What does quantum information theory tell us about thermodynamics?

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The role of information in physics

Information Theory

Claude Shannon
1916 – 2001

information is physical

Physics

Wolfgang Pauli
1900– 1958

The role of information in physics

Information Theory

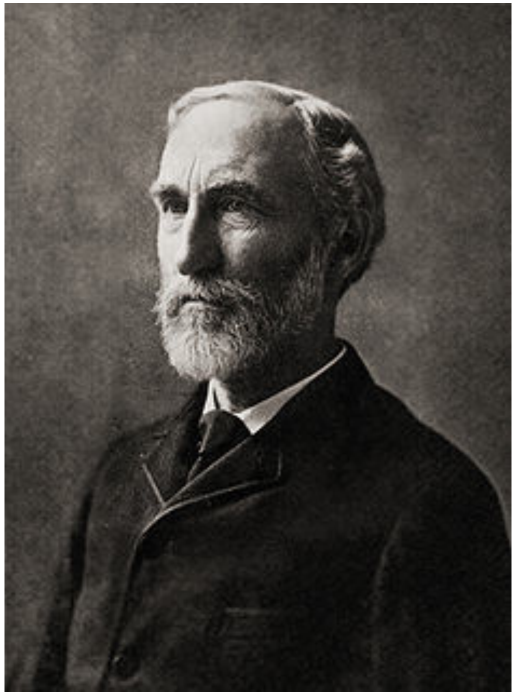
Claude Shannon
1916 – 2001

$$H = -\sum_i p_i \log_2 p_i$$

information is physical

information-theoretic view on physics

Physics



Josiah W. Gibbs
1839 – 1903

$$S = -k_B \sum_i p_i \ln p_i$$

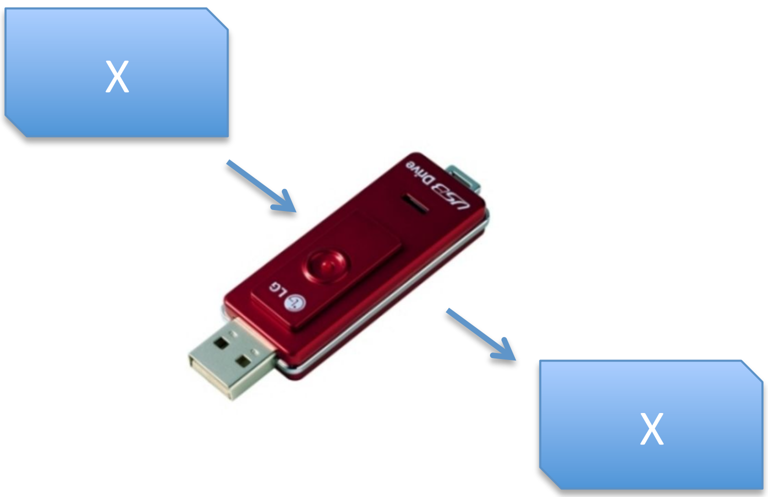
Operational significance of entropy

Information Theory

Entropy quantifies the capacity of a device to store, transmit, or process information.

Example: information storage

$$S(X) = r_{\text{compress}}(X)$$

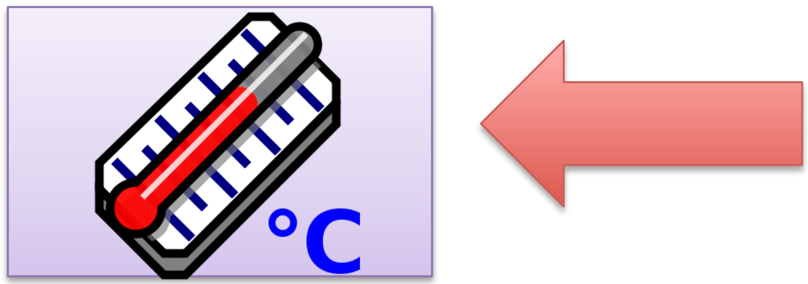


Physics

Entropy quantifies thermodynamic properties of a system.

(Defining) Example: Heat flow

$$dS = \frac{\delta Q}{T}$$



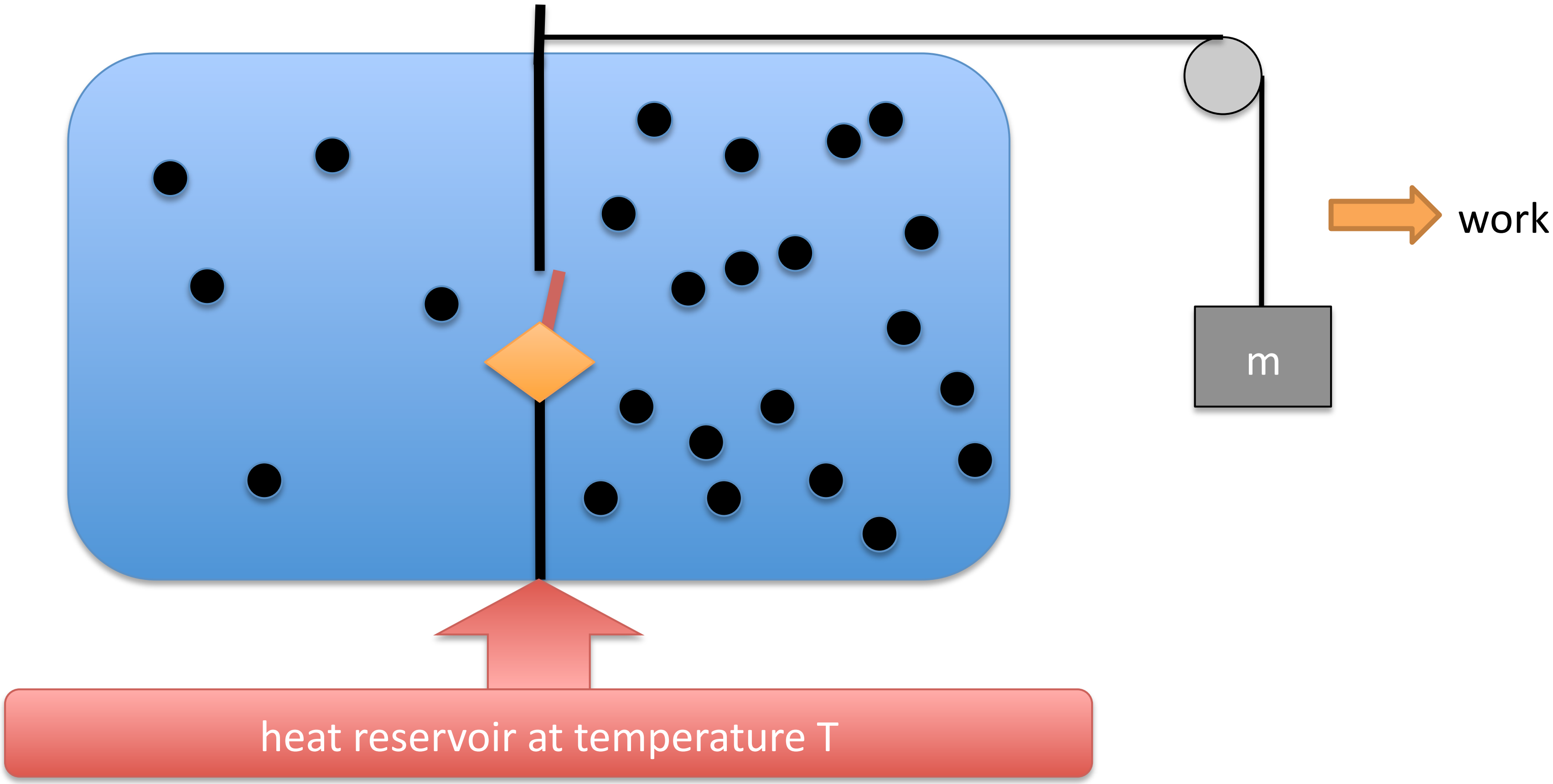
The second law of thermodynamics

"The total entropy of the system and the environment cannot decrease."

A broken egg **could** repair itself - it would just be incredibly unlikely!

Maxwell's demon

Maxwell's demon corresponds to a perpetuum mobile of the second kind.



On the impact of “Intelligent Beings”

840

Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen.

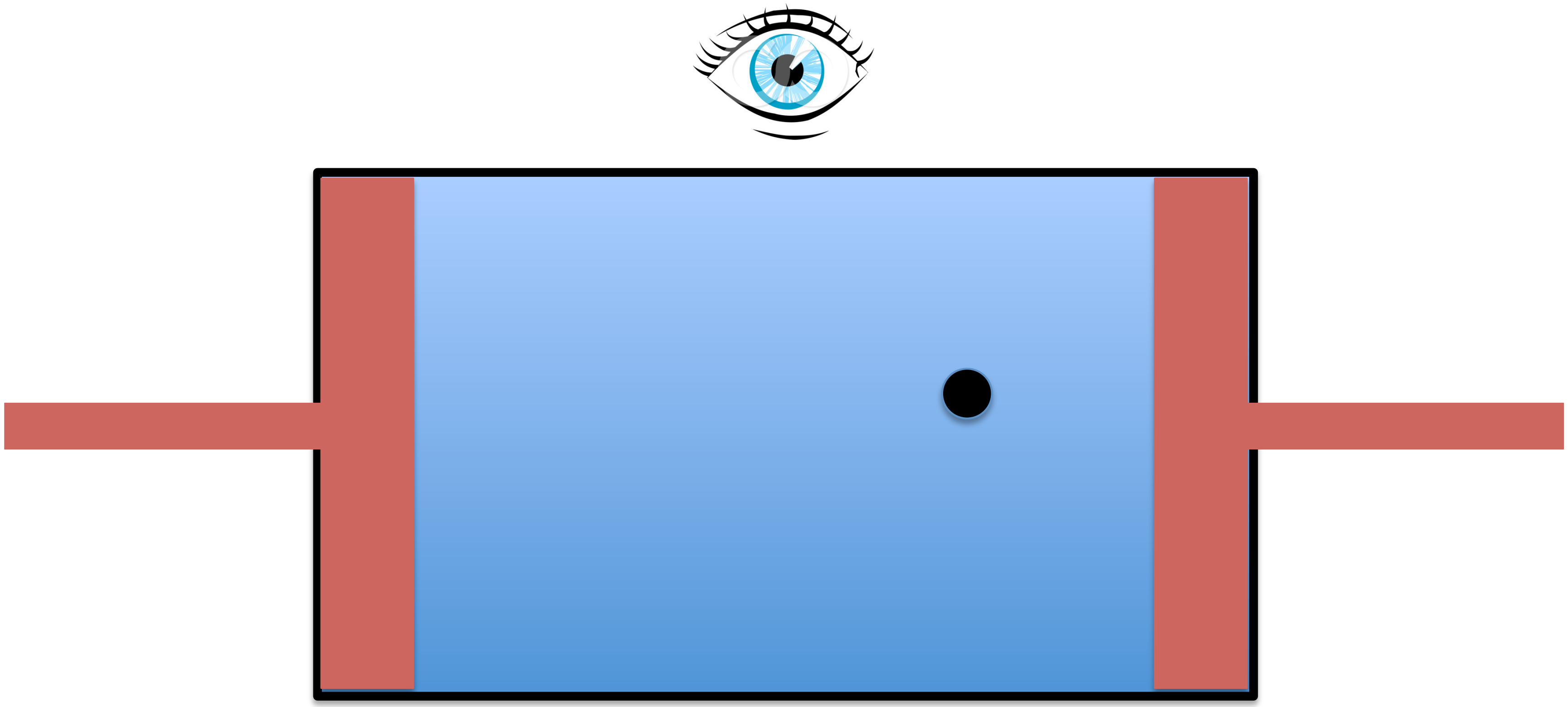
Von **L. Szilard** in Berlin.

Mit 1 Abbildung. (Eingegangen am 18. Januar 1928.)

Es wird untersucht, durch welche Umstände es bedingt ist, daß man scheinbar ein Perpetuum mobile zweiter Art konstruieren kann, wenn man ein Intellekt besitzendes Wesen Eingriffe an einem thermodynamischen System vornehmen läßt. Indem solche Wesen Messungen vornehmen, erzeugen sie ein Verhalten des Systems, welches es deutlich von einem sich selbst überlassenen mechanischen System unterscheidet. Wir zeigen, daß bereits eine Art Erinnerungsvermögen, welches ein System, in dem sich Messungen ereignen, auszeichnet, Anlaß zu einer

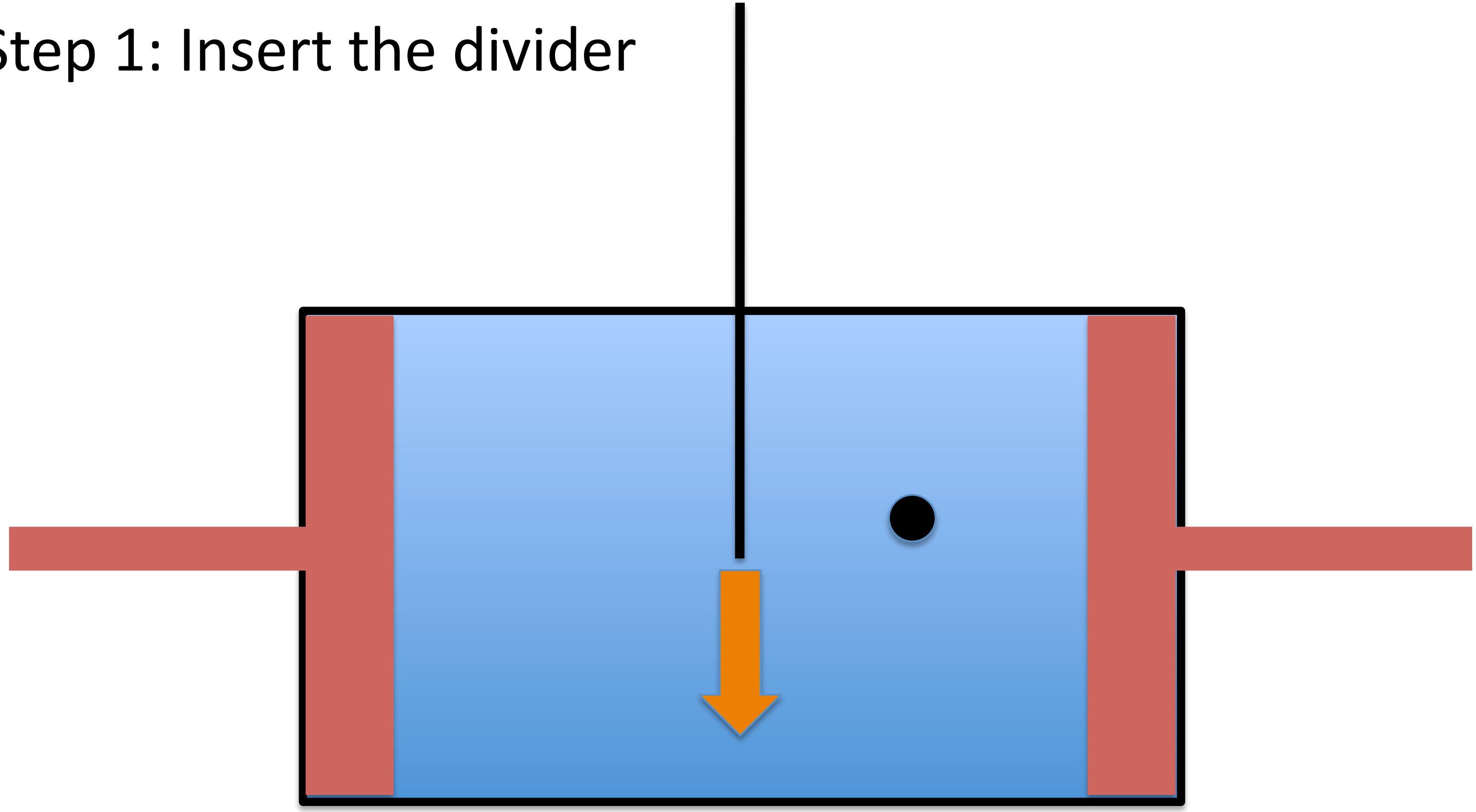
Szilárd’s engine

Setup (following Bennett’s description)



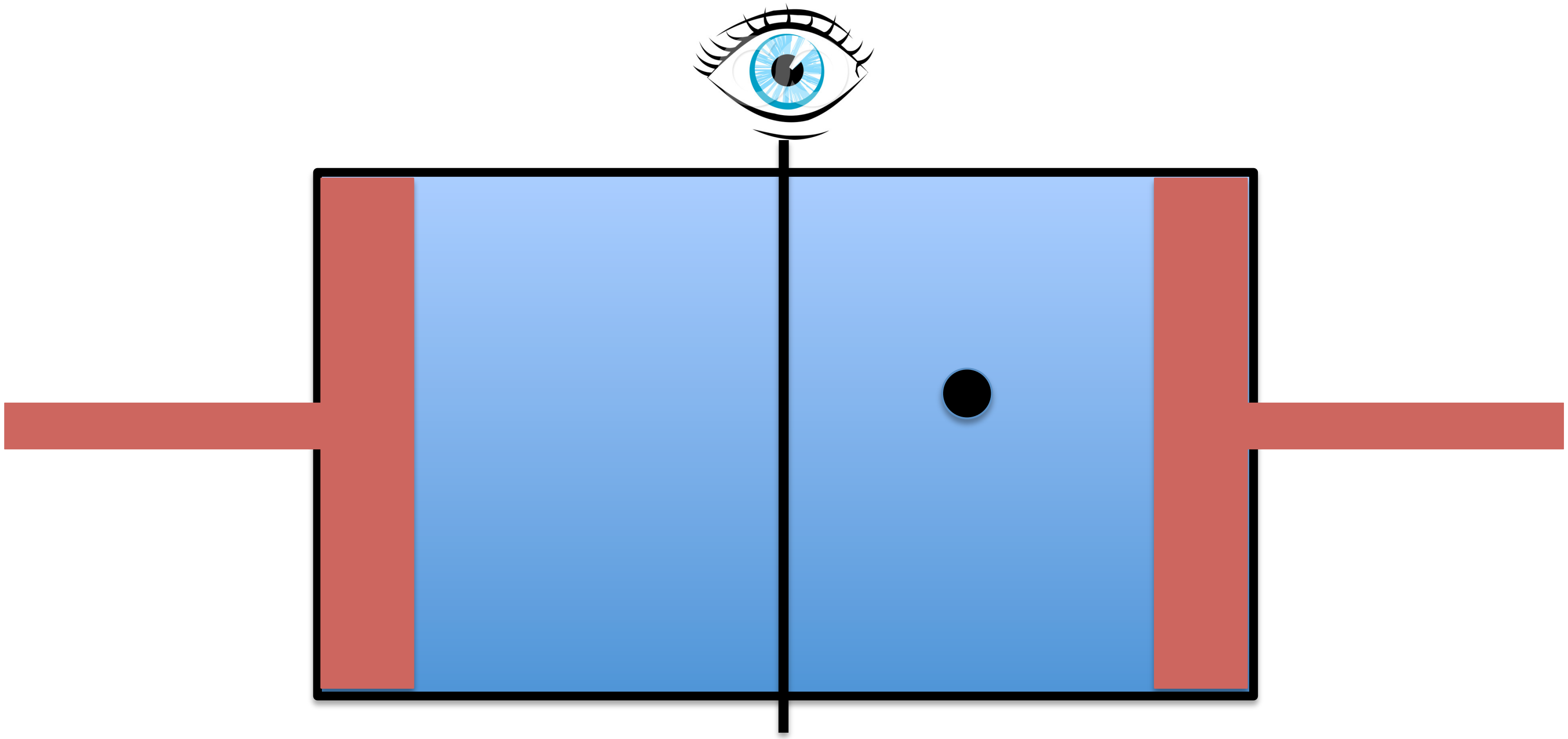
Szilárd's engine

Step 1: Insert the divider



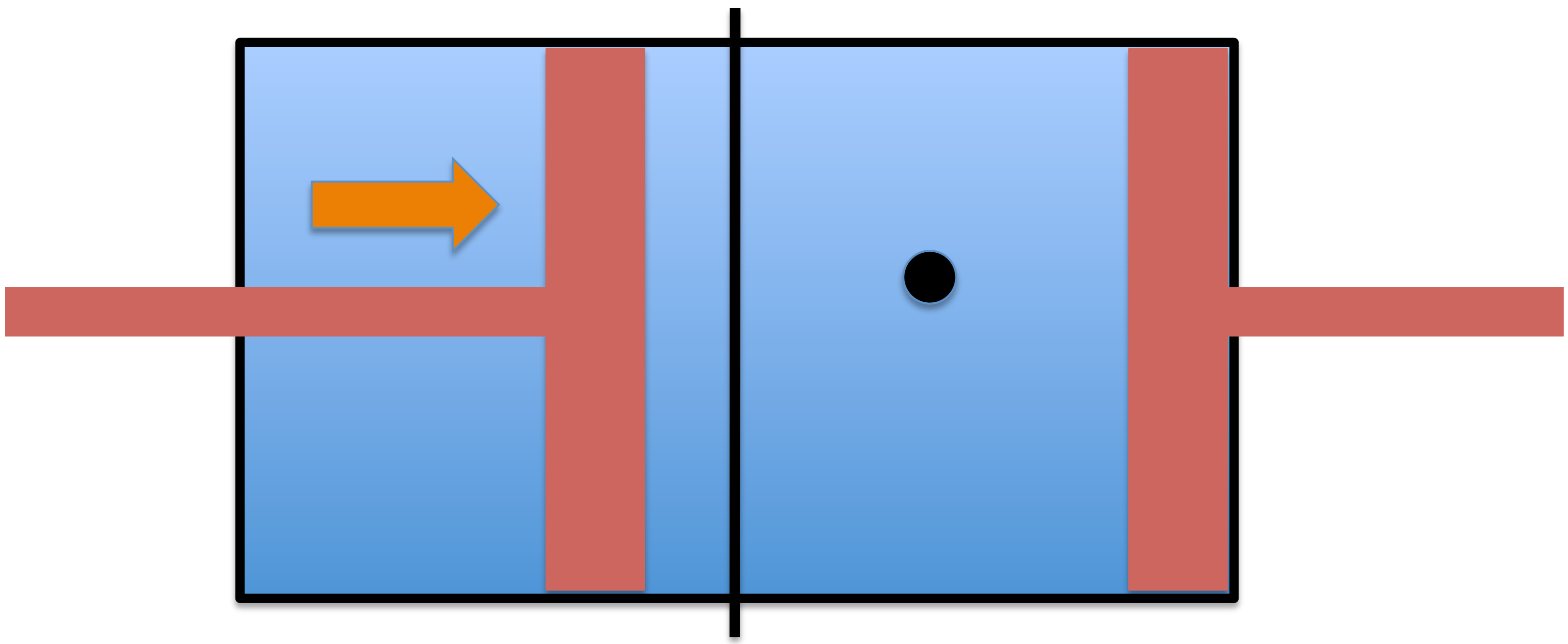
Szilárd's engine

Step 2: Position measurement



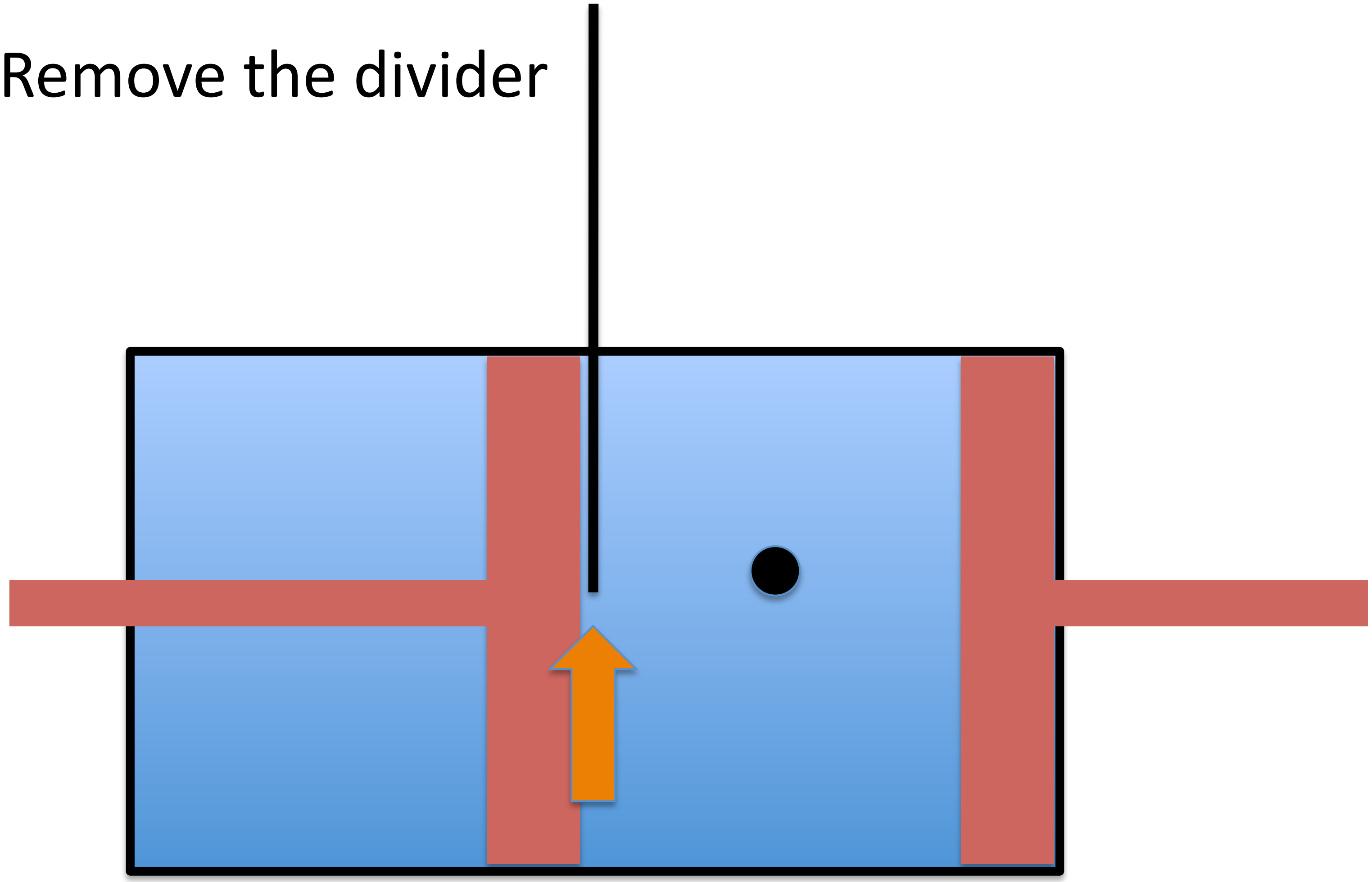
Szilárd's engine

Step 3: Compression stroke



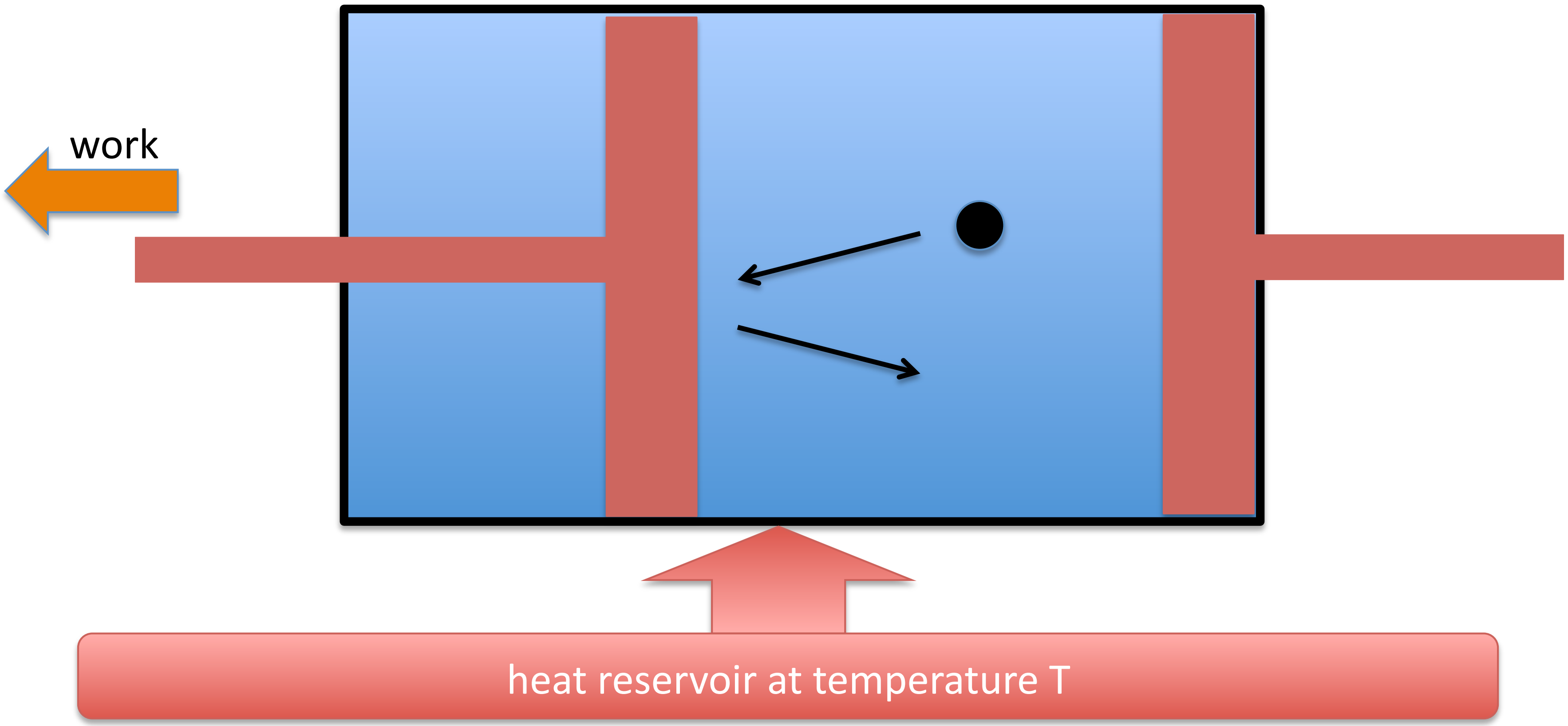
Szilárd's engine

Step 4: Remove the divider



Szilárd's engine

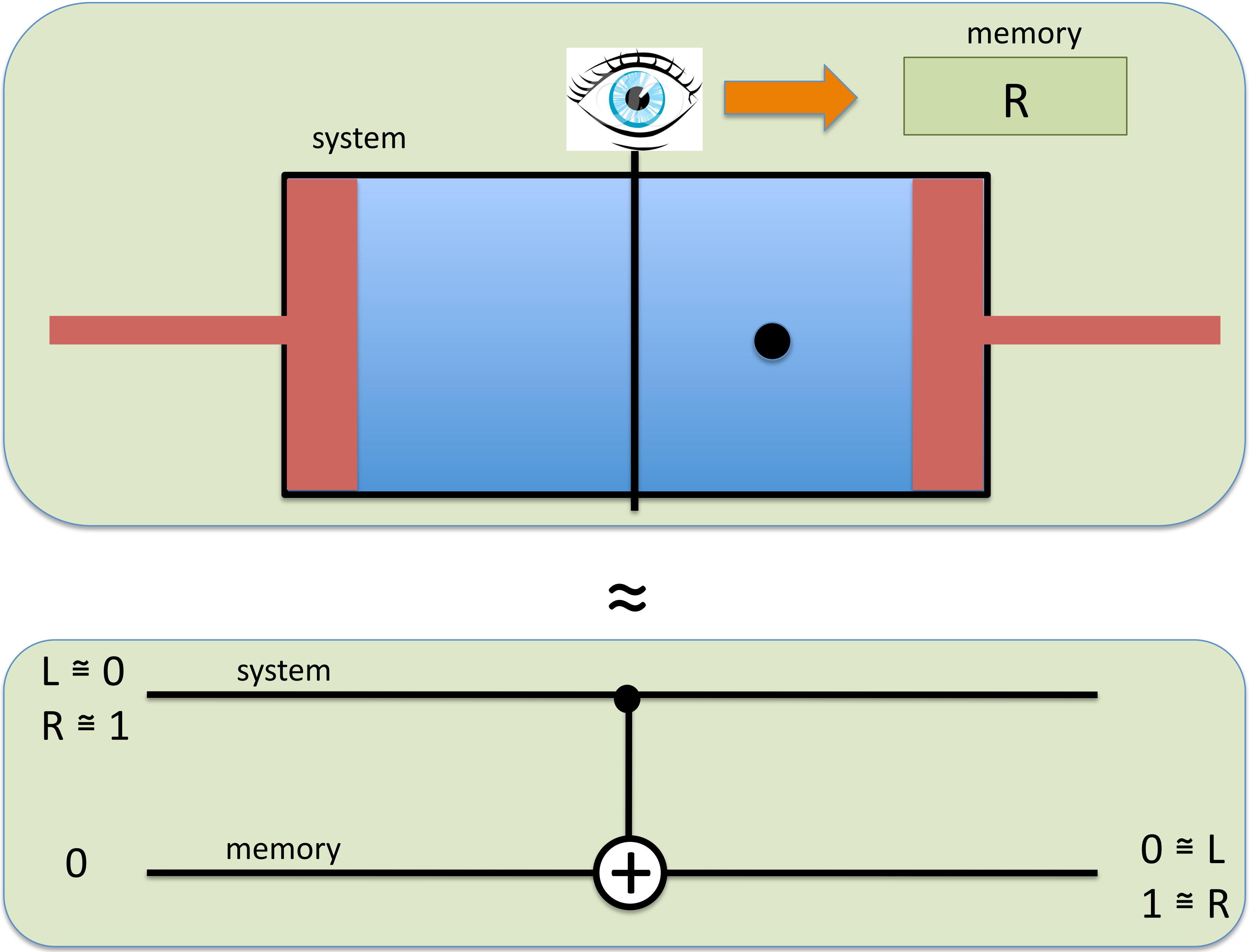
Step 5: Power stroke; converts heat into mechanical work



Bennett's argument

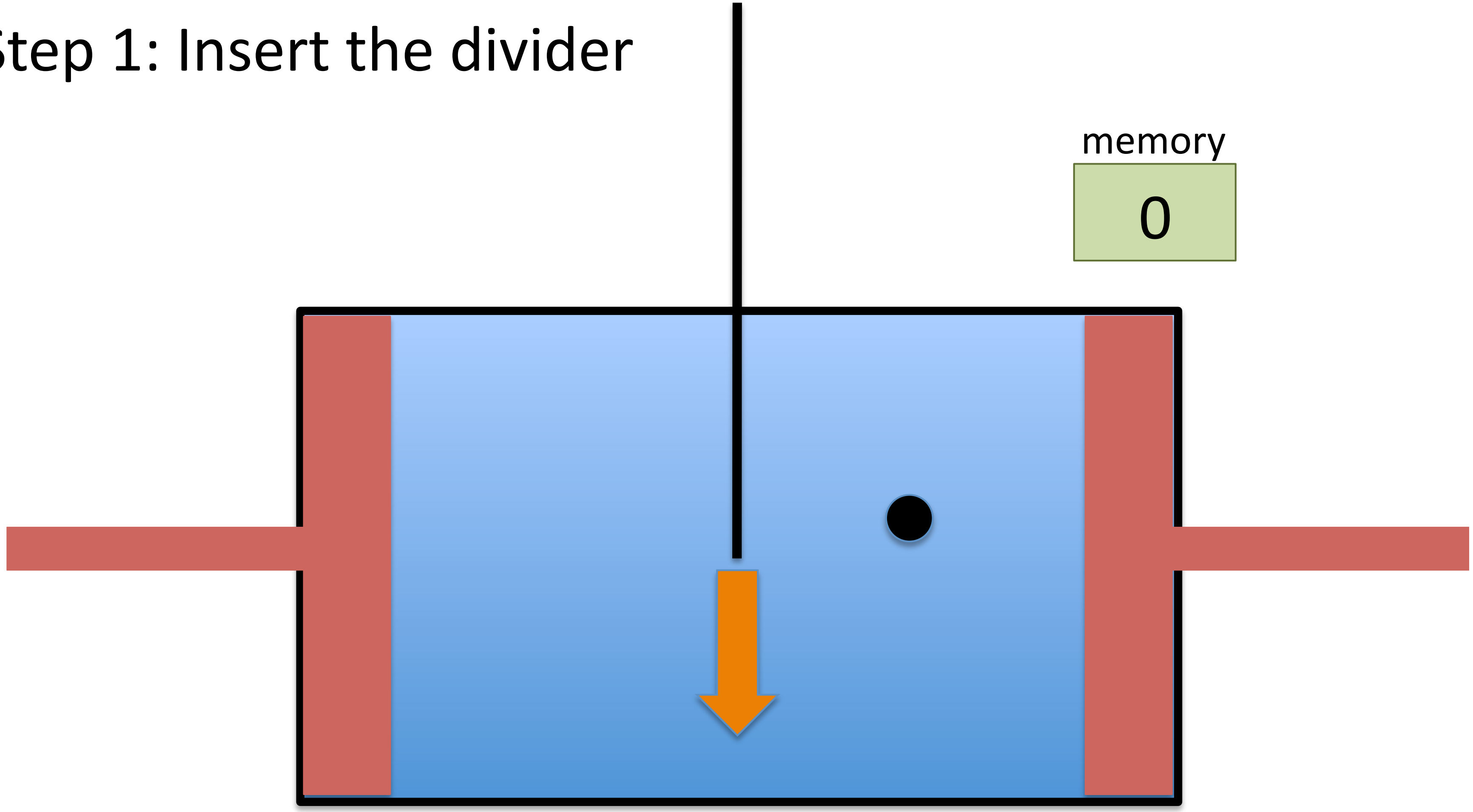
Charles H. Bennett

"The measurement process can be implemented as a reversible operation and can therefore be carried out without any energy expenditure."


memory
R
system
≈
L ≅ 0
R ≅ 1
system
0
memory
0 ≅ L
1 ≅ R

Szilárd's engine

Step 1: Insert the divider



Szilárd's engine

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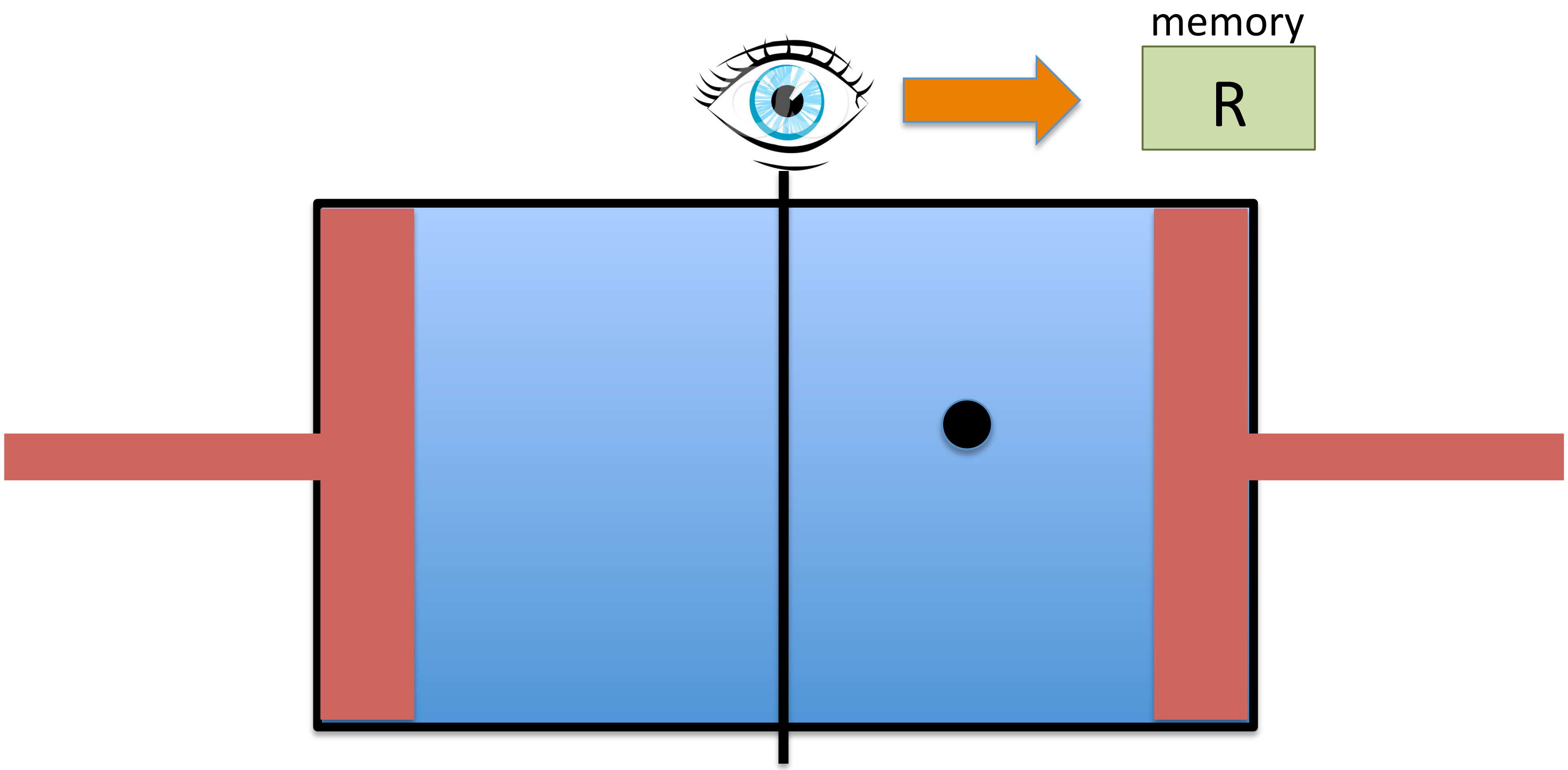

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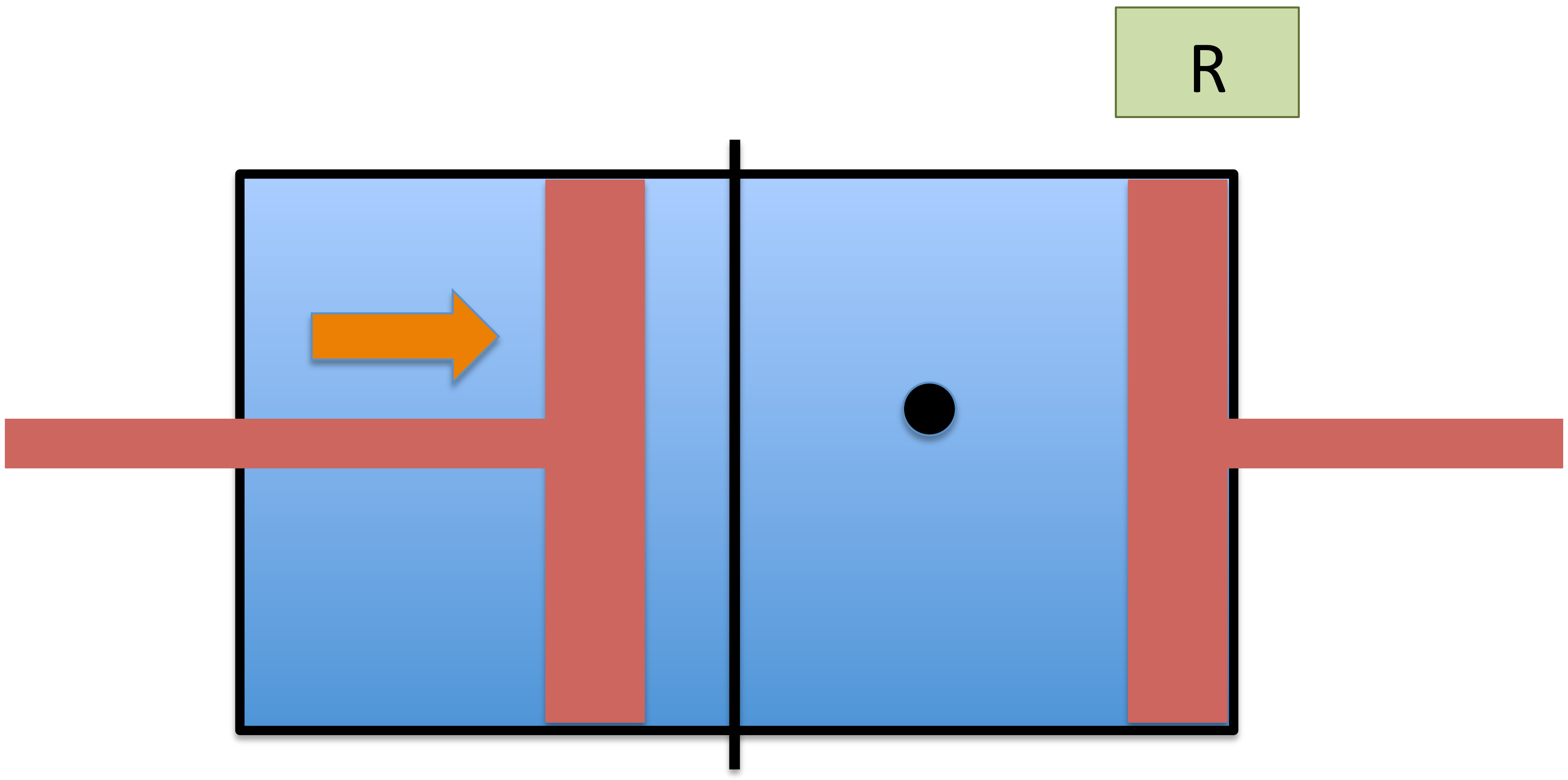

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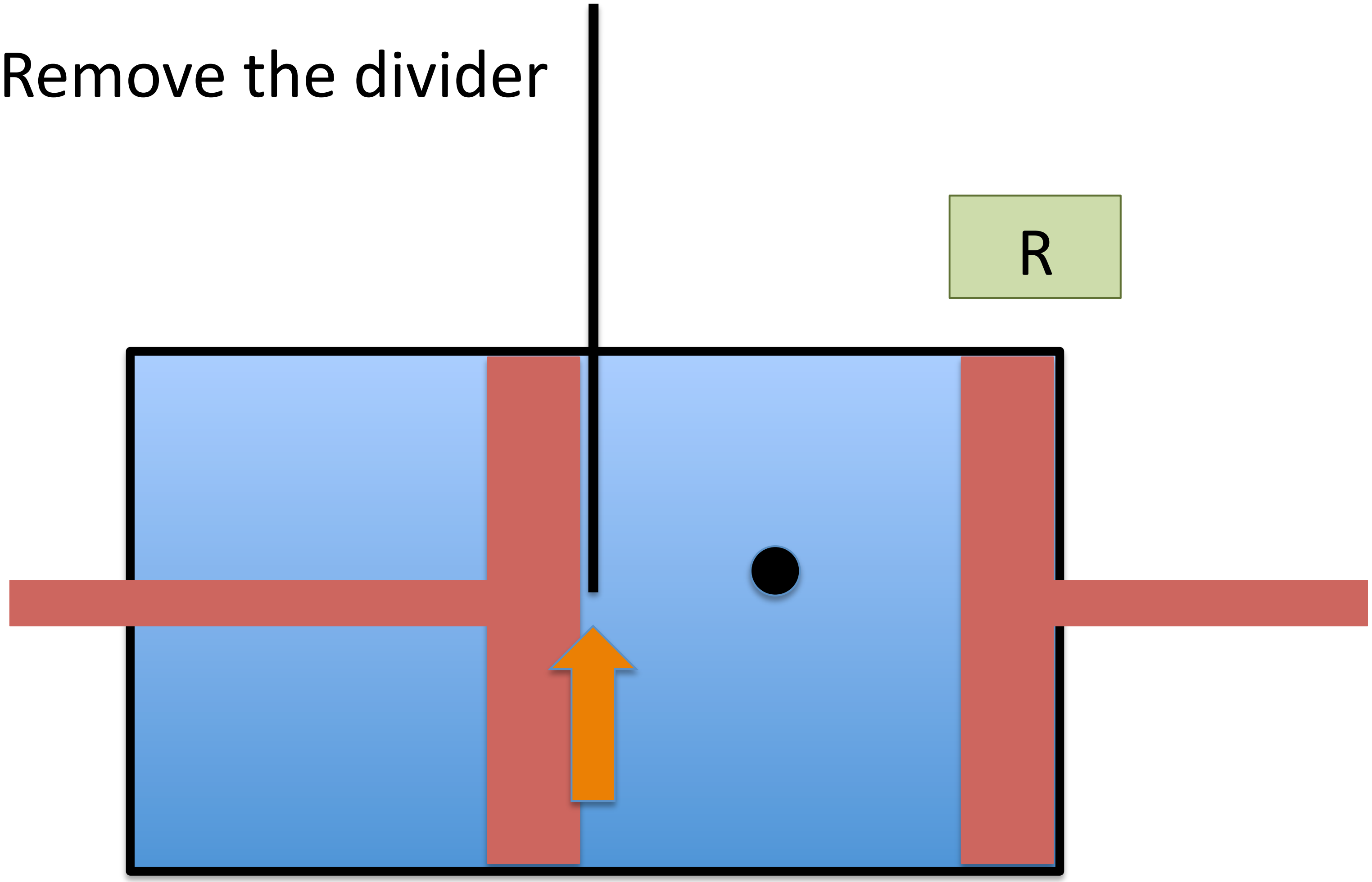

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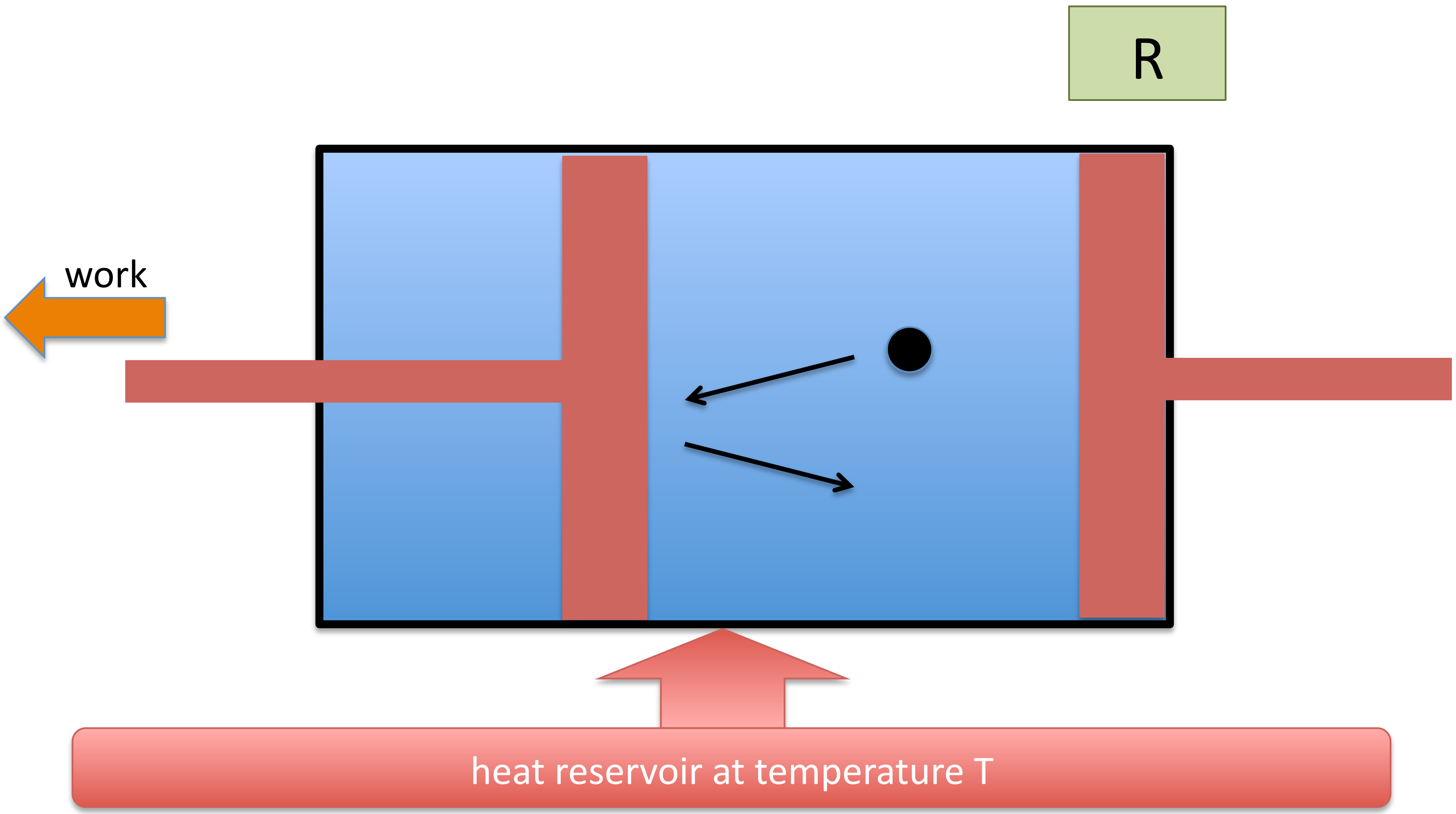


Landauer's Erasure-Principle

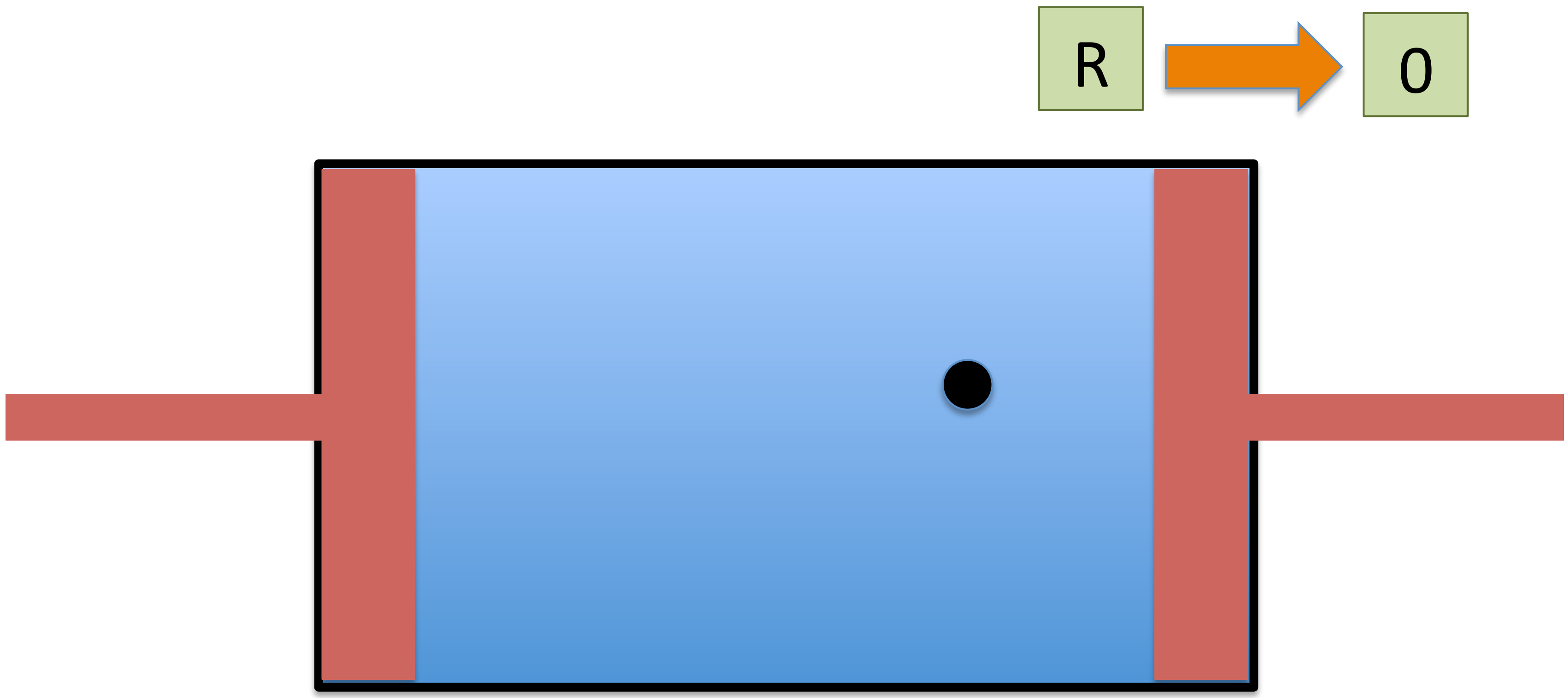
Rolf Landauer
1927- 1999

"The erasure of a bit must be accompanied by an entropy increase of the environment by 1 k_B, and a corresponding heat flow to the environment."

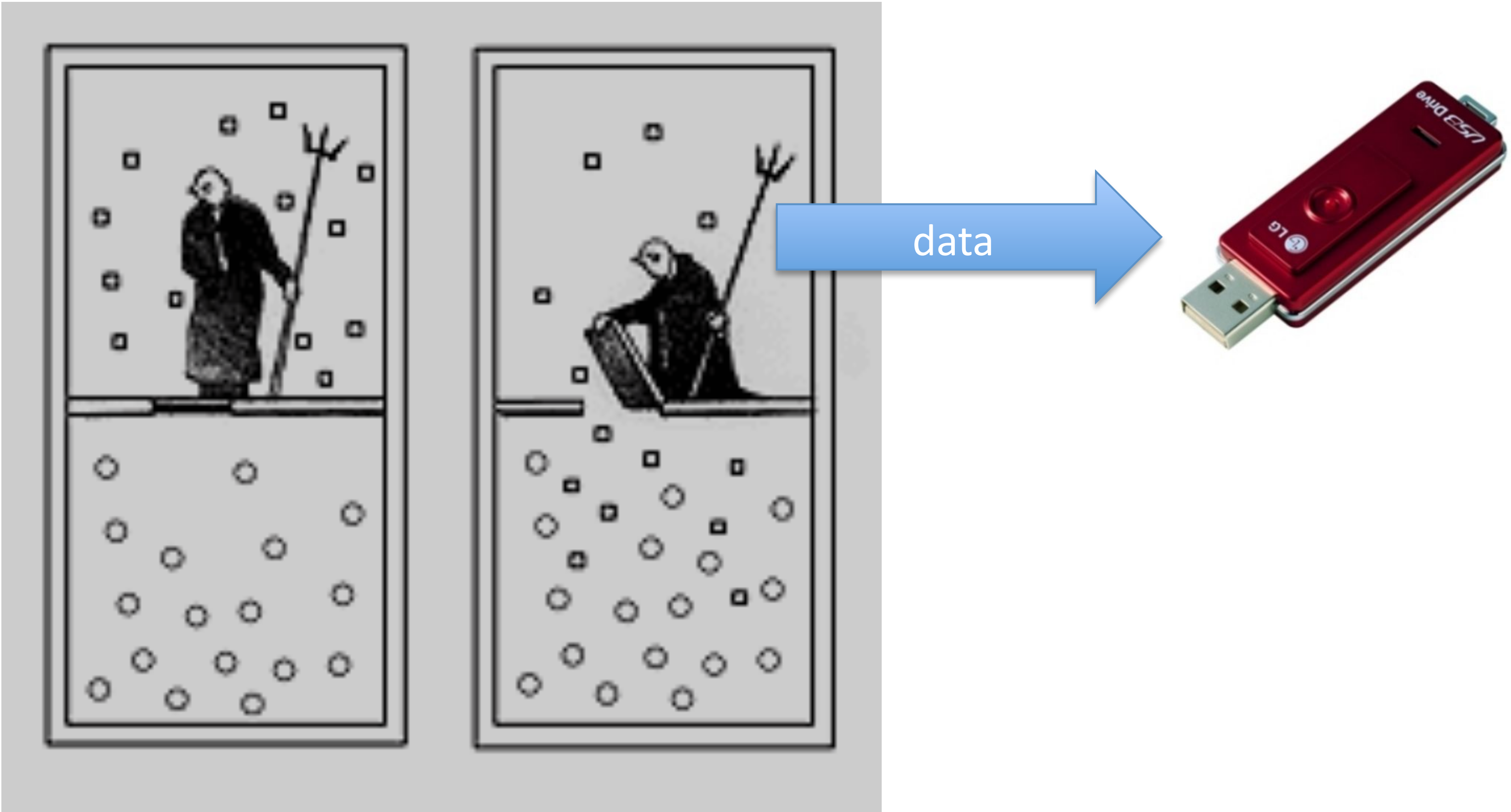
k_B : Boltzmann constant
$k_B \approx 8.61\times10^{-5}$ eV/K

Information erasure as a costly process

Step 6: Erase memory content; requires work



Maxwell's Demon



Could be built (in principle), but would accumulate data that needs to be erased.

Landauer's Erasure-Principle

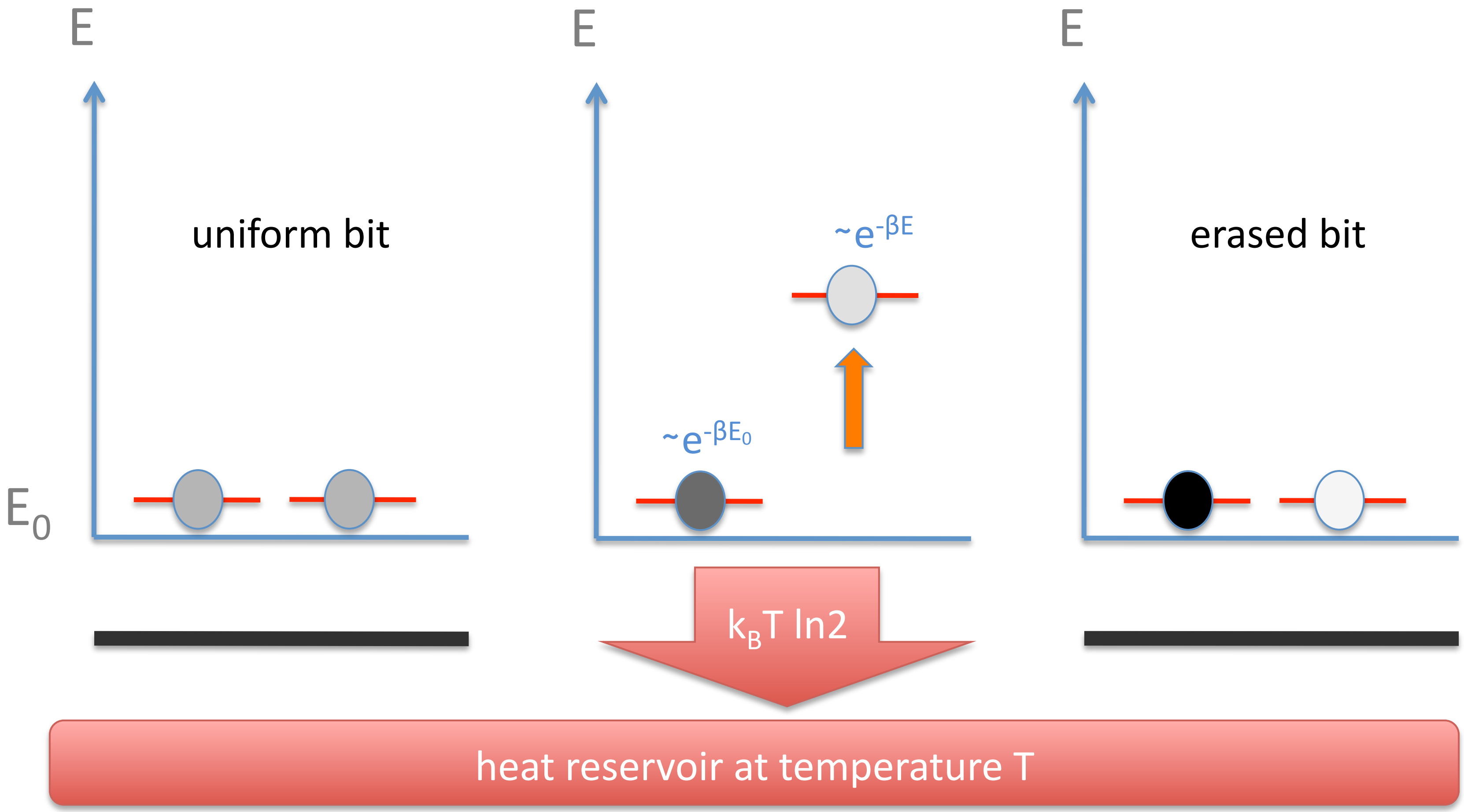
"The erasure of 1 bit requires the expenditure of

$$k_B T \ln 2$$

work."

The principle is a necessary condition for the second law of thermodynamics to hold.

Erasure of 1 uniform bit



see [R. Alicki, M. Horodecki, P. Horodecki, R. Horodecki]

Landauer's Erasure-Principle

"The erasure of n bits requires the expenditure of

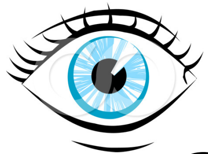
$$n\, k_B T \ln 2$$

work."

Underlying assumption: The values of the bits are, before erasure, uniformly distributed and unknown.

Observer-dependent Landauer Principle

X

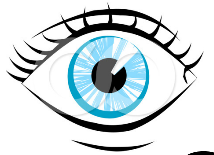


O

How much work does an observer O (who may have knowledge about X) require to erase X?

Observer-dependent Landauer Principle

X



O

The amount of work, $W(X|O)$, required by an observer O to erase a value X, is given by

$$W(X|O) = k_B T \ln 2 \, H_{max}(X|O)$$

where $H_{max}(X|O)$ is the (smooth) max-entropy.

[O. Dahlsten, R. Renner, E. Rieper, V. Vedral, 2010]

Observer-dependent Landauer Principle

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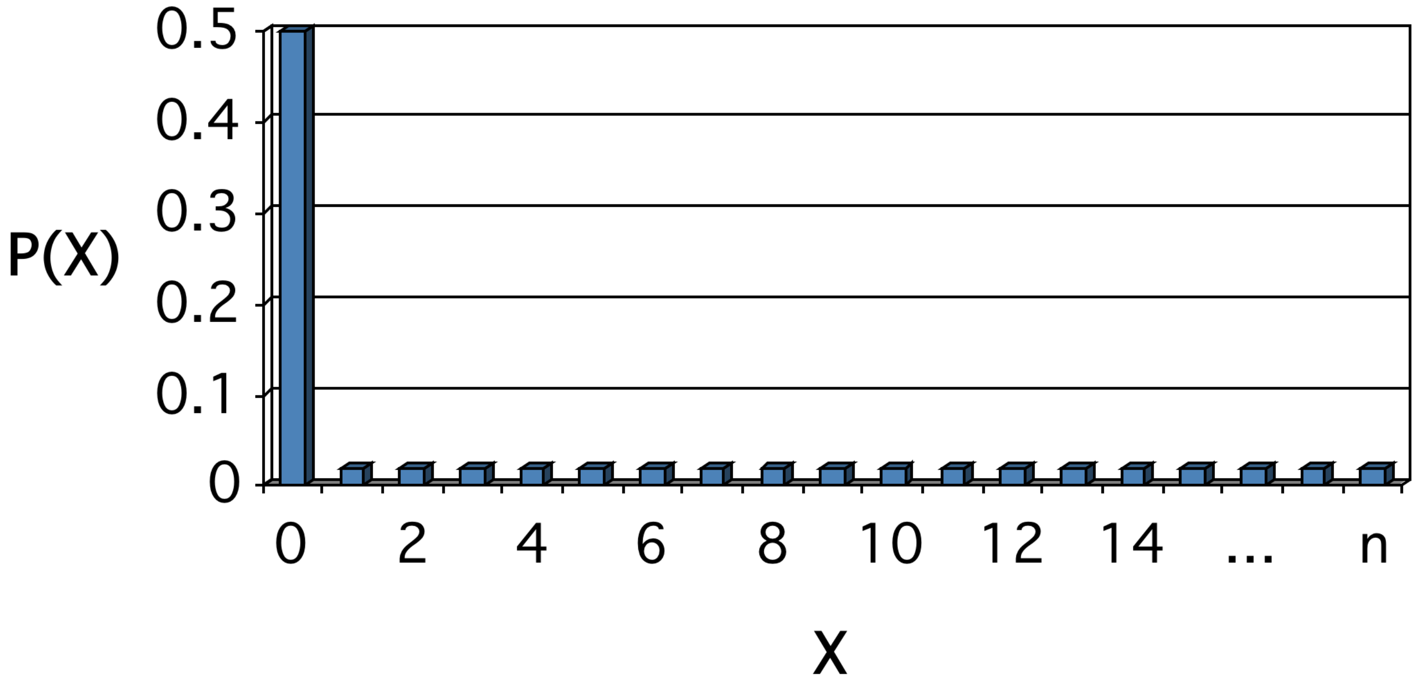
Remark: In the special case where many identical system are erased, this reduces to

$$W(X|O) = k_B T \ln 2 \; H(X|O).$$

[J. Oppenheim, M. Horodecki, P. Horodecki, R. Horodecki, 2002]

Smooth min-/max-entropies

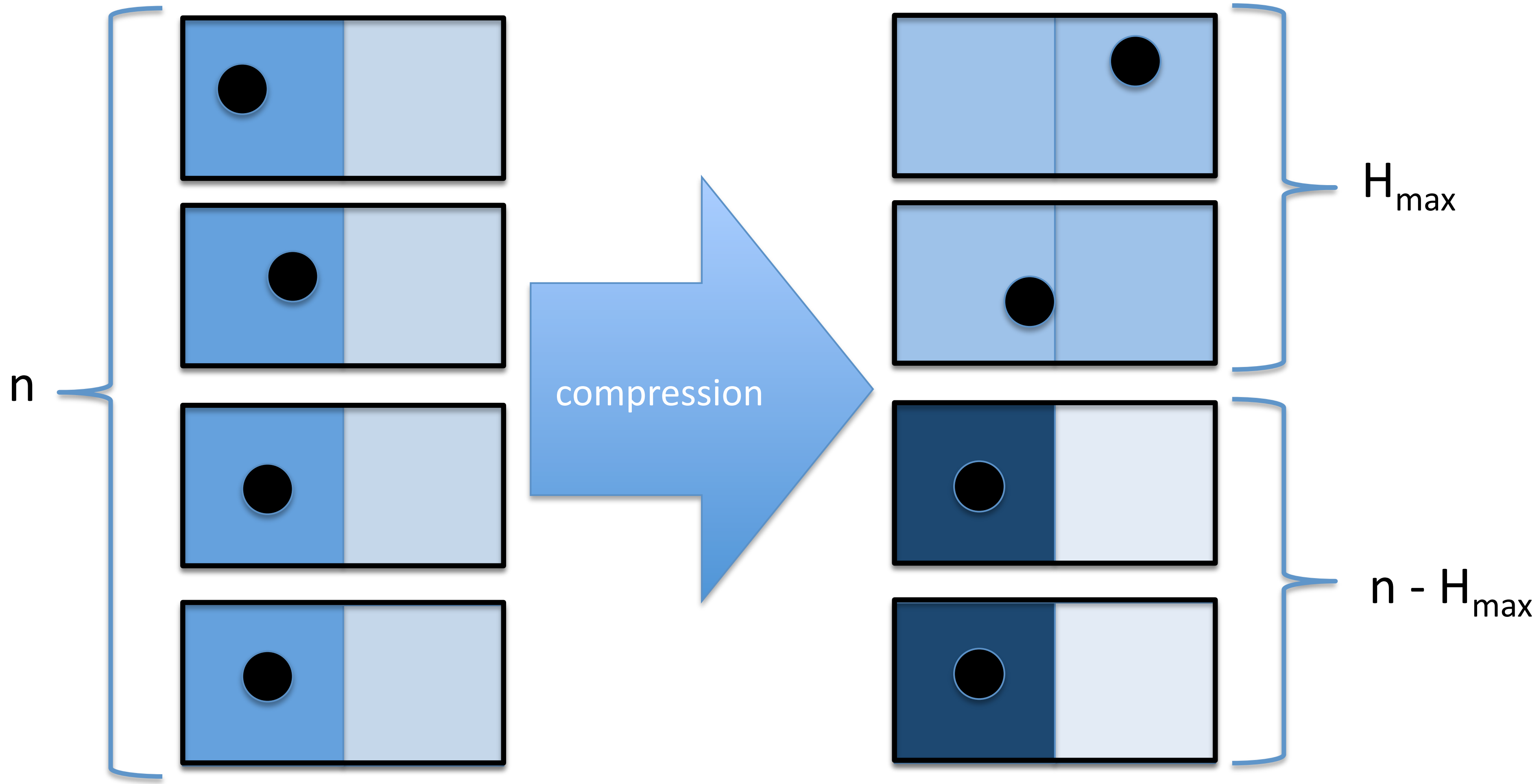


$H(X) = 1 + \frac{1}{2} \log n$

$H_{max}(X) \approx \log n$

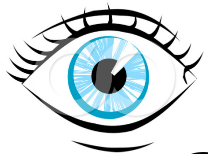
$H_{min}(X) = 1$

Erasure of several bits (proof idea)



Observer-dependent Landauer Principle

X



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Entropy as a measure of ignorance

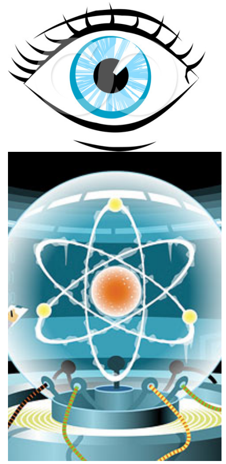
X (n random bits)

O

$H_{max}(X\|O) = n$	observer ignorant about X
$H_{max}(X\|O) = 0$	observer knows X

What about more general observers?

X



O

Entropy as a measure of ignorance

X (n random bits)

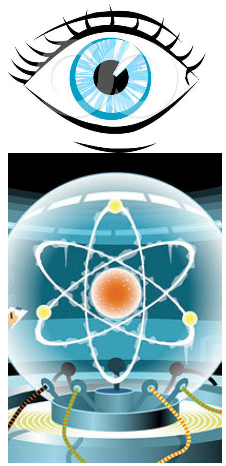
O

$H_{max}(X\|O) = n$	observer ignorant about X
$H_{max}(X\|O) = 0$	observer knows X
$H_{max}(X\|O) = -n$	observer is entangled with X *

* Negative entropies have an operational significance in the context of data compression [Horodecki, Oppenheim, Winter, *Nature* **436**:673-676; 2005].

Landauer's principle
with quantum observers

X



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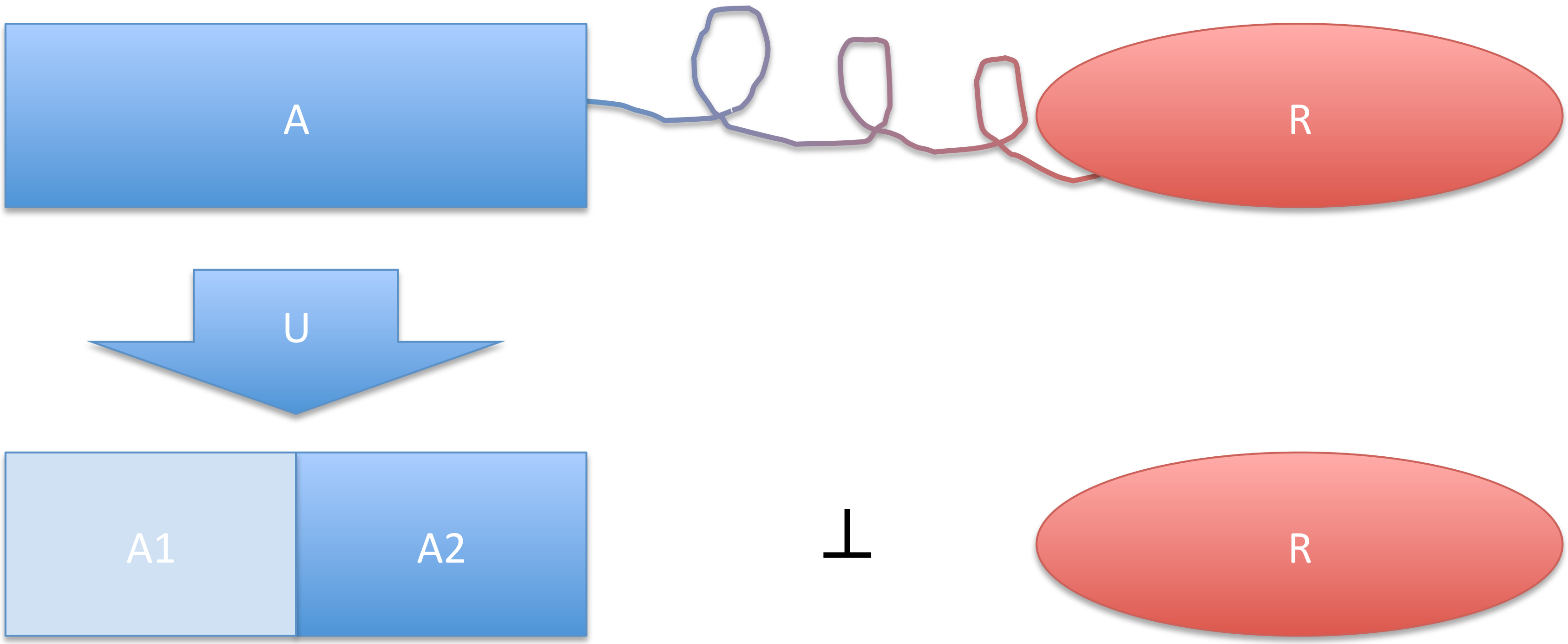
$$W(X|O) = k_B T \ln 2 \; H_{max}(X|O).$$

[del Rio, Aberg, Dahlsten, RR, Vedral, arXiv:1009.1630; 2010]

Proof based on “decoupling”

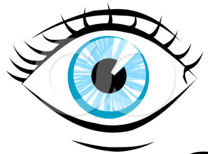


[M. Horodecki, J. Oppenheim, A. Winter, Nature 436:673-676; 2005]

[F. Dupuis, PhD thesis, arXiv:1004.1641]

What does this tell us about thermodynamic entropy?

X



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[del Rio, Aberg, Dahlsten, RR, Vedral, arXiv:1009.1630; 2010]

Information theory tells us something about thermodynamics:

Since the entropy $H_{max}(X|O)$ may be negative, erasure of information may lead to a gain of usable energy.

Conclusions

- Relation between "energy cost" and "uncertainty".

$$W(X|O) = k_B T \ln 2 \; H_{max}(X|O)$$

- In the general "single-shot" scenarios, the relevant uncertainty measure is the max-entropy H_{max} rather than the von Neumann entropy H.

- The energy required to erase a system X may be negative, if the conditional entropy of X is negative.

Thanks for your attention

arXiv:0908:0424

arXiv:1009:1630