OPTIMAL CONTROL AT THE QUANTUM SPEED LIMIT

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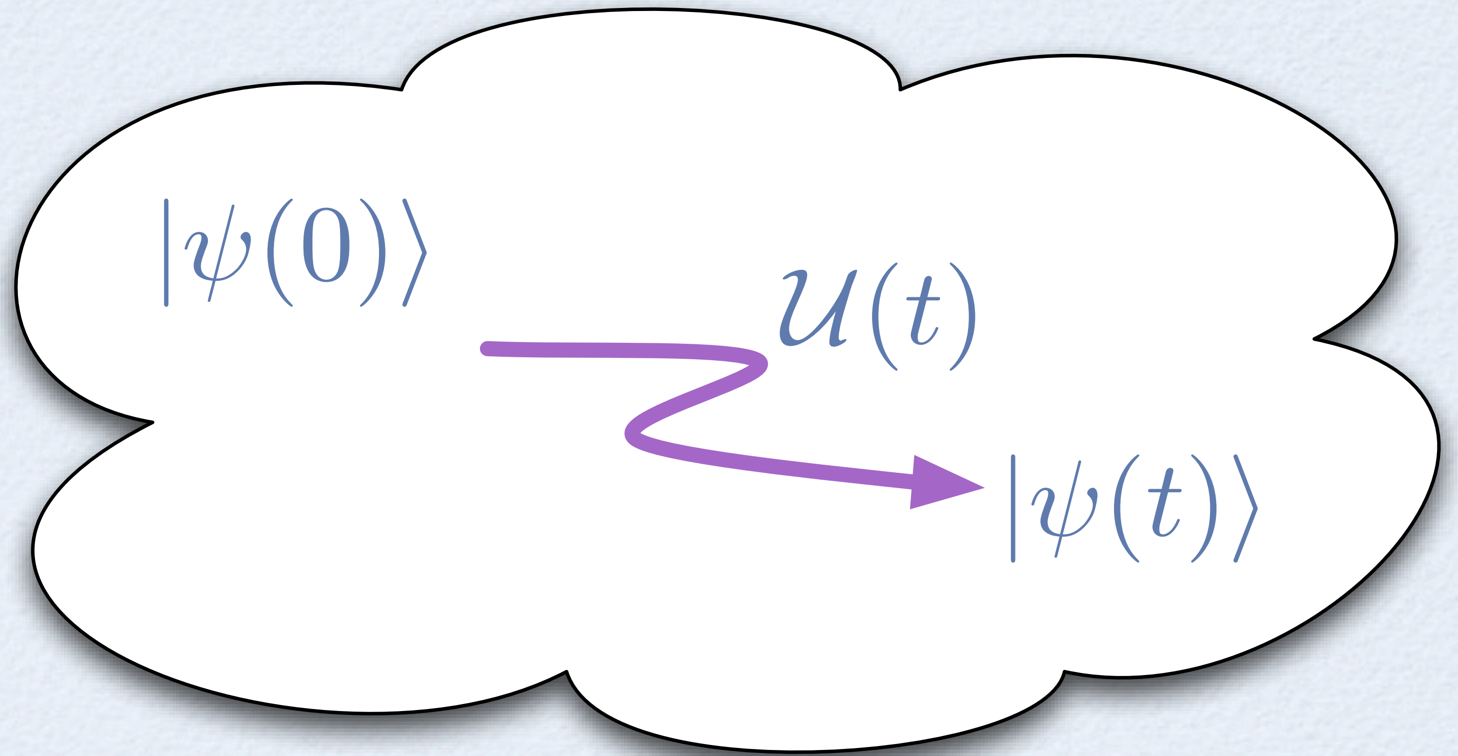
- Phys. Rev. Lett. 99, 170501 (2007)
- Phys. Rev. Lett. 103, 240501 (2009)
- arXiv:1011.6634
- unpublished

OUTLINE

- Optimal Quantum Control
- Optimal control and the speed limit
- Crossing a quantum phase transition
- Exp measure of the speed limit with BECs
- Conclusions

Quantum Information Protocols



OPTIMAL CONTROL I

Initial state	Hamiltonian	Evolved state
$\|\psi_0\rangle$	$H(u_1(t), u_2(t), \ldots)$	$\|\psi(T)\rangle$

Task: minimize $$1 - \mathcal{F} = 1 - |\langle\psi_{goal}|\psi(T)\rangle|^2$$

with the wave-fuction evolving with the Schroedinger equation

Krotov Method: an iteration in which the “pulses” $u_i(t)$ are updated and the wave function evolves according to the new Hamiltonian.

Qubits & Quantum Gates

Local Hamiltonian

$$\mathcal{H}(t) = E\sigma_z + J_1(t)\sigma_x$$

Interaction Hamiltonian

Time-dependent interaction

$$\mathcal{H}_{i,j}(t) = J_2(t)\sigma_x^i\sigma_x^j$$

Quantum Gate

Simple guess and system constraints VS desired gate

$$\mathcal{U}_{i,j} = e^{-\imath\int \mathcal{H}_T(t)dt/\hbar}$$

OPTIMAL CONTROL II

Functional to be minized:

$$\mathcal{L} = 1 - \mathcal{F} + 2\mathrm{Re} \int_0^T dt \left(\langle \dot{\psi} | + i \langle \psi | H \right) | \chi \rangle$$

where χ is a Lagrange multiplier

We seek for a stationary point of the functional using the steepest descent method in the space of the “pulses” $u_i(t)$.

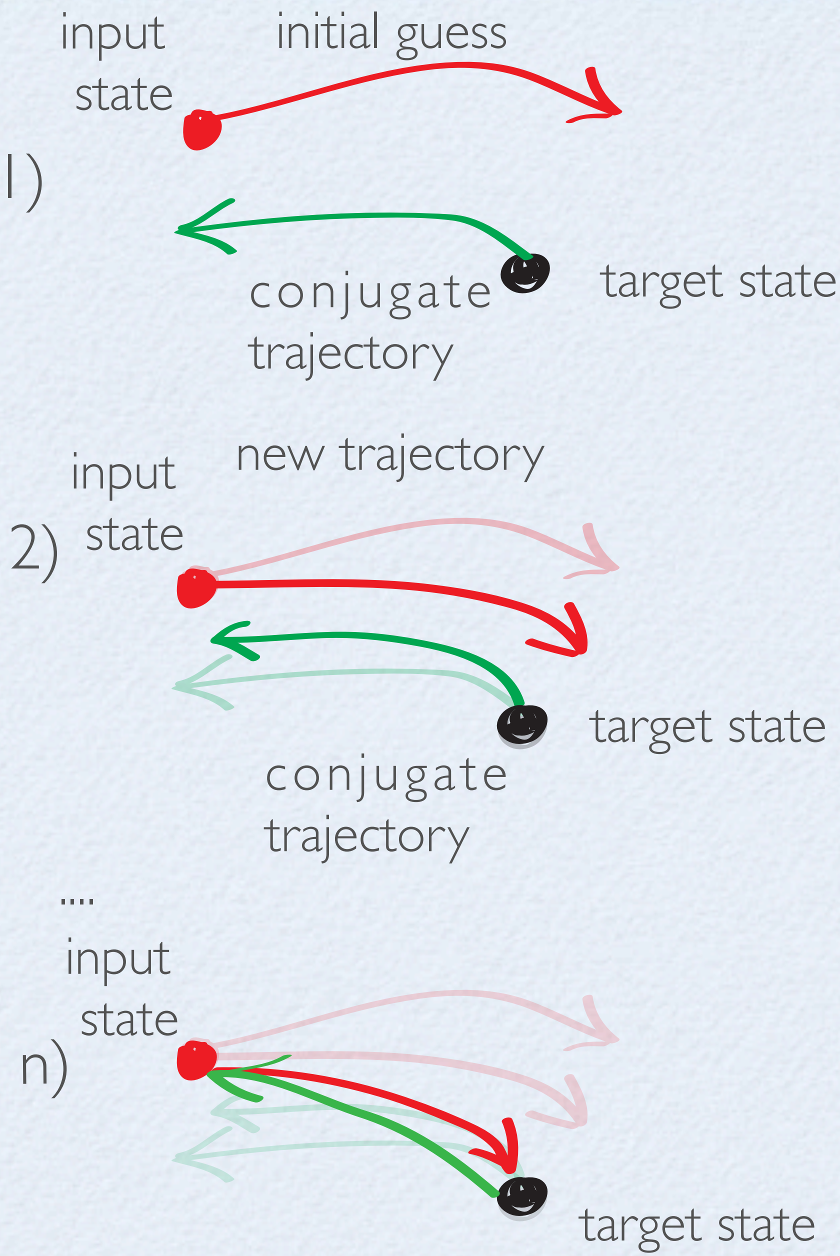
KROTOV ALGORITHM

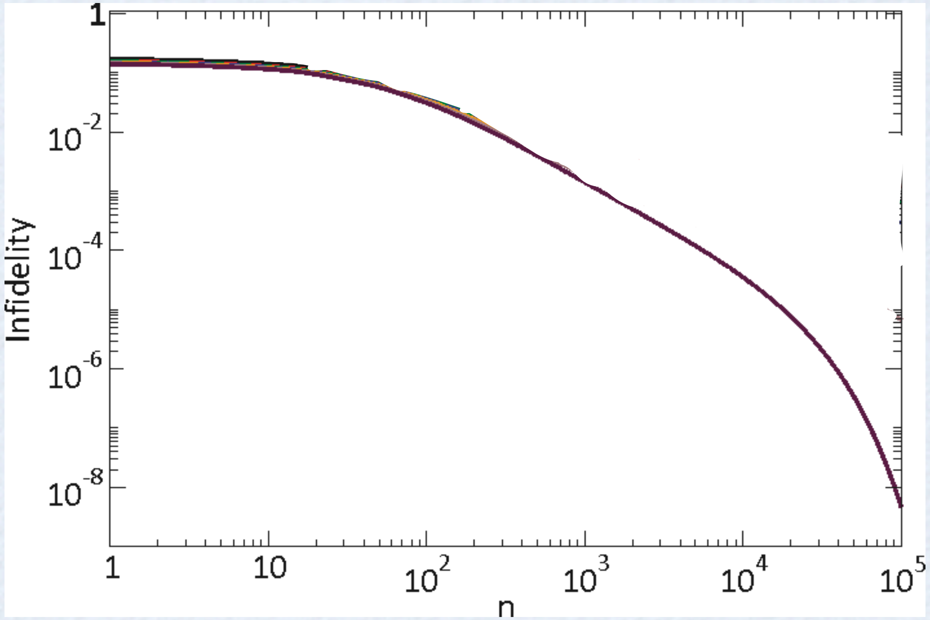
1. Choose a guess for the pulses $u_i(t)$
2. Evolve the wave function $|\psi(0)\rangle$ to $|\psi(T)\rangle$
3. Set $|\chi(T)\rangle = \langle\psi_{goal}|\psi(T)\rangle\,|\psi_{goal}\rangle$ and evolve it back to $|\chi(0)\rangle$
4. At each time step update the pulses according to:

$$u_i'(t) = u_i(t) + 2\mu(t)\,\mathrm{Im}\,\langle\chi(t)|\,\partial H/\partial u_i\,|\psi(t)\rangle$$

5. Evolve χ with the old pulses and ψ with the new ones
6. Iterate steps 3 - 5 until reaching convergence

Krotov Method

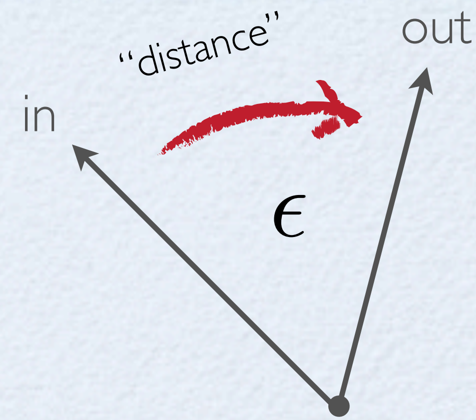




number of iterations of the algorithm

Quantum Speed Limit

Determine the Minimum time required for a quantum state to evolve to a different one placed at a certain distance from it.



$$E = \langle \Psi | H | \Psi \rangle$$

Initial energy

Initial state

$$\Delta E = \sqrt{\langle \Psi | (H - E)^2 | \Psi \rangle}$$

Energy variance

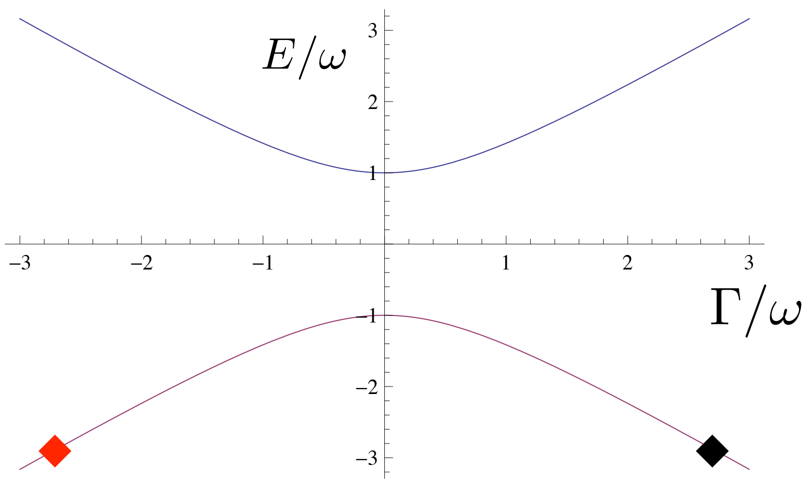
$$\mathcal{T}_\epsilon(E, \Delta E) \equiv \max\left(\alpha(\epsilon) \frac{\pi\hbar}{2E}, \beta(\epsilon) \frac{\pi\hbar}{2\Delta E} \right)$$

K. Bhattacharyya, JPA (1983)
P. Pfeifer, PRL (1993)
N. Margolus and L.B. Levitin, Physica D (1998)
V Giovannetti, S Lloyd, L Maccone, PRA (2003)
Carlini et al PRL (2006)

LIMITS TO OPTIMAL CONTROL

$$H[\Gamma(t)] = \begin{pmatrix} \Gamma(t) & \omega \\ \omega & -\Gamma(t) \end{pmatrix}$$

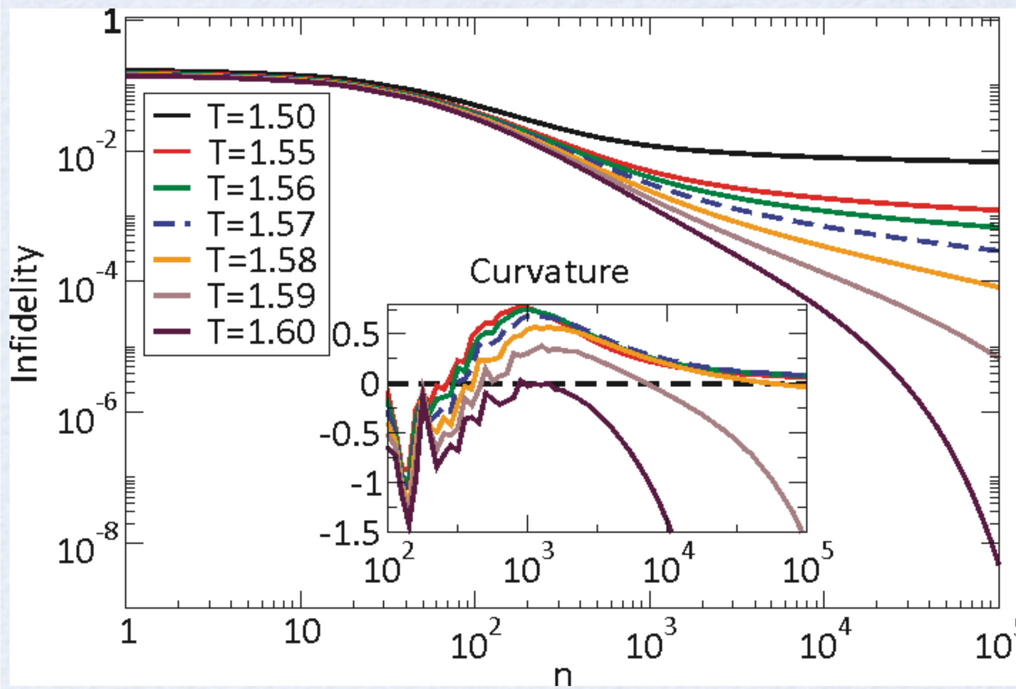
control pulse



input state

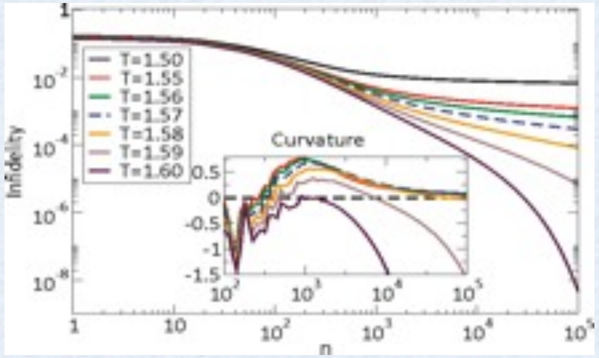
target state

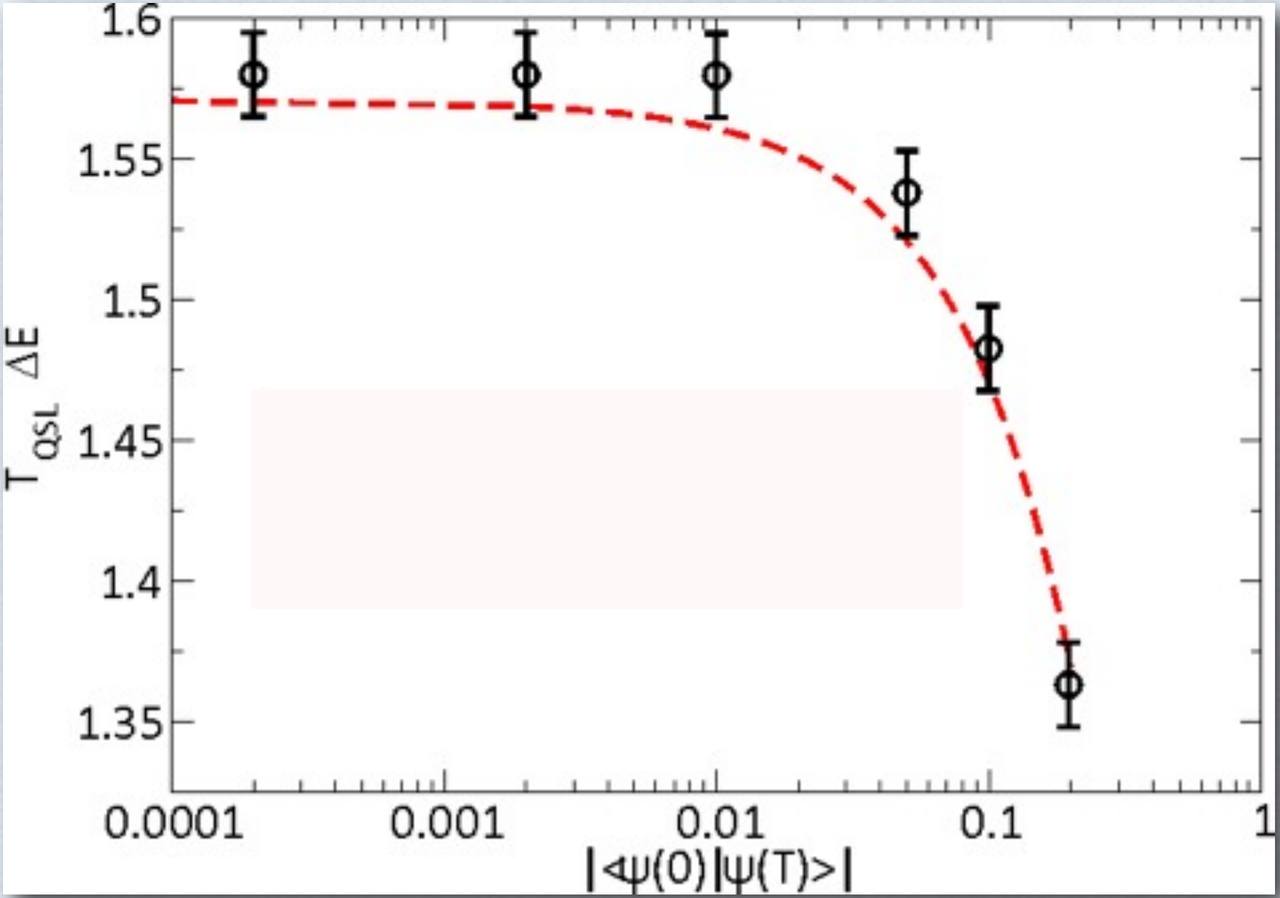
$$\Gamma(T) = -\Gamma(0)$$



If NO solutions are found we conclude that T is below the quantum speed limit time of the problem.

Limits to optimal control

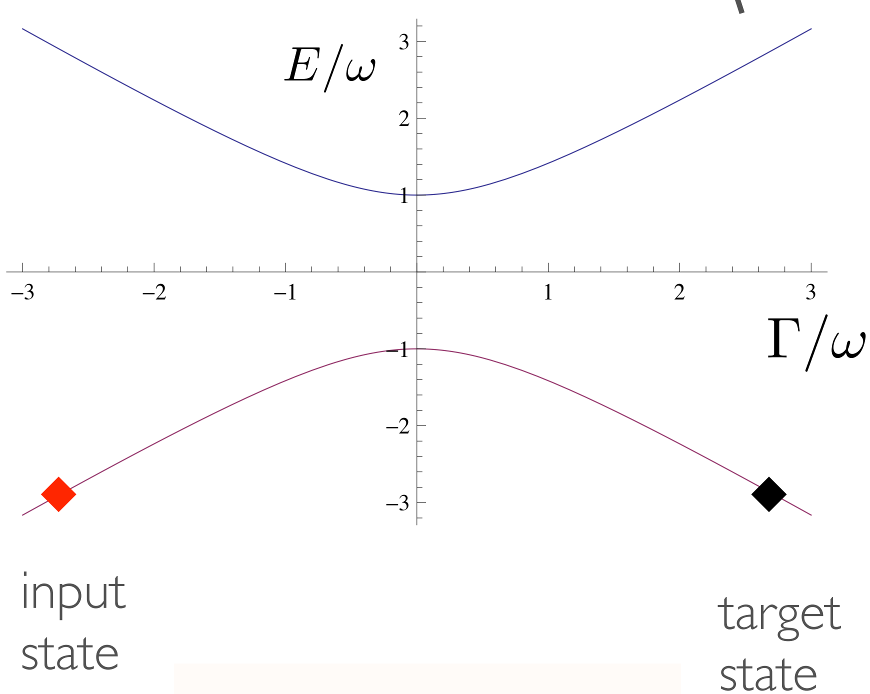


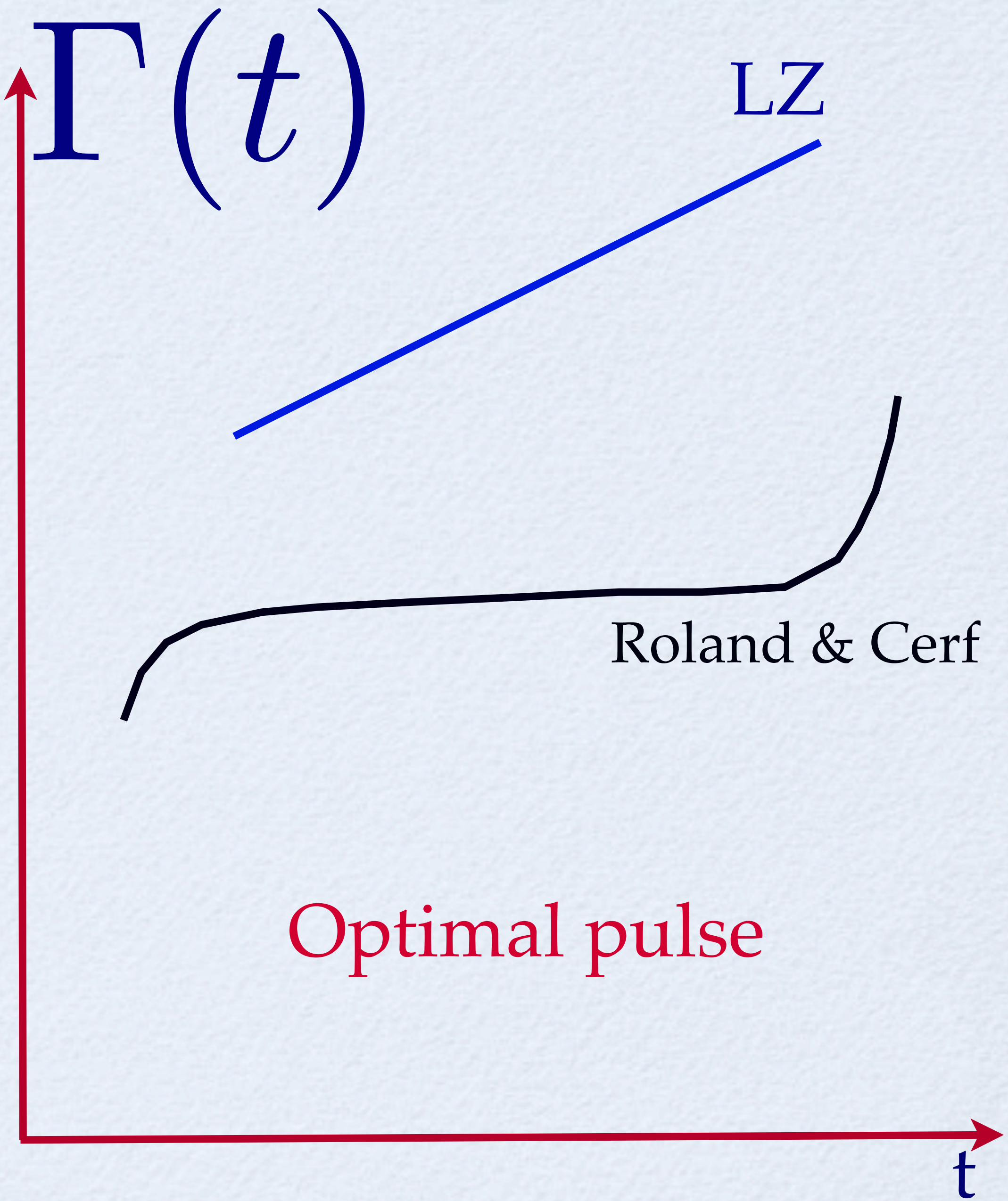


$$T_{\mathrm{QSL}} \simeq \frac{1}{\omega} \arccos |\langle \psi(0)|\psi(T)\rangle|$$

EXP MEASURE OF THE QSP

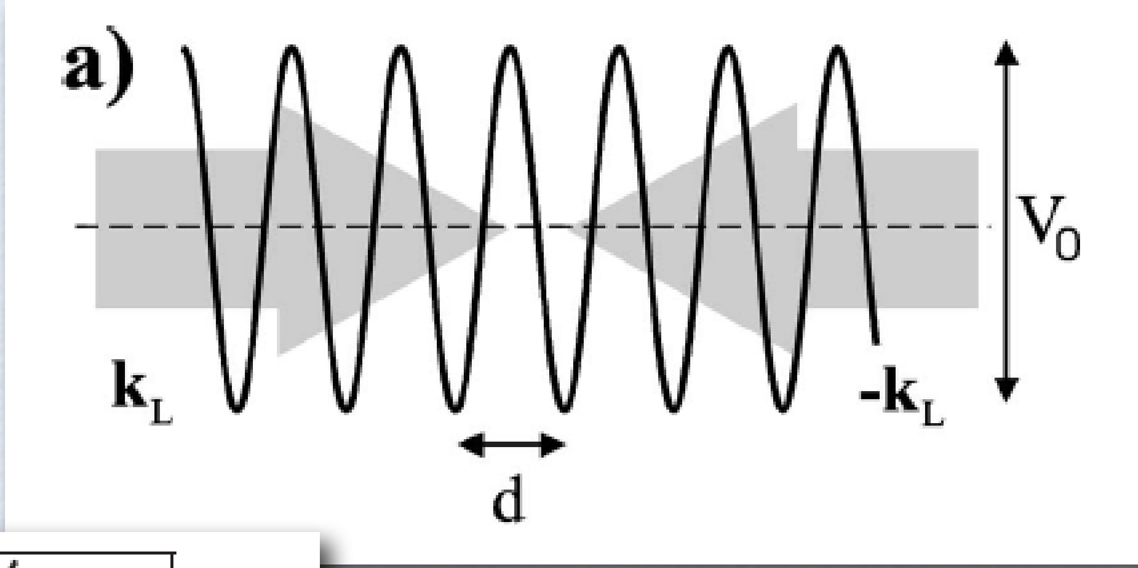
$$H[\Gamma(t)] = \begin{pmatrix} \Gamma(t) & \omega \\ \omega & -\Gamma(t) \end{pmatrix}$$

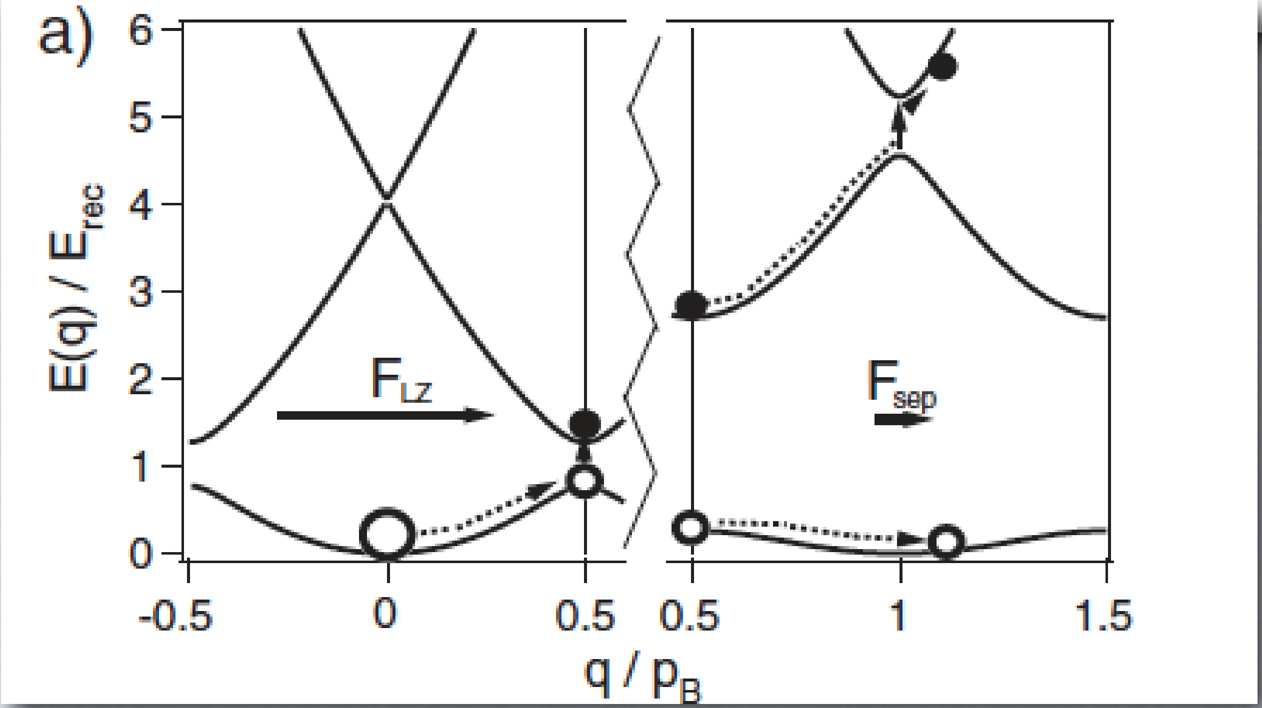




EXP MEASURE OF THE QSP

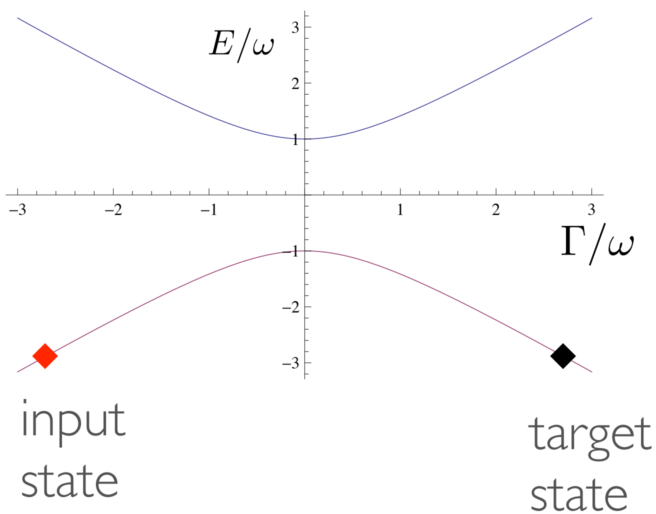
Landau-Zener transition in BEC in an optical lattice





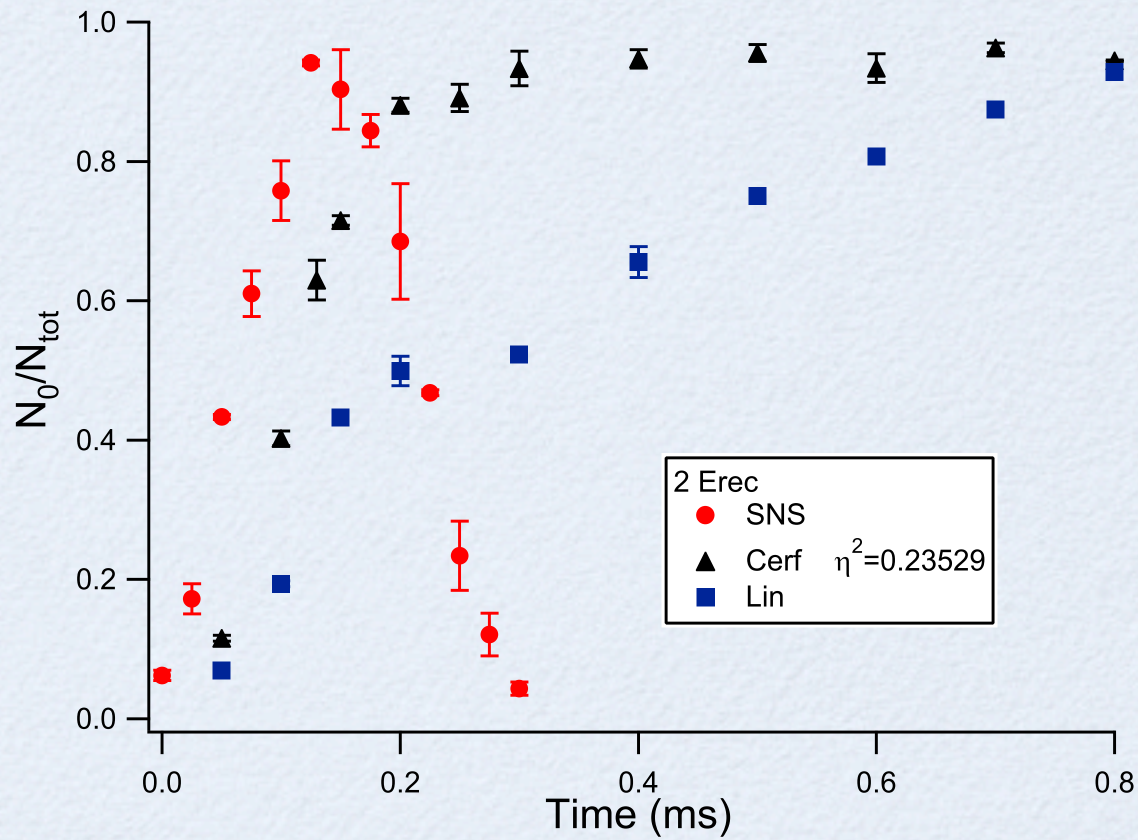
EXP MEASURE OF THE QSP

$$H[\Gamma(t)] = \begin{pmatrix} \Gamma(t) & \omega \\ \omega & -\Gamma(t) \end{pmatrix}$$

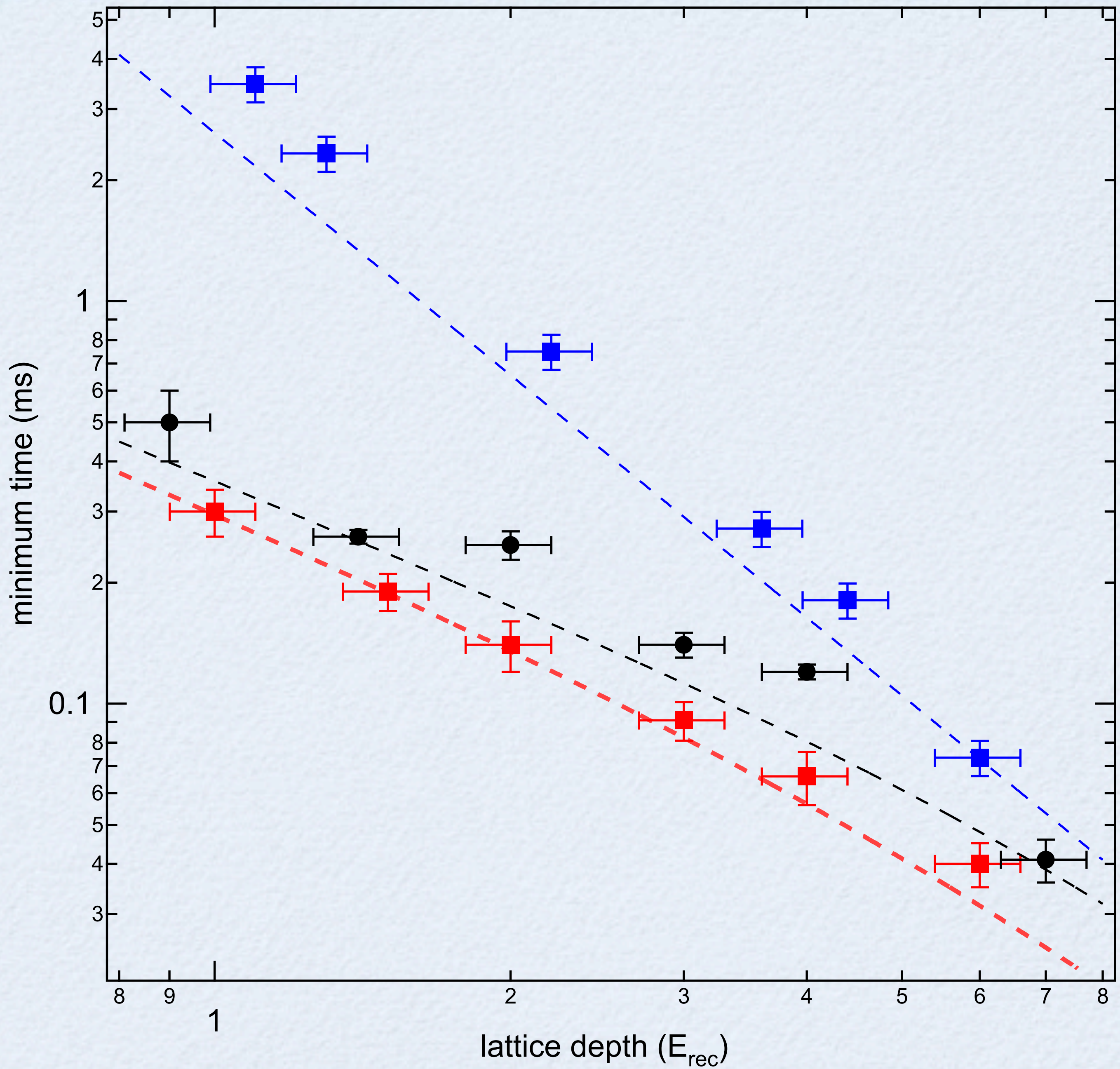


$$\Gamma(T) = -\Gamma(0)$$

control pulse

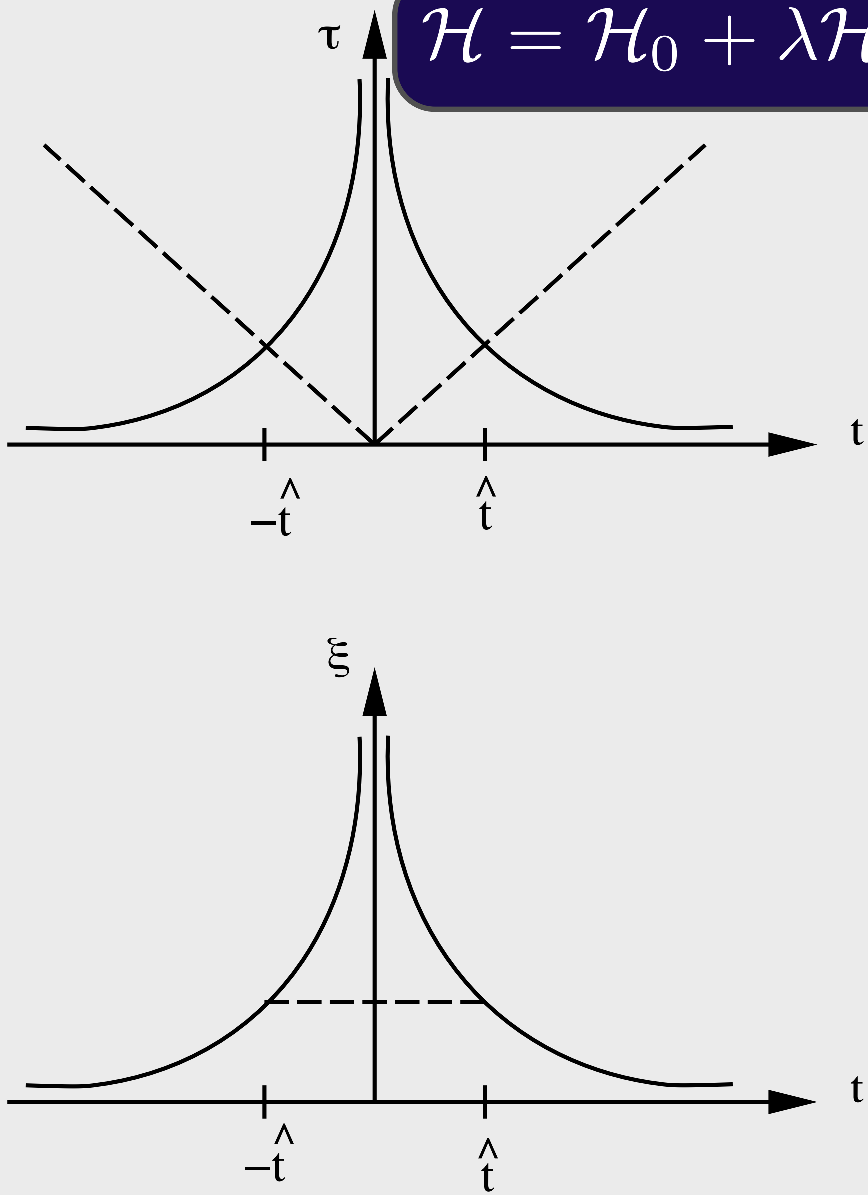


EXP MEASURE OF THE QSP



Adiabatic dynamics and QPTs

$$\mathcal{H} = \mathcal{H}_0 + \lambda \mathcal{H}_1$$



$$\lambda - \lambda_c = vt$$

The adiabatic approximation breaks down when

$$\frac{\dot{\lambda}}{\lambda} \sim \tau$$

$$\rho_{def} \sim \hat{\xi}^{-d} \sim v^{\frac{d\nu}{z\nu+1}}$$

$$\mathcal{E}_{res} \sim \mathrm{J}\rho_{def}$$

SPEED LIMIT AND QPT

Is it possible to cross a phase transition without generating any defect ?

Speed Limit and QPT

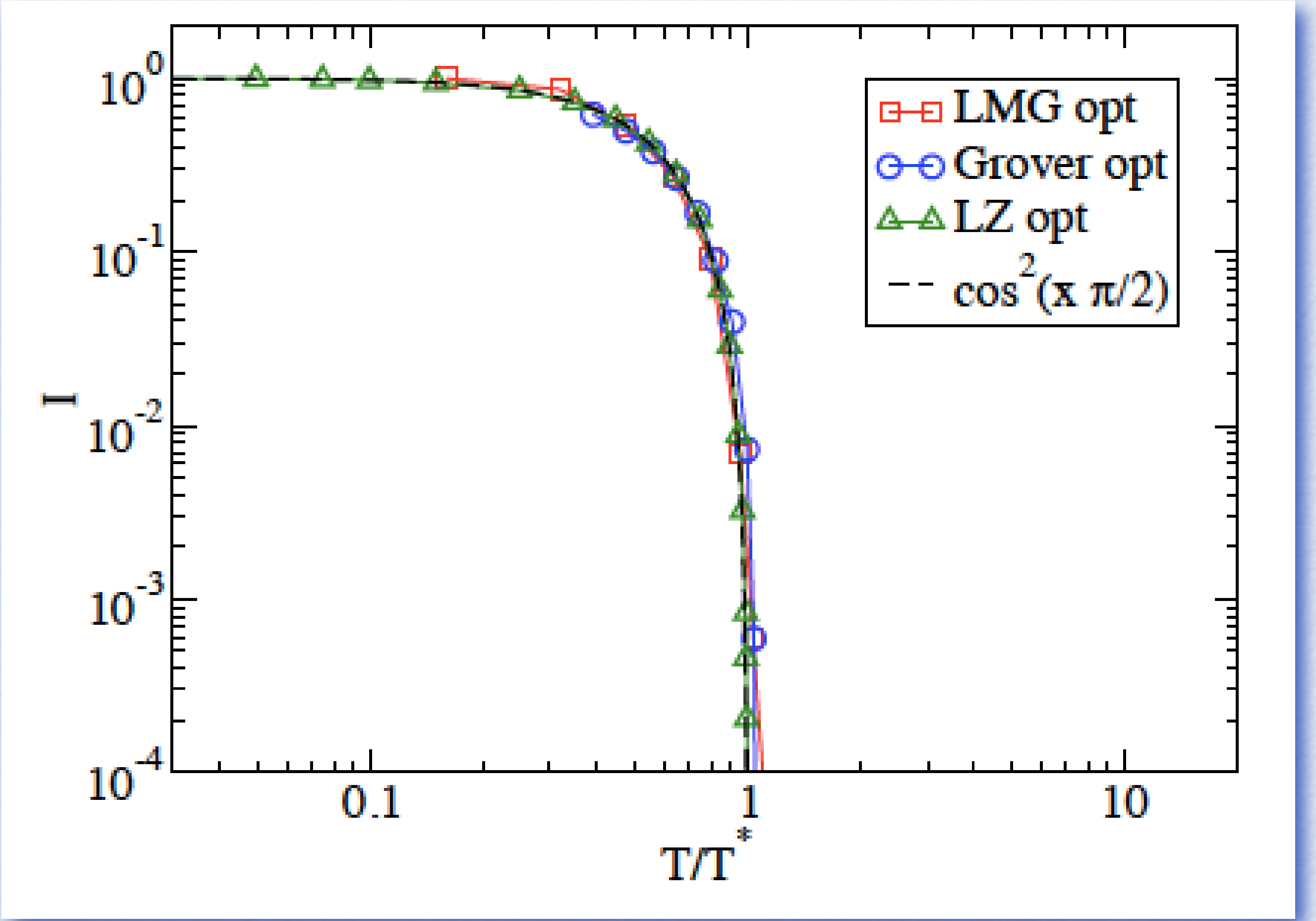
The Models

Model	H	$\|\psi_i\rangle$	$\|\psi_G\rangle$	Δ
GSA	$(1-\Gamma(t))(1-\|\psi_i\rangle\langle\|\psi_i\|)+\Gamma(t)(1-\|\psi_G\rangle\langle\|\psi_G\|)$	$\sum_i^N \|i\rangle/\sqrt{N}$	$\|10...0\rangle$	$N^{-1/2}$
LMG	$-(N^{-1})\sum_{i<j}^N \sigma_i^x\sigma_j^x - \Gamma(t)\sum_i^N \sigma_i^z$	$\|\uparrow ... \uparrow\rangle_z$	$\|\leftarrow ... \leftarrow\rangle_x, \|\rightarrow ... \rightarrow\rangle_x$	$N^{-1/3}$
LZ	$\Gamma(t)\sigma^z + \omega\sigma^x$	$\|\uparrow\rangle_z$	$\|\downarrow\rangle_z$	Δ

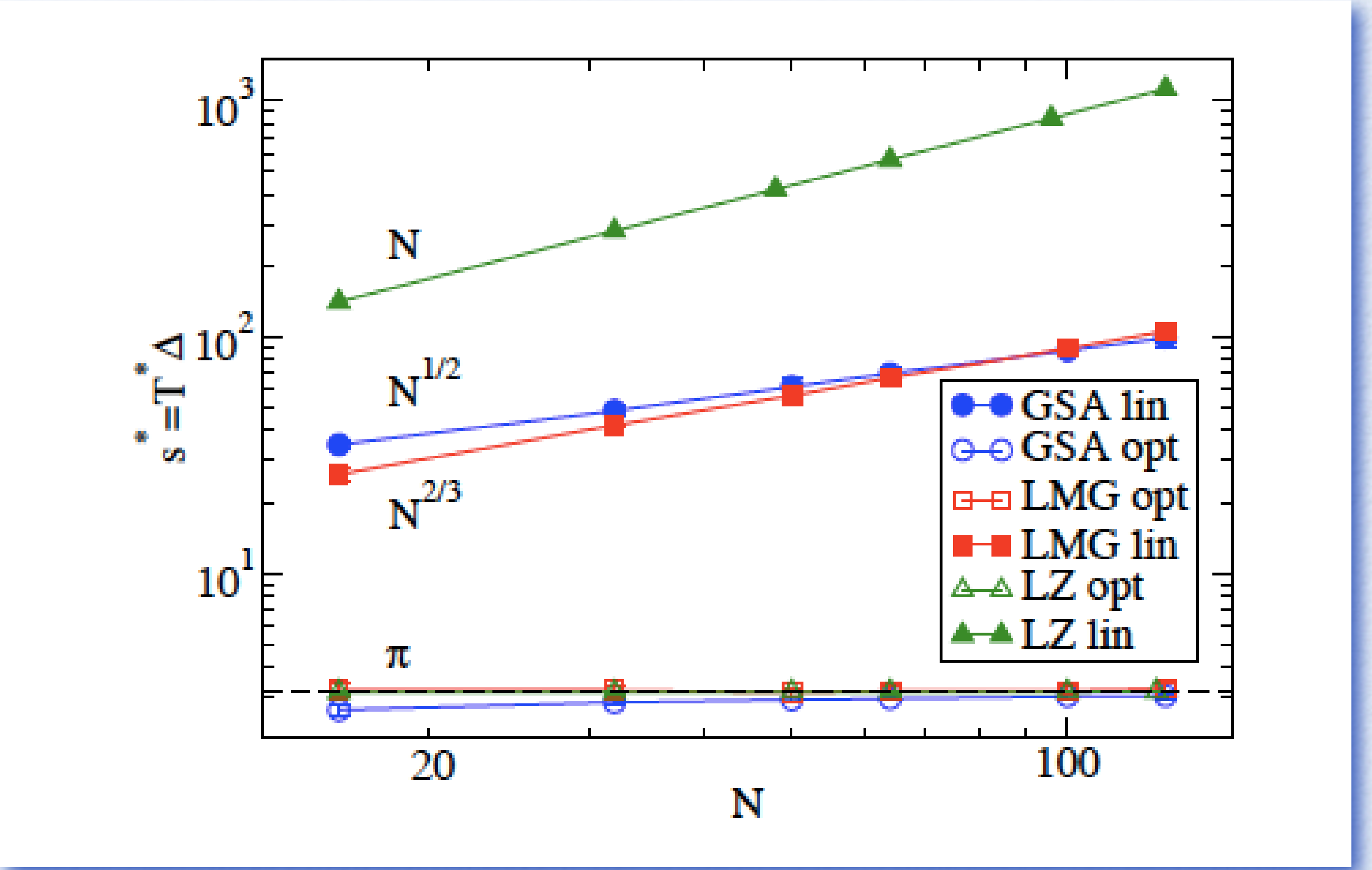
Infidelity

$$I(T) = 1 - |\langle\psi_G|\psi(T)\rangle|^2$$

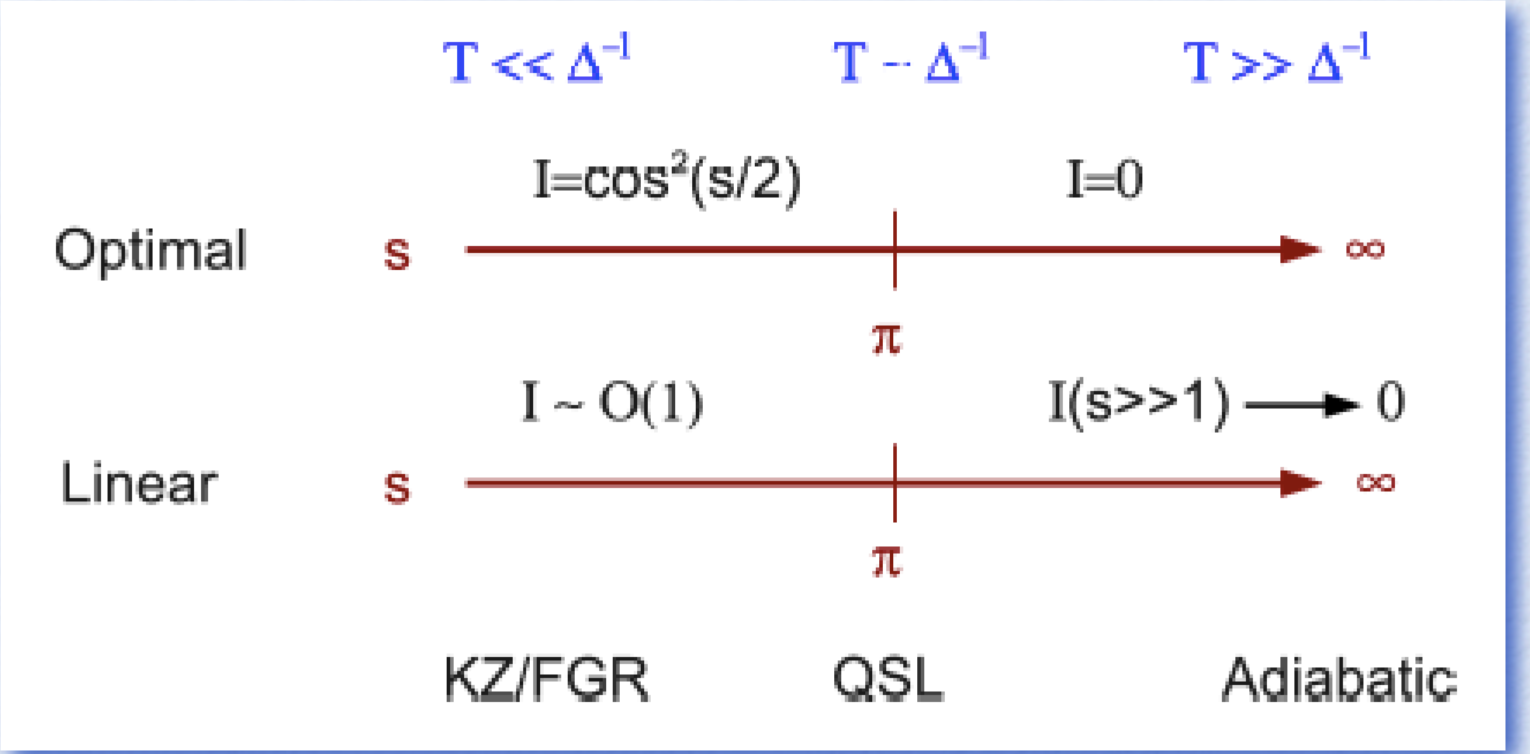
SPEED LIMIT AND QPT



Speed limit and QPT



Speed limit and QPT



Conclusions & outlook

- Quantum Optimal Control is a fundamental tool for Quantum Information Protocols
- “Efficiency” related to the quantum speed limits
- Optimal control crossing a QPT