Many-Body physics meets Quantum Information

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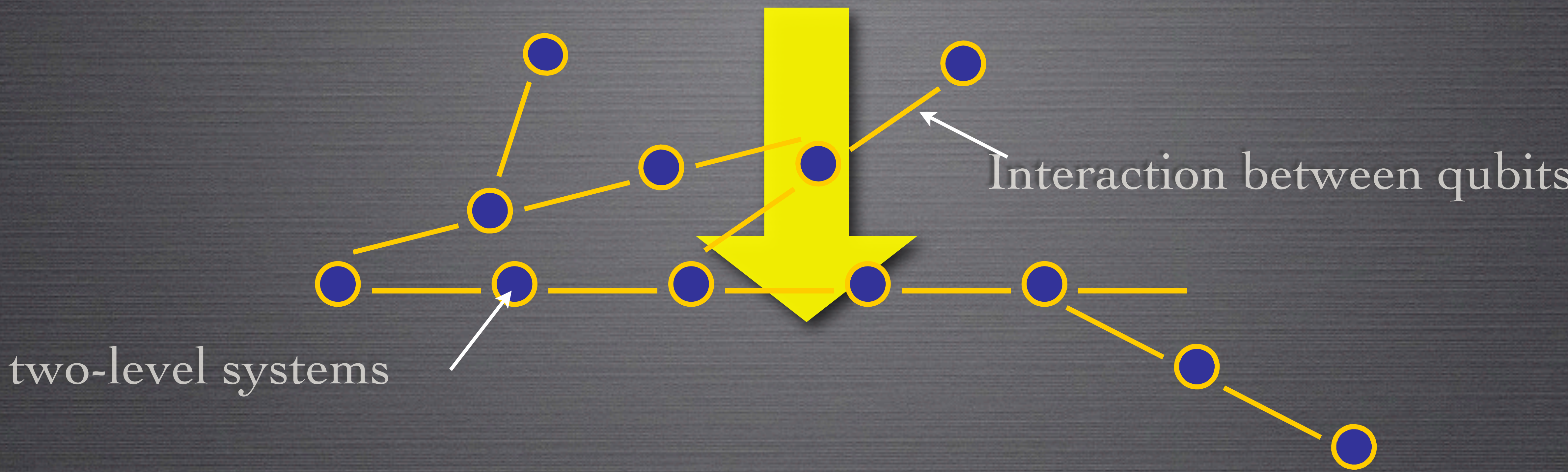
Quantum Computers



Many-Body Systems

Controlled in the ...

- preparation
- evolution
- measurement

Links between Quantum Information & Statistical Mechanics

- Quantum Information tools in Condensed Matter
- New methods to study Many-Body problems
- Adiabatic quantum comp. vs Kibble-Zurek mechanism

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Review:
L. Amico, et al Rev.Mod.Phys. **80**, 517 (2008)

Entanglement

If

$$|\Psi\rangle_{ab} \neq |\psi\rangle_a |\chi\rangle_b$$

then the state is entangled

ENTANGLEMENT as a RESOURCE
It is believed to be the main ingredient of computational speed-up in quantum information protocols

Entanglement in Condensed Matter

- Spin Systems
- Superconductivity
- Quantum Hall Effect
- ….

- Characterization of condensed phases

- Collective phenomena in Quantum Information and Quantum Communication

How to measure entanglement

- Entanglement between two spins in the network (bipartite)

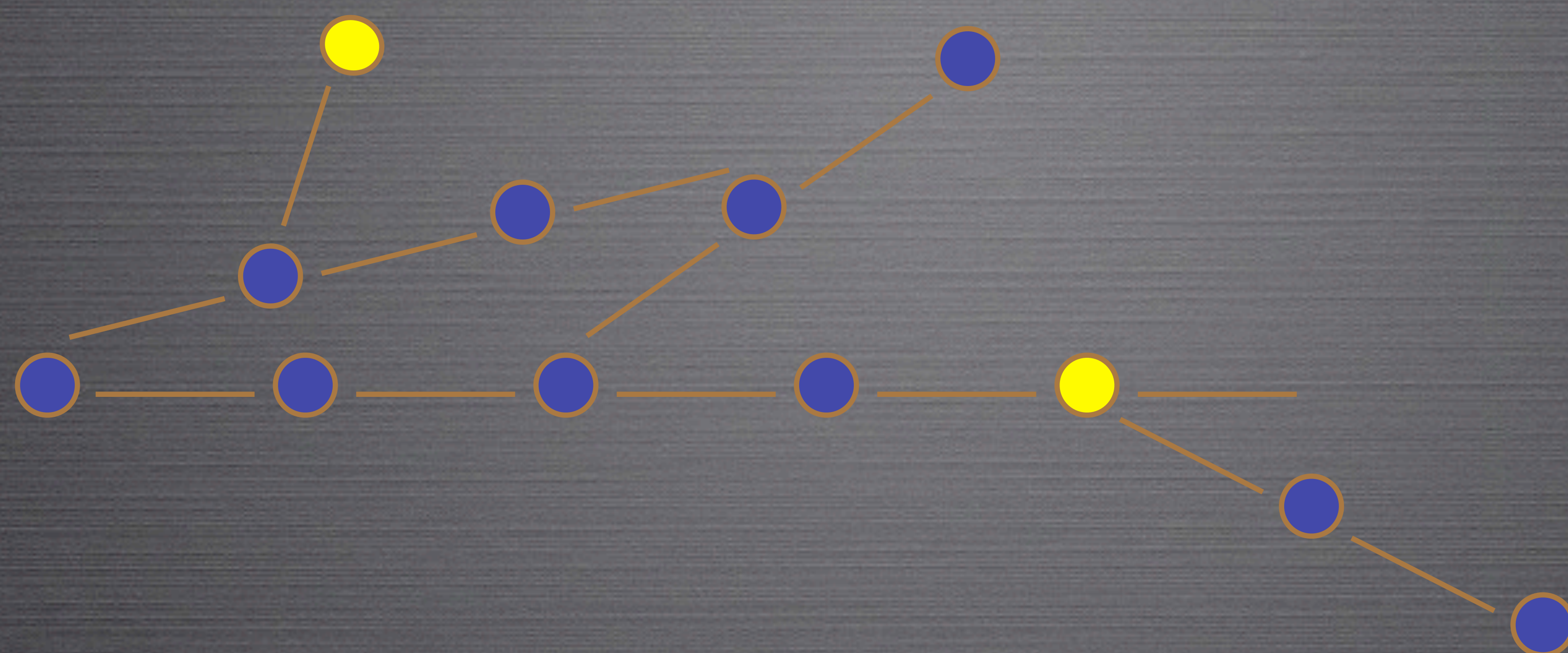
- Multipartite entanglement

- Block entropy

- Localizable entanglement

- ...

Bipartite entanglement $\neq$ Correlation

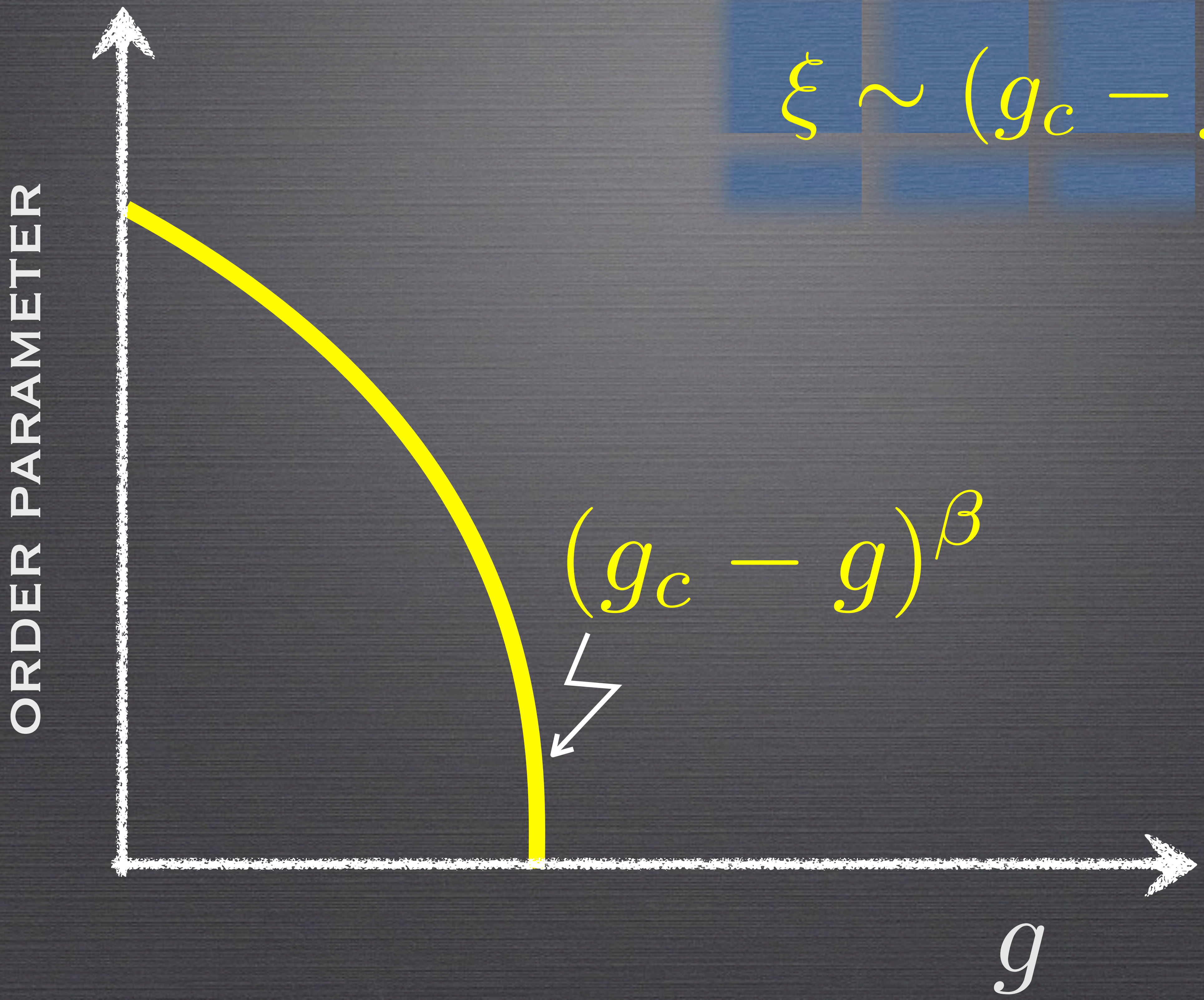


$\rho(i,j)$

The state of the two selected spins is mixed

$$|\uparrow\uparrow\uparrow\cdots\uparrow\rangle$$

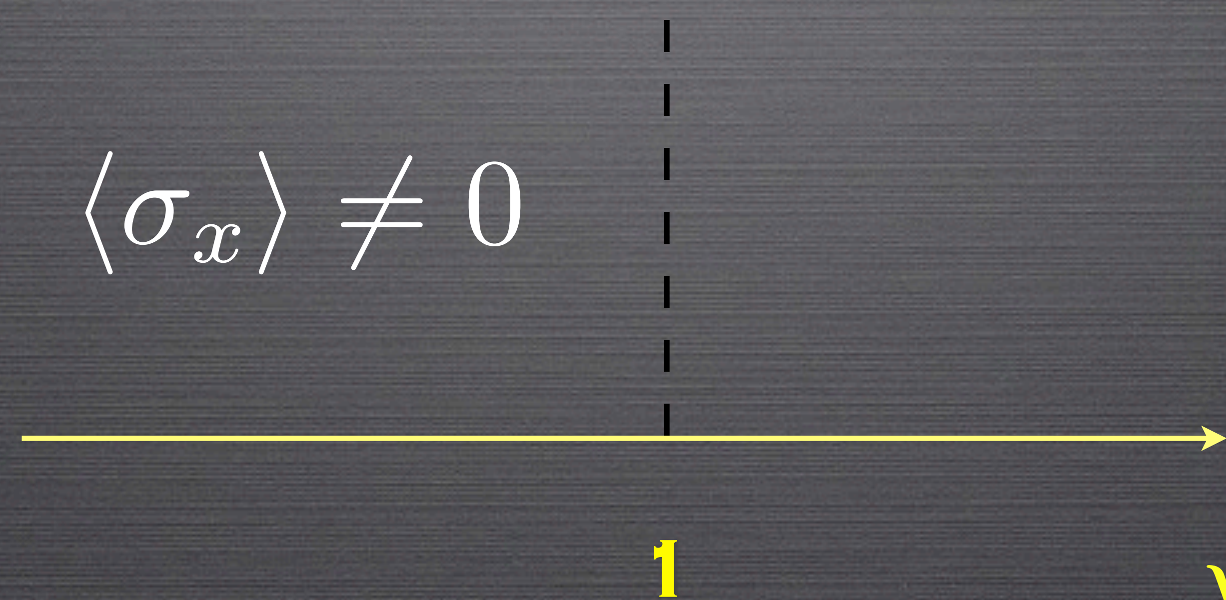
$$\xi \sim (g_c - g)^{-\nu}$$



Ising chain in a transverse field

$$H = -\frac{J}{2}\sum_{i=1}^{N}(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y - h\sum_{i=1}^{N}\sigma_i^z$$

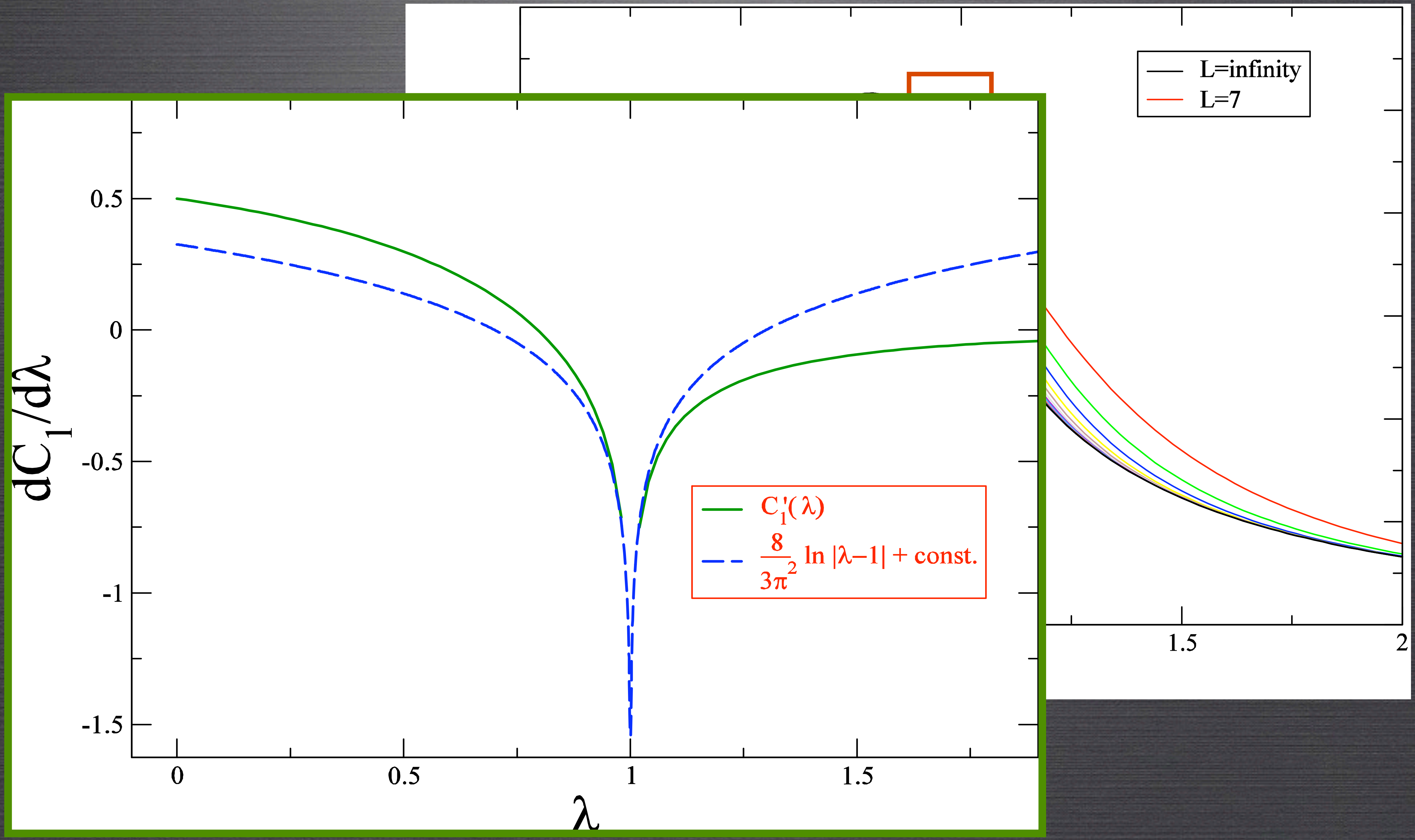
$$\langle \sigma_x \rangle \neq 0$$



$$\lambda = \frac{J}{2h}$$

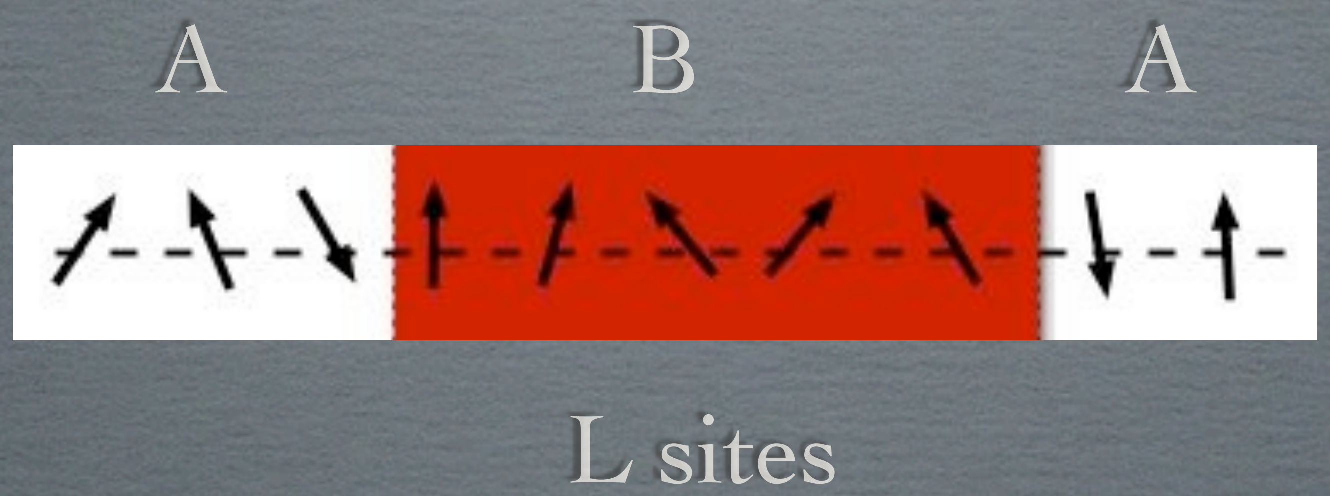
Exact solution - free fermions

Next neighbor entanglement



Block entropy

Ground State: $|\Psi_{GS}\rangle$

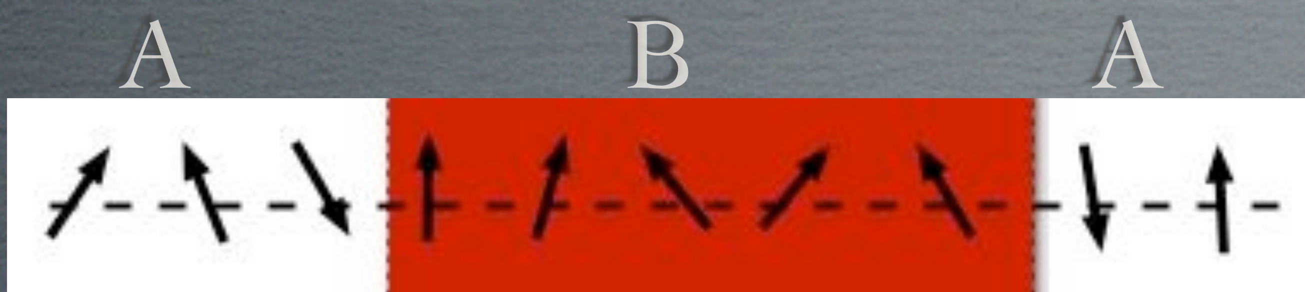


$$S(\rho_L) \equiv -tr(\rho_L log \rho_L)$$

$$\rho_L = tr_{N-L}(|\Psi_{GS}\rangle\langle\Psi_{GS}|)$$

Block entropy

$$S(\rho_L) \equiv -tr(\rho_L log \rho_L)$$



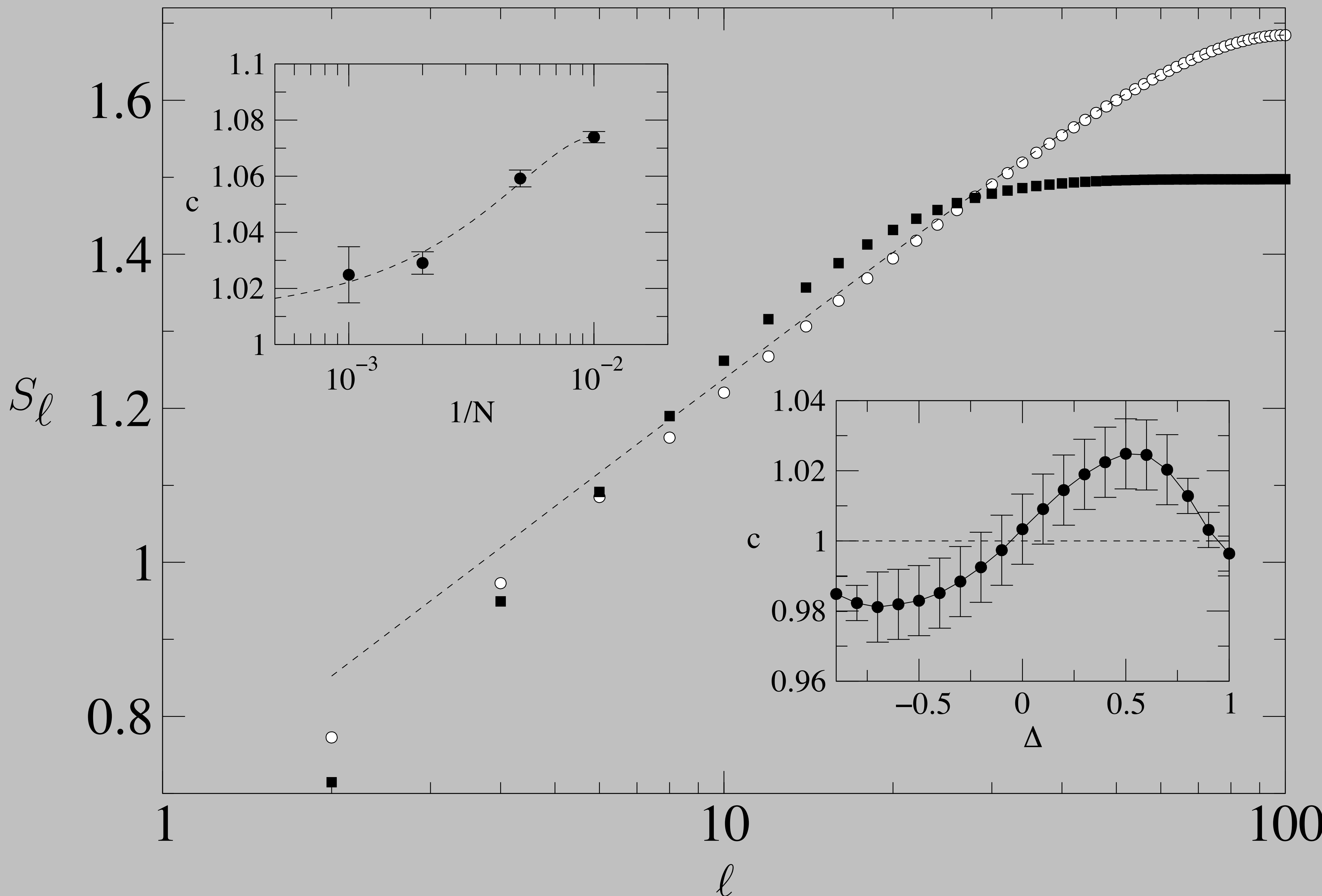
$$S_L \sim \frac{c}{6} \log_2 L$$ critical chain

$$S_L \sim \frac{c}{6} \log_2 \xi$$ non-critical chain

Area's law in d dim

$$S_B \sim L^{d-1}$$

Entropy scaling



Links between Quantum Information & Statistical Mechanics

- Quantum Information tools in Condensed Matter

- New methods to study Many-Body problems

- Adiabatic quantum comp. vs Kibble-Zurek mechanism

Reviews:
J. Dziarmaga, Adv. Phys. **59**, 1063 (2010)
A. Dutta et al, Rev. Mod Phys. (2011)

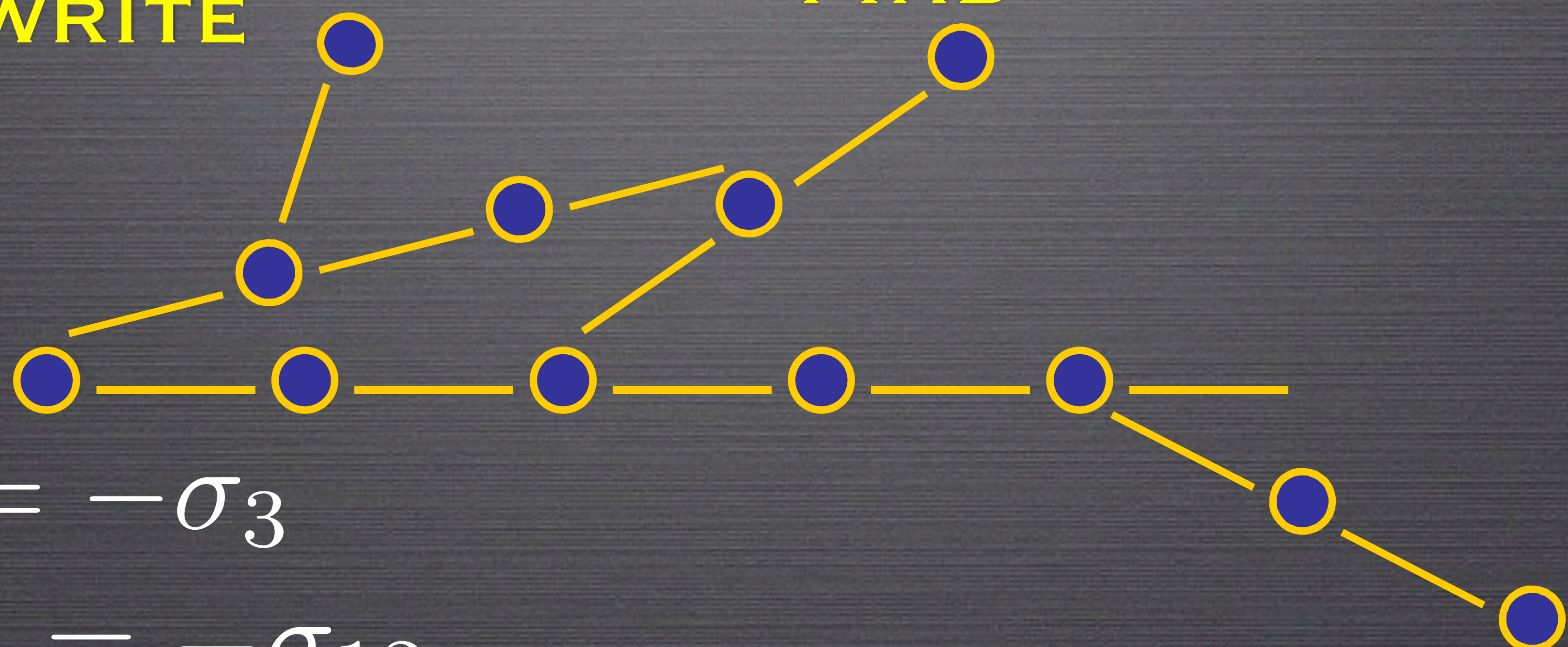
Adiabatic quantum computation

E. Fahri, J. Goldstone and S. Gutmann '00

$$\mathcal{H}_f$$

"Easy" to write

The ground state may still be very difficult to find



$$\sigma_1 = \sigma_2 = -\sigma_3$$

$$\sigma_{25} = \sigma_{32} = -\sigma_{12}$$

$$\sigma_{16} = \sigma_{44} = -\sigma_1$$

$$\dots$$

$$h_{ijk} = \sigma_i^z \sigma_j^z \sigma_k^z + \cdots$$

Adiabatic quantum computation

E. Fahri, J. Goldstone and S. Gutmann '00

$\mathcal{H}_i$ The ground state is known

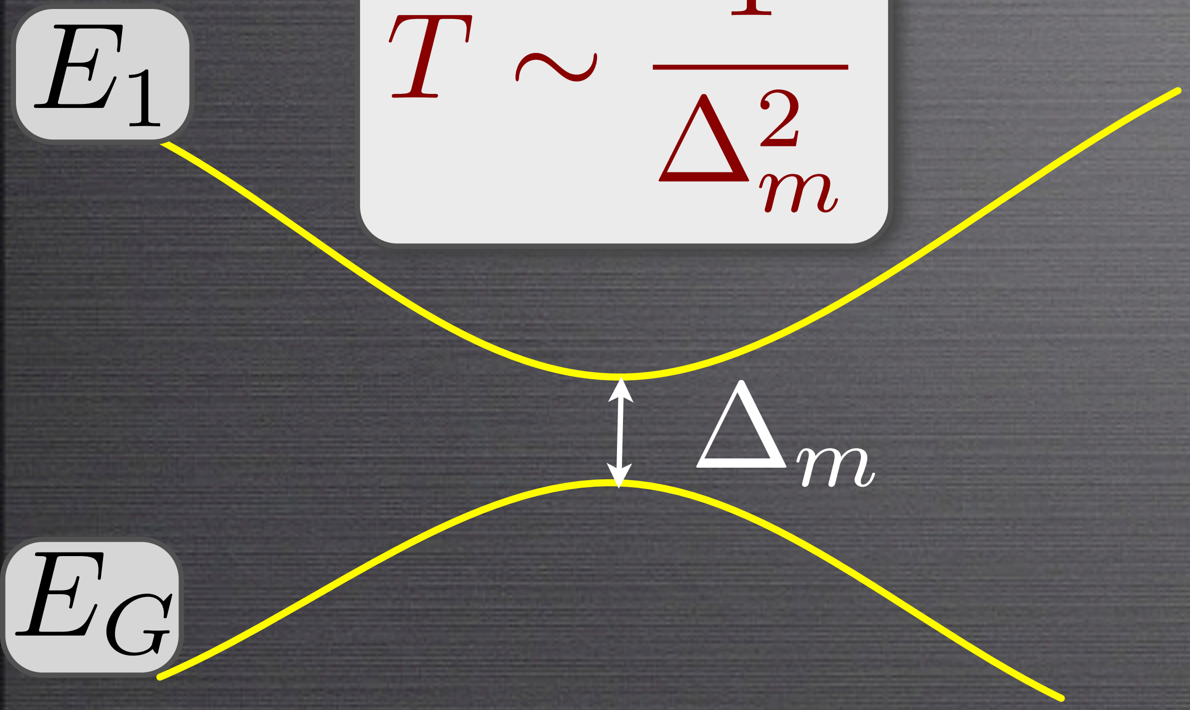
$\mathcal{H}_f$ The ground state is the solution to our problem

Adiabatic evolution

$$\mathcal{H}(t) = \frac{T-t}{T}\mathcal{H}_i + \frac{t}{T}\mathcal{H}_f$$

Adiabatic quantum computation

$$T \sim \frac{1}{\Delta_m^2}$$



$$\Delta_m \sim \frac{1}{N^\eta}$$ Easy problem

$$\Delta_m \sim e^{-N^\eta}$$ Difficult problem

Quantum speed up ⟷ Quantum critical point

Topological Defect Formation

Signatures of phase transitions which have occurred in the early universe by determining the density of defects left in the broken symmetry phase as a function of the rate of quench.

Kibble '76, Zurek '85

Topological Defect Formation

Simulation of phase transitions in the early universe in condensed matter systems (superfluids and Josephson junctions)

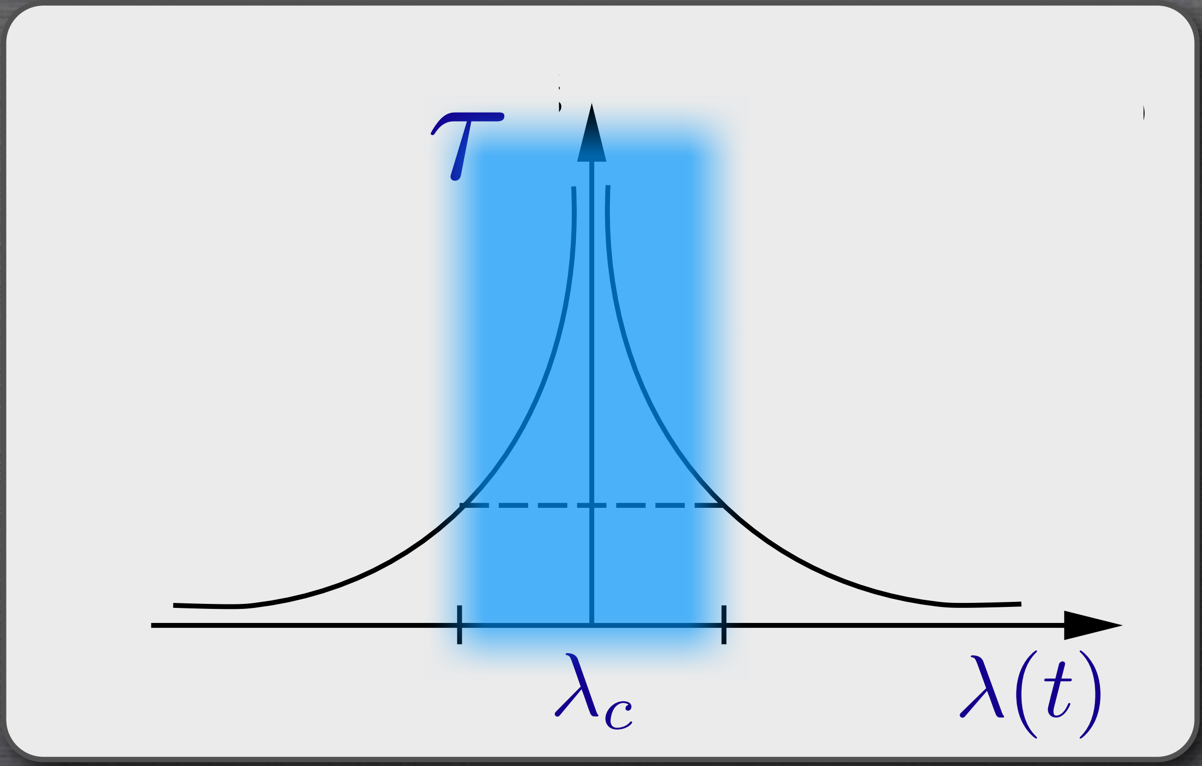
Th: Zurek '85-'88
Exps:Bauerle et al '96,Ruutu et al'96}

Extension to quantum phase transitions

Zurek, Dorner, Zoller '05
Polkovnikov '05

...

Adiabatic Dynamics Close to a Critical Point



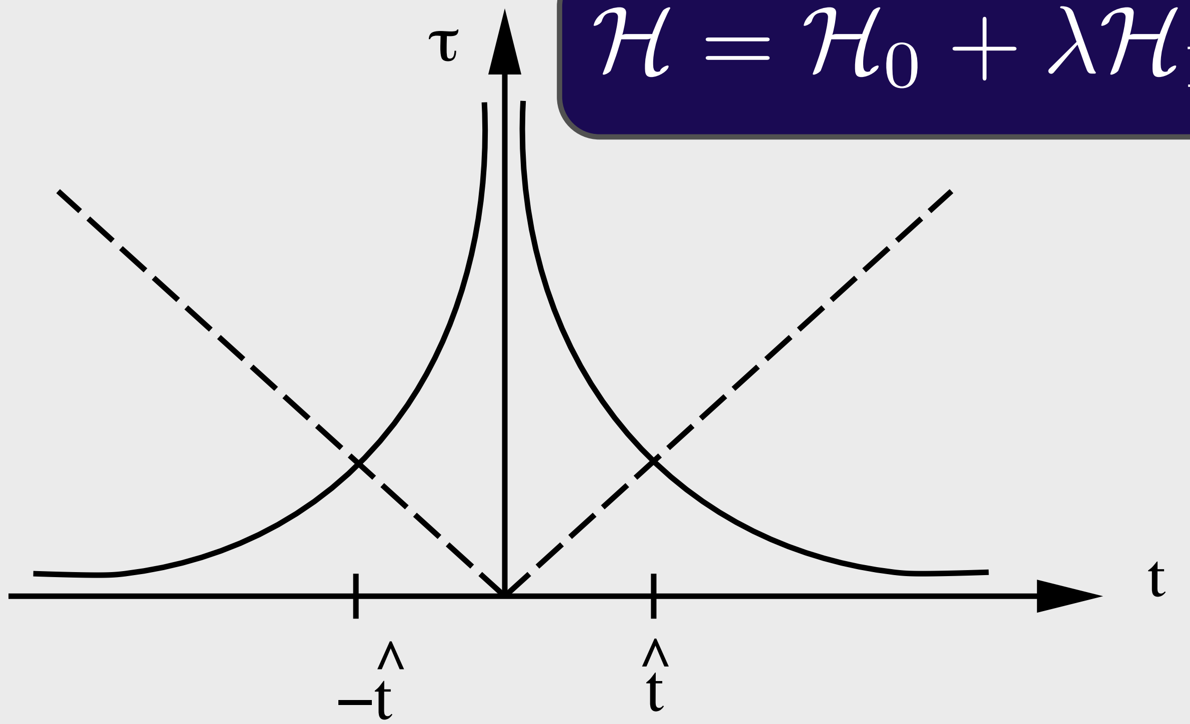
- How effective is it to execute a given computational task by slowly varying in time the Hamiltonian of a quantum system?

- Is it possible to find the ground state of a classical system by slowly annealing away its quantum fluctuations?

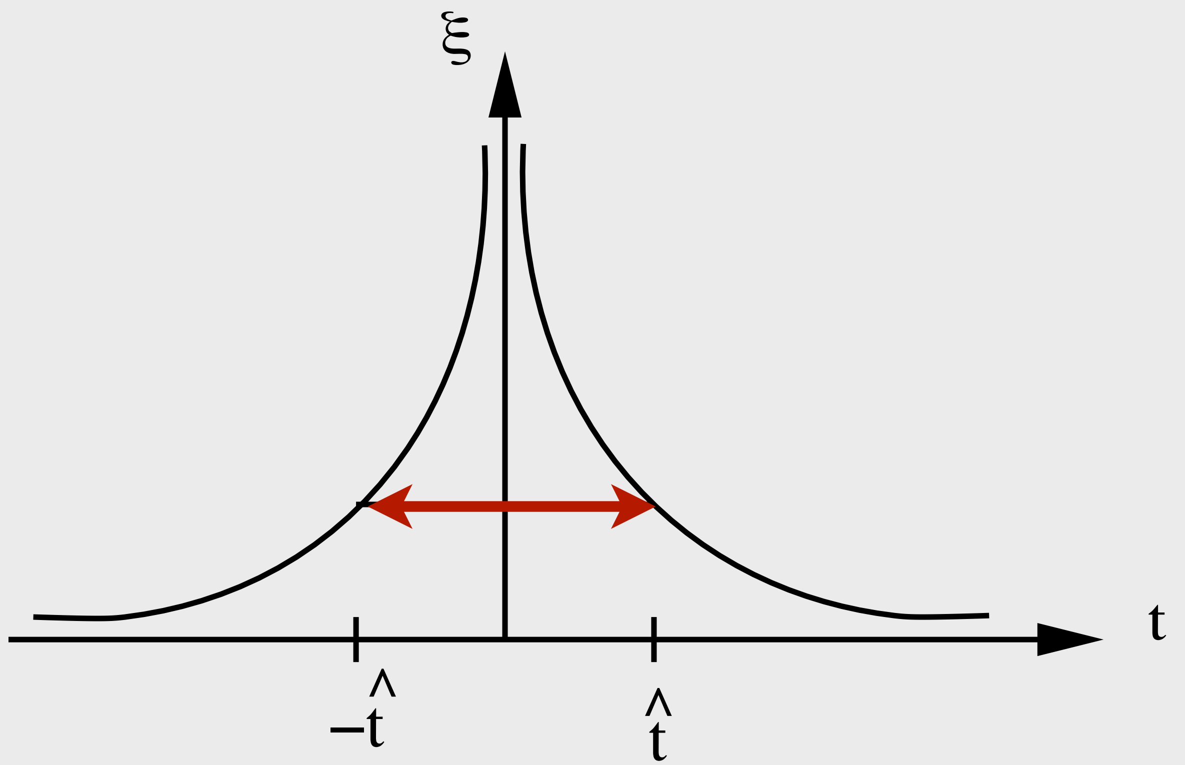
- What is the density of defects left over after a passage through a continuous (quantum) phase transition?

Defect density

W. Zurek '85
W. Zurek, U. Dorner and P. Zoller '05
A. Polkovnikov '05



$$\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{H}_1$$



$$\lambda - \lambda_c = vt$$

The adiabatic approximation breaks down when

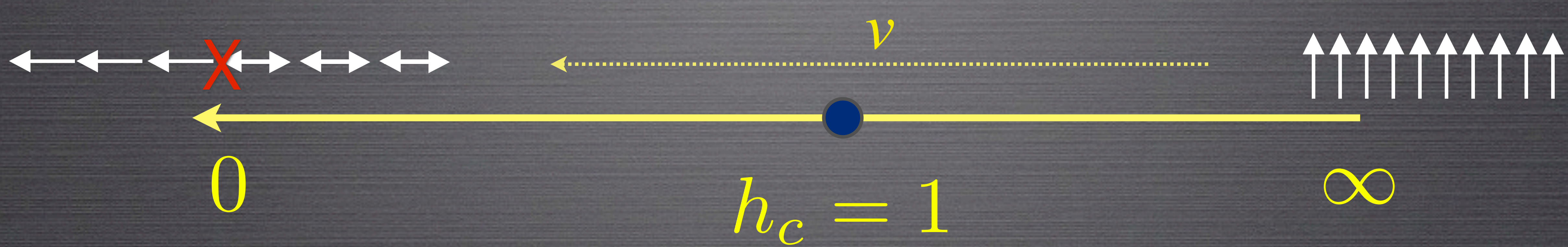
$$\frac{\dot{\lambda}}{\lambda} \sim \tau$$

$$\rho_{def} \sim \hat{\xi}^{-d} \sim v^{\frac{d\nu}{z\nu+1}}$$

$$\mathcal{E}_{res} \sim \mathrm{J}\rho_{def}$$

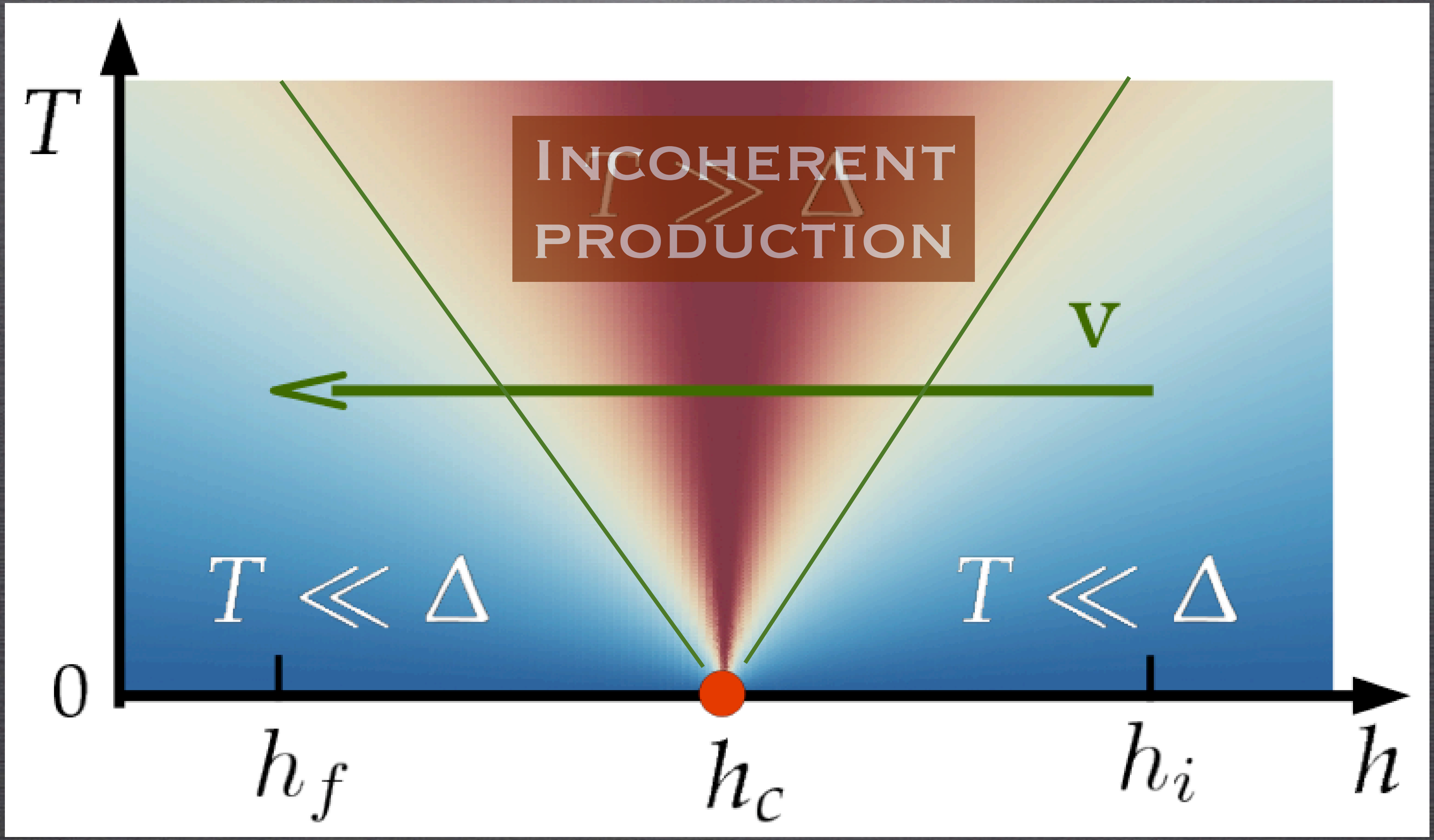
1D Ising Model

$$H = -\frac{J}{2}\sum_{j}^{N}\{\sigma_j^x\sigma_{j+1}^x + h(t)\sigma_j^z\}$$



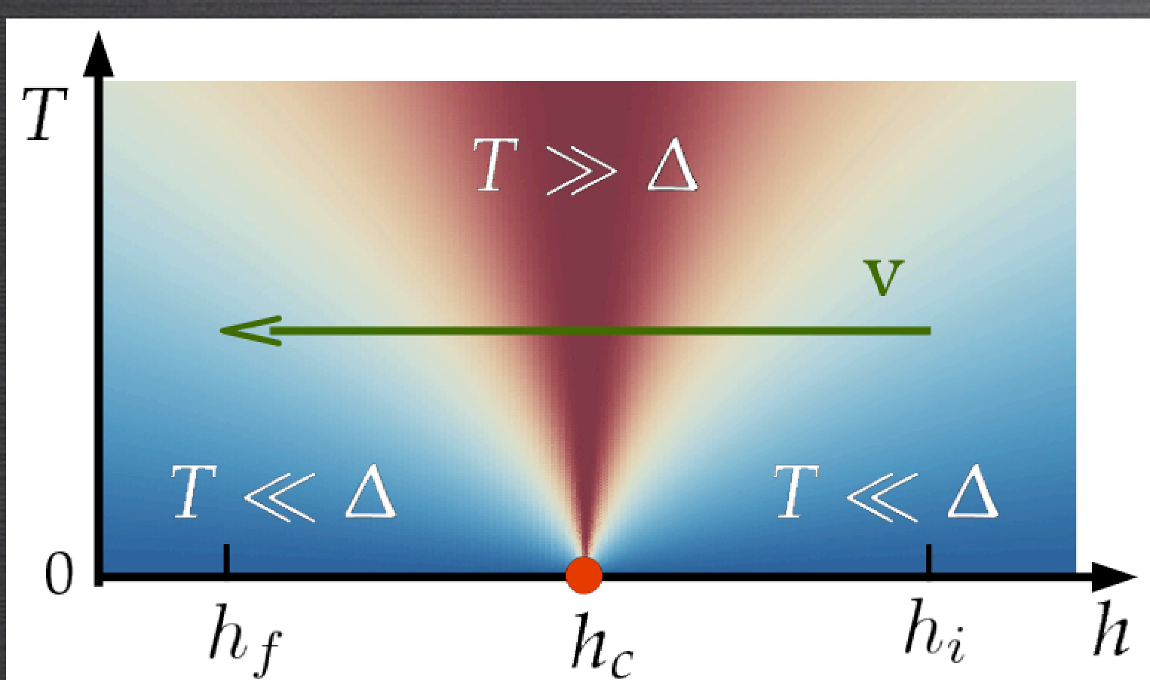
$$\mathcal{E}_{res} \sim \sqrt{v}$$

Quantum Critical Region



“Incoherent” Defects

- ✓ Density of defects $\mathcal{E} \simeq \mathcal{E}_{KZ} + \mathcal{E}_{inc}$
- ✓ The bath does not influence the system for $T \ll \Delta$
- ✓ Relaxation in the critical region $\tau_r^{-1} \propto \alpha T^{\theta}$



$$t_{QC} = 2T^{1/\nu z} v^{-1}$$

$$\mathcal{E} = \int \frac{d^d k}{(2\pi)^d} \mathcal{P}_k$$

$$\frac{d}{dt}\mathcal{P}_k = -\frac{1}{\tau}\left[\mathcal{P}_k - \mathcal{P}_k^{th}(h_c)\right]$$

"Incoherent" Defects

$$\mathcal{E}_{inc} \propto \alpha v^{-1} T^{\theta + \frac{d\nu+1}{\nu z}}$$

$$v_{cross} \propto \alpha^{\frac{\nu z+1}{\nu(z+d)+1}} T^{\left(1+\frac{(\theta-1)\nu z}{\nu(z+d)+1}\right)\left(1+\frac{1}{\nu z}\right)}$$

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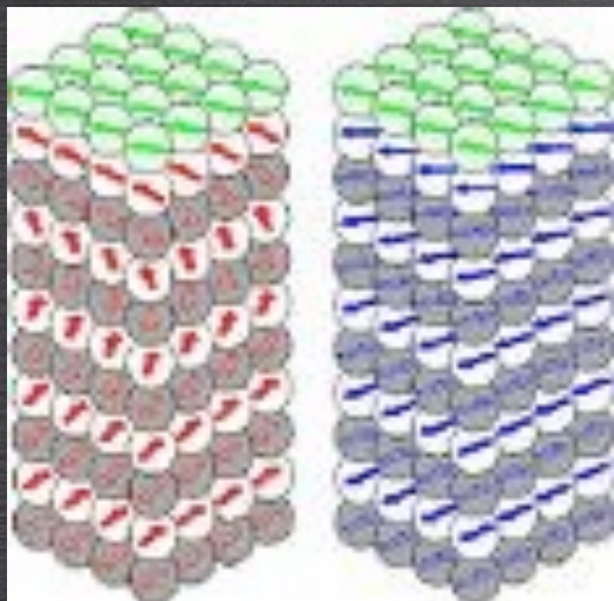
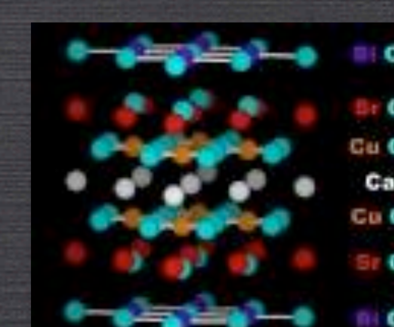
Review:
J.I. Cirac and F. Verstraete, J. Phys. A: Math. Theor. **42**, 504004 (2009)



$$|\Psi >= \sum_{\{i_\alpha\}} C(i_1, i_2, \ldots, i_N)|i_1, i_2, \cdots .i_N >$$

$i_\alpha = 1, \ldots, d$

Exponential Number of Parameters!



Variational Approach

Educated guess of the ground state wavefunction

EXAMPLE:

Gutzwiller approximation of the Bose-Hubbard Model

$$\mathcal{H} = U \sum_i n_i(n_i - 1) - \mu \sum_i n_i - t \sum_{<ij>} a_i^\dagger a_j$$

$$|\Psi> = \prod_i \left(\sum_{n_i} e^{-\kappa(n_i - \bar{n})} |n_i> \right)$$

κ

Variational ansatzs
&
Quantum Information

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Briegel, Cirac,
Eisert, Hastings,
Latorre, Plenio,
Verstraete,Vidal,
...
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New insight on variational wave-functions from Quantum Information

- General schemes for efficient computation of a variational functions (MPS, PEPS, MERA, ...)

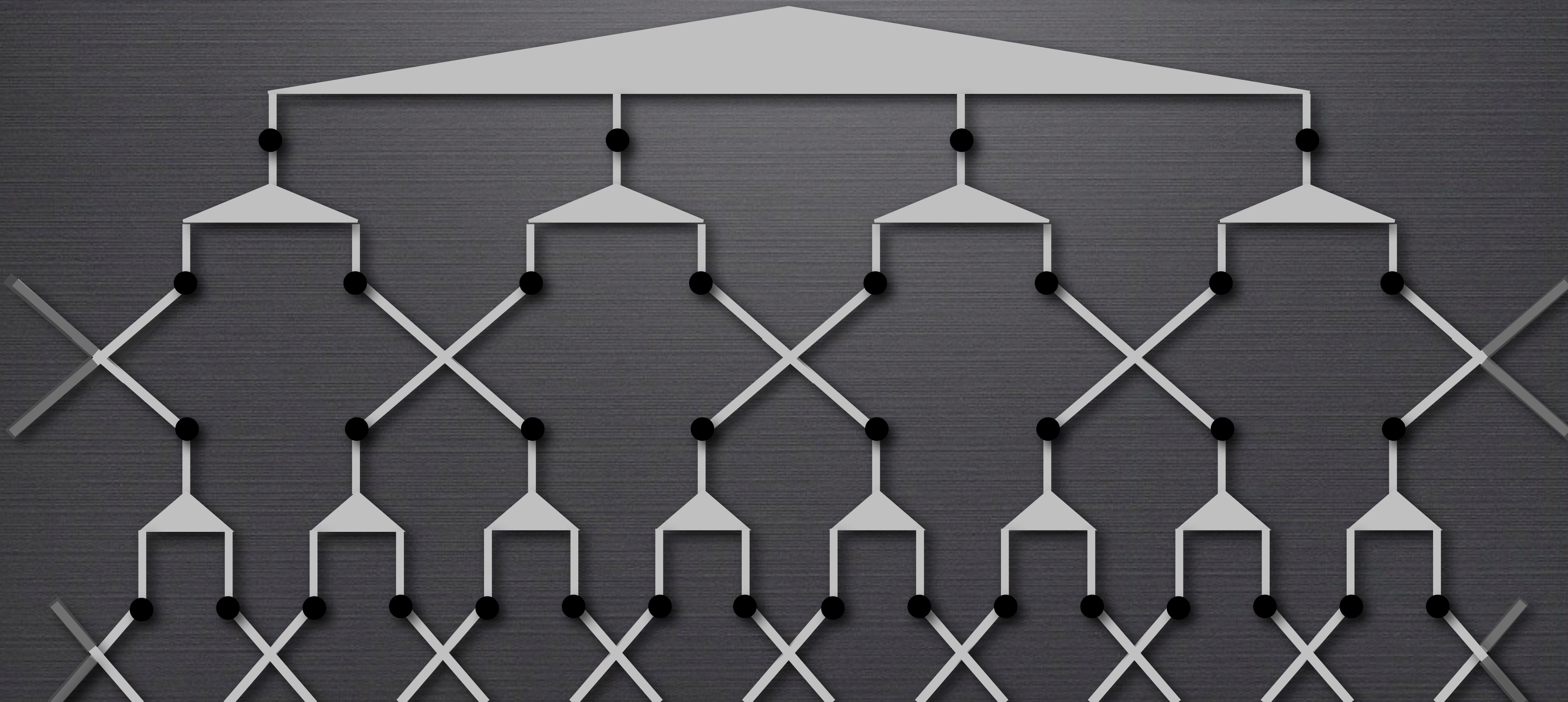
- Account for entanglement properties (crucial for critical systems)

- Extensions to time-dependent situations, finite temperatures,...

Multiscale Entanglement Renormalization Ansatz

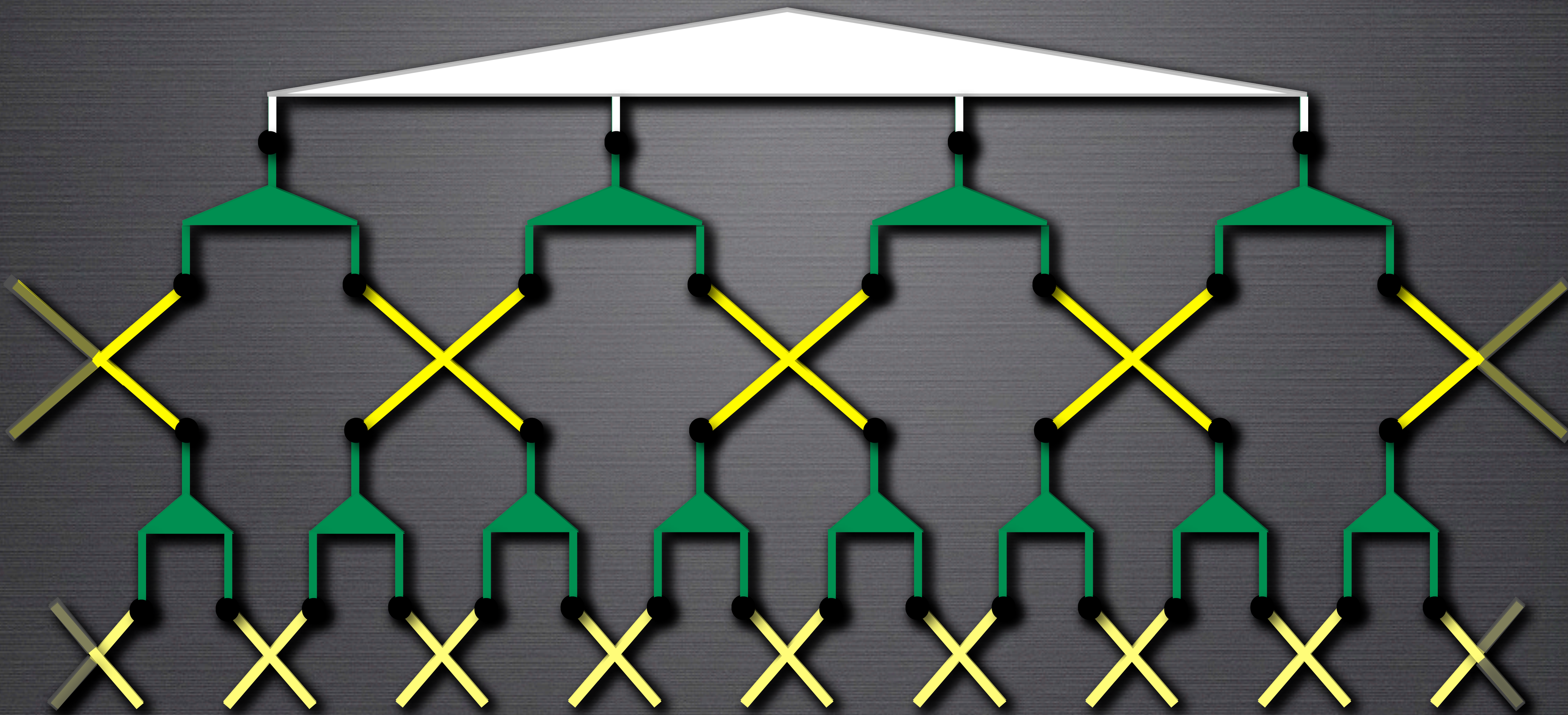
$$|\Psi\rangle = \sum_{\{i_\alpha\}} C(i_1, i_2, \ldots, i_N)|i_1, i_2, \cdots .i_N\rangle$$

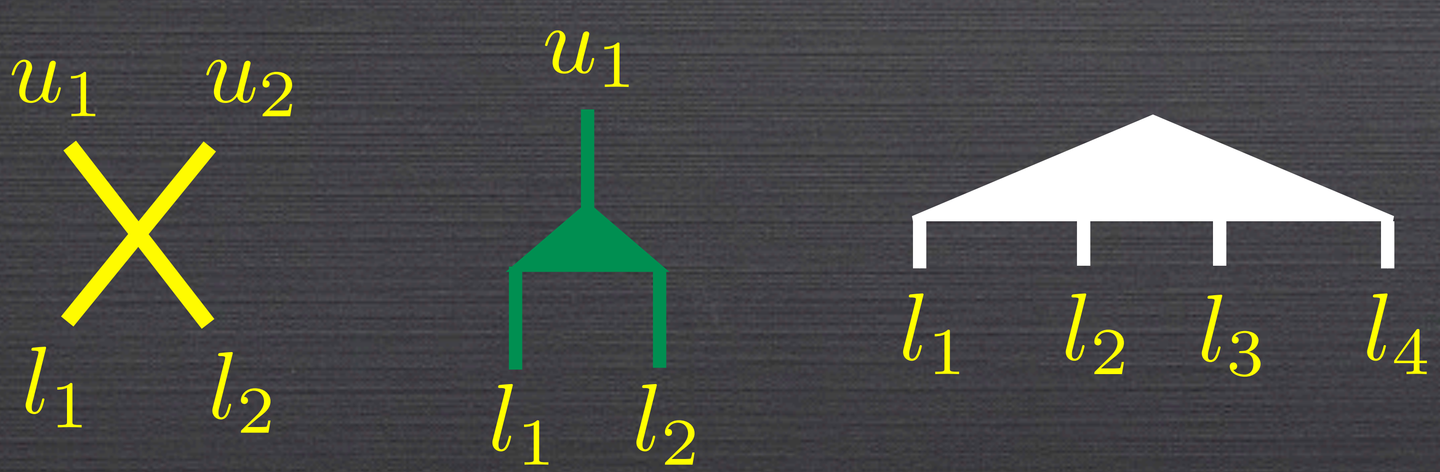
G. Vidal (2005-2009)

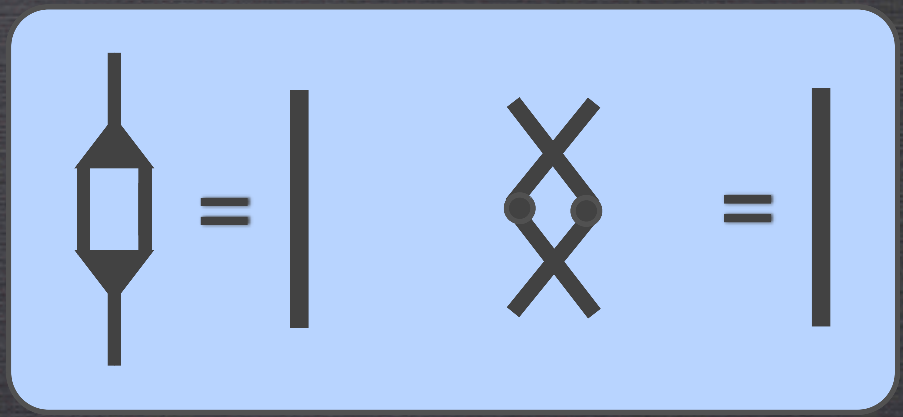


MERA: Definitions

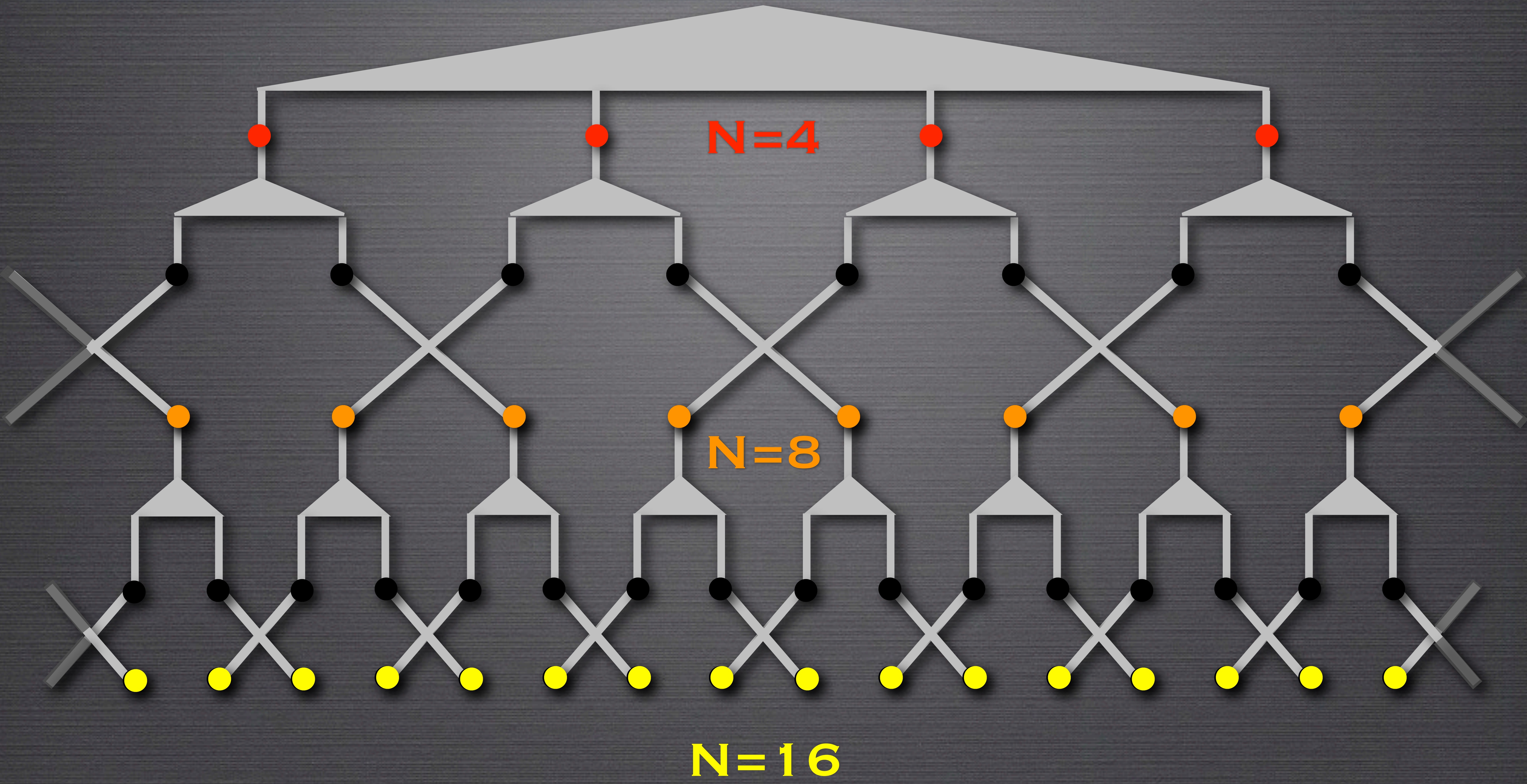
G. Vidal (2005–2009)





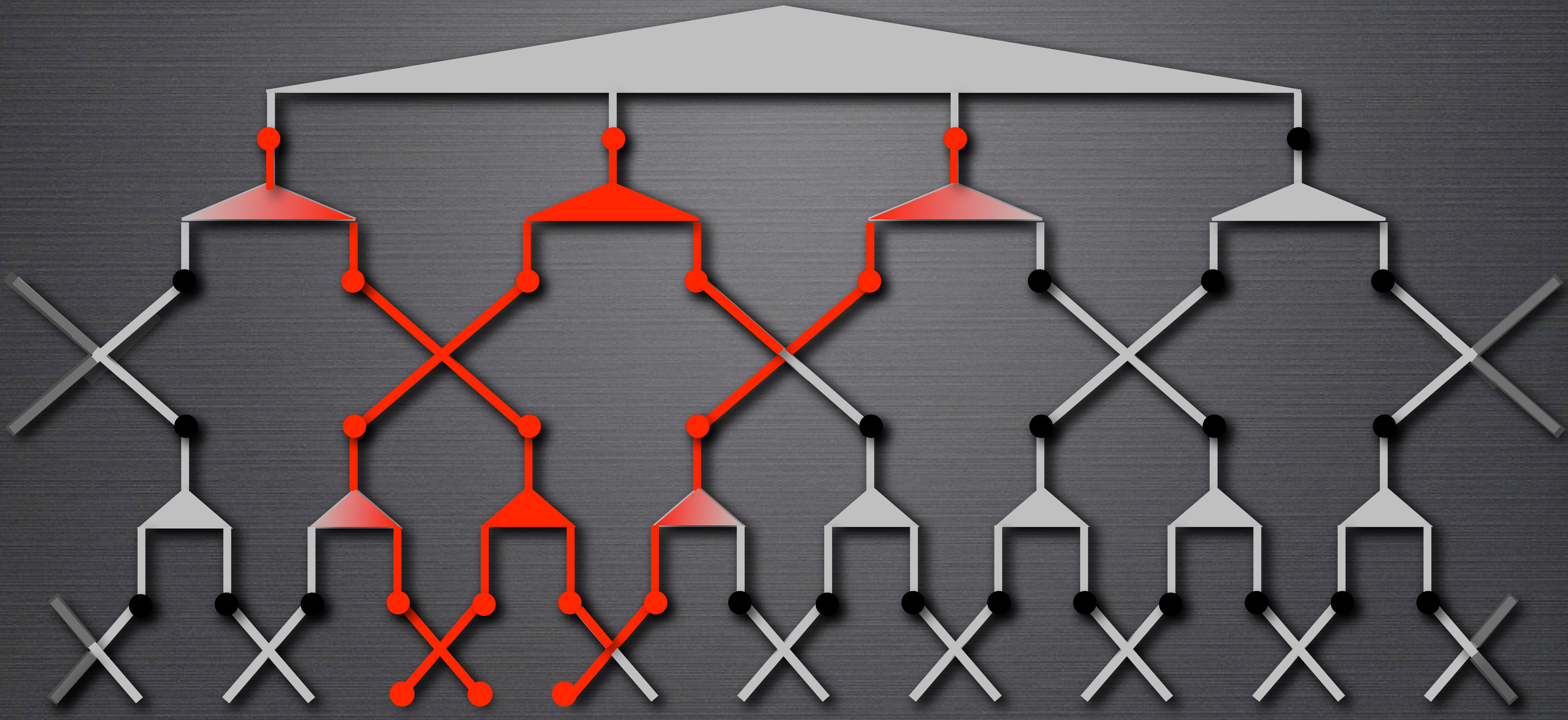


Real Space Renormalization



$$|\Psi_{i_M}(M \text{ sites})\rangle \rightarrow |\Psi_{\mathrm{i_{M+1}}}(\mathrm{M/2\ sites})\rangle$$

Causal Cone



Critical exponents
Ising Model

$$H = \sum_j J\sigma_j^x \sigma_{j+1}^x + \sum_j B\,\sigma_j^z$$

$$< \sigma_\alpha \sigma_\alpha > - < \sigma_\alpha >< \sigma_\alpha >$$

α	ν_α^{exact}	ν_α^{num}	ε
x	0.25	0.2509	0.36 %
y	2.25	2.2544	0.19 %
z	2	2.0939	4.48 %

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