END OF SEVERAL QUANTUM MYSTERIES

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Mundaka-Upanishad (<1000 B.C):

“Which is that, when known, ALL becomes known?”

Plan:

Main Result: Einstein Locality is valid in the physics of Spatially Separated Systems – EPR Systems – and Correlations result from Classical Conservation Laws encoded in a Shared variable at Source.

1) The EPR argument regarding SSS, according to Einstein (and not EPR)

2) SSS treated in local hidden variable theories, by Bell: the deviation

3) Rigorous logical Implications of experimental results

4) Quantum correlations and Classical Conservation Laws – an insightful result

5) Going beyond Bell and Proof of perfect Einstein locality in quantum correlations (or how nature does it preserving Einstein locality)

Spatially separated Systems: (SSS)
The case of two ‘spin-half’ particles:

$$\Psi_S = \frac{1}{\sqrt{2}}\left(|+1\rangle_1 |-1\rangle_2 - |-1\rangle_1 |+1\rangle_2\right)$$

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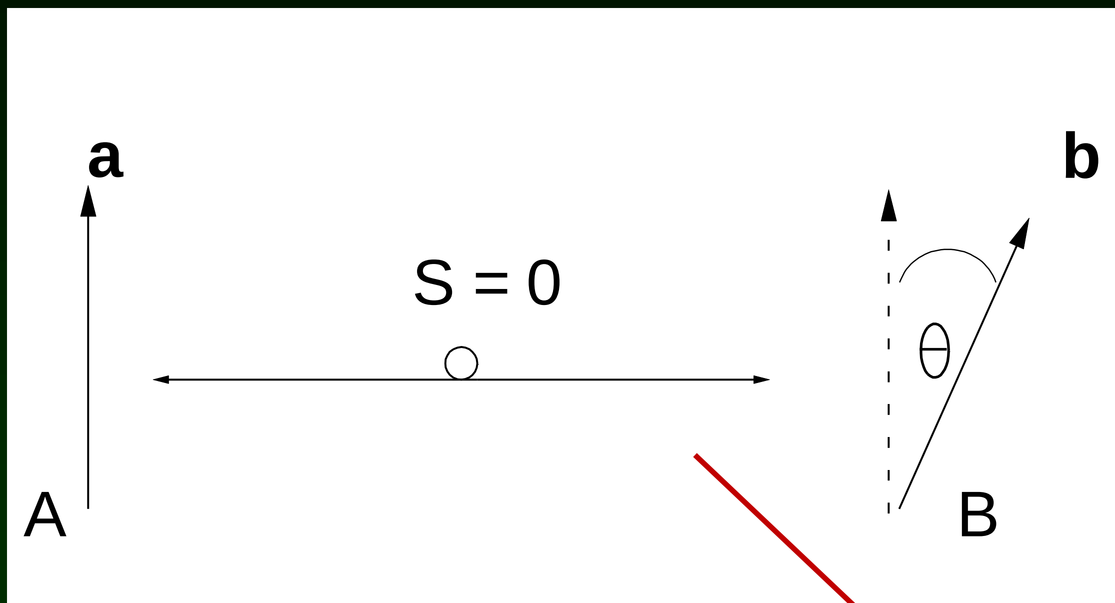

Big-B

$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i$$

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EPR argument as described by Einstein?

Excerpts from Einstein's letter to Popper (reproduced in Logic of Scientific Discovery) explaining his view that the wave-function description is incomplete:

"Should we regard the wave-function whose time dependent changes are, according to Schrödinger equation, deterministic, as a *complete* description of physical reality,…?

The answer at which we arrive is the wave-function should not be regarded as a complete description of the physical state of the system.
We consider a composite system, consisting of the partial systems A and B which interact for a short time only.

We assume that we know the wave-function of the composite system *before* the interaction – a collision of two free particles, for example – has taken place. Then Schrodinger equation will give us the wave-function for the composite system *after* the interaction.

Assume that now (after the interaction) an optimal measurement is carried out upon the partial system A, which may be done in various ways, however depending on the variables which one wants to measure precisely – for example, the momentum or the position co-ordinate. Quantum mechanics will then give us the wave-function for the partial system B, and it will give us *various wave-functions that differ, according to the kind of measurement which we have chosen to carry out upon A.*

Now it is unreasonable to assume that the physical state of B may depend upon some measurement carried out upon a system A which by now is separated from B (so that it no longer interacts with B); and this means that the two different wave-functions belong to one and the same physical state of B. Since a *complete* description of a physical state must necessarily be an *unambiguous* description (apart from superficialities such as units, choice of the co-ordinates etc.) it is therefore not possible to regard the wave-function as the *complete* description of the state of the system."

Anything beyond this in the EPR Phys. Rev. paper is superfluous and irrelevant as far as Einstein's point is concerned.

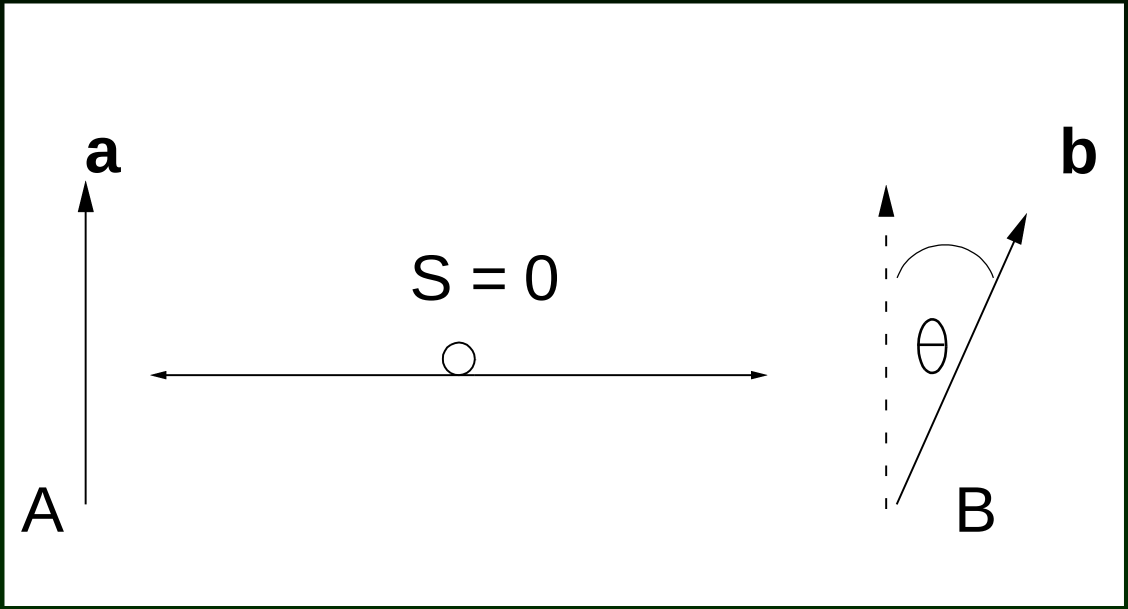
In particular there is no reference or wish regarding a possible completion of QM using some classical statistical hidden variables.

It will also be completely wrong to think that the EPR or anybody who understood core of QM for that matter, argued for a theory in which the main feature of QM - superposition of states – is thrown out and replaced by classical Newtonian mechanics applied to some ensemble:

Then how did we end up with this mess called Local Hidden Variable Theories ?!

The case of two 'spin-half' particles:

$$\Psi_S = \frac{1}{\sqrt{2}}\left(|+1\rangle_1 |-1\rangle_2 - |-1\rangle_1 |+1\rangle_2\right)$$



$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i \quad : A_i, B_i = \pm 1$$

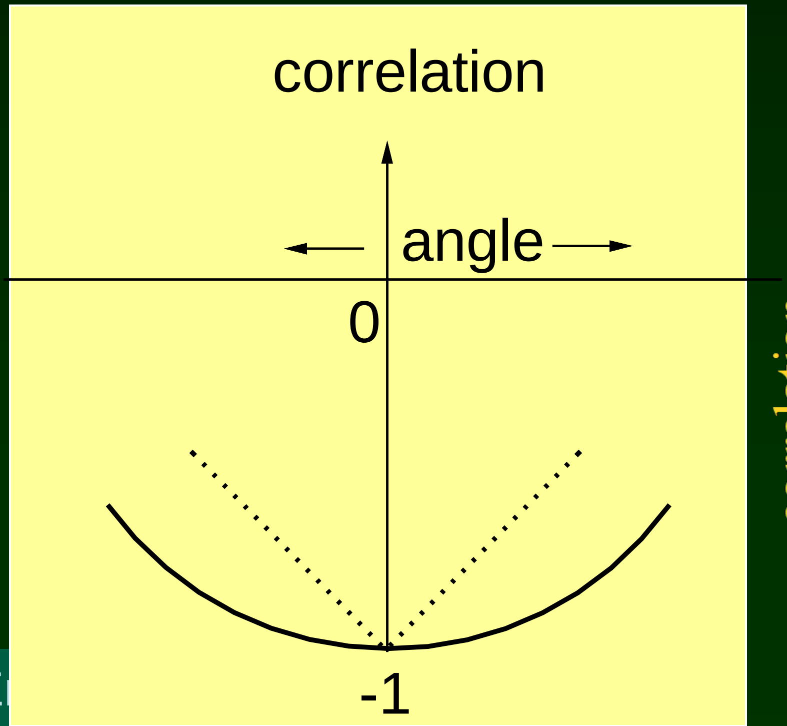
Important input

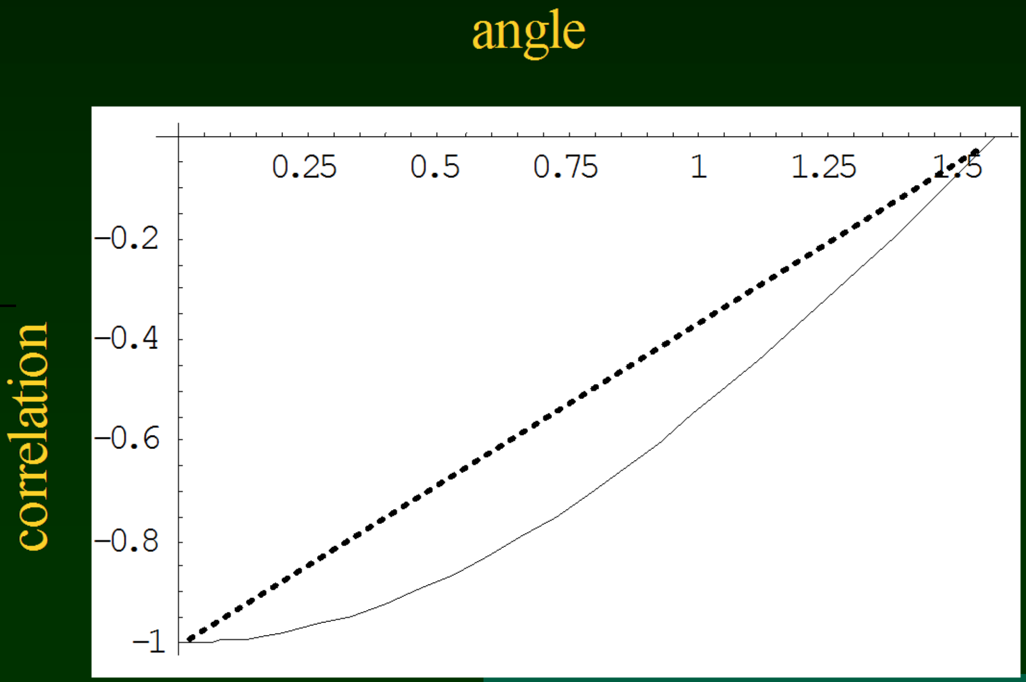
Quantum Mechanics: $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} = -\cos\theta$

$$P(\vec{a},\vec{b})_{QM} = \langle \Psi_S | \sigma_1 \cdot \vec{a} \otimes \sigma_2 \cdot \vec{b} | \Psi_s \rangle = -\vec{a} \cdot \vec{b}$$

$$P(\vec{a},\vec{b})_{Bell} = \int A(\vec{a},h) B(\vec{b},h) \rho(h) dh$$

The essence of Bell's theorem is that these two correlation functions have distinctly different dependences on the angle between the settings of the apparatus (difference of about 30% at specific angles).





Bell's inequalities and theories of correlations

Do experimental results indicate (let alone 'prove') nonlocality?
Entanglement=Nonlocality?!

Beliefs:

1) Experimental results prove that there is nonlocality (violation of Einstein locality)

2) Local Hidden Variable Theories are theoretically valid (no inconsistencies with known physical principles)

"Now it is unreasonable to assume that the <u>physical state of B </u>may depend upon some measurement carried out upon a system A which by now is separated from B"

So, what does the experimental confirmation of the violation of Bell's inequality imply as valid theoretical statements that are logically rigorous?

1) Quantum mechanics is validated as a good theory of correlations…

2) OR…a classical hidden variable theory in which statistically distributed valued of the HV determine measurement outcomes is validated as a good theory of correlations to replace QM provided there is violation of Einstein locality.

The common mistake is to mix the two and claim that experiments prove nonlocality or that Experiments prove QM is nonlocal !

examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

If nonlocal influence are allowed then <u>any classical theory</u> (of the coin tossing type) can be made to reproduce whatever correlations one demands!

Hence the strict logical implication of the experimental results is that a classical theory of the type Bell considered can be a valid theory of microscopic phenomena, replacing QM, IF one allows nonlocality as an additional feature.

This then takes away the uniqueness of quantum theory, contrary to the common belief.

Entanglement = Nonlocality ?

If one describes phenomena involving entanglement using a naïve classical statistical theory, then one needs nonlocality (influence outside the light cone – violation of Einstein locality).

A look at what Bell did to get the inequalities, to spot a deviation from the grand plan of 'completing quantum mechanics':

$$P_B(\vec{a},\vec{b}) = \int \rho(h)dh\ A(\vec{a},h)B(\vec{b},h), \quad \int \rho(h)dh = 1$$

Since $A(\vec{a}) = -B(\vec{a})$ and $P_B(\vec{a},\vec{a}) = -1$, Bell wrote

$$P_B(\vec{a},\vec{b}) = -\int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)$$

$$P_B(\vec{a},\vec{b}) - P_B(\vec{a},\vec{c}) = -\int \rho(h)dh\,[A(\vec{a},h)A(\vec{b},h) - A(\vec{a},h)A(\vec{c},h)]$$

$$= -\int \rho(h)dh\,[A(\vec{a},h)A(\vec{b},h) - A(\vec{a},h)A(\vec{b},h)A(\vec{b},h)A(\vec{c},h)]$$

$$= \int \rho(h)dh\ A(\vec{a},h)A(\vec{b},h)[A(\vec{b},h)A(\vec{c},h) - 1]$$

$$\left|P_B(\vec{a},\vec{b}) - P_B(\vec{a},\vec{c})\right| \le \int \rho(h)dh\ [1 + A(\vec{b},h)B(\vec{c},h)] = 1 + P_B(\vec{b},\vec{c})$$

$$P_B(\vec{a},\vec{b}) = \int \rho(h)dh\ A(\vec{a},h)B(\vec{b},h), \quad \int \rho(h)dh = 1$$

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Simultaneous definite values for quantum mechanically non-commuting observables

Clearly not part of a program to complete QM by adding additional features to QM.

A physically correct program of completing QM should never have simultaneous values for 'conjugate' observables before measurement – that is not consistent with even basic wave-particle duality.

CSU, Proc. SPIE Photonics 2007

Quantum correlations and Classical Conservation Laws

Assumption: Fundamental conservation laws related to space-time symmetries are valid on the average over the quantum ensemble and measurements are made with finite number of discrete outcomes. (conservation check is not possible event-wise)

Result: Unique two-particle and multi-particle correlation functions can be derived from the assumption of validity of conservation laws alone. Interestingly, they are identical to the ones derived using formal quantum mechanics with appropriate operators and states.

In particular the quantum correlation functions relevant for experiments have been derived from classical conservation law for angular momentum valid over the ensemble.

CSU, Europhys. Lett, 2005, Pramana-J.Phys (2006)

A	B
-1	+1
-1	-1
+1	+1
-1	+1
+1	-1
-1	+1
-1	+1
+1	-1
-1	+1
+1	+1
-1	+1
+1	-1
+1	+1
+1	-1
+1	+1
-1	+1
+1	-1

→

A (reordered)	B (reordered)
+1	+1
+1	-1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
-1	+1
-1	-1
-1	-1
-1	+1
-1	+1
-1	+1
-1	+1
-1	+1

A (reordered)	B (reordered)
+1	+1
+1	-1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
+1	+1
+1	-1
-1	+1
-1	-1
-1	-1
-1	+1
-1	+1
-1	+1
-1	+1
-1	+1

Average Angular Momentum/($\hbar$/2)

$$= \frac{1}{N_{A+}} \sum_i +1 = +1$$

Average Angular Momentum/($\hbar$/2)

$$= \frac{1}{N_{A-}} \sum_i -1 = -1$$

What are the AVERAGE angular momenta at B for the two sub-ensembles?

$$\text{For } L_A = +1, \quad L_B = -\cos(\theta)$$

$$\text{For } L_A = -1, \quad L_B = +\cos(\theta)$$

Correlation functions for the sub-ensembles:

$$P(\vec{a},\vec{b})_{A_i=+1} = \frac{1}{N}\sum_i A_i B_i = \frac{+1}{N}\sum_i B_i \equiv L_B$$

$$P(\vec{a},\vec{b})_{A_i=-1} = \frac{1}{N}\sum_i A_i B_i = \frac{-1}{N}\sum_i B_i$$

$$P(\vec{a},\vec{b}) = \frac{1}{N}\sum_i A_i B_i = \frac{1}{2}\left(P(\vec{a},\vec{b})_{A=+1} + P(\vec{a},\vec{b})_{A=-1}\right)$$

$$= \frac{1}{2}\left(\langle B_i \rangle_{A_i=+1} - \langle B_i \rangle_{A_i=-1}\right) = \frac{1}{2}\left(L_{B(A=+1)} + L_{B(A=-1)}\right) = -\cos(\theta)$$

$$P(\vec{a},\vec{b})_{CL} = \left(L_{B(A=+1} - L_{B(A=-1}\right)/2 = -\cos(\theta)$$

This is the causally necessary consequence of the conservation law.
We have the theory independent correlation function.

Fundamental Conservation Laws {F(p,q,s...)=0} $\Rightarrow$
Quantum Mechanical Correlation Functions $\{C_{QM}(\theta_i)\}$

A correlation function with a different functional form is incompatible with the conservation laws: they can be physically realized only by violating a fundamental conservation law!

1) Correlation functions of quantum mechanics are direct consequence of the CLASSICAL conservation laws arising in space-time symmetries (fundamental conservation laws), applied to ensembles.

2) Any theory that has a correlation function different from the ones in QM is incompatible with the fundamental conservation laws and space-time symmetries, and therefore it is unphysical. Local hidden variable theories fall in this class. Bell's inequalities can be obeyed (in the general case) only by violating a fundamental conservation law, making them redundant in physics.

1) No less, no more

2) Closing loopholes will improve agreement with QM!
(better tally with conservation principle)

CSU, Europhys. Lett, 2006, Pramana-J.Phys (2006)

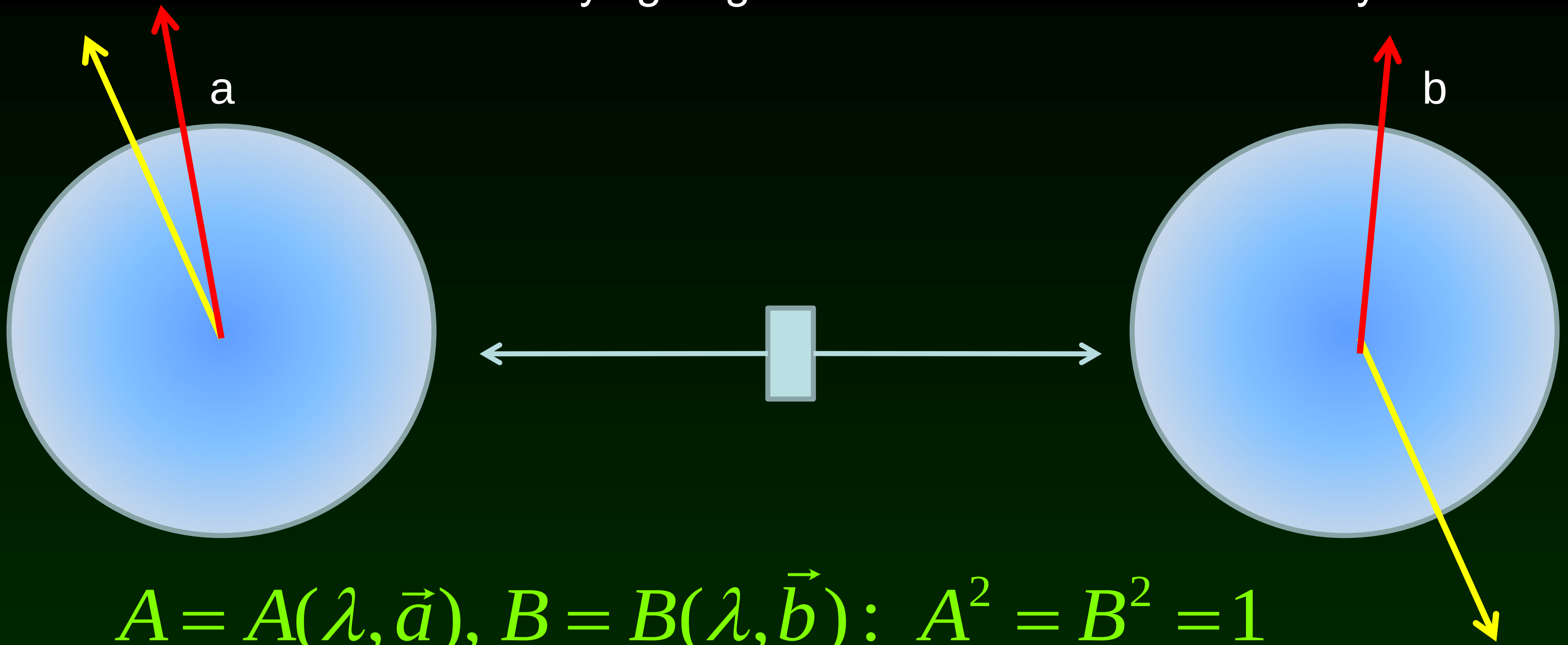
1) No experiment to date proves violation of Einstein locality

2) Quantum correlations functions are direct consequence of conservation laws, just as in the case of classical correlations.

Now I prove the key result that the observed correlations of microscopic physical systems (like the spin-1/2 singlet in QM) are realized in nature preserving strictly Einstein locality.

In other words, the correlations arise from a 'quantum-compatible variable' that is shared between the particles during interaction or break-up (at source), related to the relevant conservation law, analogous to the case in classical situation and differing in a crucial way.

Bell's scheme of trying to get correlations local realistically:



$$A = A(\lambda, \vec{a}),\ B = B(\lambda, \vec{b}):\ A^2 = B^2 = 1$$

$$\text{Outcome: Sign}(\vec{\lambda} \cdot \vec{a}) \text{ and Sign}(\vec{\lambda} \cdot \vec{b})$$

This prescription will reproduce P(a,b) for some angles, and the perfect correlation at zero relative angle. But, this does not reproduce the QM correlation.

What was missing in the LHV approach?

1) Explicit discarding of features associated with ‘wave-particle duality’

2) Incompatibility with fundamental conservation laws

3) Trying to get perfect determinism in individual local measurement when the EPR query did not criticize that aspect – (trying to solve the local quantum measurement problem as well!)

4) In short LHV theories tried to reach the higher goals by working with an inferior theory!

$$\text{Outcome: Sign}(\vec{\lambda}\cdot\vec{a}) \text{ and Sign}(\vec{\lambda}\cdot\vec{b})$$

My approach to address the issues (2000-2004):

1) Notice that conservation constraints and wave-particle duality hold the key.

2) Notice that the conservation constraint directly reflects as a phase constraint for multi-particle systems

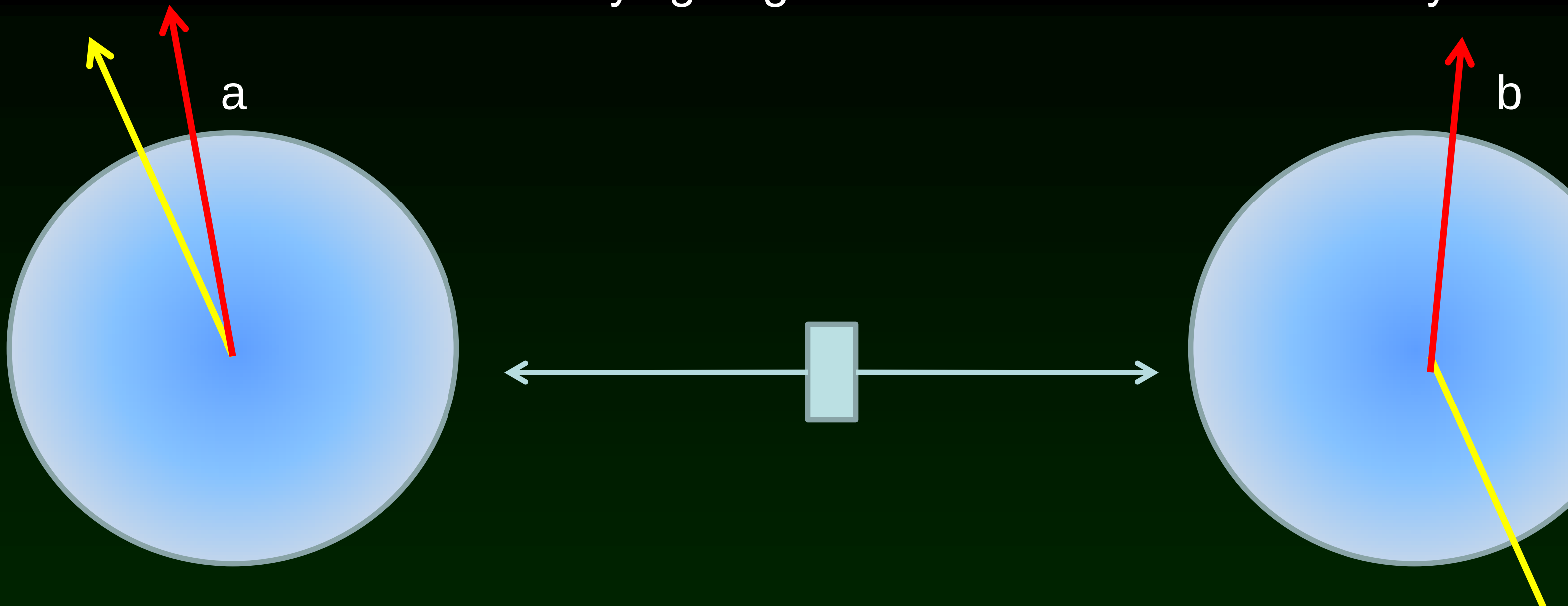
$$\text{Conservation law: } p_1 + p_2 = 0 \rightarrow \exp\frac{i}{\hbar}(p_1 x_1 + p_2 x_2)$$

The assertion was that a local phase constraint (relative phase being fixed, while individual phases are random) at the source or interaction point determines the correlations, and that Einstein locality is preserved.

But involved a prescription for calculating the correlation function that was not rigorously justified.

Unnikrishnan, Current Science (2000), Found. Phys. Lett **15**, 1-25 (2002), Ann. Fondation L. de Broglie (2002)…

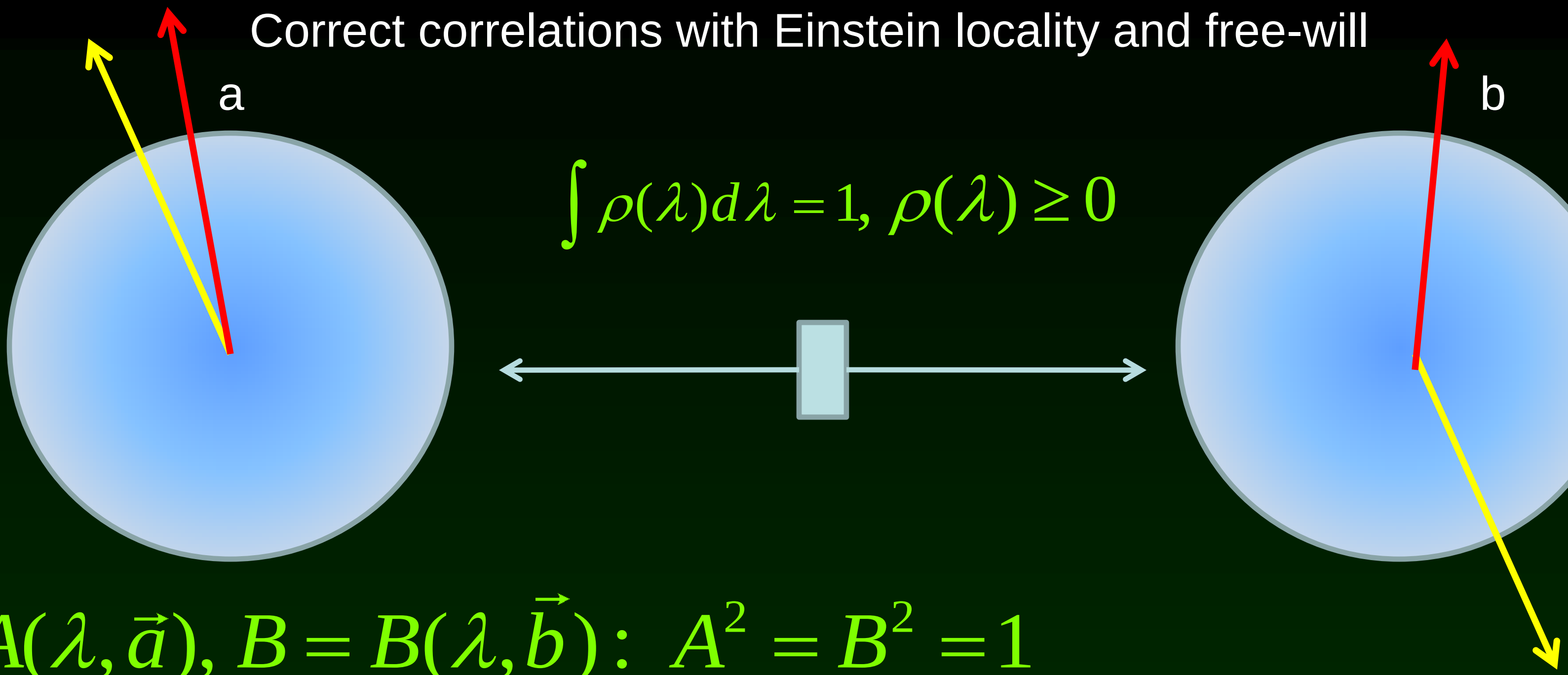
Bell's scheme of trying to get correlations local realistically:



Outcome: $\mathrm{Sign}(\vec{\lambda}\cdot\vec{a})$ and $\mathrm{Sign}(\vec{\lambda}\cdot\vec{b})$

Contrast with conservation law: $s_1 + s_2 = 0 \rightarrow \exp\frac{i}{\hbar}(s_1\theta_1 + s_2\theta_2)$

Correct correlations with Einstein locality and free-will



$$\int \rho(\lambda)\, d\lambda = 1, \ \rho(\lambda) \geq 0$$

$$A = A(\lambda, \vec{a}),\ B = B(\lambda, \vec{b}):\ \ A^2 = B^2 = 1$$

Outcomes: A=$(\vec{\lambda} \cdot \vec{a})$ and B=$(-\vec{\lambda} \cdot \vec{b})$

$$(\vec{\lambda} \cdot \vec{a})^2 = (\vec{\lambda} \cdot \vec{b})^2 = 1$$

Correlation: $-\left\langle (\vec{\lambda} \cdot \vec{a})\ (\vec{\lambda} \cdot \vec{b}) \right\rangle_{\lambda}$

Correlation: $\left\langle (\vec{\lambda}\cdot\vec{a})(\vec{\lambda}\cdot\vec{b}) \right\rangle_{\lambda}$

$$\left(\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3\right)\left(\lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3\right)$$

$$= \lambda_1^2 a_1 b_1 + \lambda_2^2 a_2 b_2 + \lambda_1^2 a_3 b_3 +$$

$$\lambda_1\lambda_2 a_1 b_2 + \lambda_1\lambda_3 a_1 b_3 + \lambda_2\lambda_1 a_2 b_1 + \lambda_2\lambda_3 a_2 b_3 + \lambda_3\lambda_1 a_3 b_1 + \lambda_3\lambda_2 a_3 b_2$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 +$$

$$\lambda_1\lambda_2 (a_1 b_2 - a_2 b_1) + \lambda_1\lambda_3 \left(a_1 b_3 - a_3 b_1\right) + \lambda_2\lambda_3 \left(a_2 b_3 - a_3 b_2\right)$$

$= \vec{a}\cdot\vec{b} + i\vec{\lambda}\cdot(\vec{a}\times\vec{b})$ with $\lambda_i^2 = 1,\ \lambda_i\lambda_j = -\lambda_j\lambda_i$ and $\lambda_i\lambda_j = i\lambda_k$

$$-\left\langle \vec{a}\cdot\vec{b} + i\vec{\lambda}\cdot(\vec{a}\times\vec{b}) \right\rangle_{\lambda} = -\vec{a}\cdot\vec{b}$$

$$\vec{a}\cdot\vec{b}+i\vec{\lambda}\cdot(\vec{a}\times\vec{b})=1Cos\theta+i\vec{\lambda}\cdot\vec{n}\,Sin\theta=\exp(i\vec{\lambda}\cdot\vec{n}\,\theta)$$

This connects up the present proof with the earlier writing (Found. Phys. Lett, 2002, for example) where I dealt with pure random phases associated with the individual particle, with a fixed relative phase arising from conservation constraint, as nature's device for showing quantum correlations preserving Einstein locality.

Conclusions:

1) Quantum correlations, Teleportation physics, Measures of entanglement, violations and decoherence etc. and all other entanglement related phenomena can be understood once it is formulated in terms of conservation constraints applied to a set of quantized observables.

2) We have discovered Quantum Compatible Shared -Variable vectors that generate the correct correlation while preserving perfect Einstein locality

3) They are fully compatible with quantum mechanical notion and requirement of of superposition and they give random local measurement outcomes ±1 as well as the correct correlation.

A simple, satisfactory, consistent and physically appealing solution to the grand puzzle originated in 1935 is now in hand.

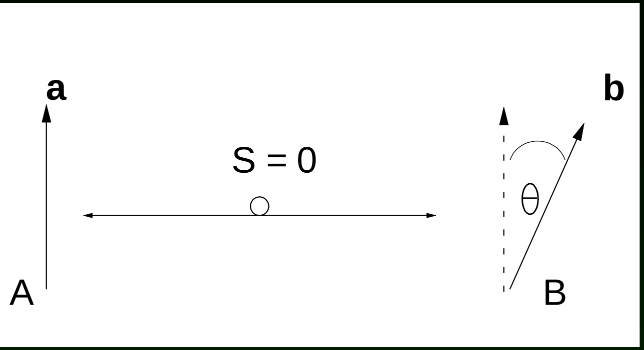
"Which is that, when known, IT becomes known?"

Higher Spins, Triplet state, GHZ etc…

Spin-S singlet:

$$+S, +(S-1), \ldots 0, \ldots -(S-1), -S$$

are the possible values



1) Create 2S+1 sub-ensembles at A

2) For sub-ensemble with average (and individual) value (S-n), the average in the direction rotated at an angle is

$$(S-n)\cos\theta$$

3) Then the average angular momentum at B for the matching sub-ensemble is

$$-(S-n)\cos\theta \quad \Rightarrow \text{Correlation function}$$

$$Av(A_i B_i) = -(S-n)^2 \cos\theta$$

Full Correlation function

$$P(\vec{a},\vec{b}) = \frac{2\sum_{n=0}^{S} -(S-n)^2 \cos\theta}{2S+1} = -\cos(\theta) S(S+1)/3$$

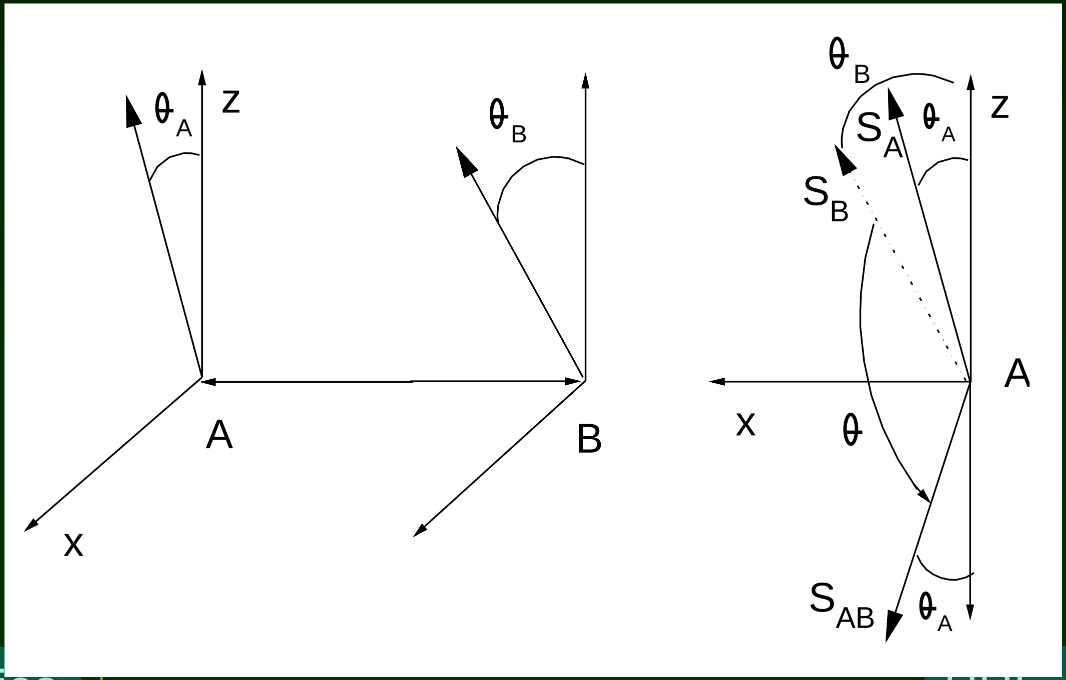
(Same as the QM correlation function!)

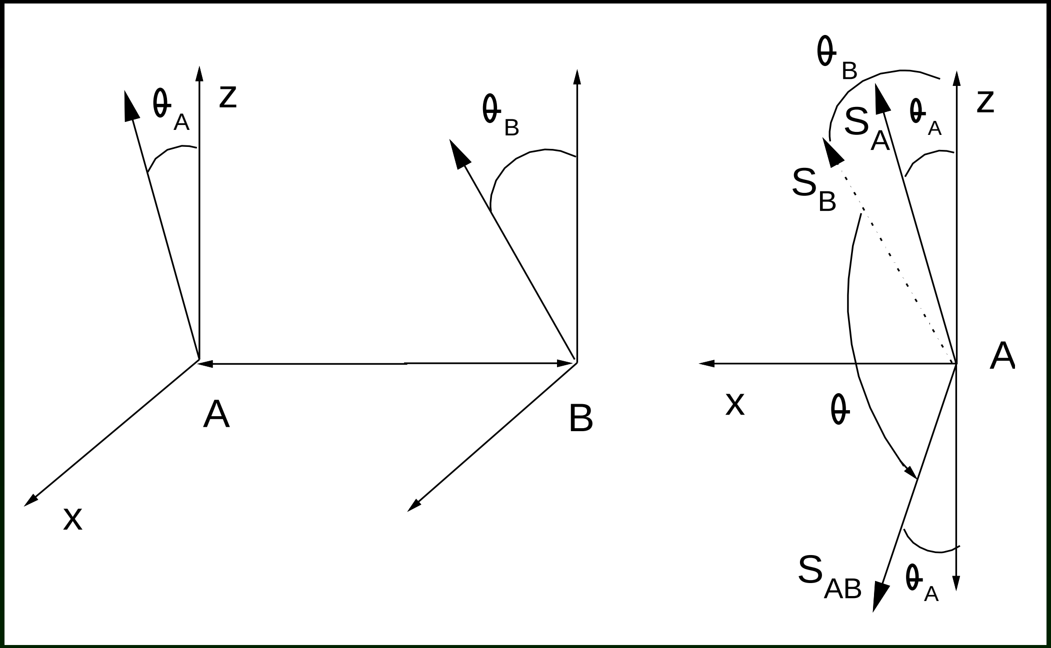
Total Angular momentum

$\sqrt{S(S+1)}$; S=1, with values of projection $m = \pm 1, 0$

Consider the m=0 case: Classically, this means that the average angular momentum along the z axis is zero, and in any direction in the x-y plane is 1 (aligned spins).

Let S_A be the average angular momentum of the +1 sub-ensemble at A.
What is the average angular momentum of the correlated sub-ensemble at B?





$$P(\vec{a},\vec{b})_{A=+1} = +1_A \times (+1)\cos\theta = \cos\left(180-(\theta_A+\theta_B)\right) = -\cos(\theta_A+\theta_B)$$

$$P(\vec{a},\vec{b})_{A=-1} = \left(-1\right)\times(-1)\cos\theta = \cos\left(180-(\theta_A+\theta_B)\right) = -\cos(\theta_A+\theta_B)$$

$$P(\vec{a},\vec{b})_{S=1,m=0} = -\cos(\theta_A+\theta_B)$$

From the conservation law

$$P(\vec{a},\vec{b})_{S=1,m=0} = -\cos(\theta_A + \theta_B)$$

From quantum mechanics:

$$\Psi_{TZ} = \frac{1}{\sqrt{2}}\left(|+1\rangle_1 |-1\rangle_2 + |-1\rangle_1 |+1\rangle_2\right)$$

$$P(\vec{a},\vec{b})_{QM} = \langle \Psi_T | (\sigma_1 \cdot \vec{a}) \left(\sigma_2 \cdot \vec{b}\right) | \Psi_T \rangle$$

$$\sigma \cdot n = \begin{bmatrix} n_3 & n_1 - in_2 \\ n_1 + in_2 & -n_3 \end{bmatrix}, \quad \Psi_{Tz} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_A \begin{pmatrix}0\\1\end{pmatrix}_B + \begin{pmatrix}0\\1\end{pmatrix}_A \begin{pmatrix}1\\0\end{pmatrix}_B\right]$$

$$P(\vec{a},\vec{b})^{QM}_{S=1,m=0} = -\cos(\theta_A + \theta_B)$$

Conservation law implies the Quantum Mechanical Correlation Function