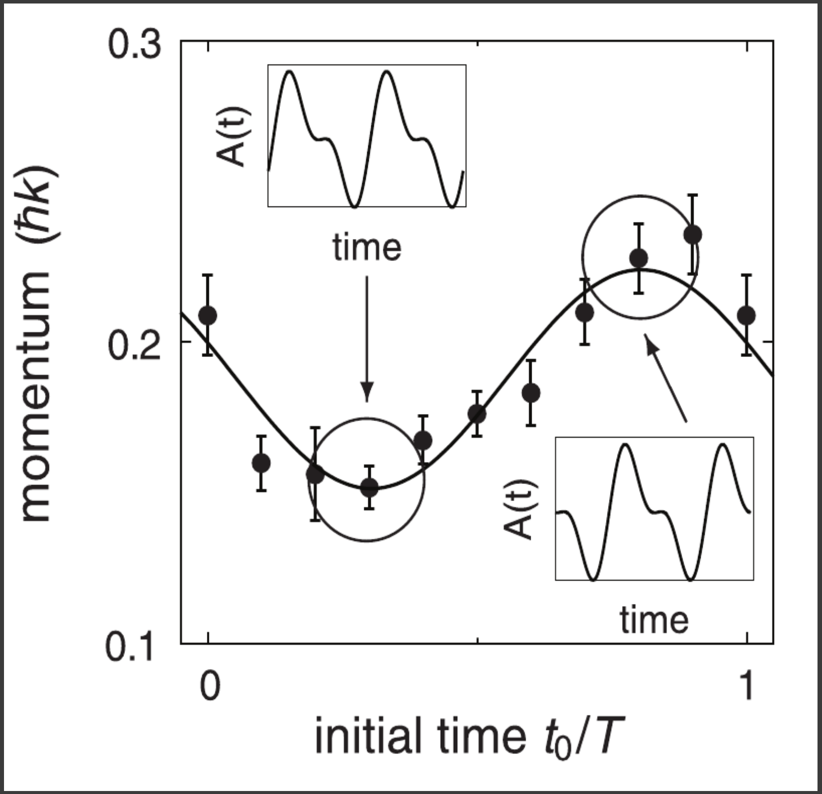


$\phi = \pi/2$

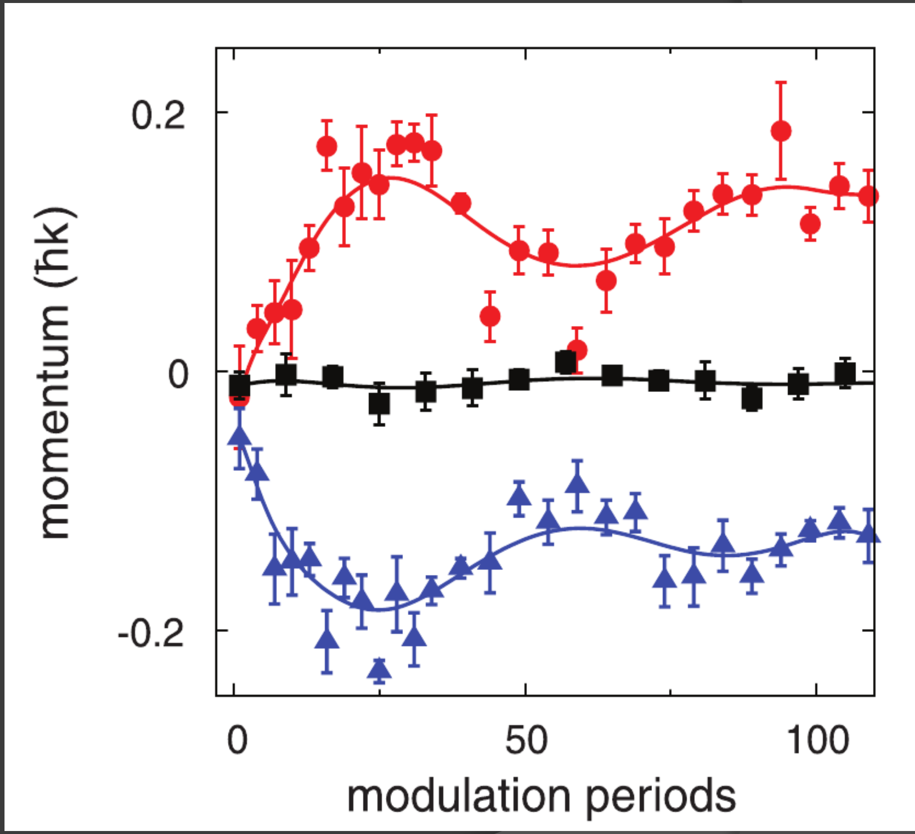
T. Salger , S. King, T. Hecking, C. Geckeler, L. Morales-Molina, M. Weitz, Science 326, 2009

Quantum effects:

Dependence on initial time



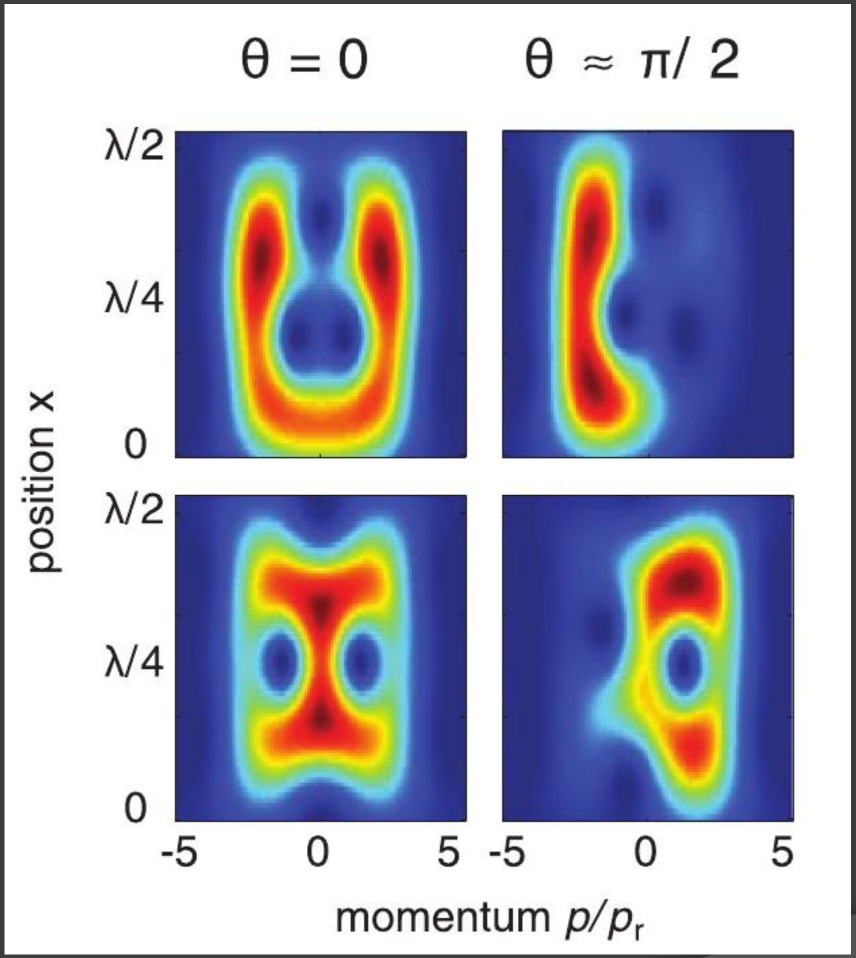
Quantum beatings



T. Salger et al Science 326,2009

cyclic eigenstates

T. Salger et al Science 326,2009

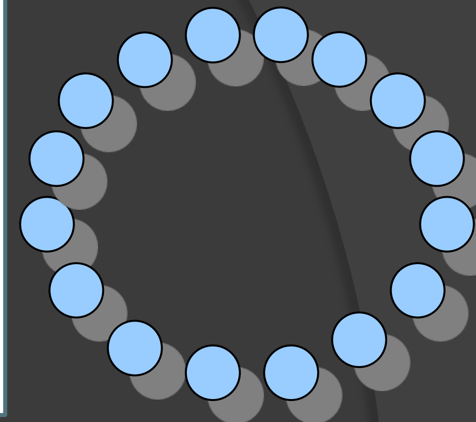


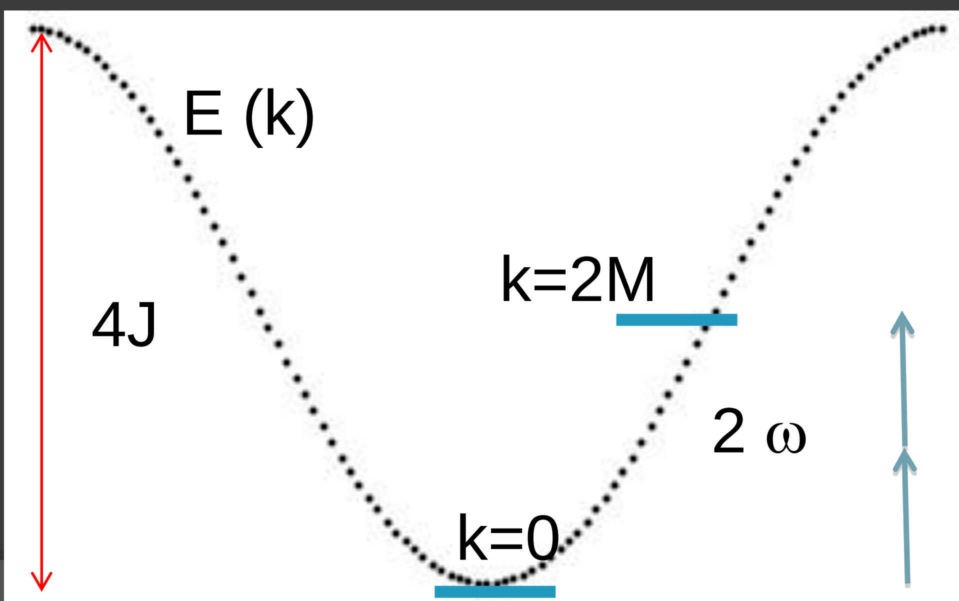
ring+resonant weak driving force

$$H(t) = H_0 + V(t)$$

$$H_0 \;=\; -J \sum_{i=1}^{L} |i\rangle\langle i+1| + |i+1\rangle\langle i|$$

$$V(i,t) \;=\; V \sin(\omega t)\left[\sin(M x_i) + \alpha \sin(2 M x_i + \phi)\right]$$

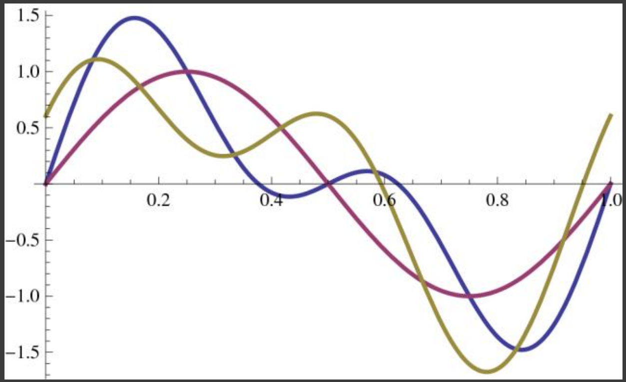




M integer < (L-1)/2
L system size

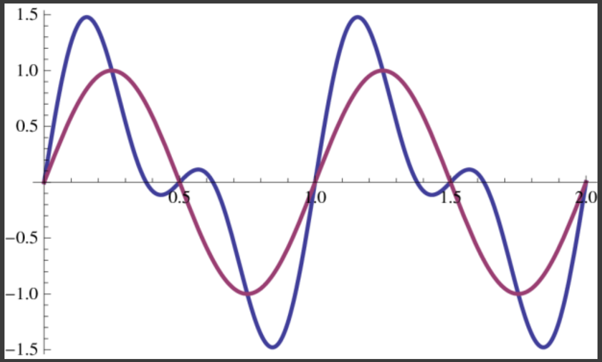
- On resonance
- Weak driving V/J<1

flashing potential does not break the temporal symmetry $\rightarrow$ no ratchet current



$V(x)=\sin(x)+\alpha \sin(2x+\phi)$

- $\alpha=0, \phi=0$
- α non-zero, $\phi=0$
- α, ϕ non-zero



$V(t)=\sin(\omega t)+\beta \sin(2\ \omega t)$

- $\beta=0$
- β non-zero

$V(x)=[\sin(x)+\alpha \sin(2x+\phi)] \sin(\omega t)$
breaks the spatial but not the temporal symmetry

C E Creffield and F. Sols, Phys Rev. Lett 103, 200601 (2009)

average current (blue line) and average current per cycle

a) M=5, L=41, ω =J, V/J =0.1 ,α=1.2 , $\phi=\pi/4$

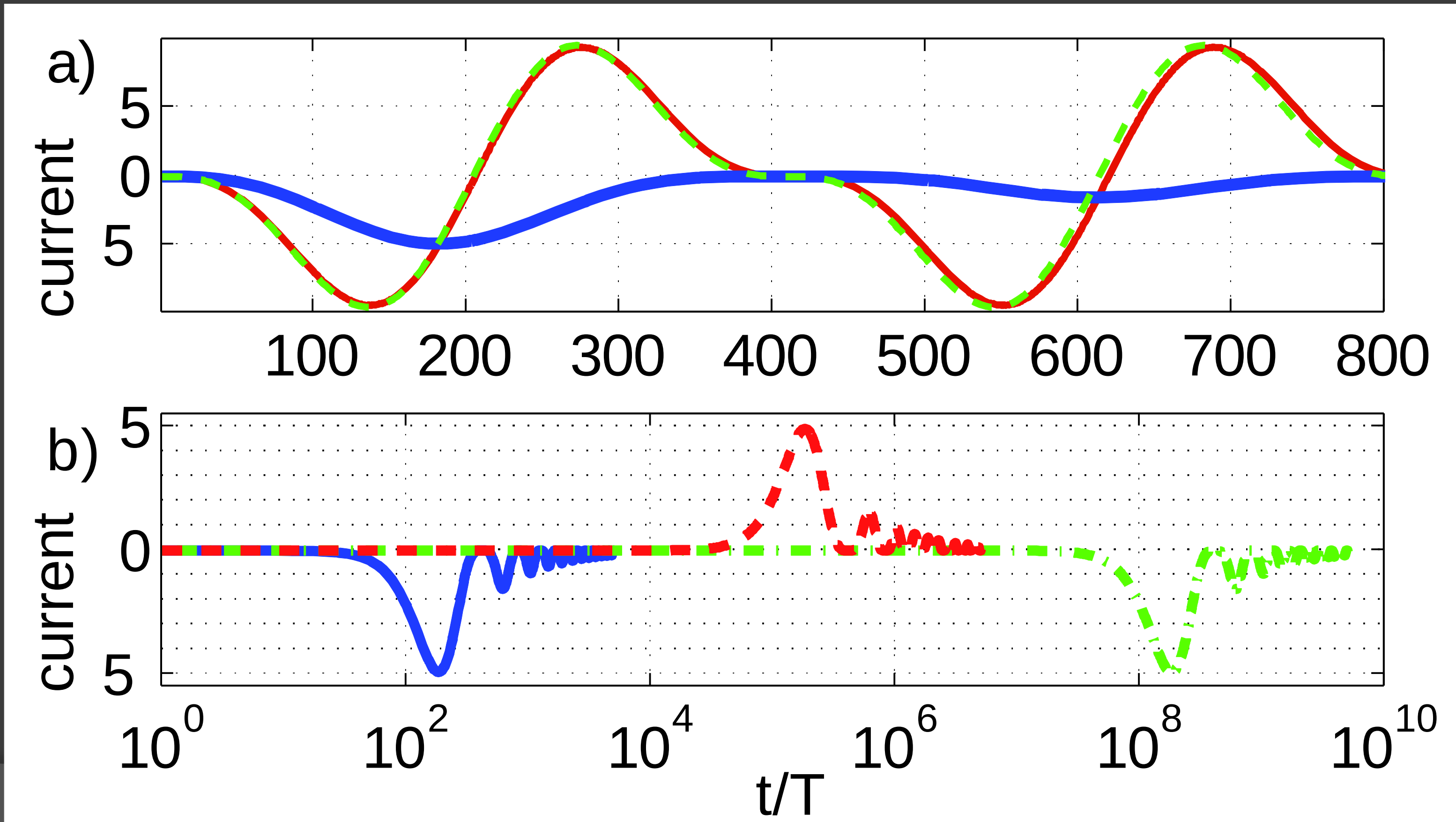
b) V/J=0.1, sqrt(10)10^{-2},10^{-3}

no asymptotic current

current changes sign with ϕ and is 0 for $\phi=0$

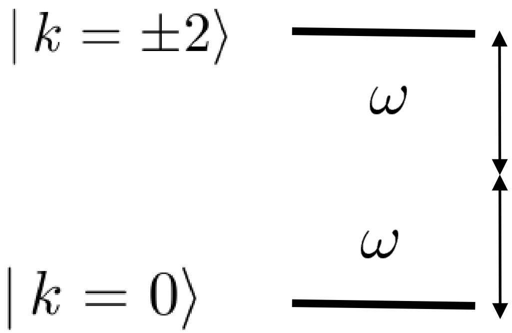
maximum average current M

tunable time-scale for the maximum current

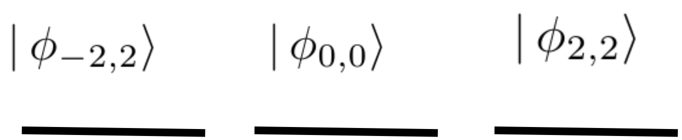


3-level model

- On-resonance



$$|\phi_{k,n}(t)\rangle = |k\rangle\,|n\rangle = |k\rangle\, e^{-in\omega t}$$



- Weak driving V/J<1

$$\mathcal{T}(\epsilon) = V + V G_0(\epsilon)\mathcal{T}$$

$$G_0(\epsilon) = \sum_j \frac{|j\rangle\langle j|}{\epsilon - \varepsilon_j^0} \quad \varepsilon_j^0 \equiv E_k - n\omega$$

$$\mathcal{T}(\varepsilon_0^0) \simeq V G_0(\varepsilon_0^0) V$$

$$\langle 0,0 \,|\, T \,|\, 0,0 \rangle = \sum_j \frac{|\,\langle j \,|\, \hat{V} \,|\, 00 \rangle\,|^2}{\varepsilon_0 - E_j}$$

$$\langle 0,0 \,|\, T \,|\, 0,0 \rangle \;=\; \frac{V^2}{8}\left(\frac{1}{\varepsilon_0 - E_{11}} + \frac{1}{\varepsilon_0 - E_{1,-1}}\right) + \frac{V^2\alpha^2}{8}\left(\frac{1}{\varepsilon_0 - E_{21}} + \frac{1}{\varepsilon_0 - E_{2,-1}}\right)$$

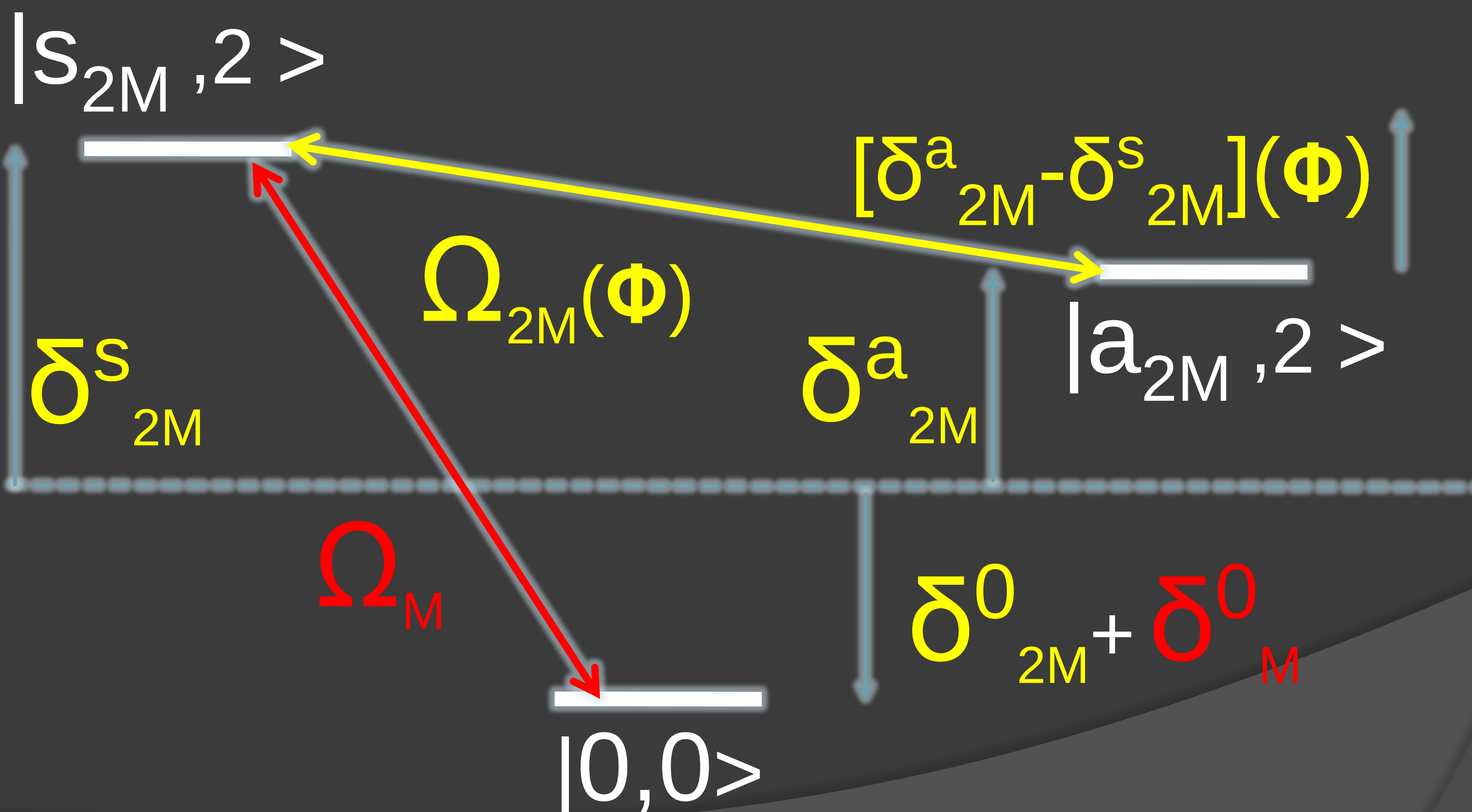
Effective matrix Hamiltonian

$$\mathcal{T} \simeq \frac{V^2}{4}\begin{pmatrix} \delta^s_{2M}(\alpha,\phi) & \Omega_M & \Omega_{2M}(\alpha,\phi) \\ \Omega_M & \delta^0_M + \delta^s_{2M}(\alpha) & 0 \\ \Omega^*_{2M}(\alpha,\phi) & 0 & \delta^a_{2M}(\alpha,\phi) \end{pmatrix}$$

Effective 3-level model

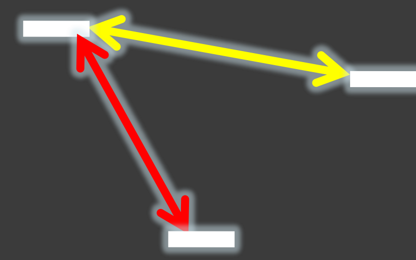
$$\mathcal{T}(\varepsilon_0^0) \simeq V G_0(\varepsilon_0^0) V$$

$$\mathcal{I}(nT) = \frac{1}{n} \sum_{j,j'} c_j c_{j'}^* \langle p \rangle_{jj'} \frac{1 - e^{-inT(\varepsilon_j - \varepsilon_j')}}{1 - e^{iT(\varepsilon_j - \varepsilon_j')}}$$



Optimal parameters

$V(x)=V[\sin(Mx)+\alpha \sin(2Mx+\phi)] \sin(\omega t)$

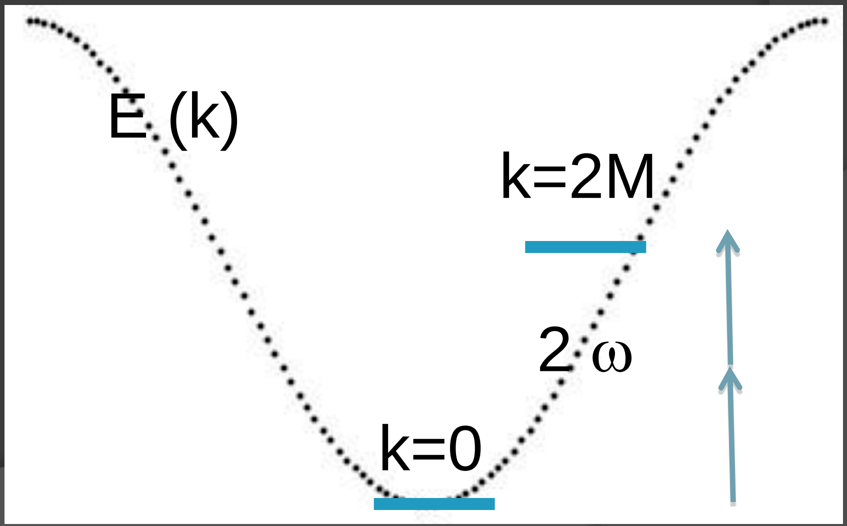


- $\alpha \sim 1$
- $\boldsymbol{\Phi} \sim \pi/4$
- $2M = k_{max}/2$
- Independent of V/J

→ time scale

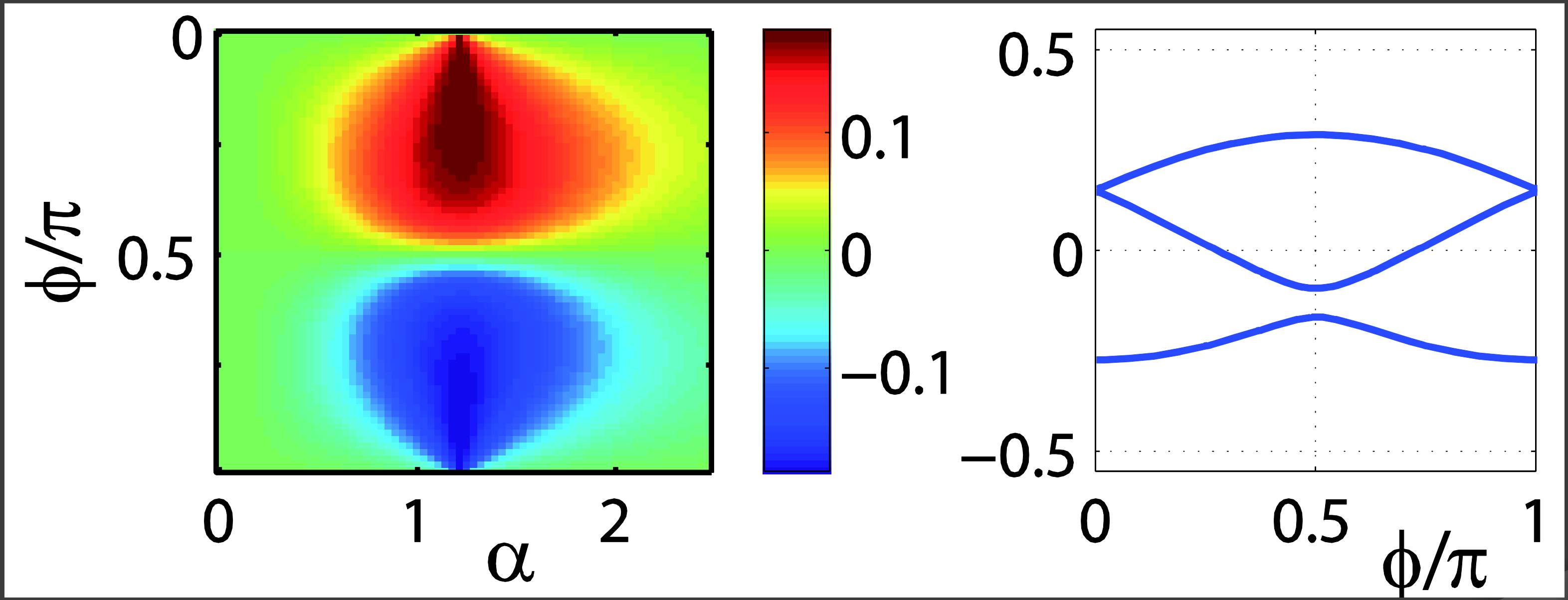
$\Omega_{2M}(\boldsymbol{\Phi}) \sim \mathrm{Cos}\ 2\boldsymbol{\Phi}$

$[\delta^a_{2M}-\delta^s_{2M}](\boldsymbol{\Phi}) \sim \mathrm{Sin}\ 2\boldsymbol{\Phi}$



Current

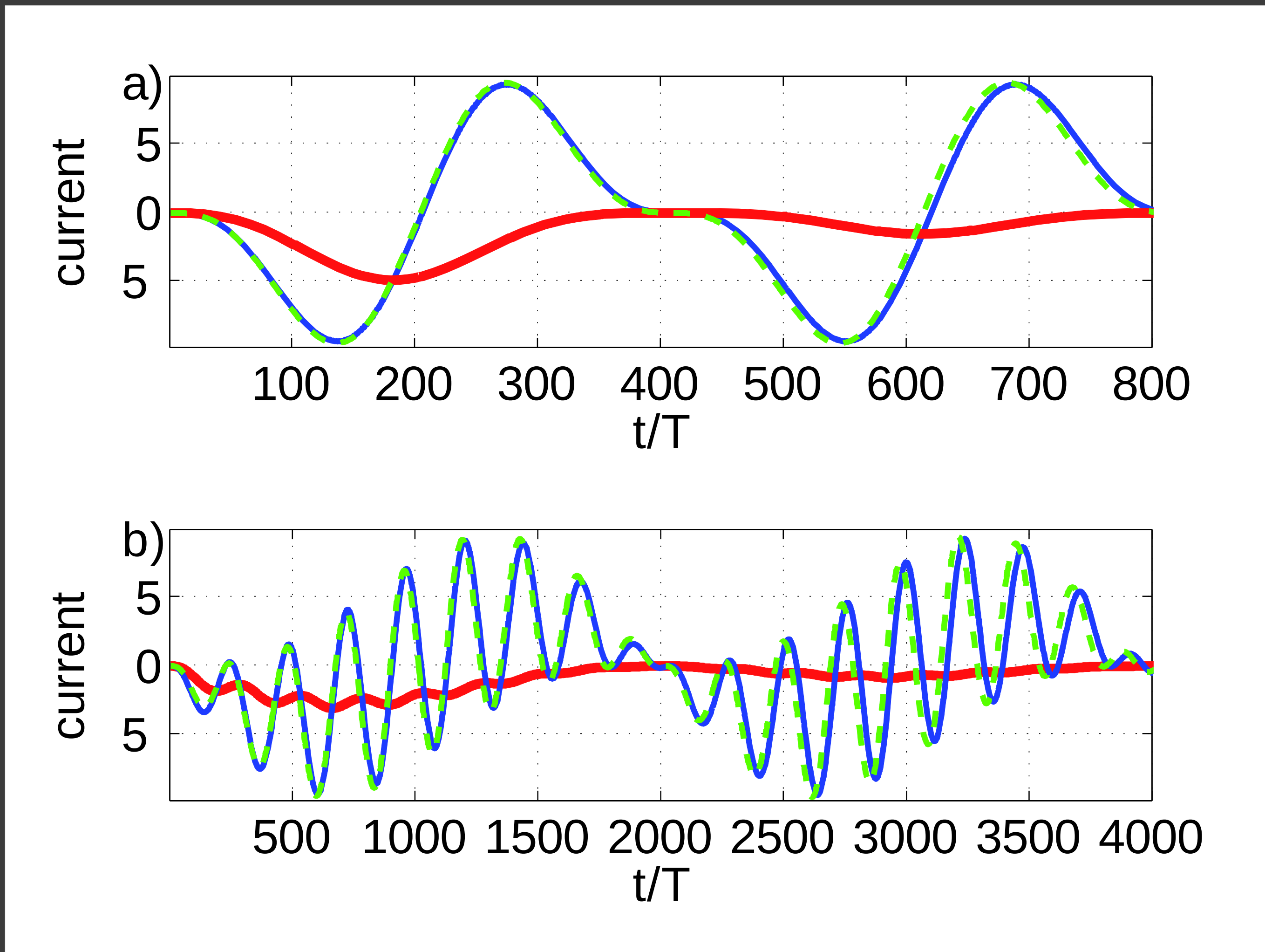
$$\mathcal{I}_m = \mathcal{C}(\alpha,\phi)\sum_{j<j'}\sin(mT\Delta\varepsilon_{jj'}),$$



- Amplitude of the current C/4M
- Quasienergies V^2/ω

Resonance (ω =J) Weak coupling (V/J =0.1) M=5 L=41

Current



- a) $\Phi/\pi=0.25$
- b) $\Phi/\pi=0.1$

resonance

$\omega = J$

weak coupling

$V/J = 0.1$

$\alpha=1.2$

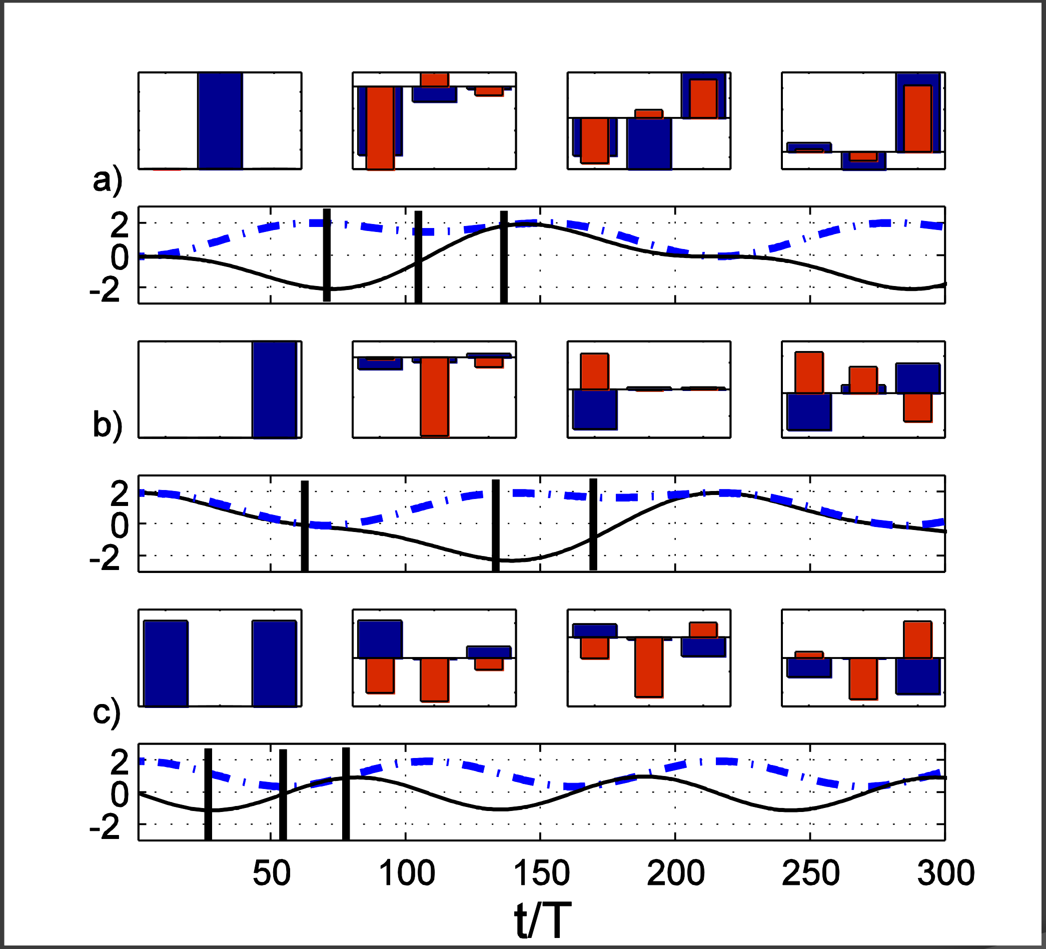
$M=5$ $L=41$

$2M = k_{max}/2$

one can obtain tunable strong average currents

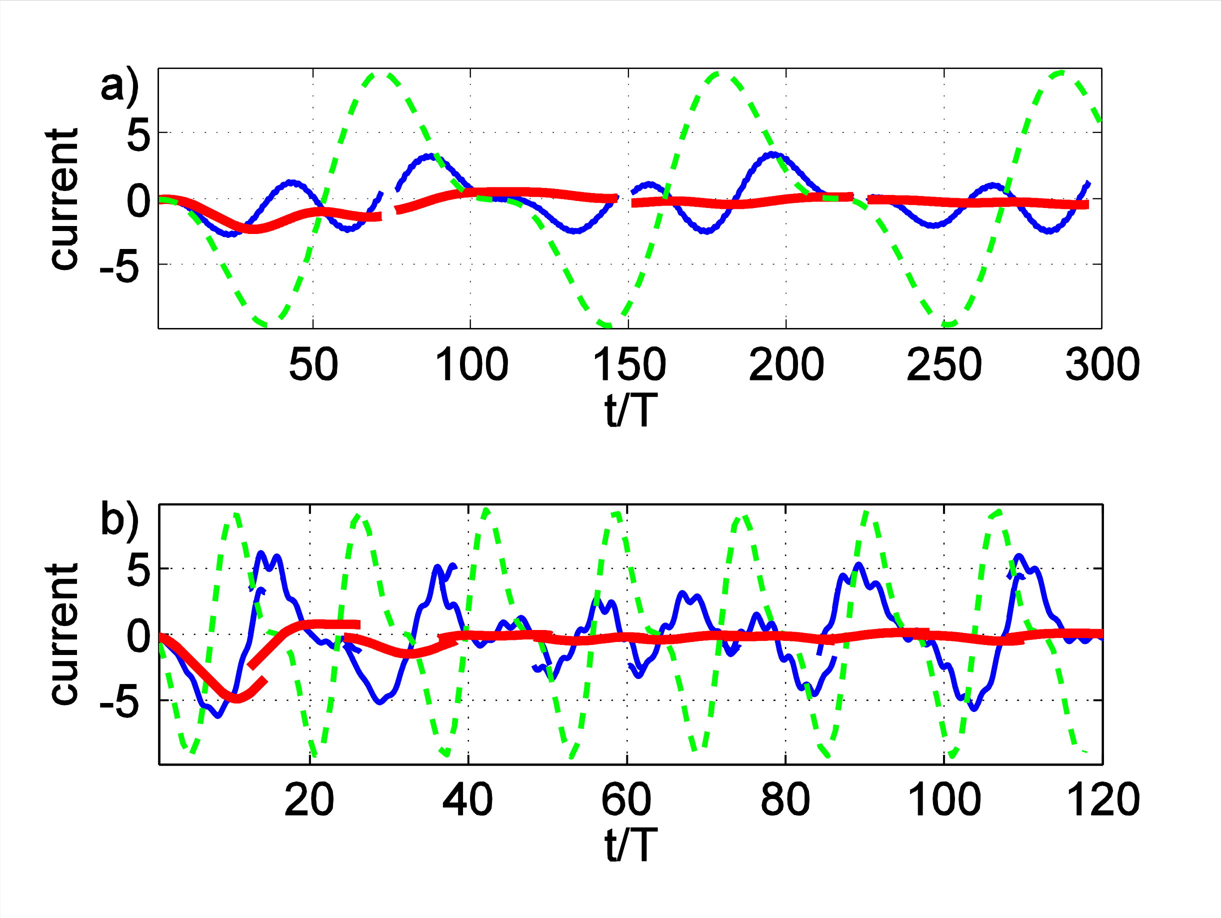
- H_0 sets the length L and energy scale J
- Tune driving to resonance $\omega = J$
- Tune α, ϕ to the optimal values
- We obtain an average current $M = k_{max}/4$
- The time scale can be tuned $t_e = 11.5J/V^2$ by controlling the driving strength.

One can control the state and the average energy in the system



- State in the momentum basis
- Average energy in the system

Robustness



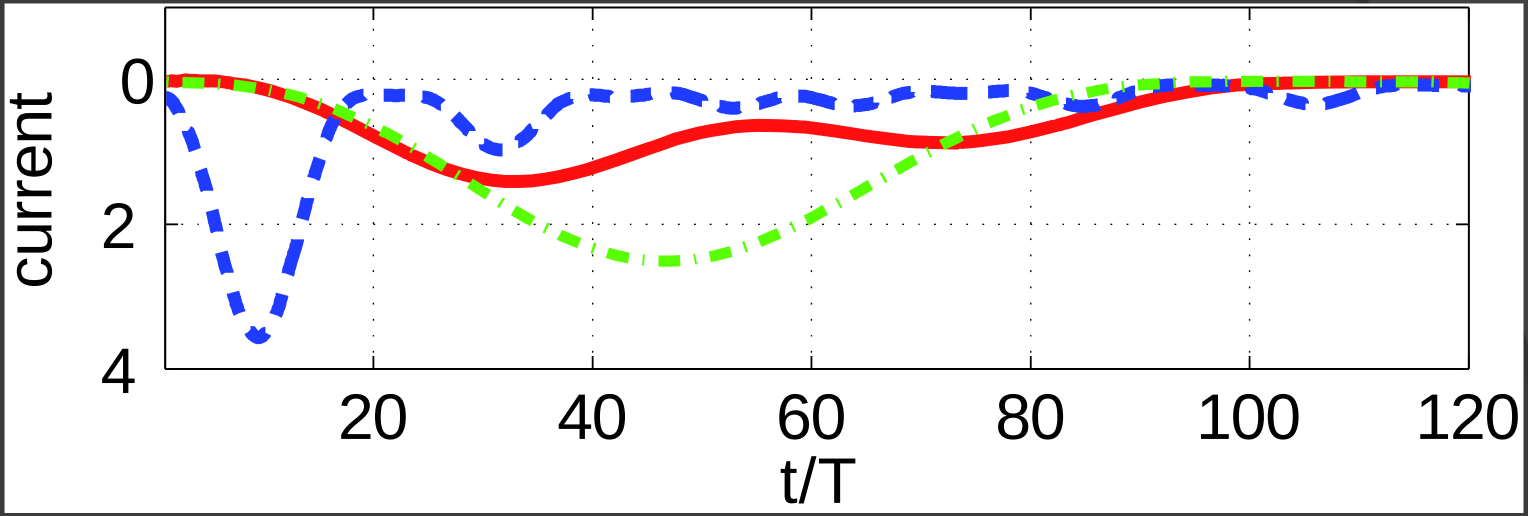
- Weak coupling condition

  a) $V/\omega=0.5$

- Resonance condition

  b) $V/\omega=0.2$

  $\omega=1.01J$



- Average current

Blue → out of resonance

Red→ stronger coupling

Green→ gaussian wave packet with zero momentum

Summary

- One can obtain strong long-lasting average currents using a weakly coupled on-resonant flashing potential
- The potential does not break the time reversal symmetry and it is therefore not a ratchet
- The current is tunable to experimentally relevant times and robust.

J.Santos, R.Molina, J. Ortigoso and MR, arXiv:1010.4523

THANKS!