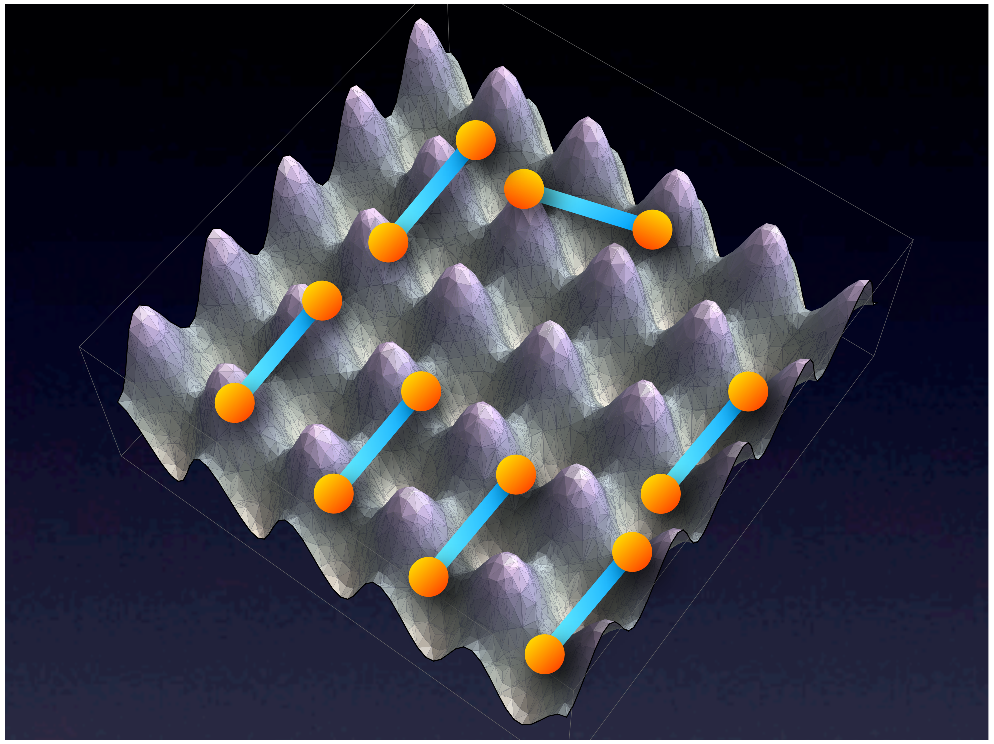
Entanglement in strongly correlated systems



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Allahabad, December 2013

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Universitat Autònoma de Barcelona

Quantum Information Group @ UAB

Identifying and characterizing quantum states of complex systems

Entanglement dynamics in strongly correlated systems

Interface between complex systems of matter and light

New techniques and tools of fundamental character

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Why strongly correlated systems are interesting?

FACTS:

1_Gradual convergence of condensed matter physics and quantum information (QI) motivated by the fact that quantum many-body systems are a natural territory of quantum entanglement, which is the basic resource in QI.

2_ The characterization of quantum correlations is crucial for understanding the structure of many-body systems.

3_The role of entanglement becomes particularly manifest in the study of quantum phase transitions (QPT) but also in topological states, deconfined criticality, etc..

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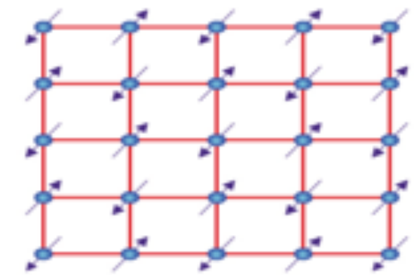
The collective behaviour leads to (local) ordering
of the ground state of the many-body systems i.e magnetism



Taken from S. Shadev, Focuss Issue

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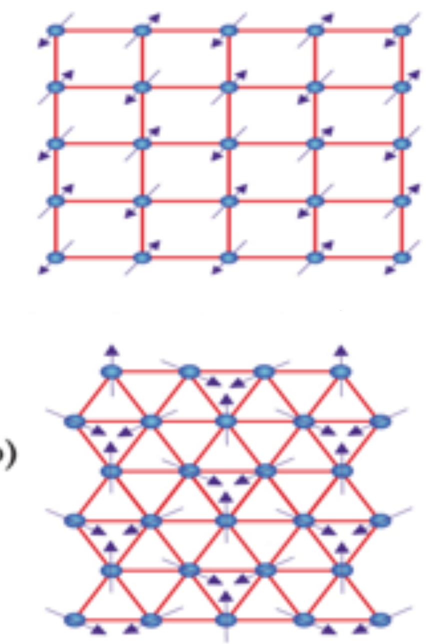

adev, Focuss Issue

(b)

The ordering is generally linked to symmetries and dimensionality
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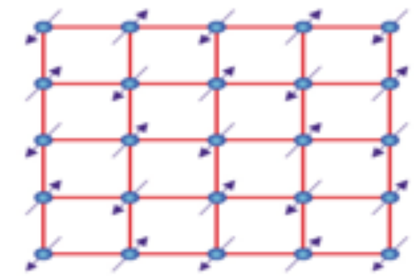

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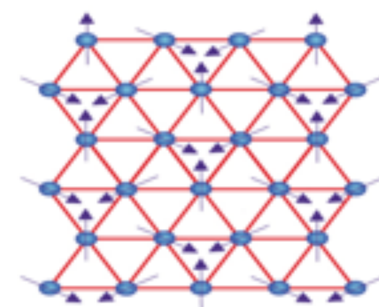
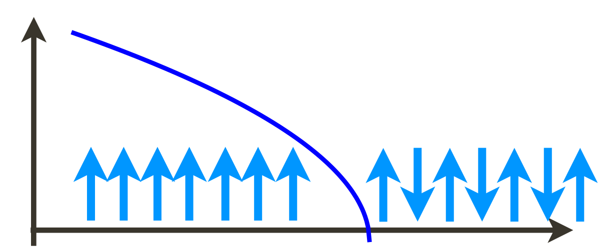


The collective behaviour displays a sudden change if the Hamiltonian is modified such that a symmetry is broken:
quantum phase transitions



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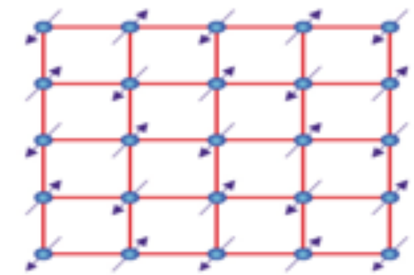

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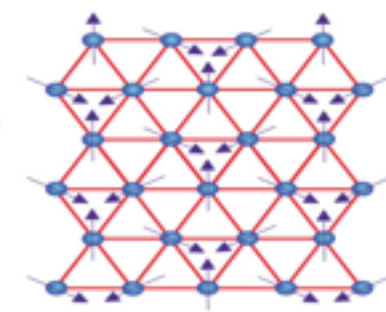

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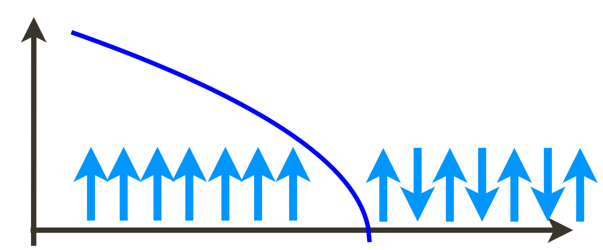
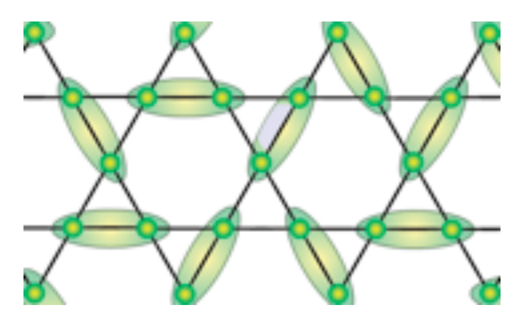


Sometimes the collective behaviour leads to a hidden global structure not associated to any symmetry breaking and with extreme exotic features: topological quantum phase transitions

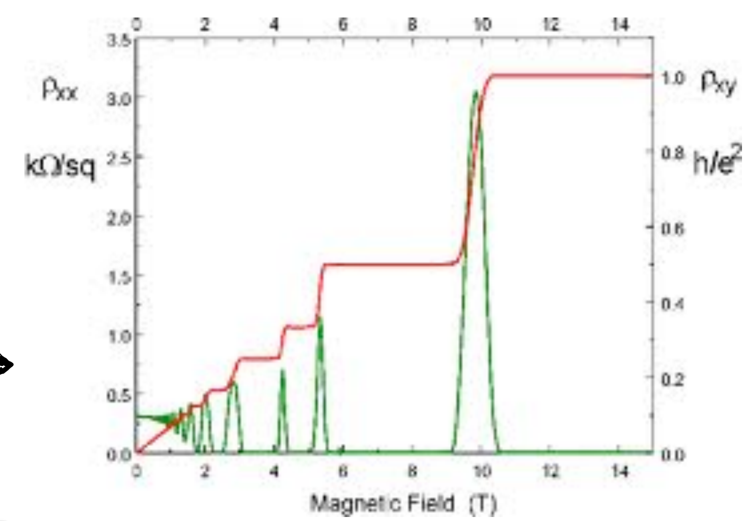


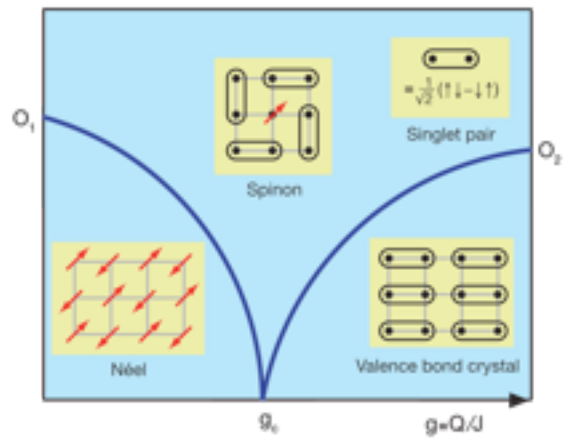
Courtesy L. Clark/University of Edinburg
Phys. Rev. Lett. **110**, 207208 (2013)

Strongly correlated systems= collective behaviour of many-body systems driven by the interactions between them

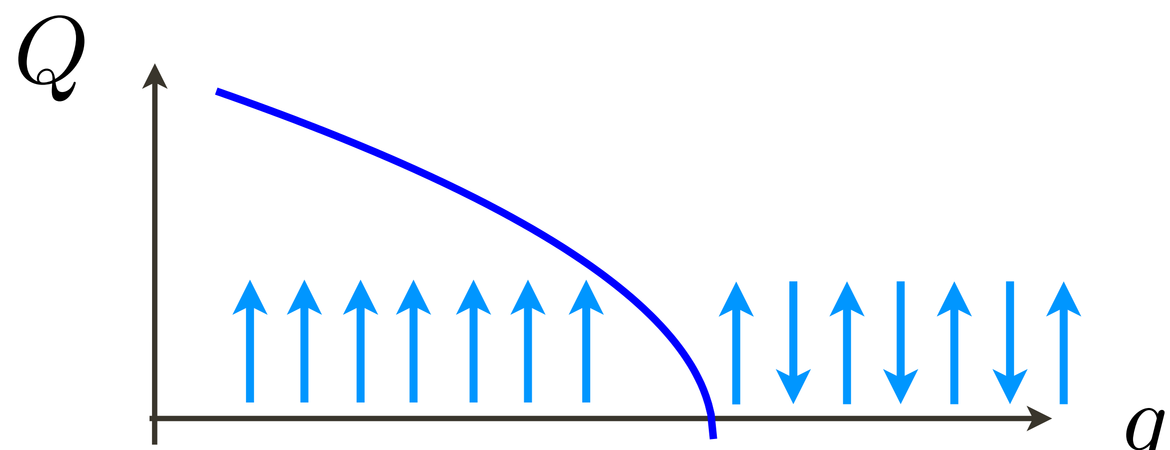
1. Strong correlations determines the physics of a **LARGE** variety of phenomena:

- Transport
- Conductivity
- Quantum Phase Transitions: cooperative quantum fluctuations of a large number of microscopic degrees of freedom
- Critical systems,
- Topological insulators
- Exotic states of matter: Majorana fermions
- High Tc-superconductivity
- Fractional Quantum Hall effect
- Deconfined criticality
- ....

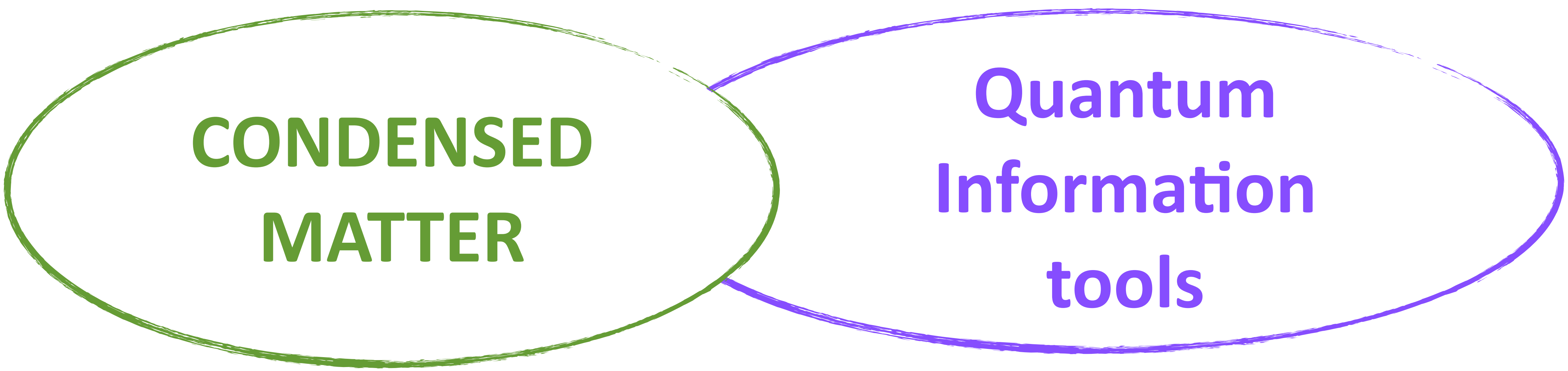




- Quantum phase transitions, order parameters, universality class, conformal field theory and all that



$$H = H_0 + gH_1$$



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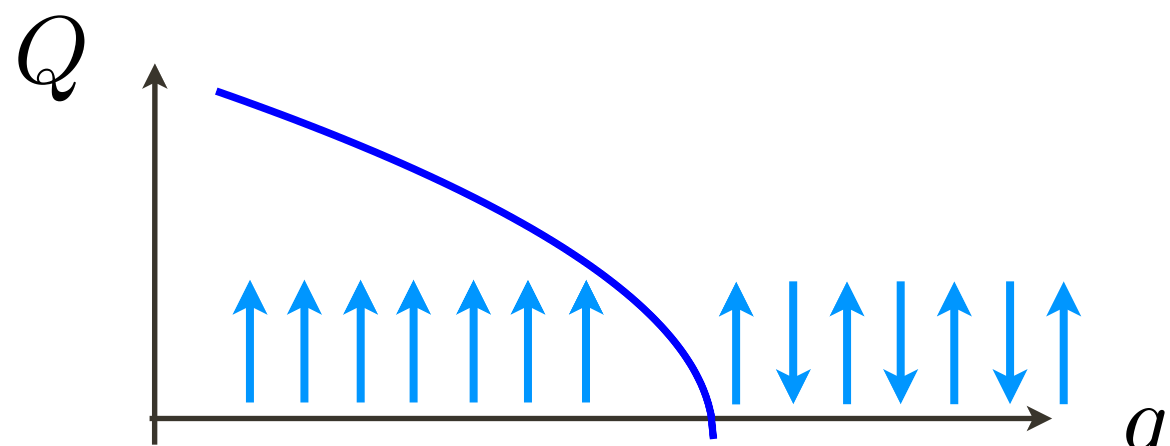

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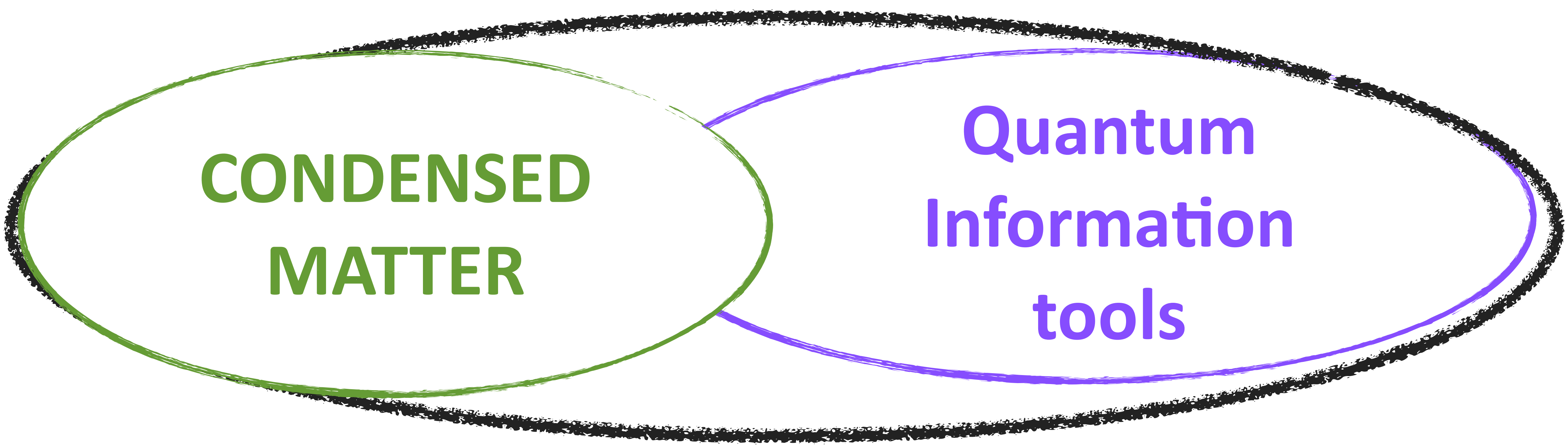
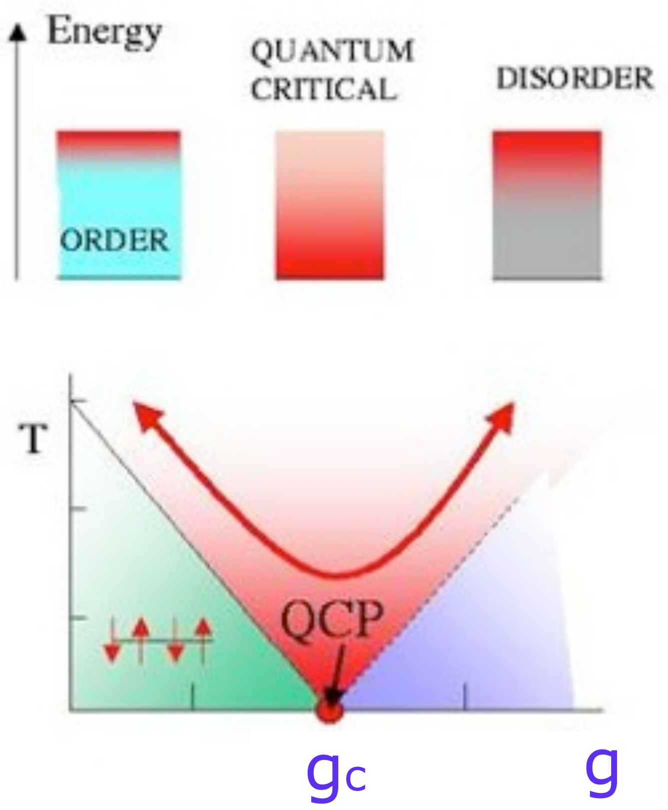


Landau-Ginzburg theory of phase transitions $H = H_0 + gH_1$



A condensed matter description of **quantum phase transitions & criticality**

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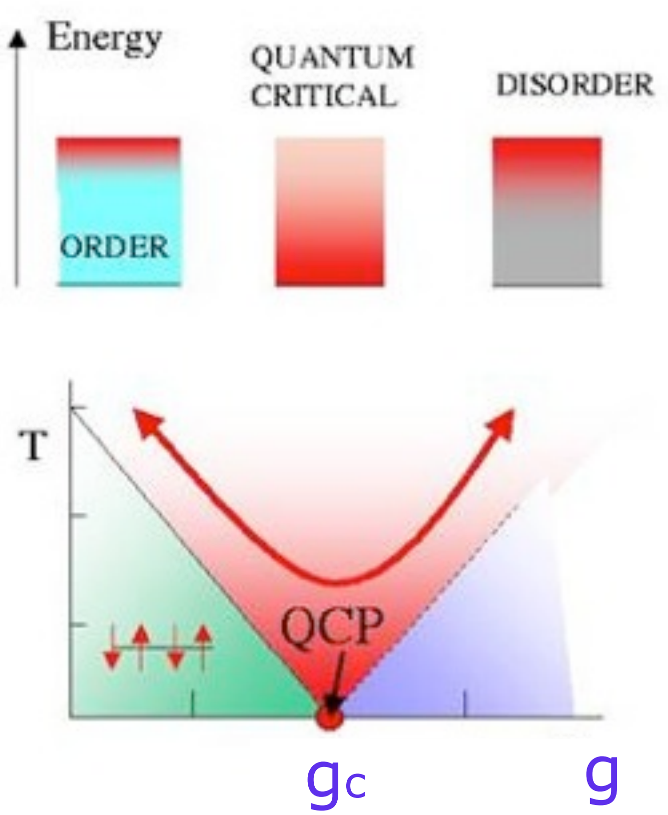

Landau paradigm:

Associated to a phase there is an order parameter **Q**,
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The order **is local** and is associated to breaking of some symmetry.
The system orders for a given value of g

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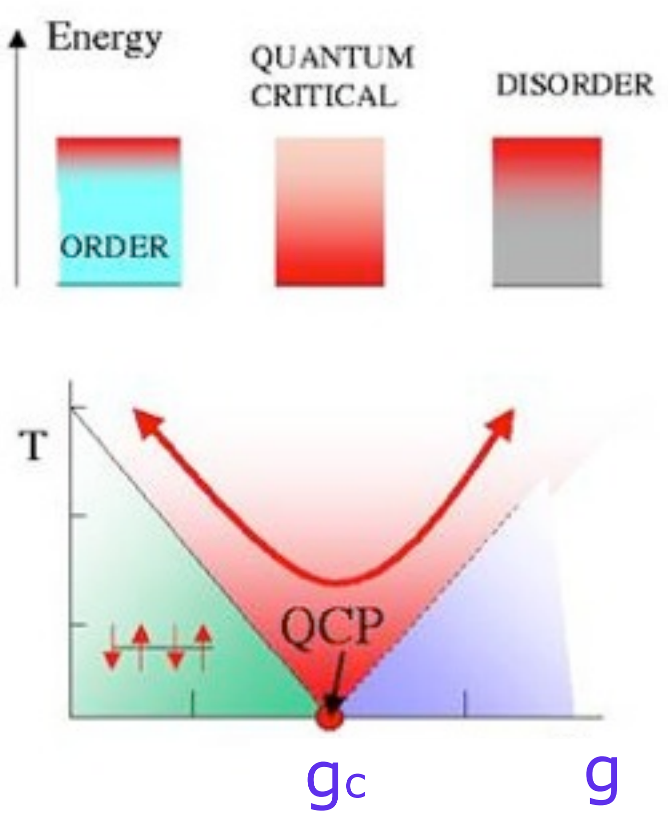

The order parameter becomes zero at the critical point $g=g_c$ (QFT) $\langle \mathbf{Q} \rangle = 0$, signaling a transition to a different (disordered) state.

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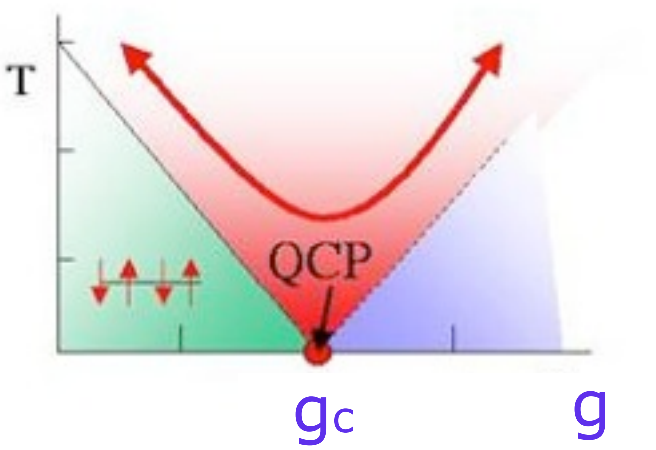

The order parameter becomes zero at the critical point $g=g_c$ (QFT) <**Q**>=0, signaling a transition to a different (disordered) state.

Near criticality, the physical quantities describing the many-body system exhibit scaling behavior, i.e. a power dependence with $|g-g_c|$. The exponents of these power laws are called critical exponents

$$A \sim |g - g_c|^{\alpha}$$

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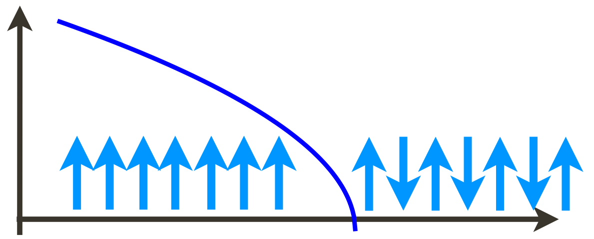
Landau-Ginzburg theory of phase transitions



In principle, the two phases on either side of the critical point **order differently (i.e one is ordered the other disordered with respect to some symmetry)** and have thus different order parameters.
The **critical exponents** could also be different above and below the critical point, **but for continuous phase transitions**, due to the scaling laws, this is not the case.

$$A \sim |g - g_c|^{\alpha}$$

This is true
* for all **classical** phase transitions (g= Temperature)
* for **many quantum phase transitions** (second order or continuous phase transitions)

BUT...there are quantum phases which cannot be described by a **local** order parameter. Neither they break a symmetry. Why?

A condensed matter description of quantum matter: Scaling, Universality, Renormalization

H. Eugene Stanley: Scaling, ur

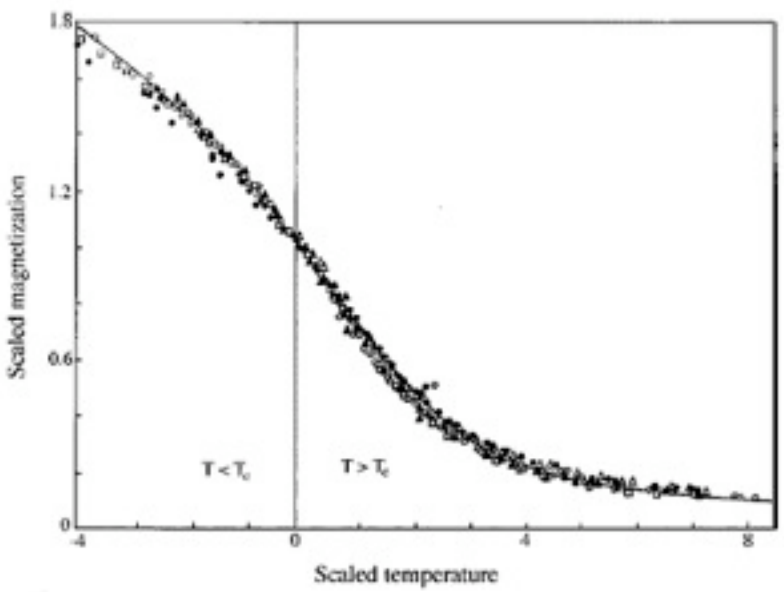


FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and Pd_3Fe. None of these materials is an idealized ferromagnet: $CrBr_3$ has considerable lattice anisotropy, EuO has significant second-neighbor interactions. Ni is an itinerant-electron ferromagnet, YIG is a ferrimagnet, and Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for all materials collapse onto a single scaling function, which is that calculated for the $d=3$ Heisenberg model [after Milošević and Stanley (1976)].

CONDENSED
MATTER

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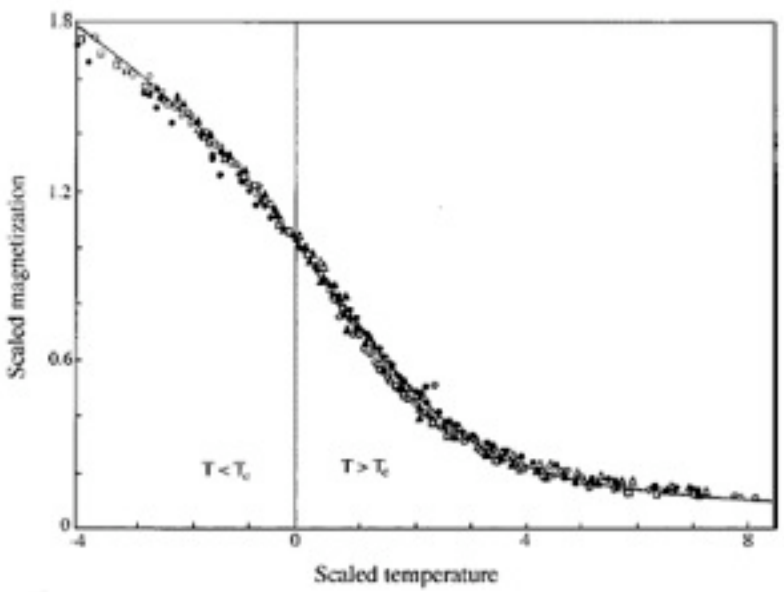

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CONDENSED MATTER

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$$Q(g, L \to \infty) \sim |g - g_c|^{\beta}$$

$$\frac{Q(g,L)}{L^{\beta}} = f\left(\frac{|g-g_c|}{L^{\alpha}}\right)$$

Finite size scaling

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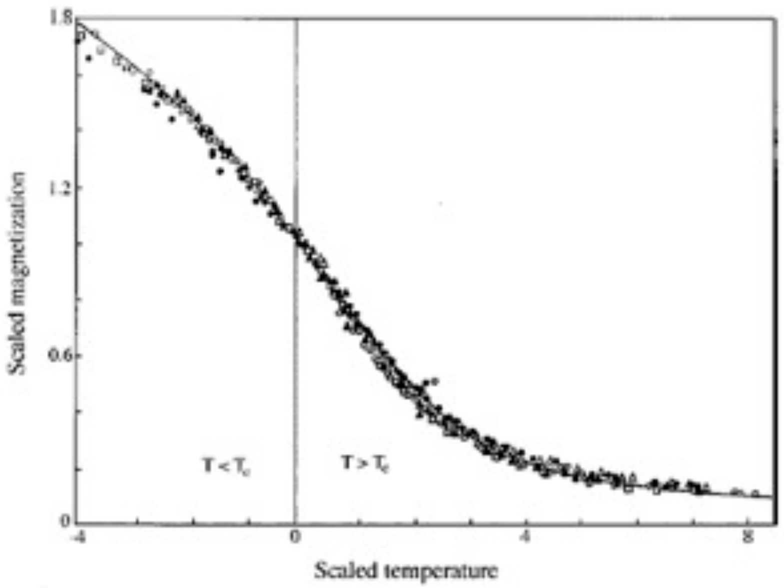

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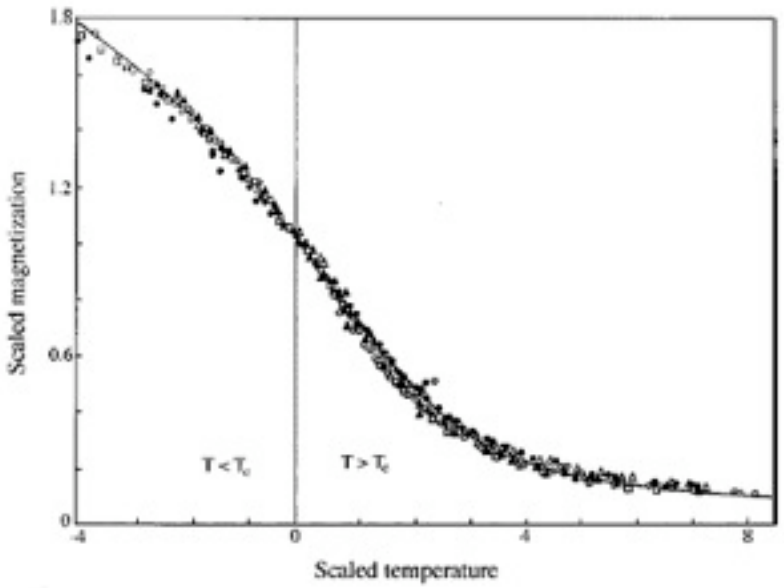

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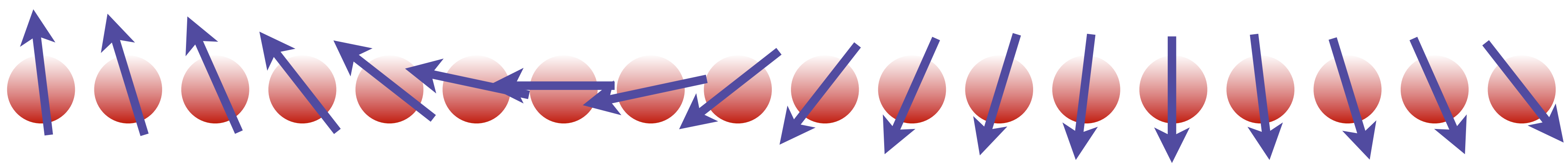
Renormalization

Wilson's essential idea is that the critical point can be mapped onto a fixed point of a suitably chosen transformation on the system's Hamiltonian.
The concept of renormalization encompasses the concepts of scaling and universality.

Paradigmatic model of strongly correlated system: spin chain



$|\psi_G\rangle$

1_ Pairwise entanglement
2_ Block entanglement
3_ Entanglement spectrum
4_ Multipartite entanglement

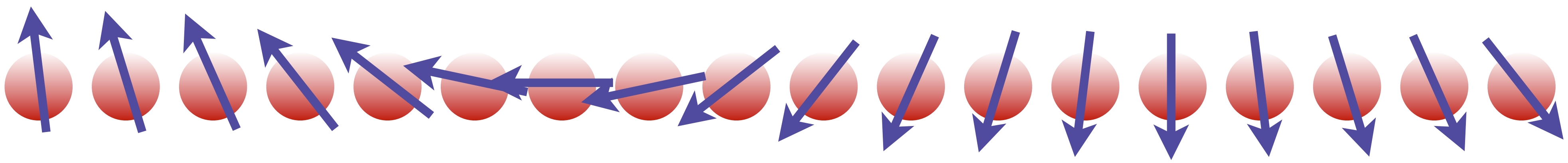
Osterhlo & Amico, 02
Nielsen & Osborne 02
Vidal, Latorre, Rico, Kitaev, PRL 2003
Calabrese & Cardy, JSTAT 2004
Jin & Korepin, JSTAT 2004
Lee & Haldane 2008
Pollmann et al. 2010

1-Entanglement in many-body systems : **pairwise entanglement**

Osterloh, Amico, 02, Nielsen, Osborne 02

I-Entanglement in many-body systems : **pairwise entanglement**

$$H_{XY} = -\frac{J}{2}\sum_i \left[(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y\right] - h\sum_i \sigma_i^z \qquad \lambda = \frac{J}{2h}$$



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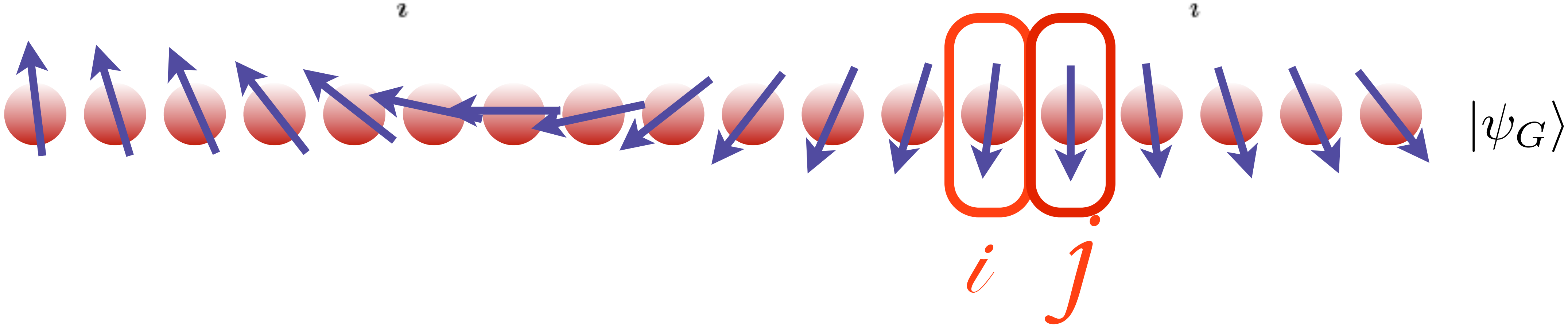

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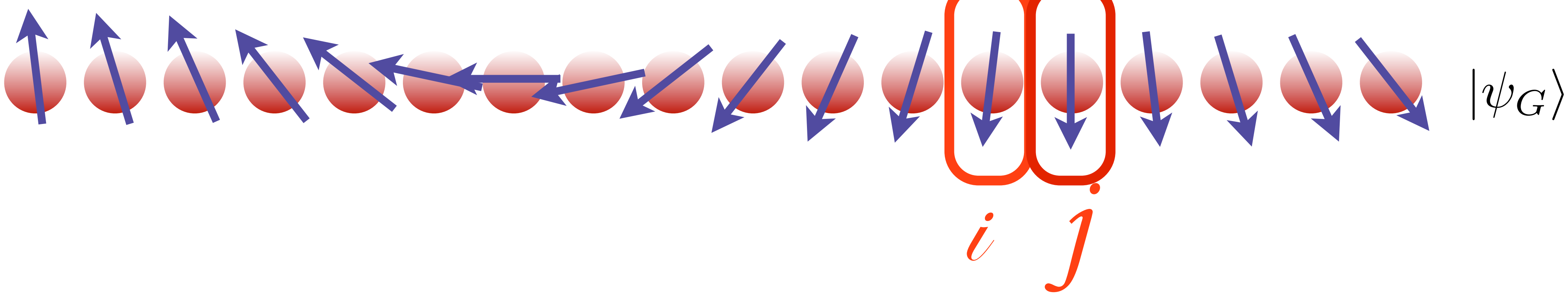

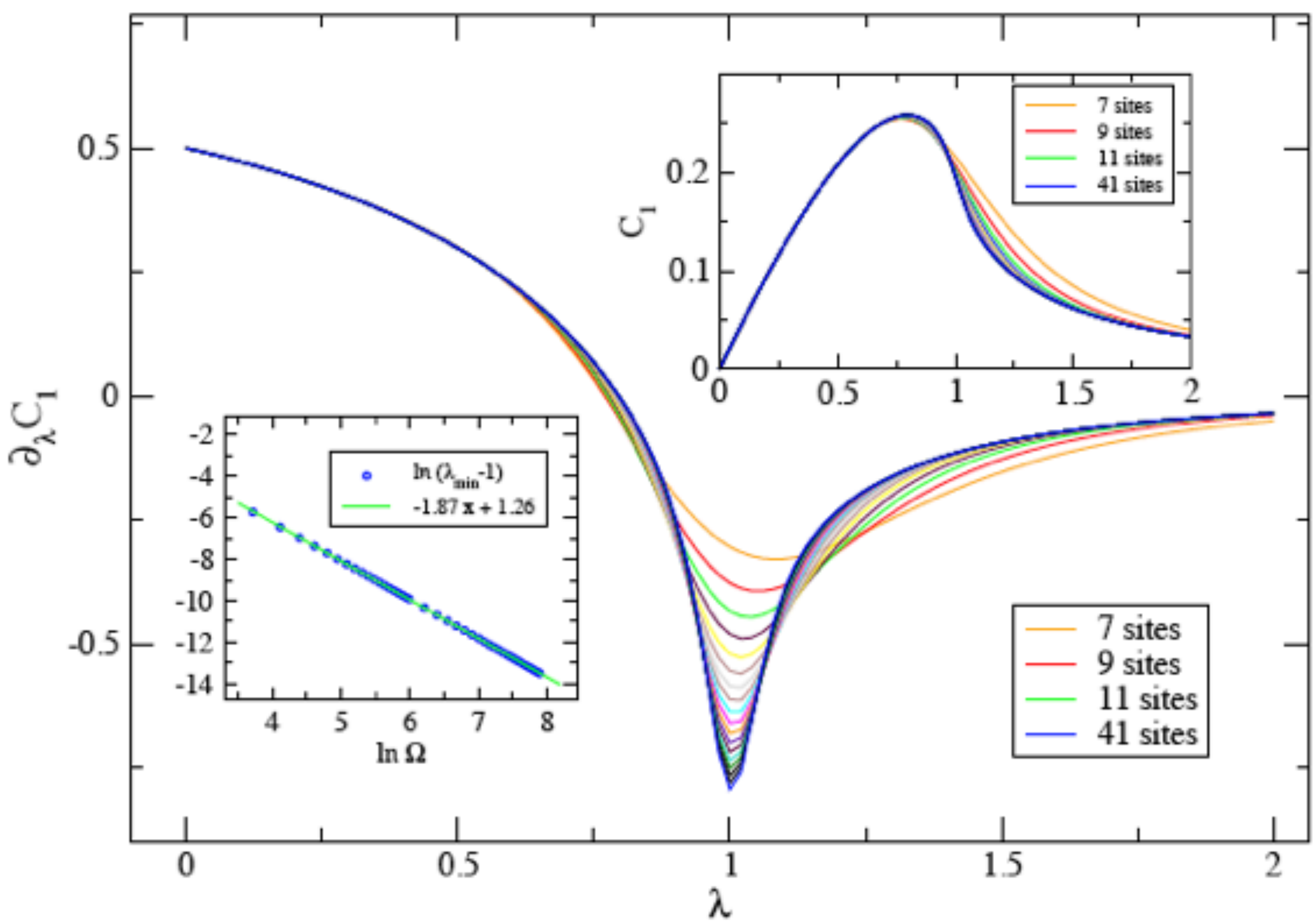

Concurrence

$$C = C(\varrho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

The λ_i are the eigenvalues $\left(\varrho^{\frac{1}{2}}\tilde{\varrho}\varrho^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$$\tilde{\varrho} = \sigma_y \otimes \sigma_y \varrho^* \sigma_y \otimes \sigma_y$$

$$\partial_\lambda C_1 = \frac{8}{3\pi^2}\ln|\lambda - \lambda_c|$$

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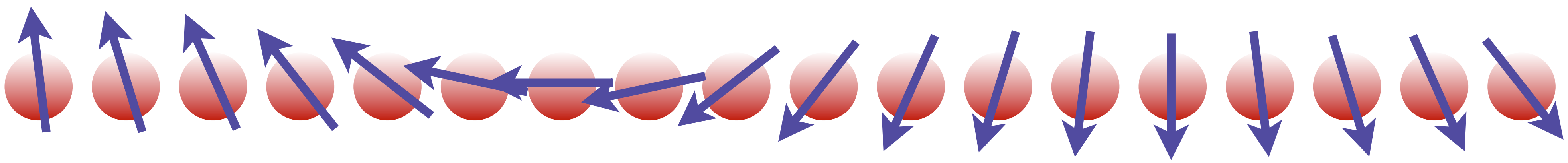

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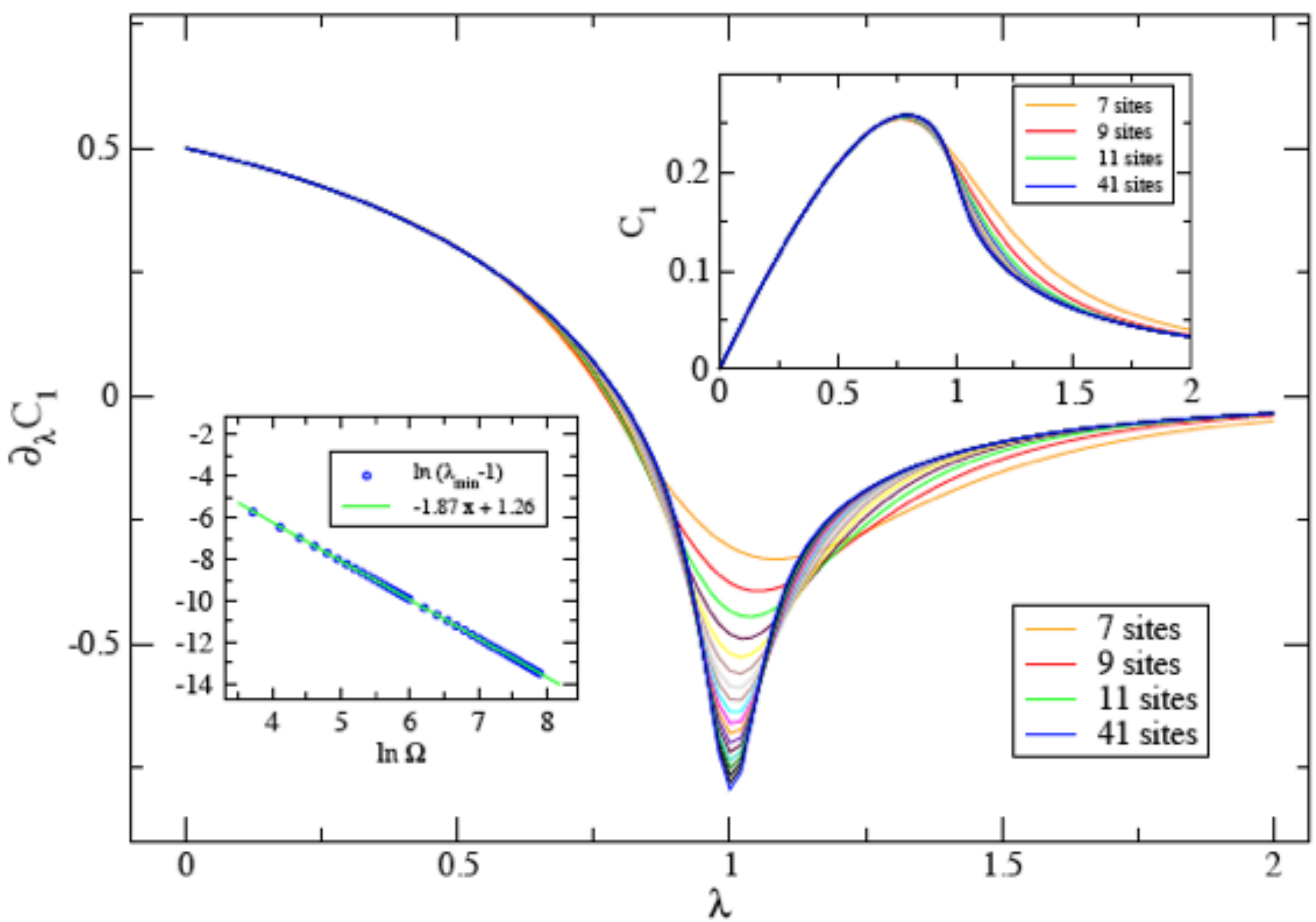

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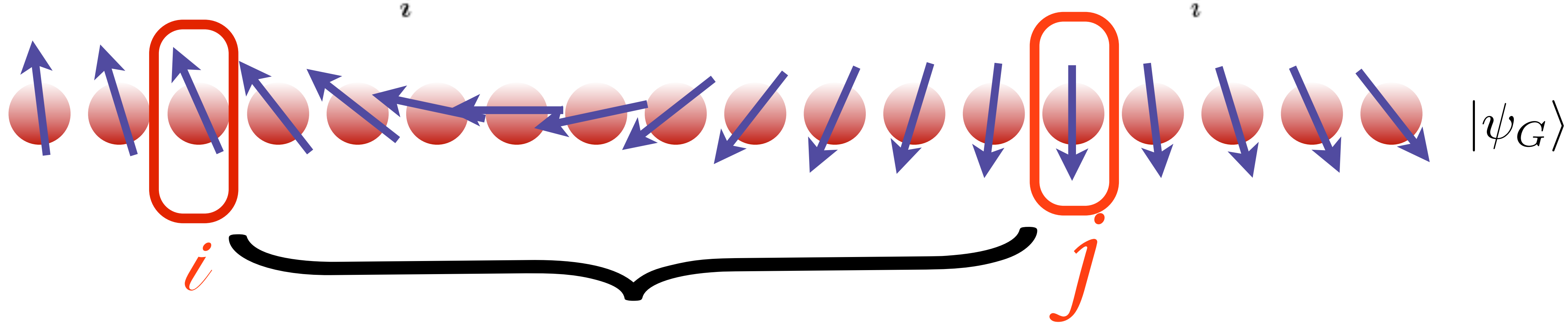

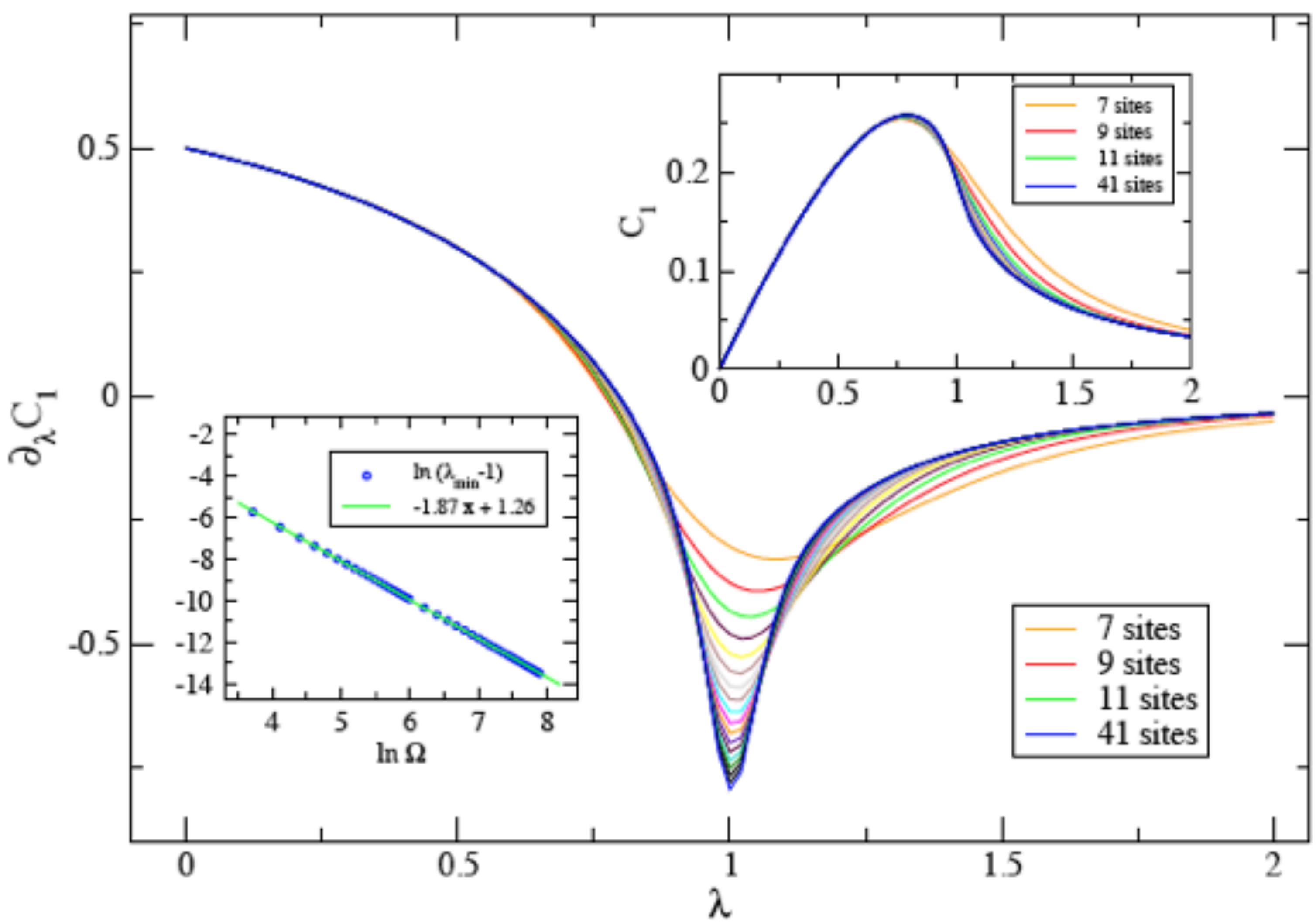

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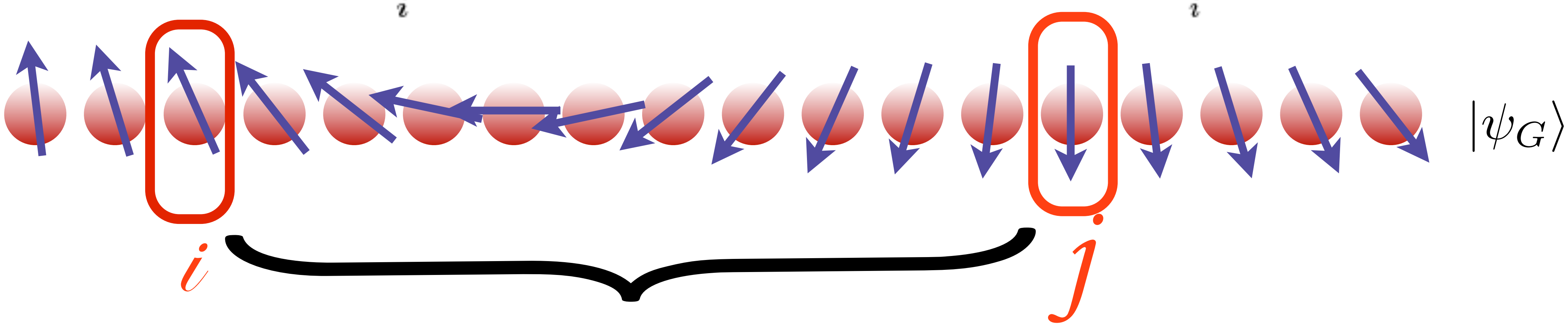

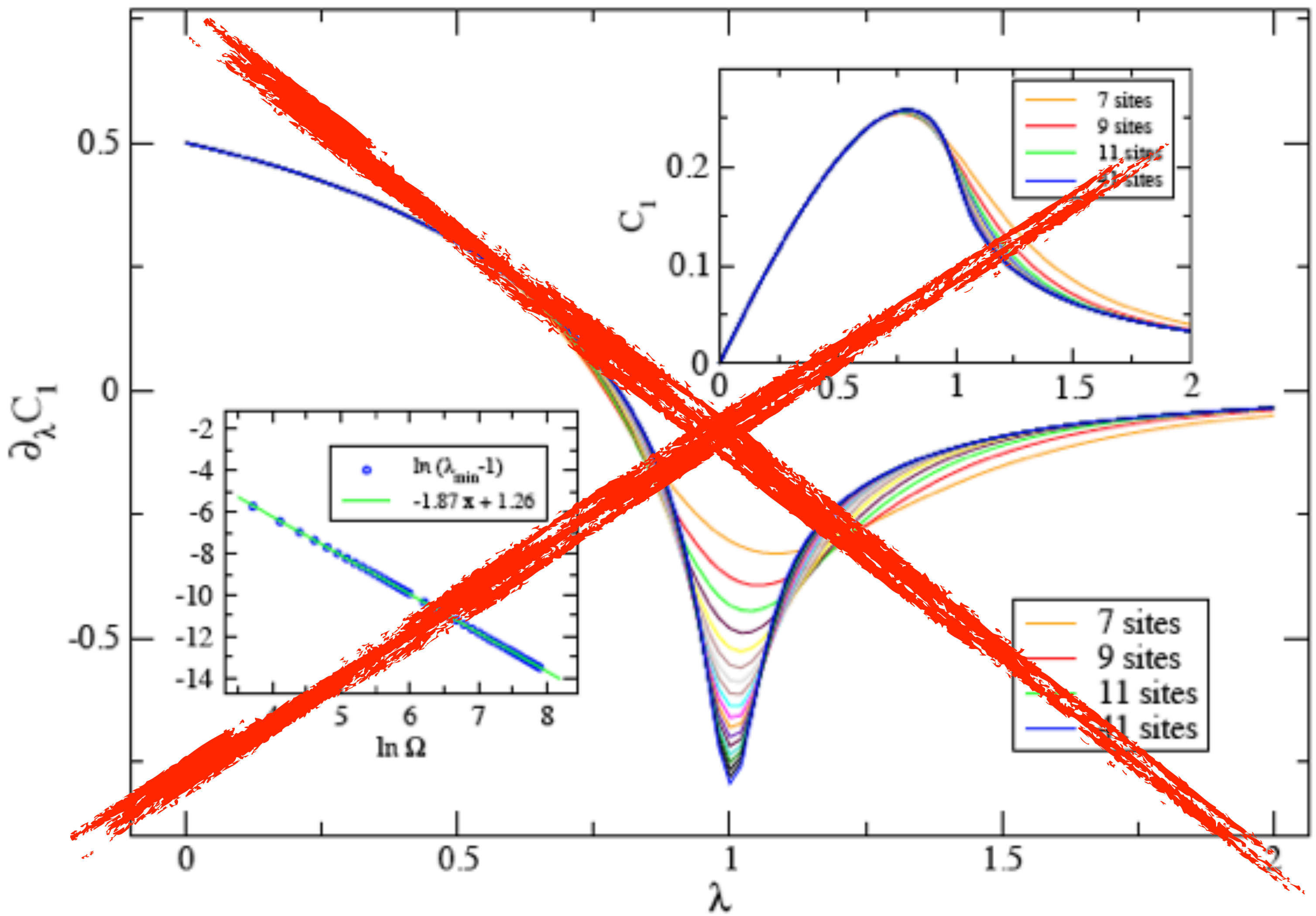

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Osterloh, Amico, 02, Nielsen, Osborne 02

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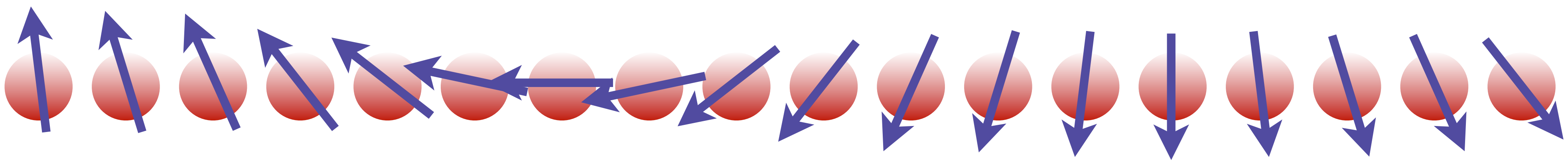
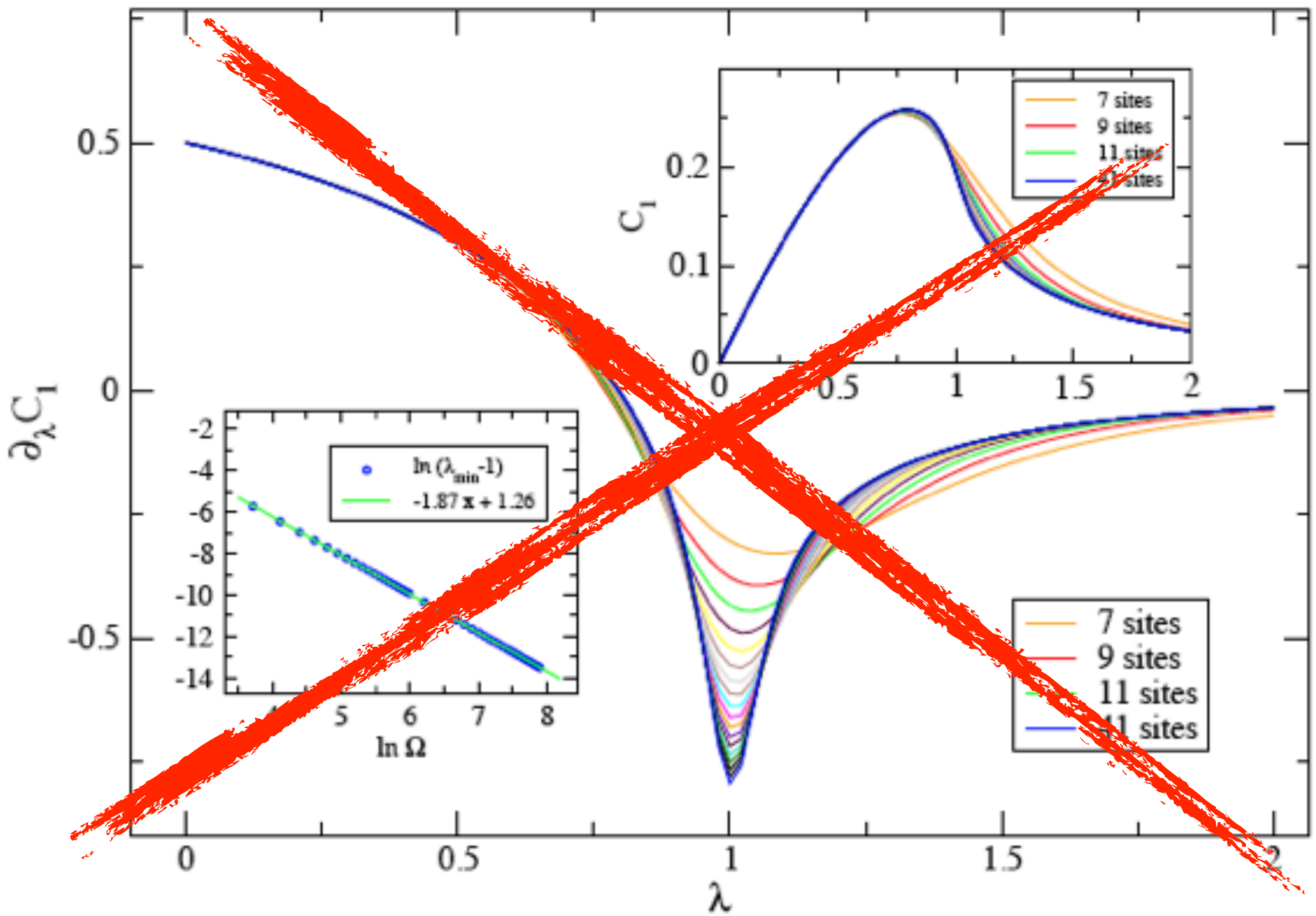
$$H_{XY} = -\frac{J}{2}\sum_i \left[(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y\right] - h\sum_i \sigma_i^z \qquad \lambda = \frac{J}{2h}$$





Concurrence

$$C = C(\varrho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

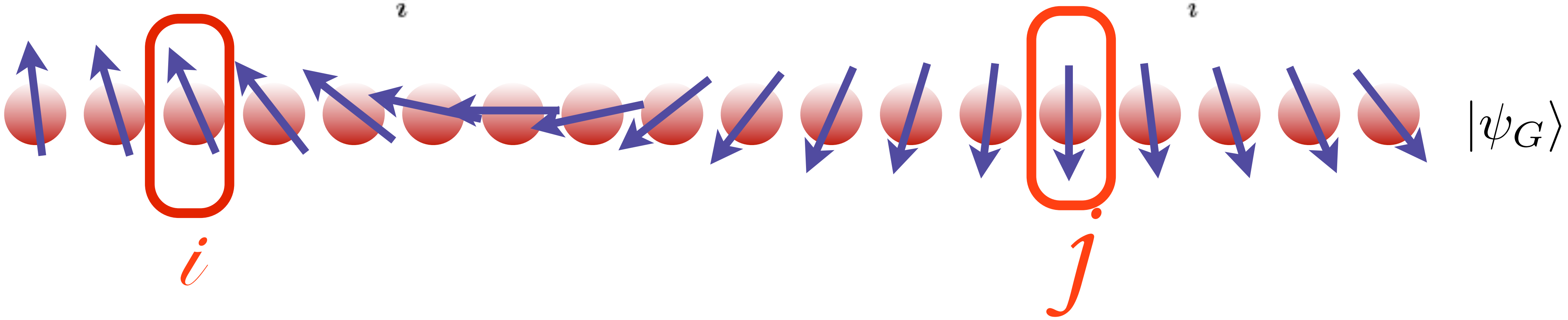
The λ_i are the eigenvalues $\left(\varrho^{\frac{1}{2}}\tilde{\varrho}\varrho^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$$\tilde{\varrho} = \sigma_y \otimes \sigma_y \varrho^* \sigma_y \otimes \sigma_y$$

$$\partial_\lambda C_1 = \frac{8}{3\pi^2}\ln|\lambda - \lambda_c|$$

Osterloh, Amico, 02, Nielsen, Osborne 02

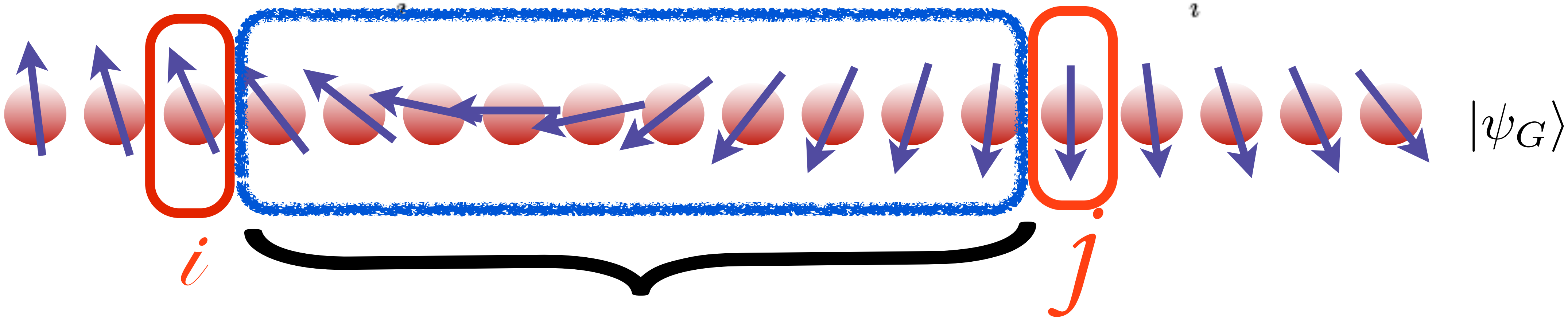
$$H_{XY} = -\frac{J}{2}\sum_i \left[(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y\right] - h\sum_i \sigma_i^z \qquad \lambda = \frac{J}{2h}$$



Verstreate & Cirac 04

2-Entanglement in many-body systems : **localizable entanglement**

$$H_{XY} = -\frac{J}{2}\sum_i \left[(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y\right] - h\sum_i \sigma_i^z \qquad \lambda = \frac{J}{2h}$$



$$U_{i+1} \otimes U_{i+2} \otimes \ldots U_{j-1}$$

Verstreate & Cirac 04

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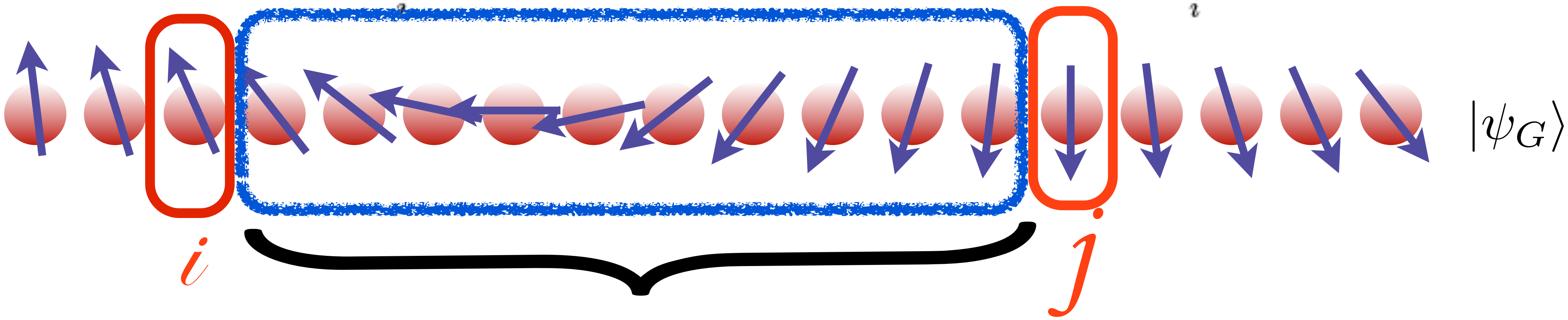

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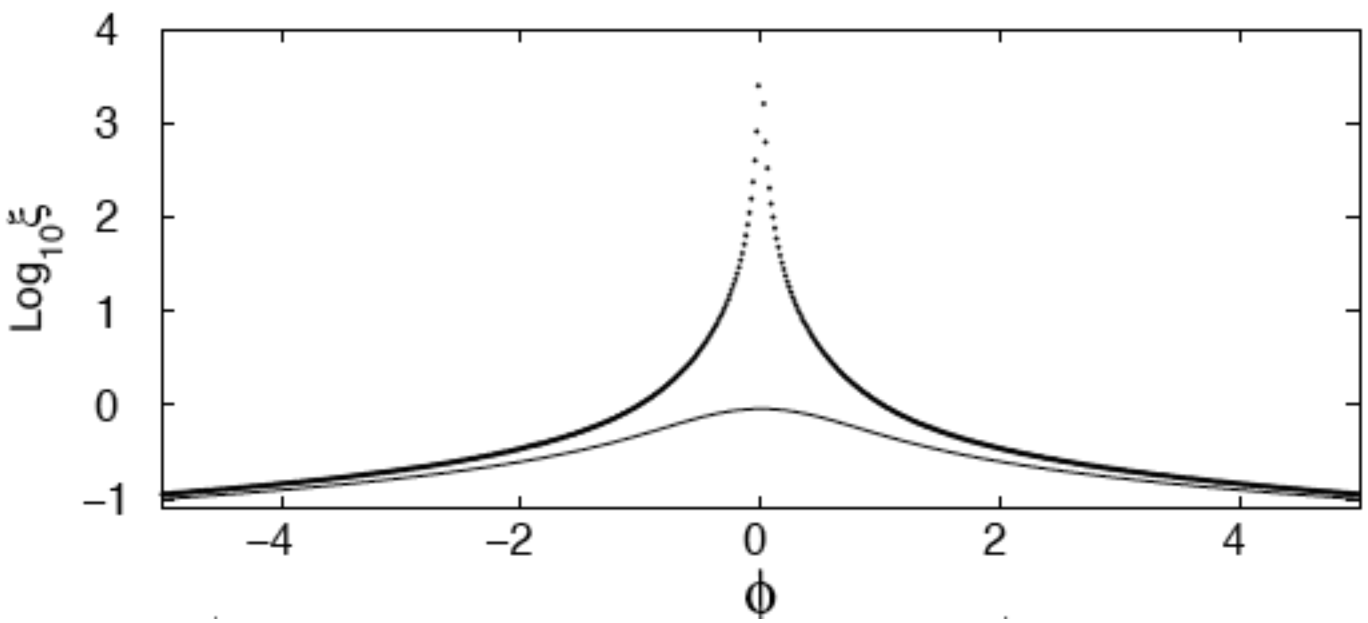


Verstreate & Cirac 04

Entanglement length:

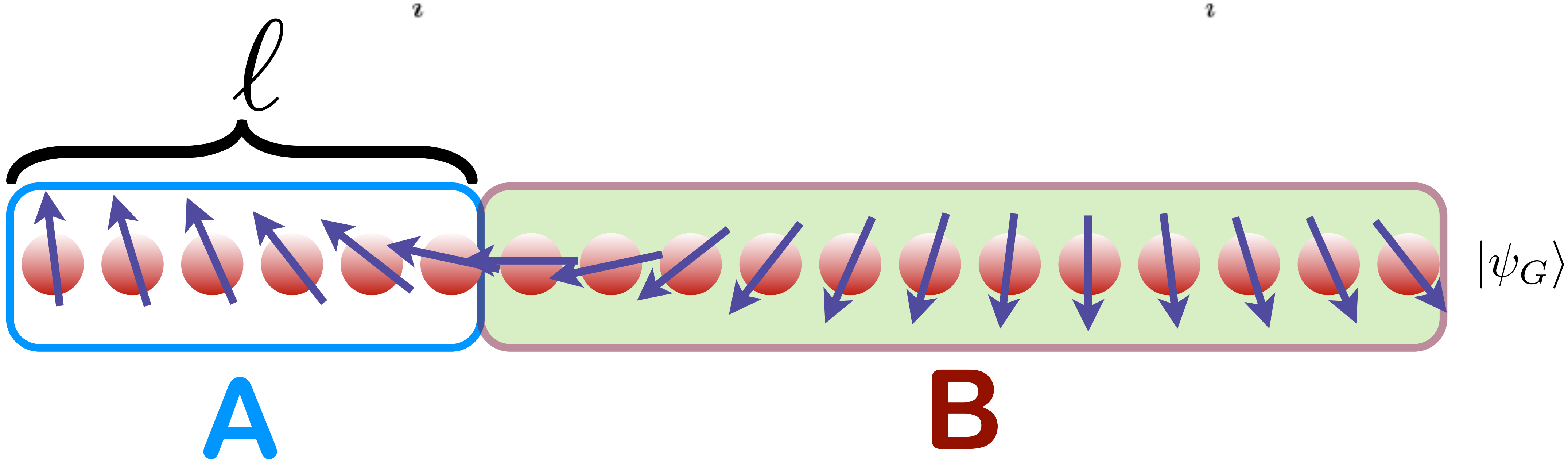
maximal distance at which is it possible to create singlets by operating locally on the other parties

$$\xi_E^{-1} = \lim_{n\to\infty}\left(\frac{-\ln E_{i,i+n}}{n}\right)$$

$$\xi_C^{-1} = \lim_{n\to\infty}\left(\frac{-\ln \langle O_i O_{i+n}\rangle}{n}\right)$$

3-Entanglement in many-body systems : **block entanglement**

$$H_{XY} = -\frac{J}{2}\sum_i \left[(1-\gamma)\sigma_i^x\sigma_{i+1}^x + (1+\gamma)\sigma_i^y\sigma_{i+1}^y\right] - h\sum_i \sigma_i^z \qquad \lambda = \frac{J}{2h}$$



Holzhey, Larsen, and Wilczek, Nucl Phys 1994
Vidal, Latorre, Rico, Kitaev, PRL 03
Calabrese & Cardy, JSTAT 04
Jin & Korepin, JSTAT 04
Peschel 04,05, Hastings 07

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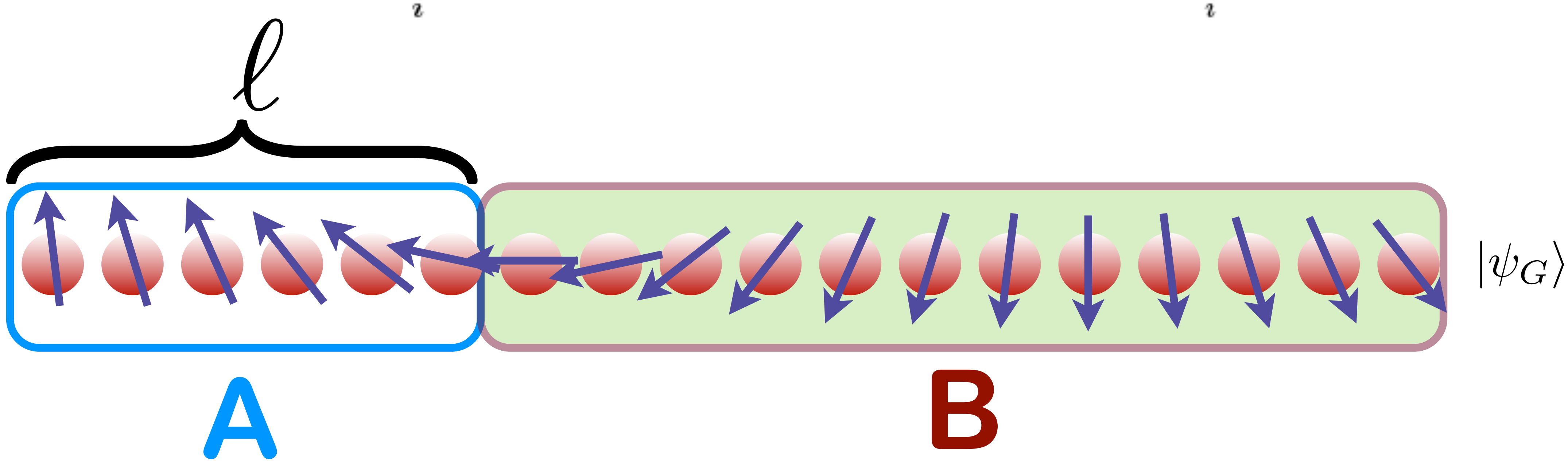

Entanglement Entropy

Holzhey, Larsen, and Wilczek, Nucl Phys 1994
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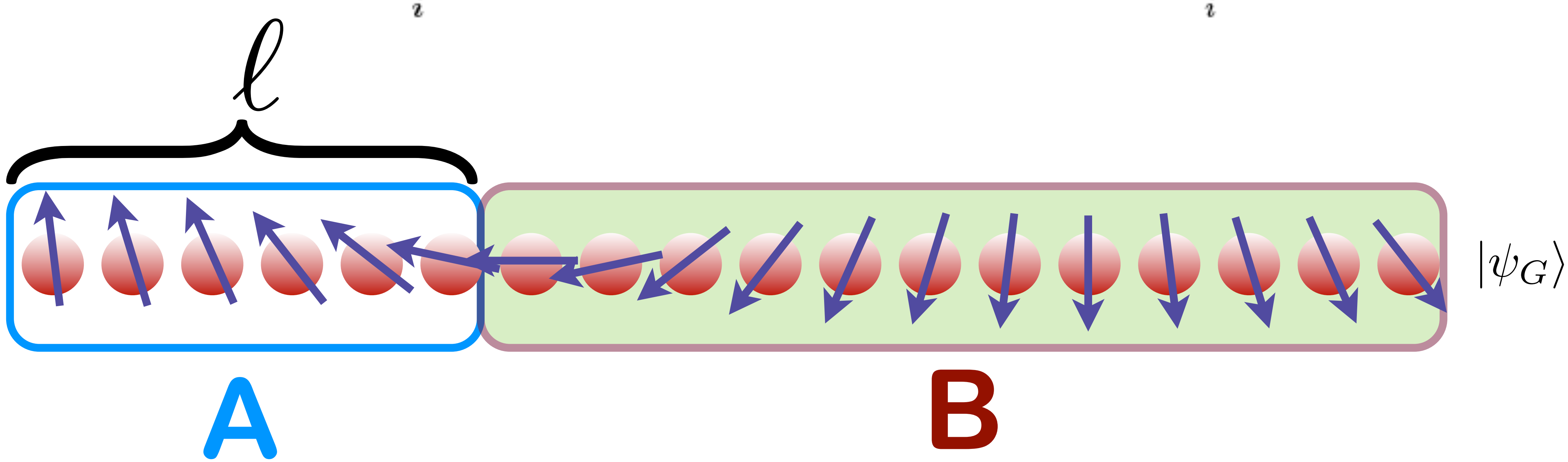

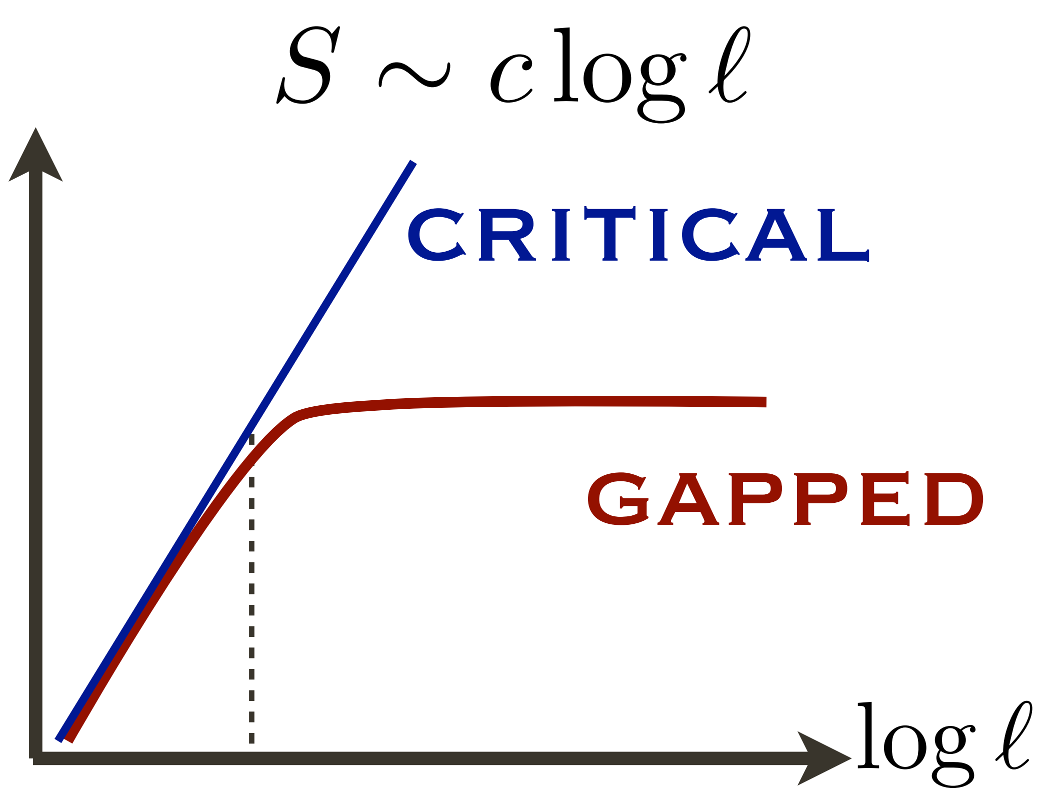

Entanglement Entropy

$$S(\rho_A) = -\mathrm{Tr}\rho_A \log\rho_A$$

Critical behaviour !

Holzhey, Larsen, and Wilczek, Nucl Phys 1994
Vidal, Latorre, Rico, Kitaev, PRL 03
Calabrese & Cardy, JSTAT 04
Jin & Korepin, JSTAT 04
Peschel 04,05, Hastings 07

Theorem 12 *Let* $|\psi_{AB}\rangle$ *be a bipartite pure state from* $\mathbb{C}^m \otimes \mathbb{C}^n$ $(m \leq n)$ *chosen at random according to the Haar measure on the unitary group, and* $\rho_A = \mathrm{Tr}_B |\psi_{AB}\rangle\langle\psi_{AB}|$ *be its subsystem acting on* $\mathbb{C}^m$*. Then*

$$\langle S(\rho_A)\rangle \approx \log m - \frac{m}{2n}. \tag{12.72}$$

Plenio 05, Cramer 06,
Verstraete, Wolf, Perez-Garcia, Cirac, 2006

Entanglement in many-body systems : **Area Law**

Theorem 12 *Let $|\psi_{AB}\rangle$ be a bipartite pure state from $\mathbb{C}^m \otimes \mathbb{C}^n$ $(m \leq n)$ chosen at random according to the Haar measure on the unitary group, and $\rho_A = \mathrm{Tr}_B |\psi_{AB}\rangle\langle\psi_{AB}|$ be its subsystem acting on $\mathbb{C}^m$. Then*

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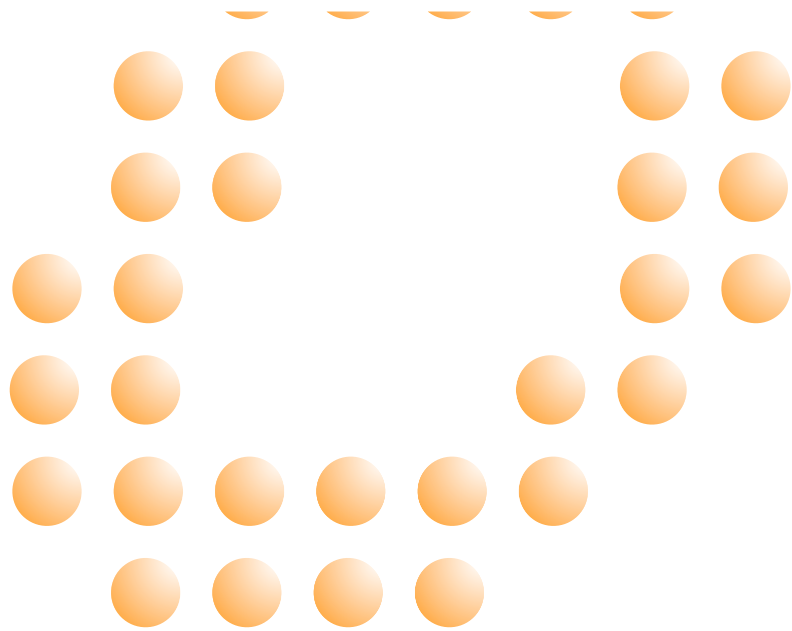

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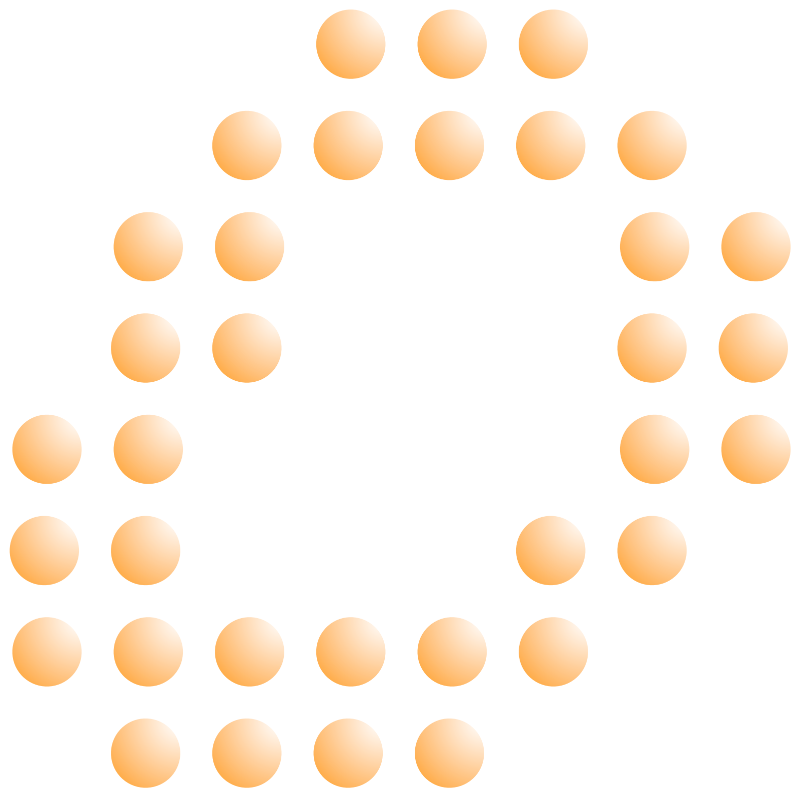
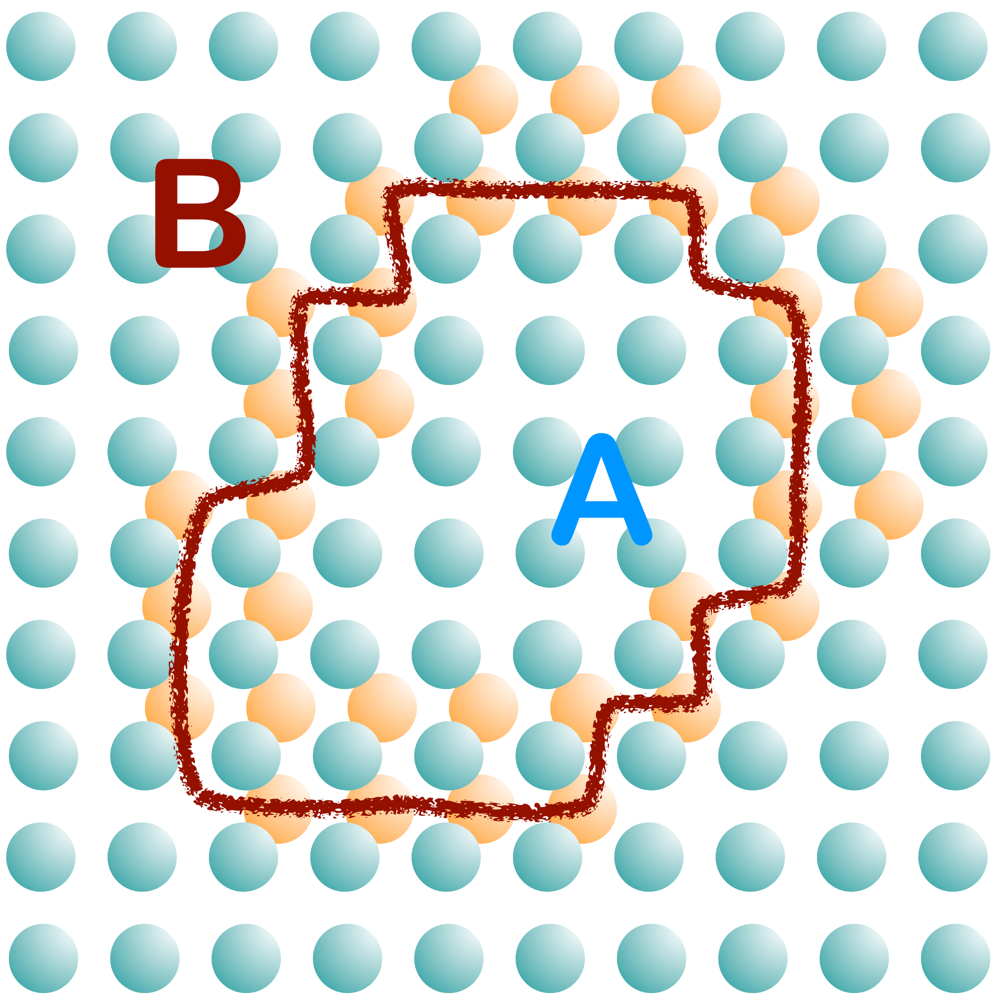


Plenio 05, Cramer 06,
Verstraete, Wolf, Perez-Garcia, Cirac, 2006

Ground states of many-body systems away from criticality obey an area law



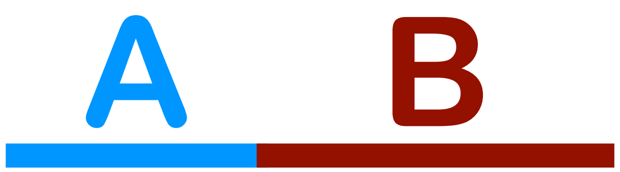
Entanglement is proportional to the area of the boundary between A and B

For a cubic lattice in D-dimensions:

$$S \sim \ell^{D-1}$$

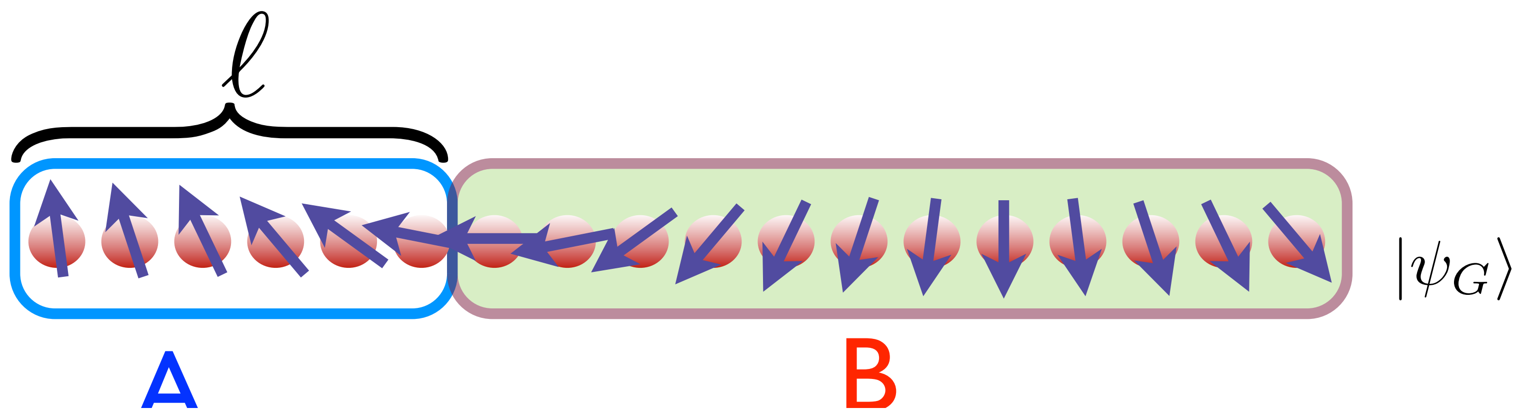
In 1D:

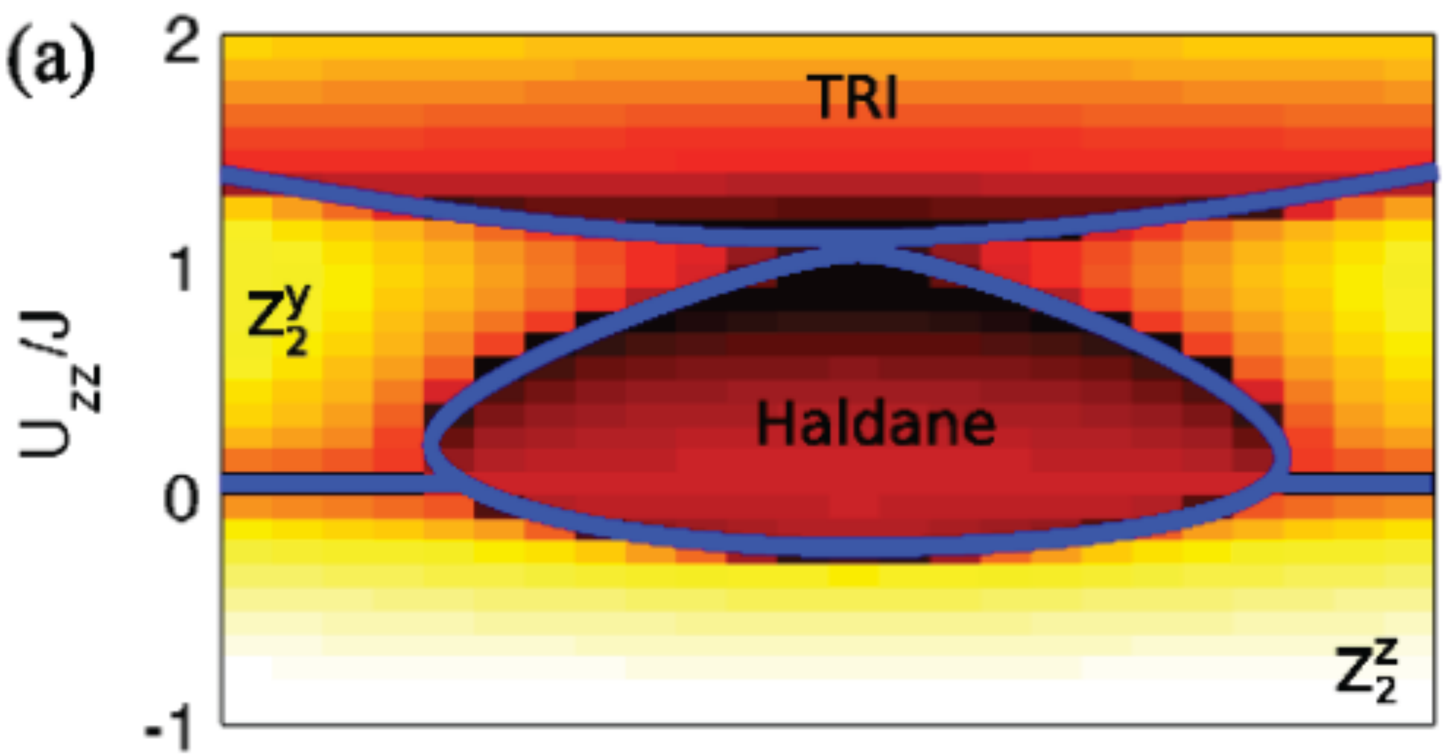
$$S \sim s_0 + \frac{c}{6} \log \ell$$



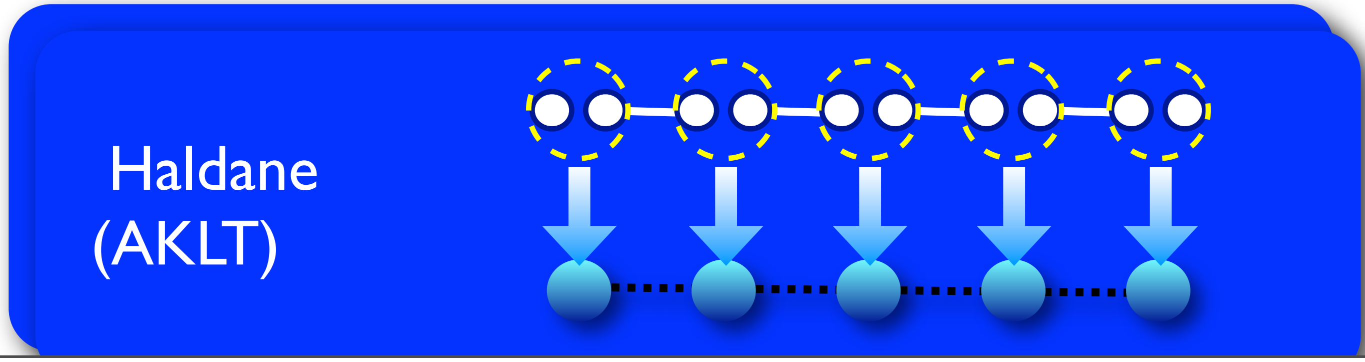
Plenio 05, Cramer 06,
Verstraete, Wolf, Perez-Garcia, Cirac, 2006

$$H_0 = J\sum_j \vec{S}_j \cdot \vec{S}_{j+1} + B_x \sum_j S_j^x + U_{zz}\sum_j (S_j^z)^2$$ (Spin 1)





Topological phase !



Li and Haldane 08
Pollman, Turner, Berg, Oshikawa '10

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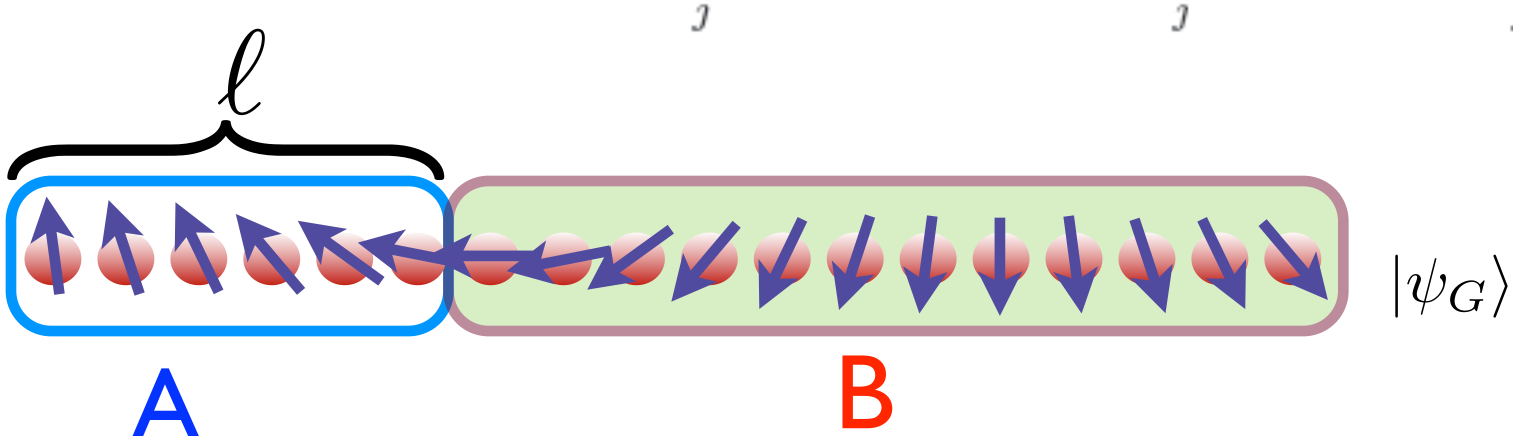
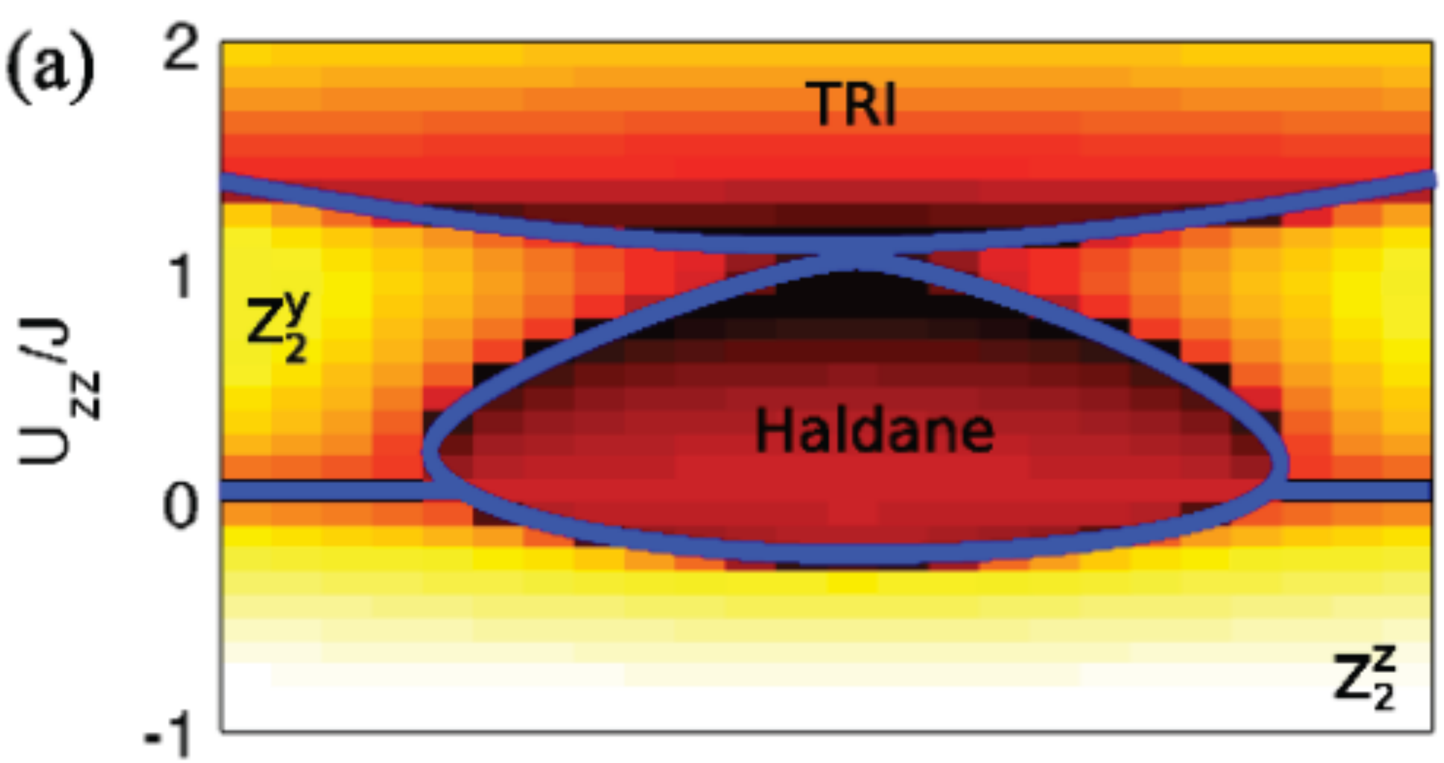
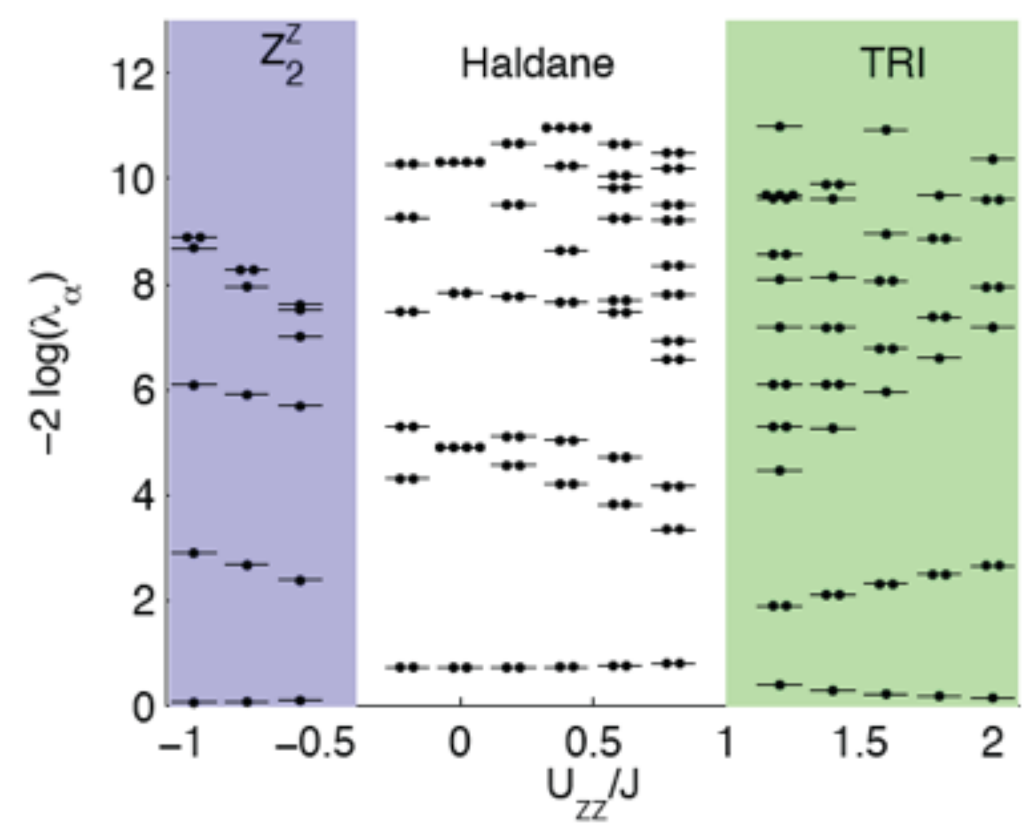


$$S(\rho_A) = -Tr\rho_A \log \rho_A = \sum \lambda_i \log(\lambda_i)$$

Entanglement spectrum





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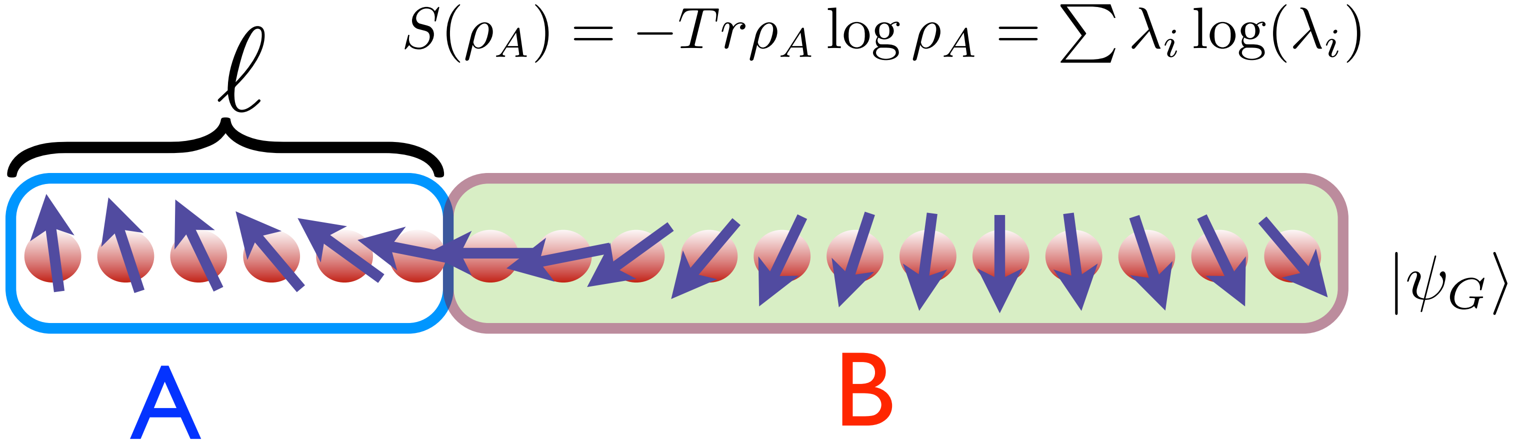
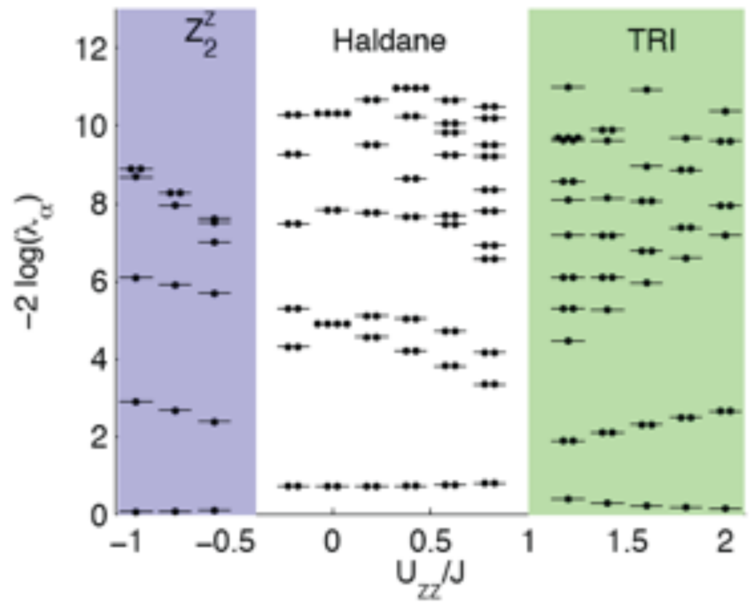


Entanglement spectrum



We focus just in the first two Schmidt eigenvalues and define the Schmidt gap as:

$$\Delta\lambda = \lambda_1 - \lambda_2$$

1-How is the Schmidt gap at criticality (QPT) ?

2-Does the Schmidt gap show scaling behavior, out of criticality?

DeChiara, Lepori, Lewenstein AS '12
Lepori Dechiara, AS '13

Quantum phase transitions, entanglement spectrum, quantum correlations, area law, Schmidt coefficients.....

1-How is the Schmidt gap at criticality (QPT) in finite systems ?

Use CFT: It depends on the central charge c and on scaling dimension of the relevant operator of the theory (CFT). In the thermodynamical limit closes!

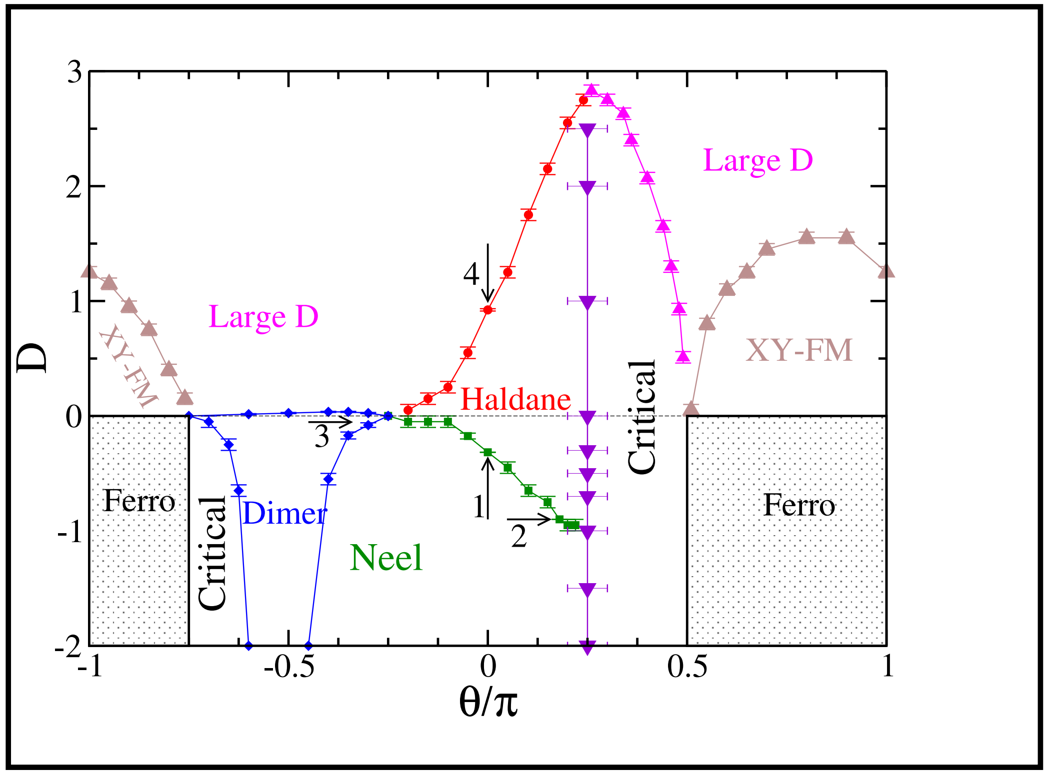
Scaling dimension of relevant operator

$$\Delta\lambda = \lambda_1 - \lambda_2 \qquad \Delta\lambda(l, g_c) = \frac{1 - q^{\alpha_1}}{Z_l(q)} \sim \frac{1 - q^{\alpha_1}}{l^{c/12}}$$

Partition function

At criticality, the largest eigenvalue of the reduced density matrix is the single copy entanglement which is half of the Von Neumann entropy, which depends only on the central charge

Entanglement in many-body systems : **Schmidt gap**



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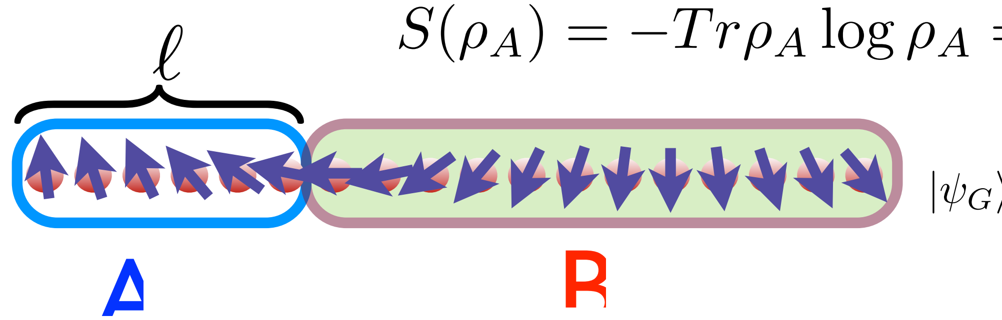
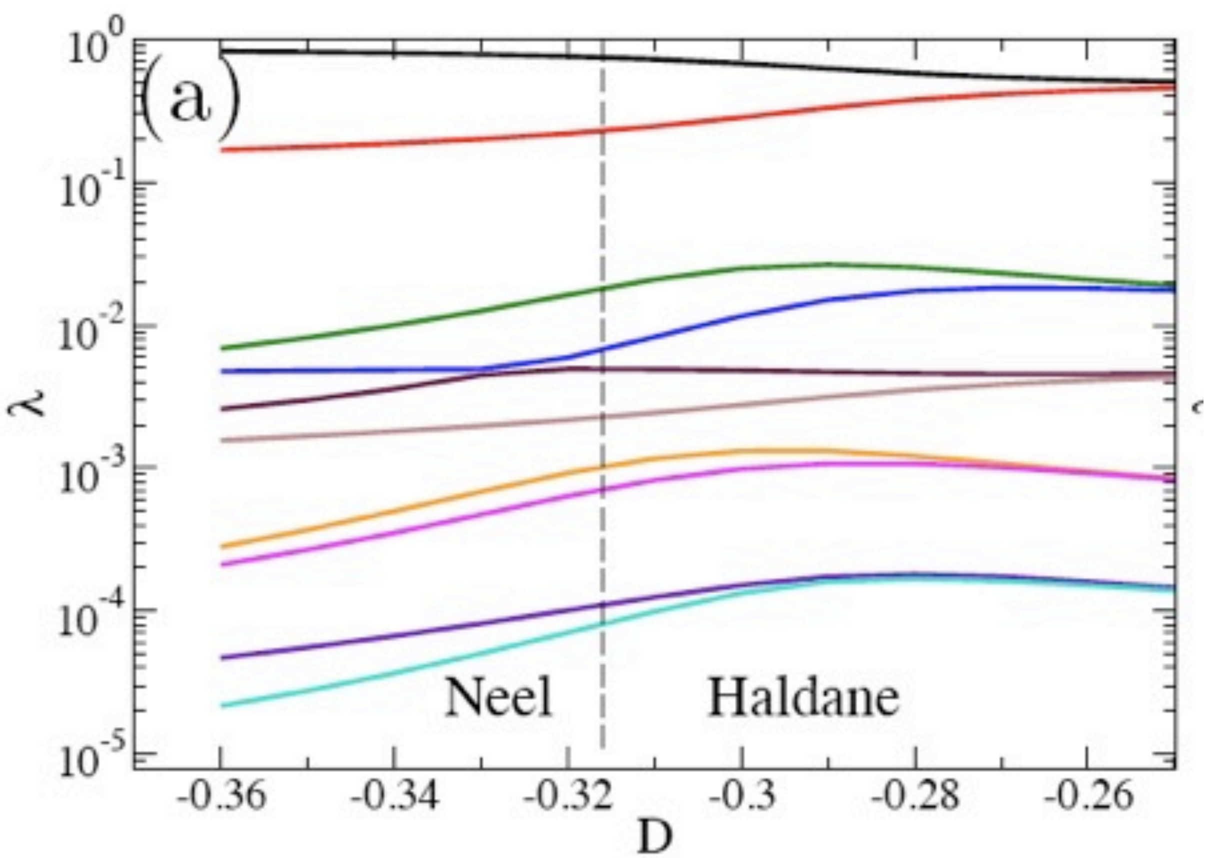
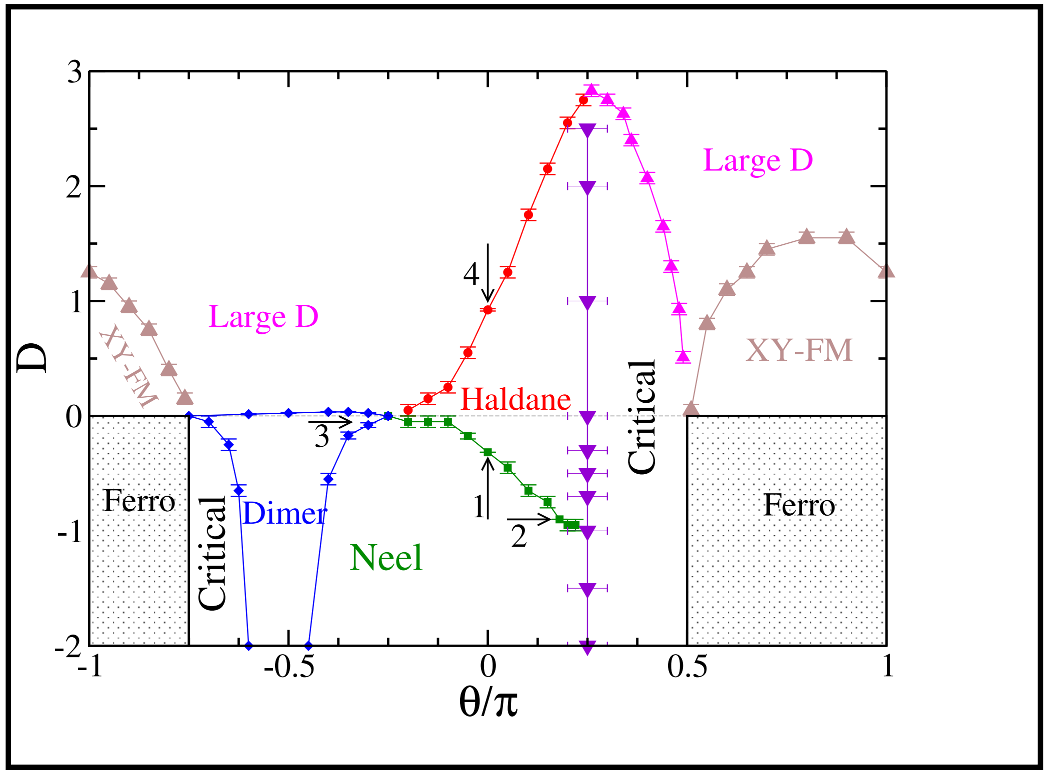


$$\Delta\lambda = \lambda_1 - \lambda_2$$

The Schmidt spectrum



Entanglement in many-body systems : **Schmidt gap**



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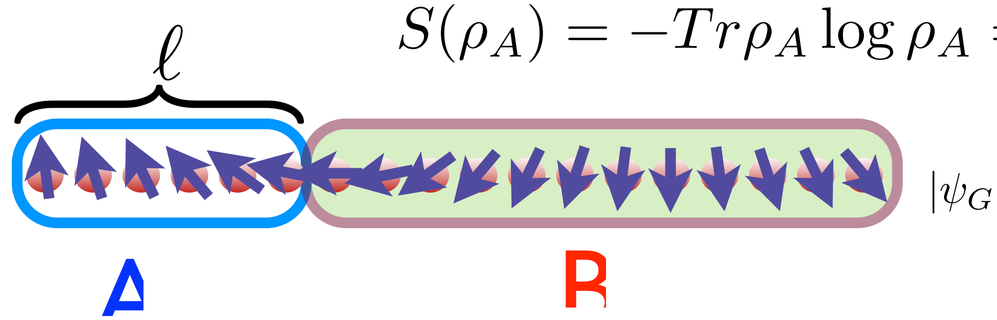
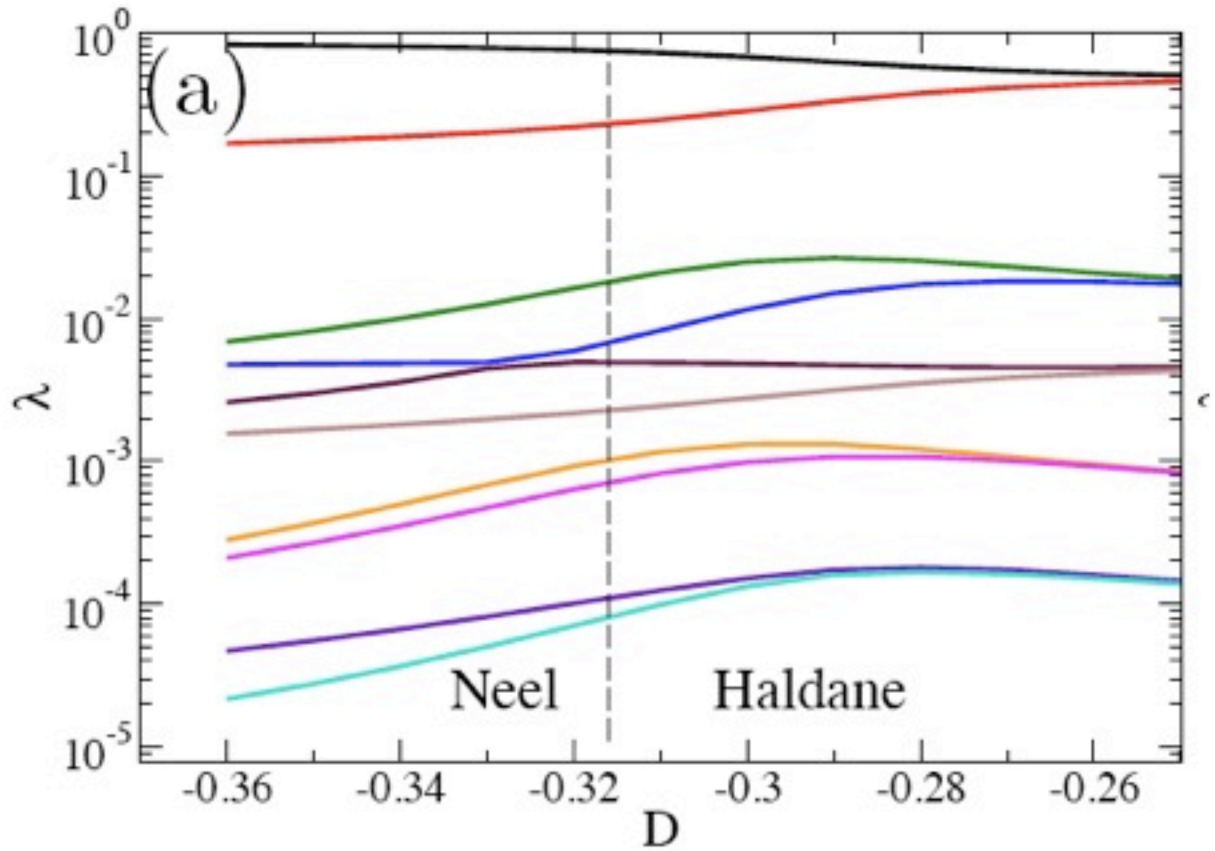
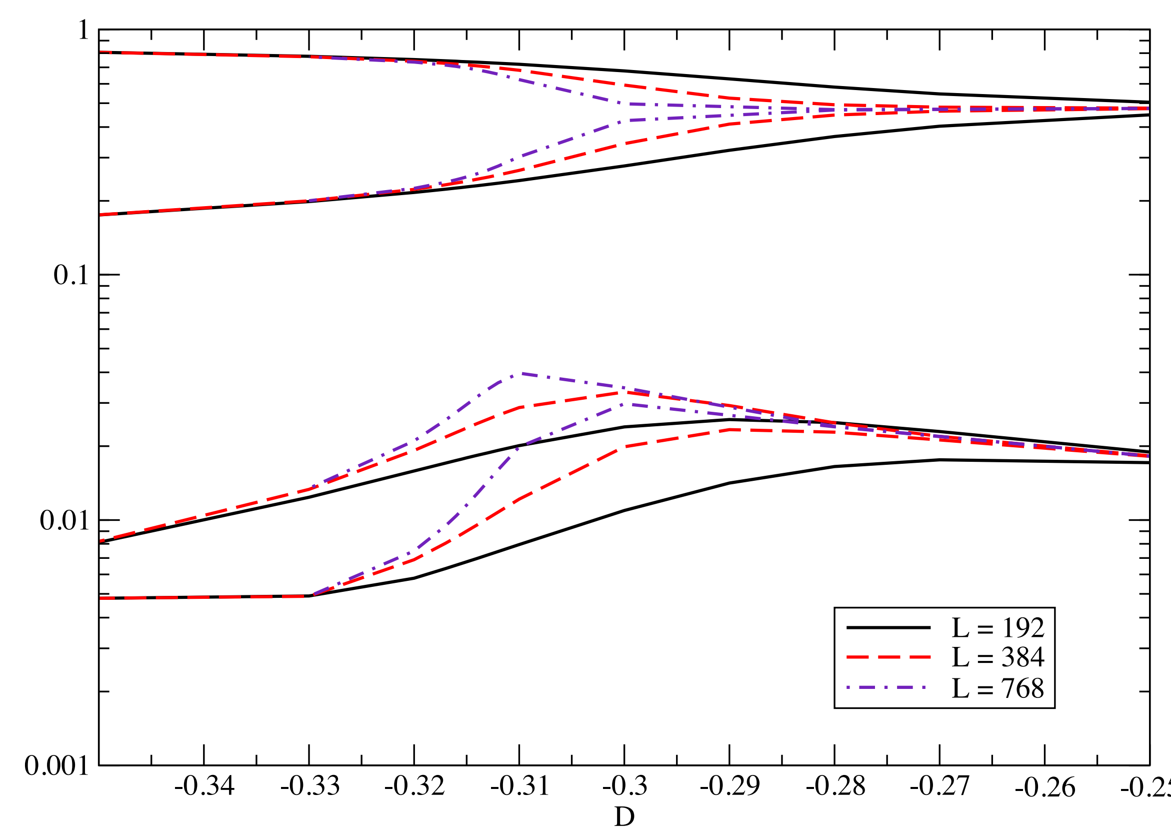


$$\Delta\lambda = \lambda_1 - \lambda_2$$

The Schmidt spectrum

λ1, λ2, λ3, λ4





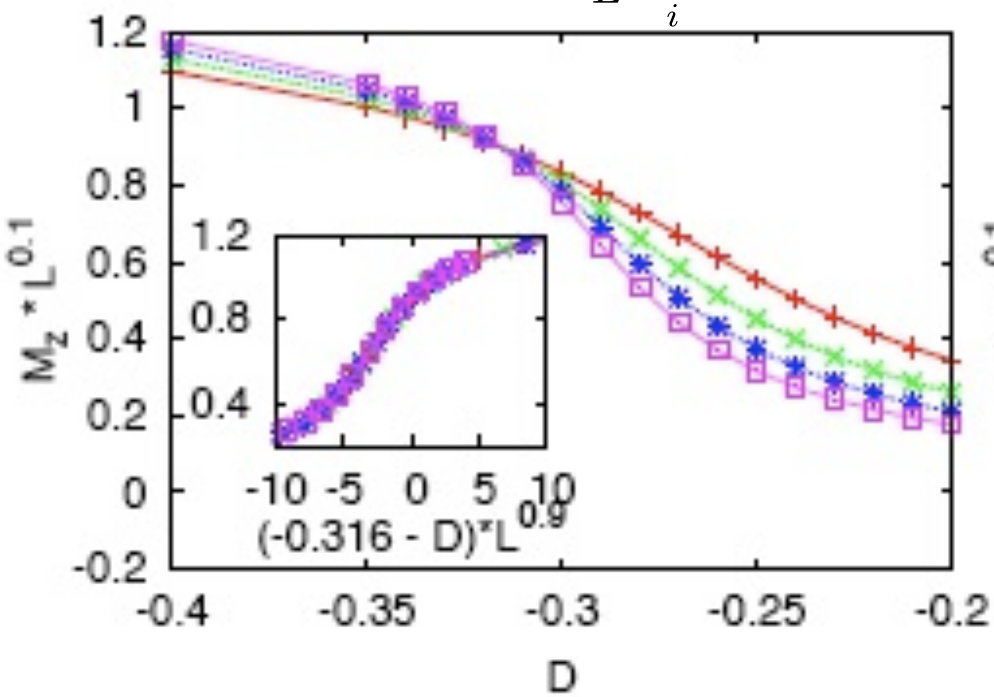
Neel-Haldane Spin 1: Scaling theory for the Schmidt gap

$$\Delta\lambda(g,L) \simeq L^{-\beta} f_{\Delta\lambda}(|g-g_c|L^{1/\nu})$$

observable	D_c	β	ν
M_z	-0.315	0.11	1.01
$\Delta\lambda$	-0.315	0.11	1.04

We compare the staggered magnetization with the Schmidt gap
Finite size scaling shows equal critical exponents for both

$$M_z = \frac{1}{L}\sum_i (-1)^i S_i^z$$



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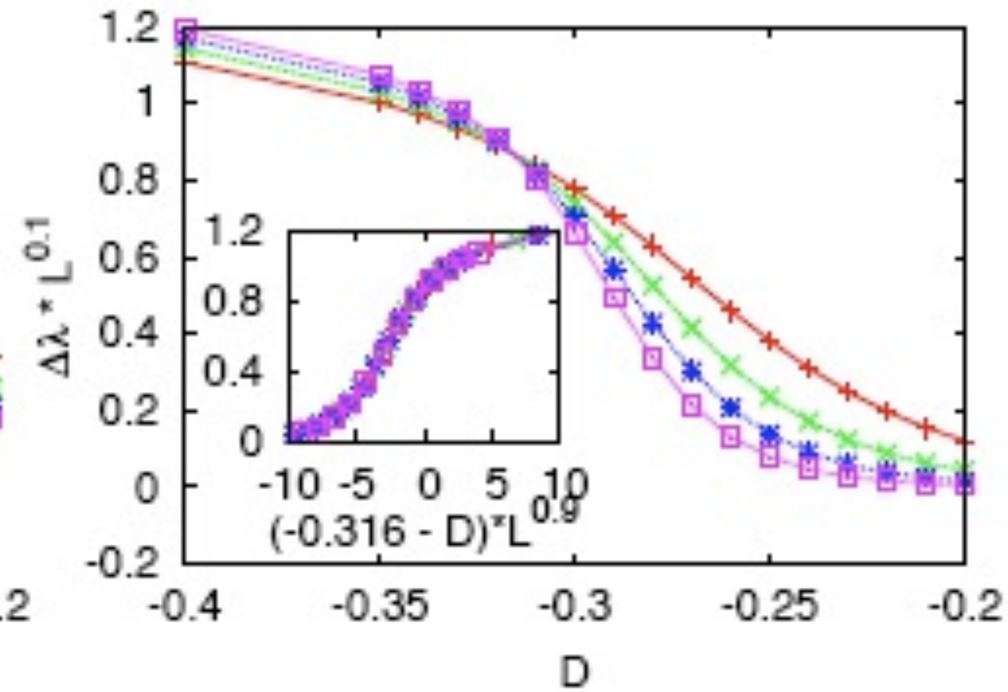


Fitting results from Schmidt gap:

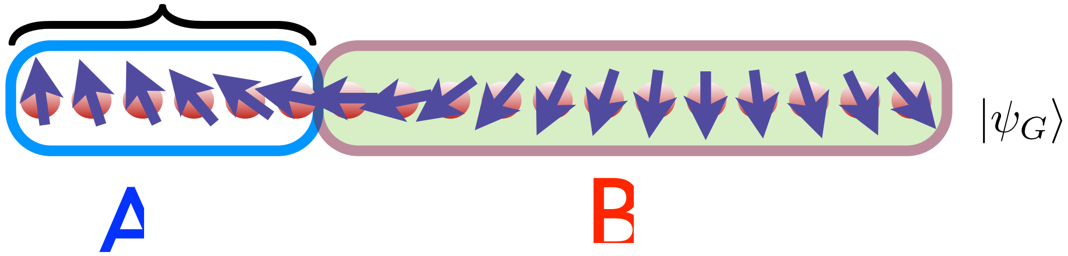
$$\beta = 0.110$$
$$\nu = 1.04$$

Ising Universality class !

DeChiara, Lepori, Lewenstein AS '12
Lepori Dechiara AS '13

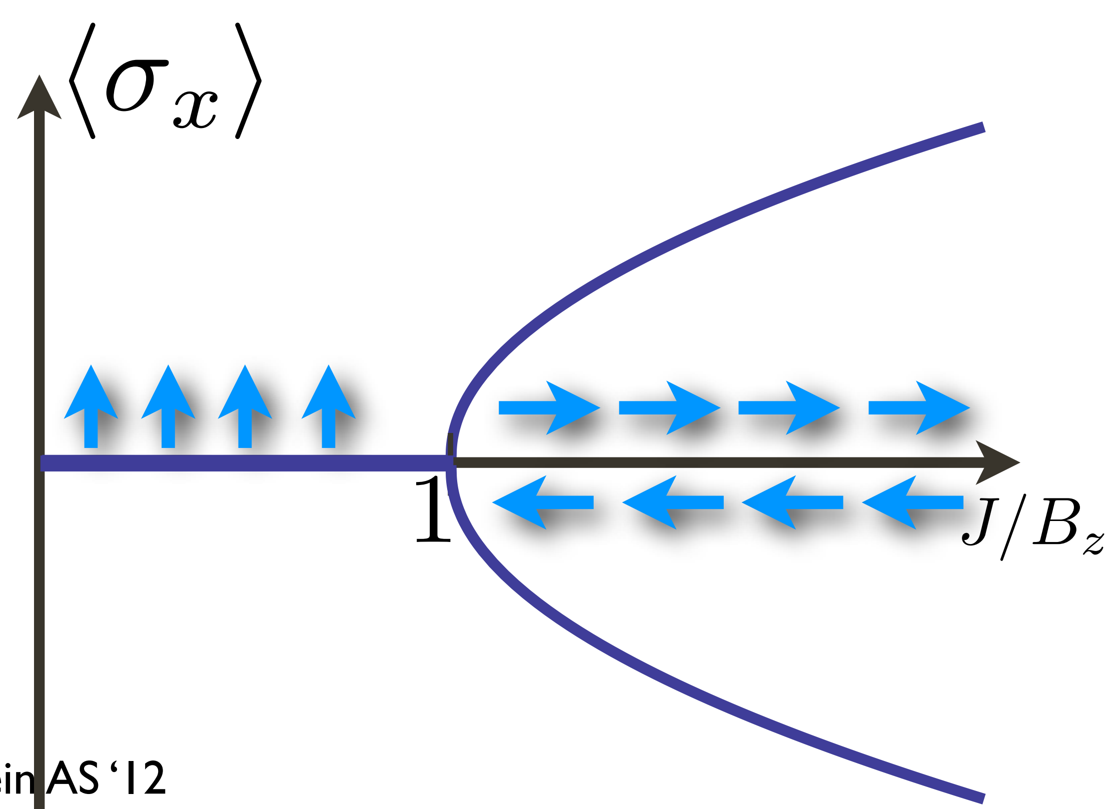
Entanglement in many-body systems : **Schmidt gap in transverse Ising model**

$$\ell \; S(\rho_A) = -Tr\rho_A \log \rho_A = \sum \lambda_i \log(\lambda_i)$$



$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$H = -J \sum_{i=1}^{L-1} \sigma_x^i \sigma_x^{i+1} - B_z \sum_{i=1}^{L} \sigma_z^i$$



DeChiara, Lepori, Lewenstein AS '12
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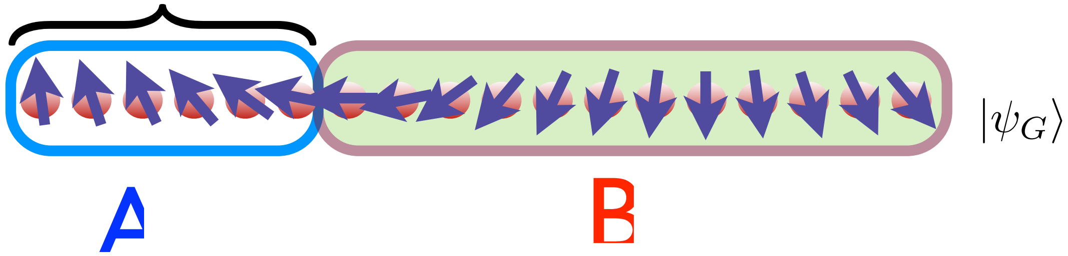

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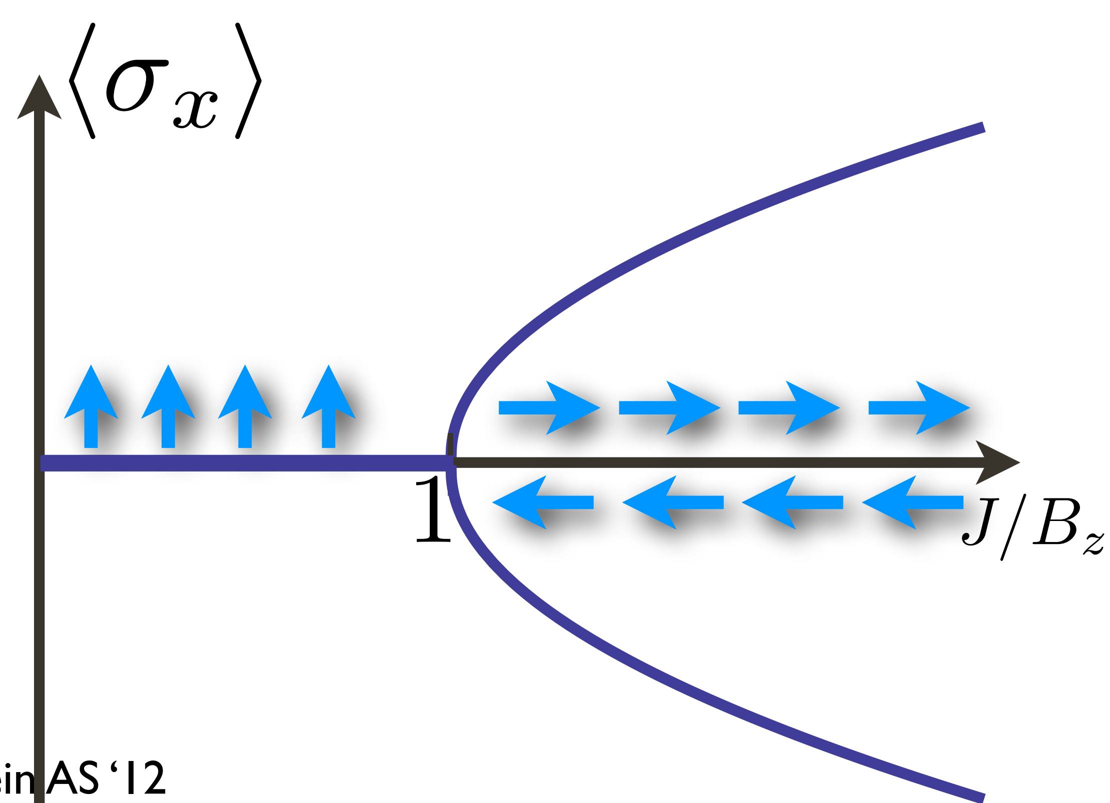

Scaling of the

$$\langle\sigma_x\rangle \sim |B_z - J|^\beta$$

critical exponent

$$\beta = 1/8$$

Correlations

$$\langle\sigma_x^i \sigma_x^{i+r}\rangle \sim e^{-r/\xi}$$

$$\xi \sim |B_z - J|^{-\nu}$$

$$\nu = 1$$

DeChiara, Lepori, Lewenstein AS '12
Lepori Dechiara, AS '13

Entanglement in many-body systems : **Schmidt gap**

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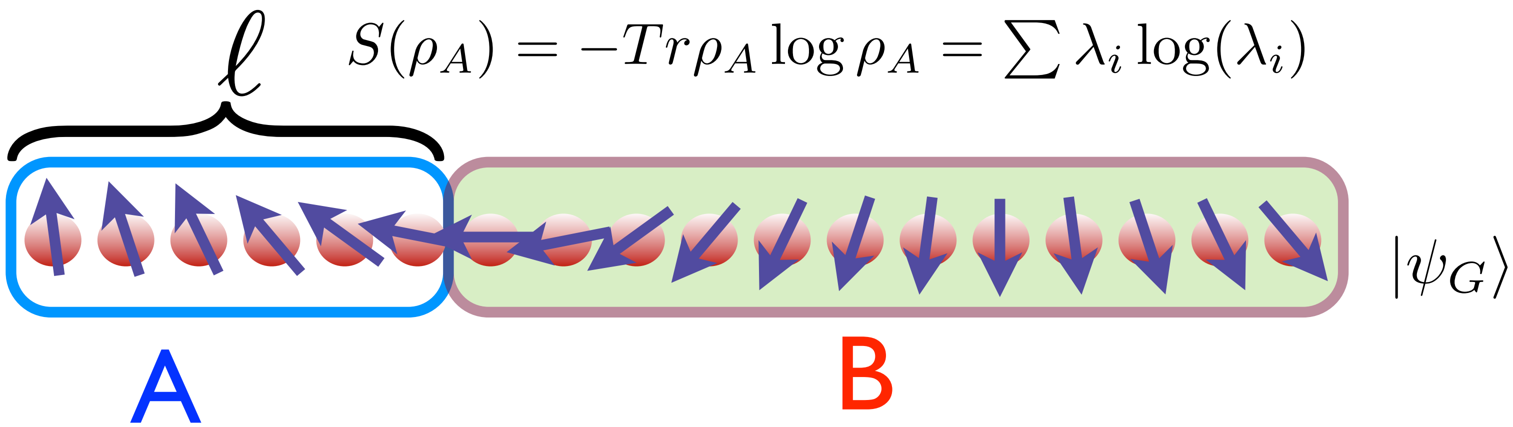

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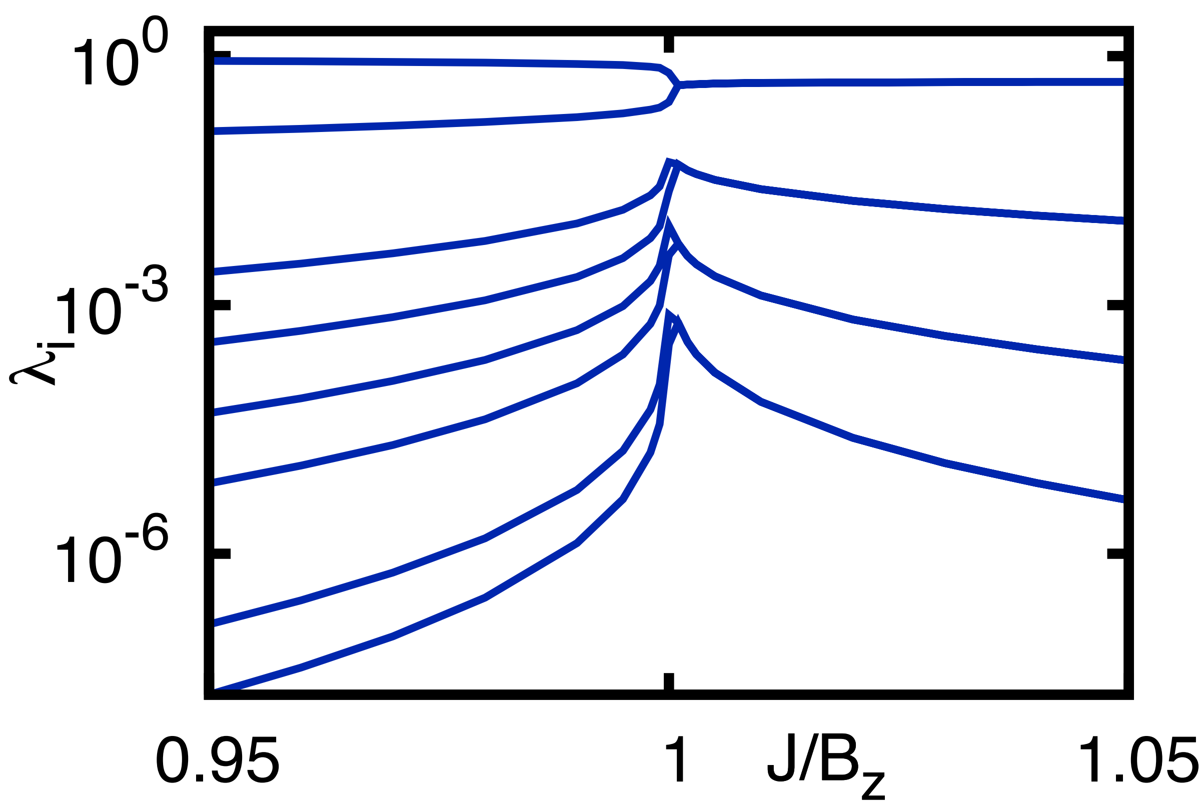

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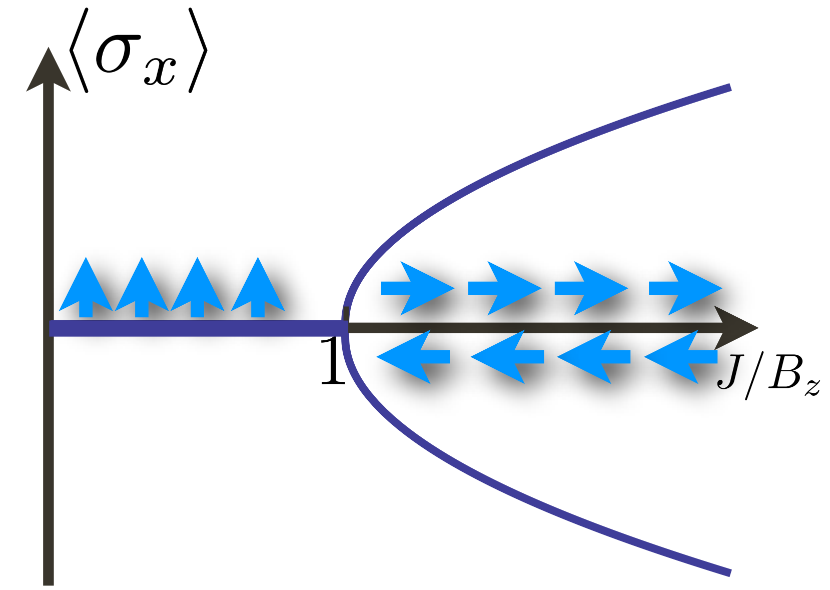

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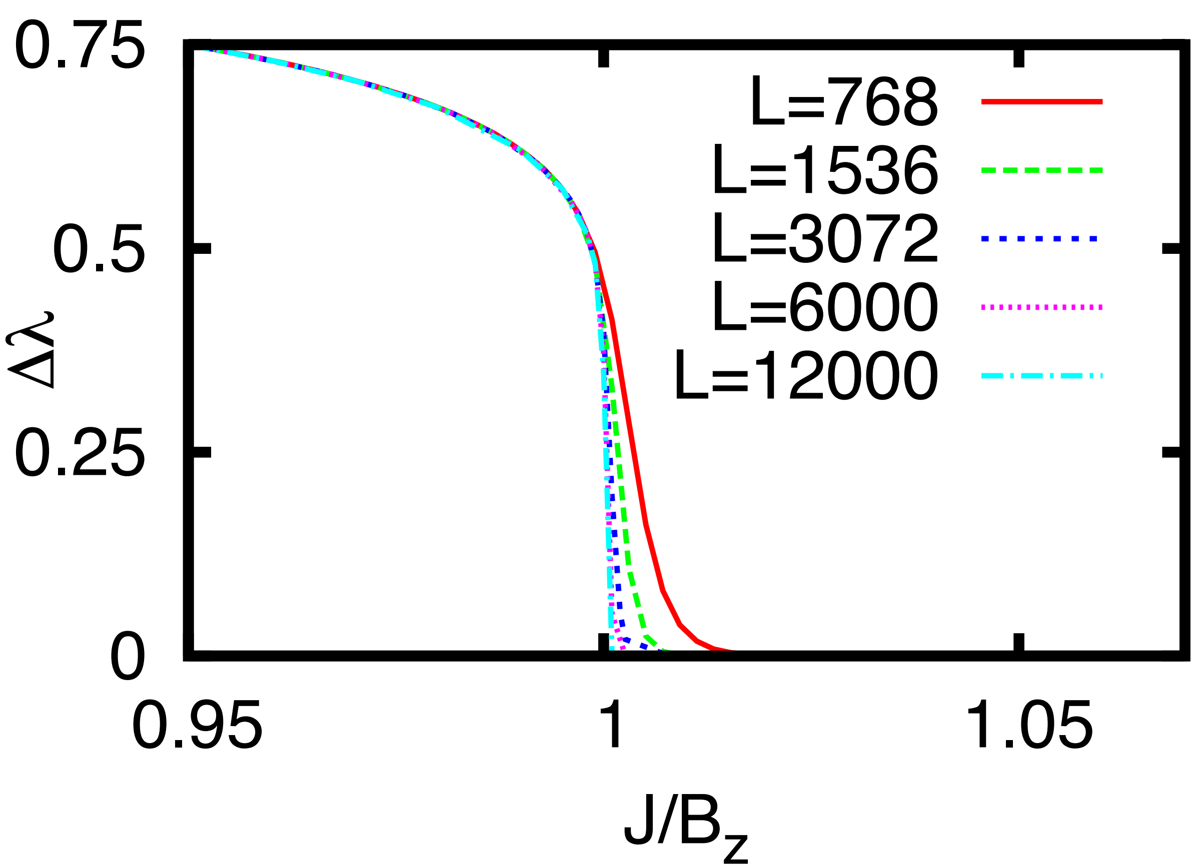

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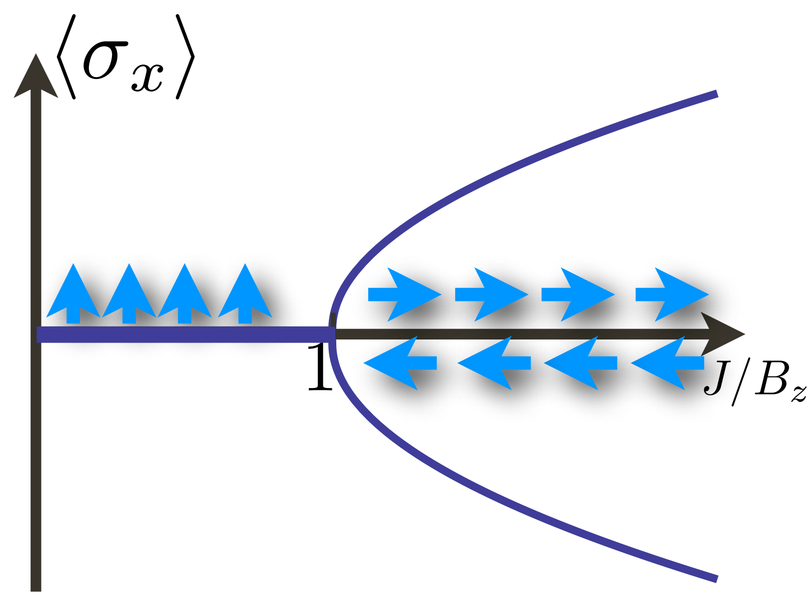

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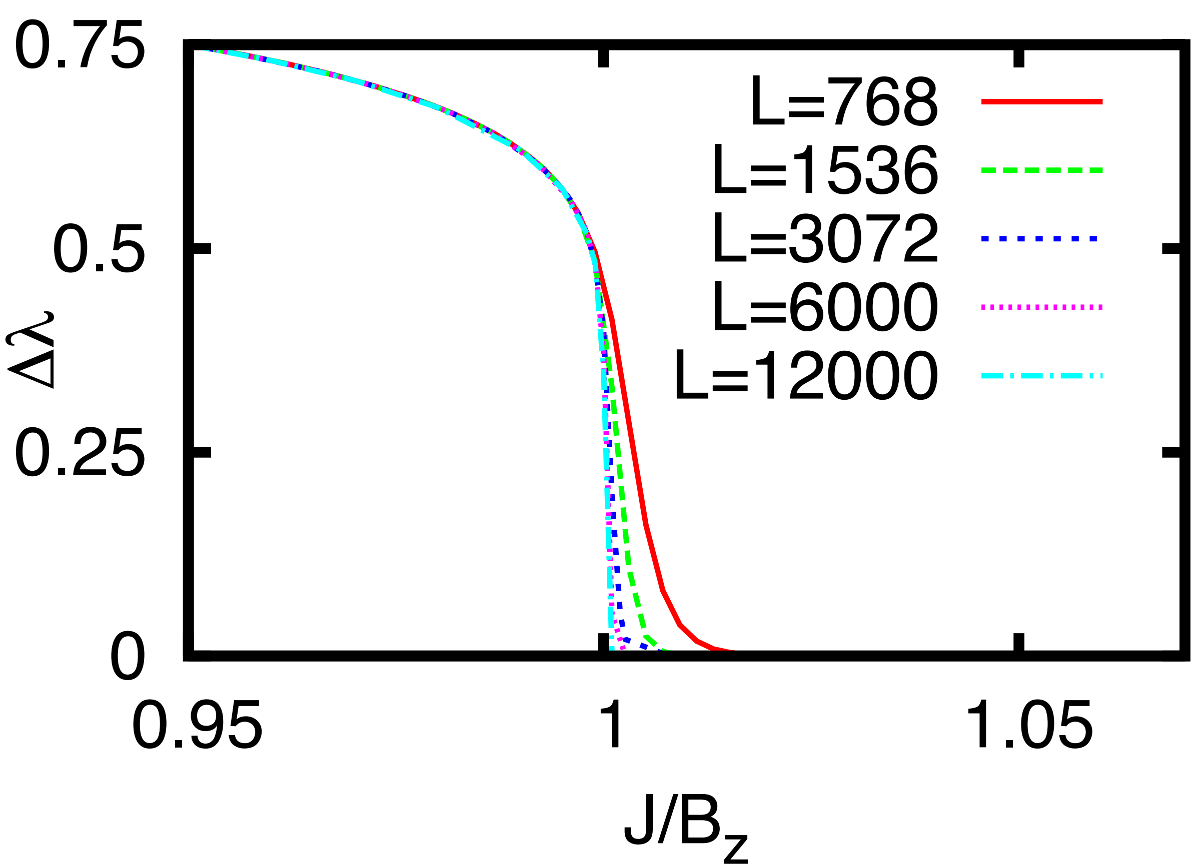

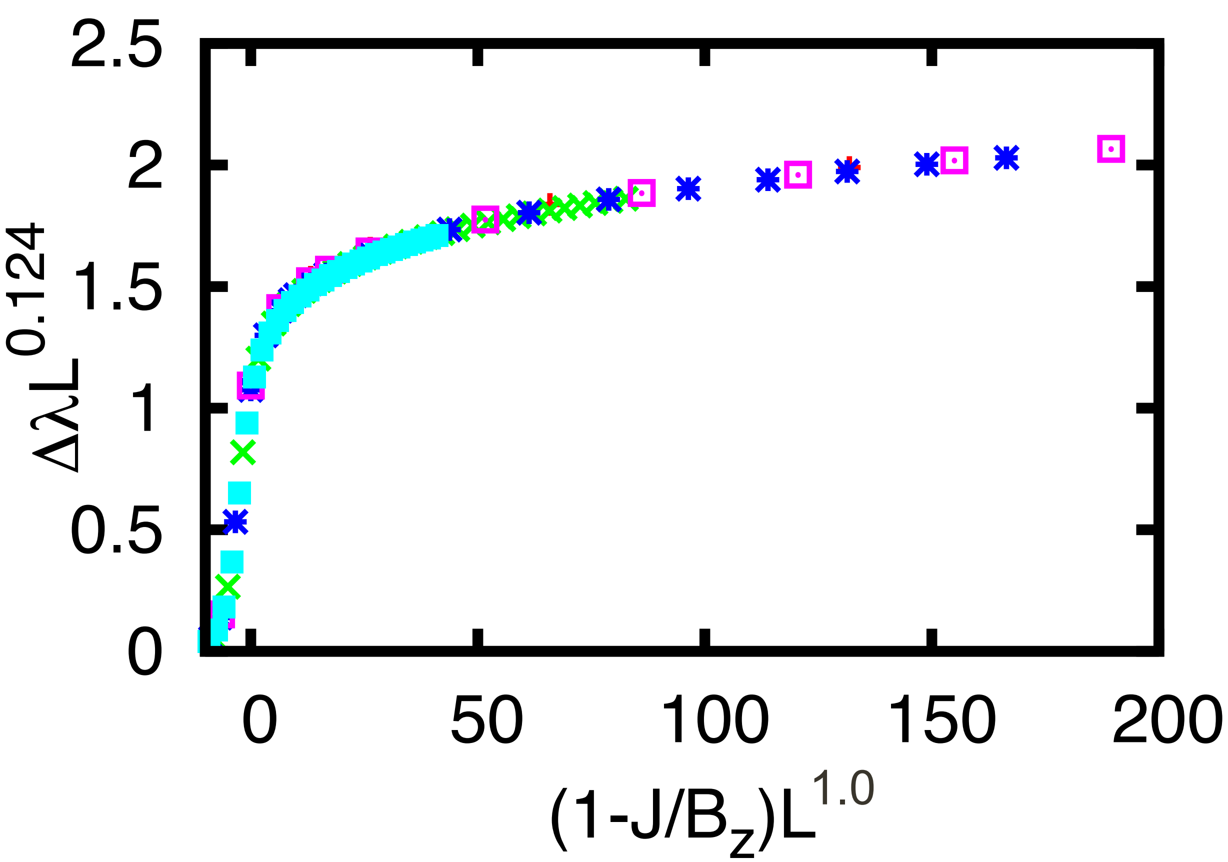

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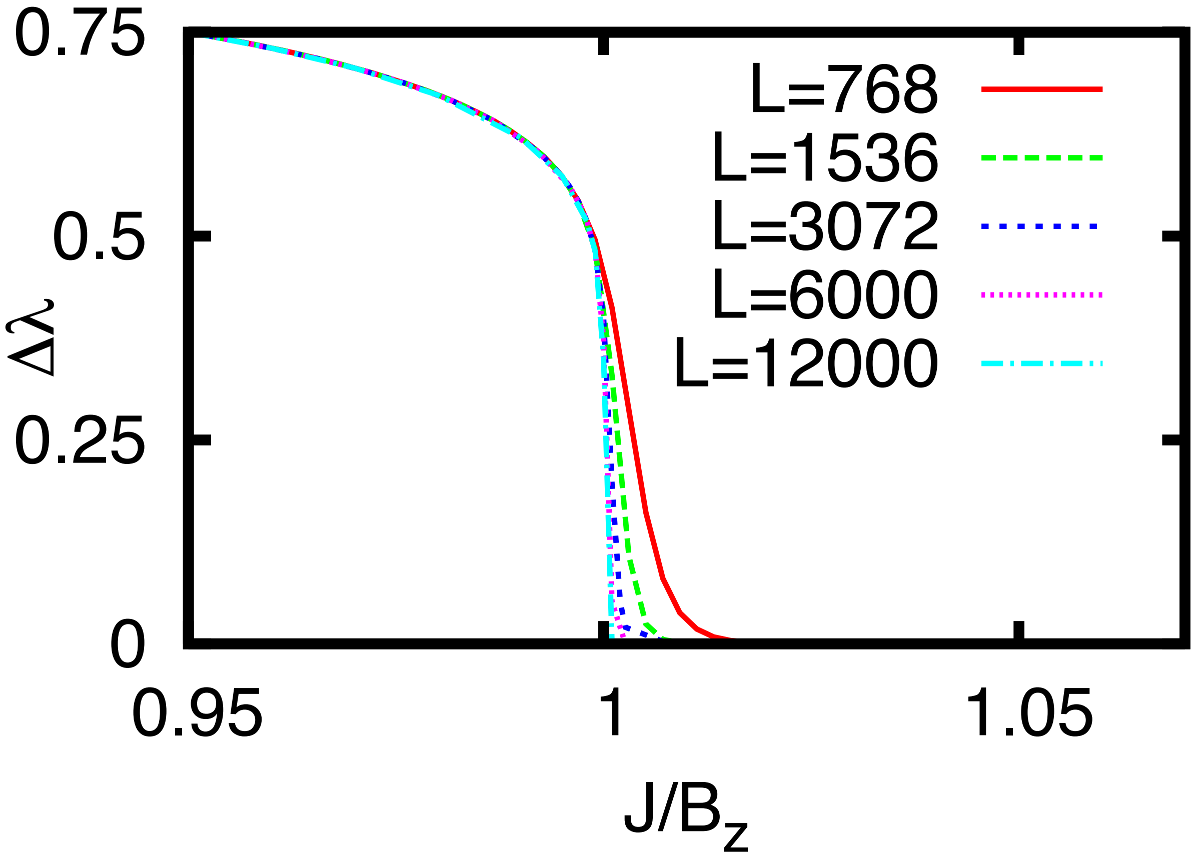
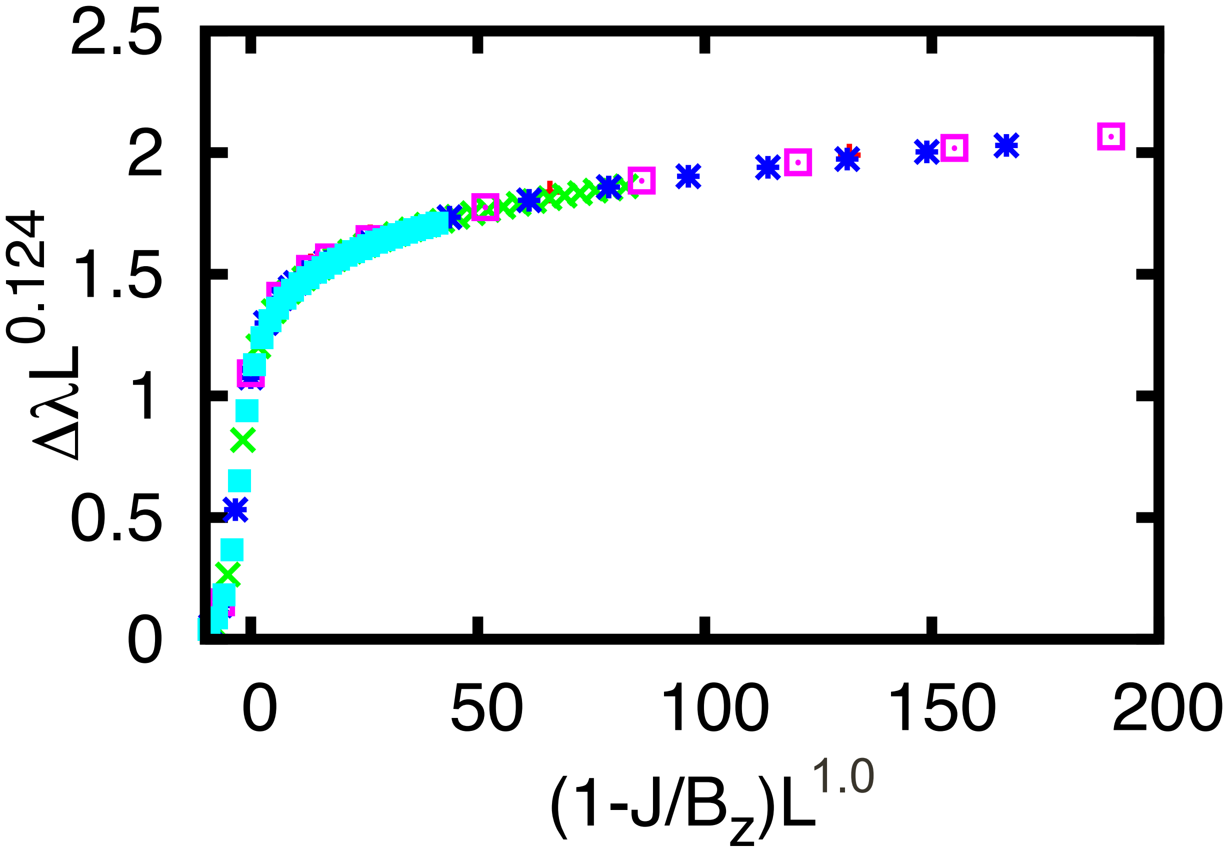
$$\Delta\lambda = \lambda_1 - \lambda_2$$

Finite Size Scaling (Fisher & Barber 1972)

$$\Delta\lambda = L^{-\beta/\nu} f[(J - B_z)L^{1/\nu}]$$

FITTING	Theory
$\beta = 0.124$	$\beta = 0.125$
$\nu = 1.00$	$\nu = 1$





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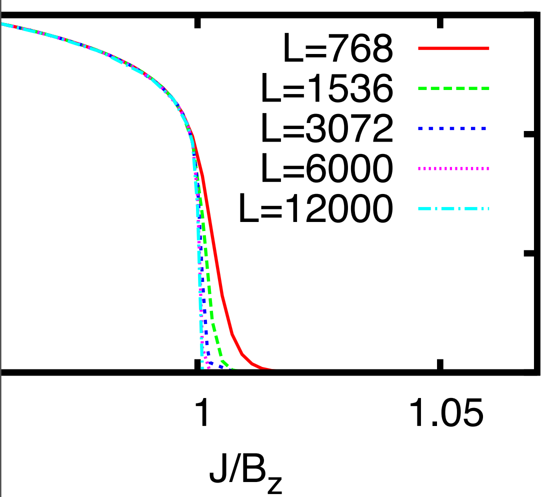
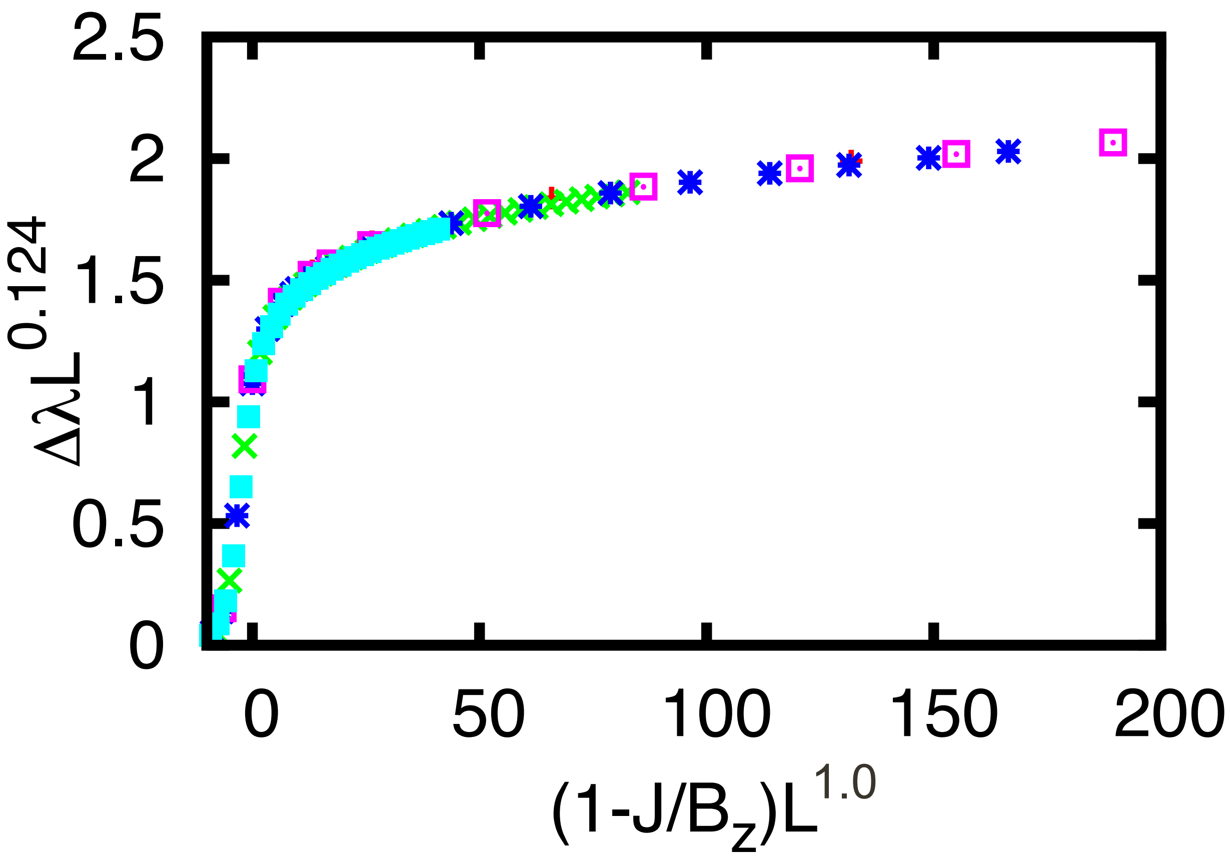
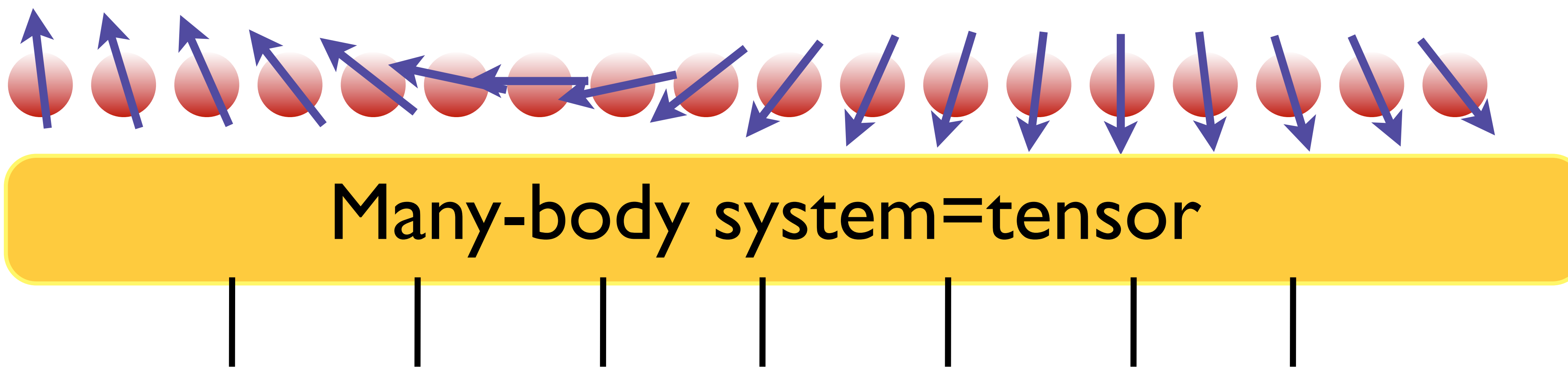
$$\Delta\lambda = L^{-\beta/\nu} f[(J - B_z)L^{1/\nu}]$$

FITTING	Theory
$\beta = 0.124$	$\beta = 0.125$
$\nu = 1.00$	$\nu = 1$





Summarize: bipartite entanglement has to say from condensed matter



We have review 4 approaches

- Pairwise entanglement detects quantum phase transitions but not criticality
- Block entanglement: detects critical behaviour and signals area laws
- Entanglement spectrum: detects topological phases
- Schmidt gap: links critical exponents and scaling with entanglement
- **Plus the whole plethora of tensor networs (PEPS,MERAS,ETC...)**

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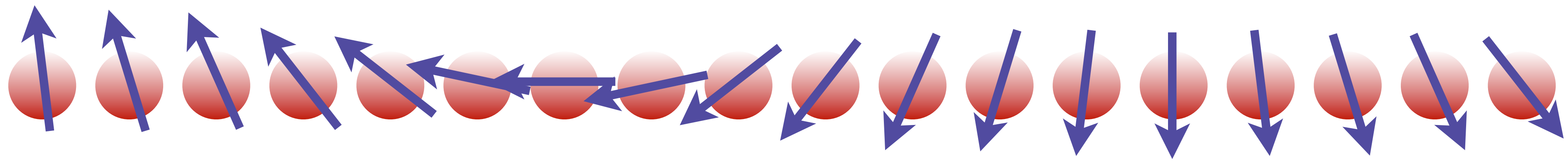

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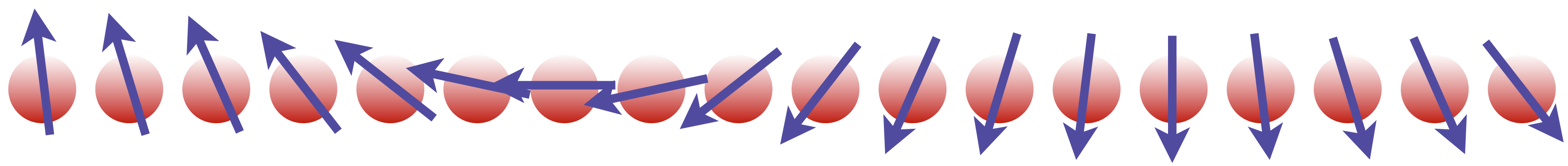
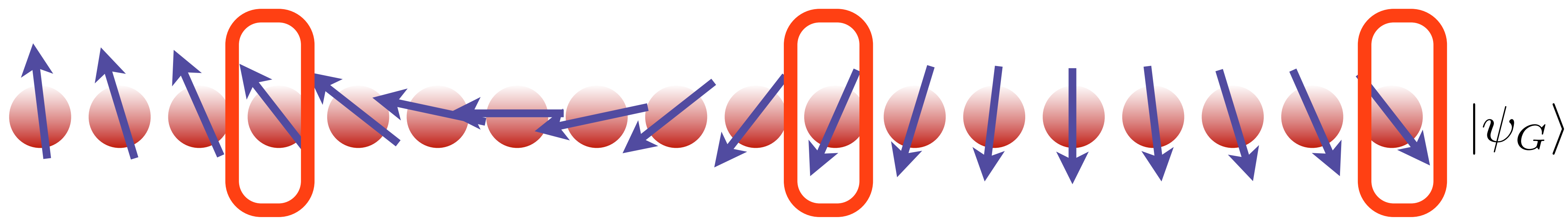


What about multipartite entanglement ?

Back to basics:



Guhne, Huber, Kraus, A. Sen (De), U. Sen, B. Hismayer, G. Toth and others

Multipartite entanglement: Rotationally invariant SU(2) 3-qubit states:

$$\left\{\rho : \forall_{\hat{\boldsymbol{n}},\theta} \left[\mathcal{D}^{j_1}_{\hat{\boldsymbol{n}},\theta} \otimes \mathcal{D}^{j_2}_{\hat{\boldsymbol{n}},\theta} \otimes \mathcal{D}^{j_3}_{\hat{\boldsymbol{n}},\theta}, \rho\right] = 0\right\} \longrightarrow \rho = \sum_{k=+,0,1,2,3} \frac{r_k}{4} R_k$$

Eggeling and Werner (2001)

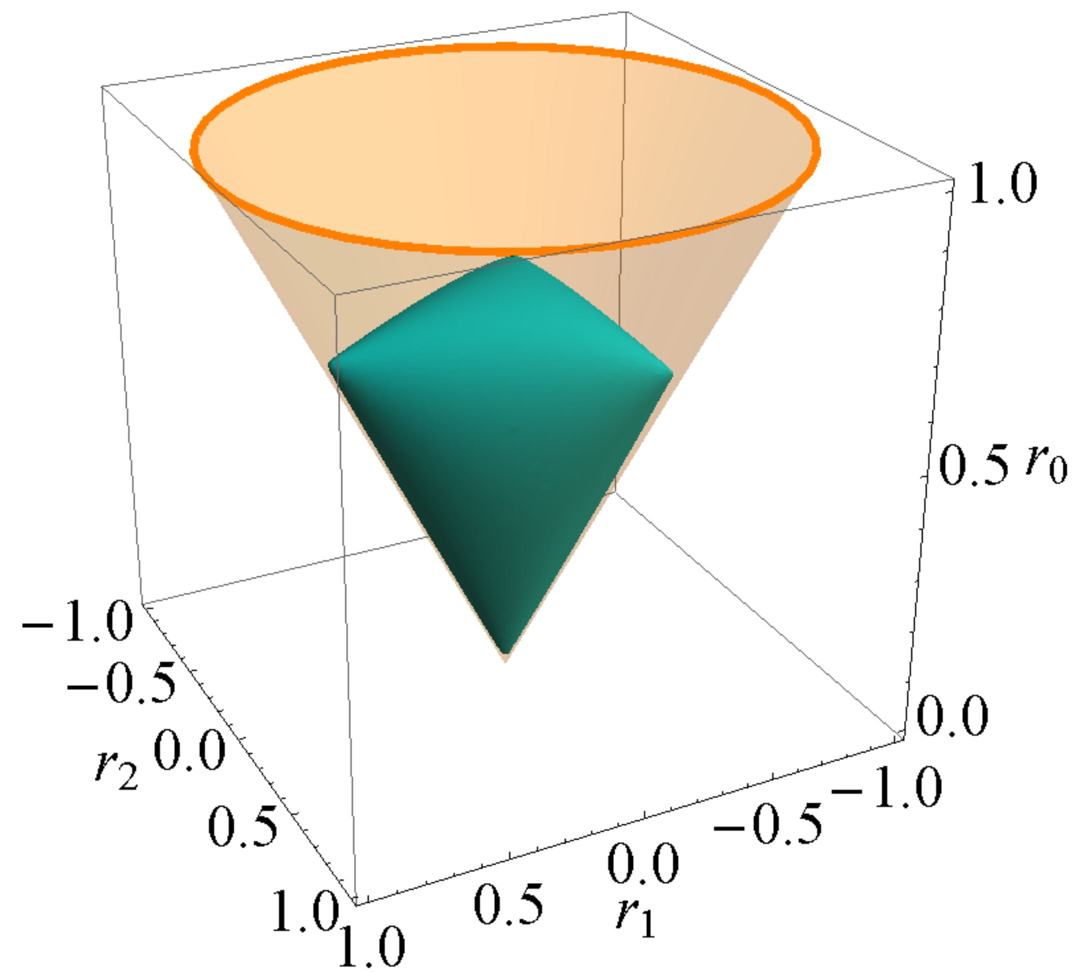
- Normalization only 4 parameters!
- For real matrices and their reductions only 3 (r_3=0)
- For 3 qutrits (spin-1) we need 13 parameters!

(i) separable states S $\quad \rho = \sum_i \lambda_i \rho_i^1 \otimes \rho_i^2 \otimes \rho_i^3$

(ii) biseparable states $\mathcal{B}$ belong to the convex hull of states separable with respect to one of the partitions 1|23, 2|13 or 3|12 denoted by $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$, respectively,

(iii) multipartite entangled states W, GHZ

$$\mathcal{S} \subset \mathcal{B} \subset \mathrm{W} \subset \mathrm{GHZ}.$$



Stasinka et al. 2013

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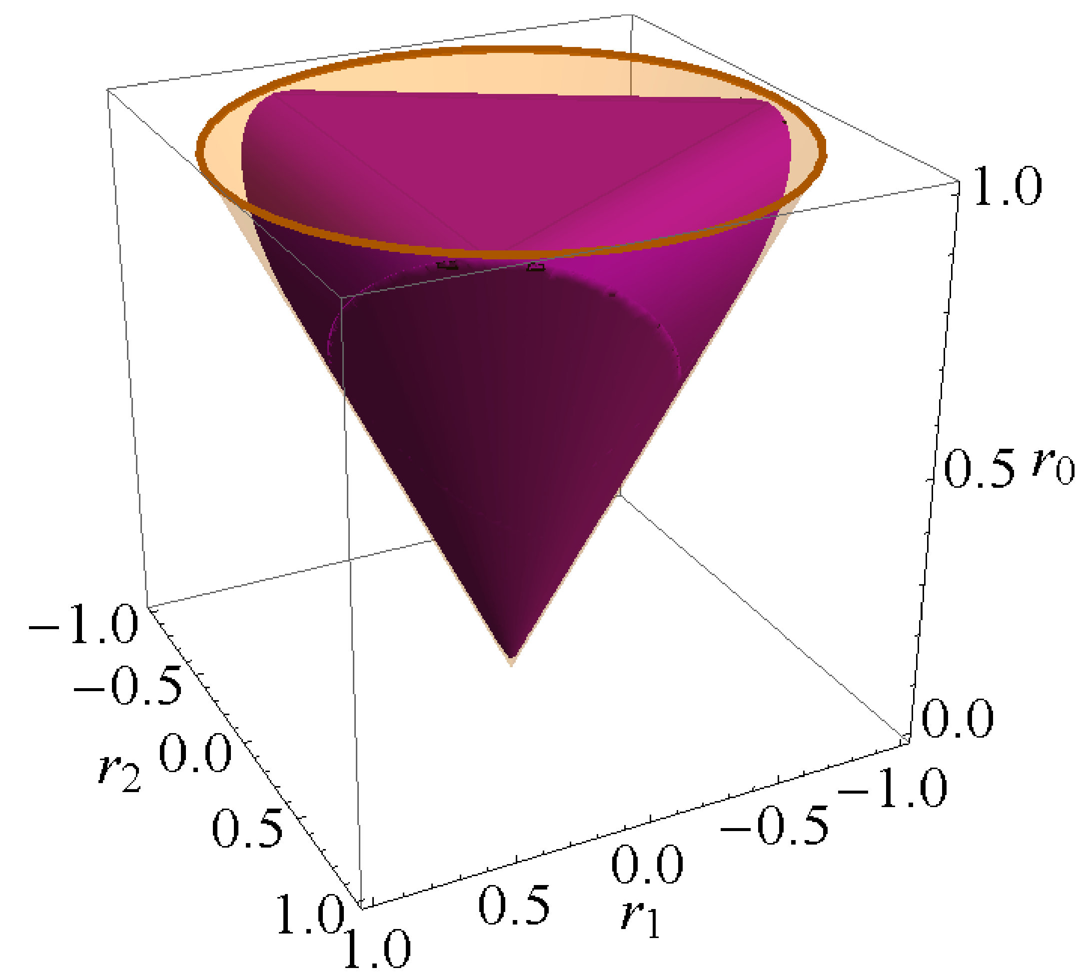

Stasinka et al. 2013

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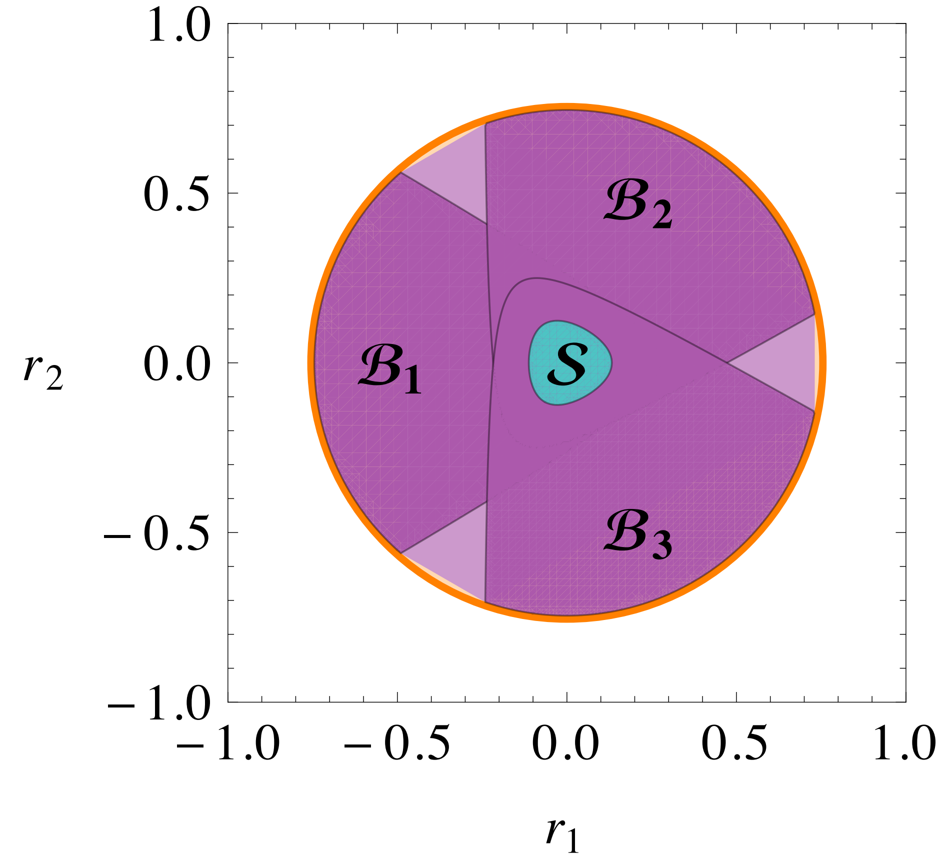
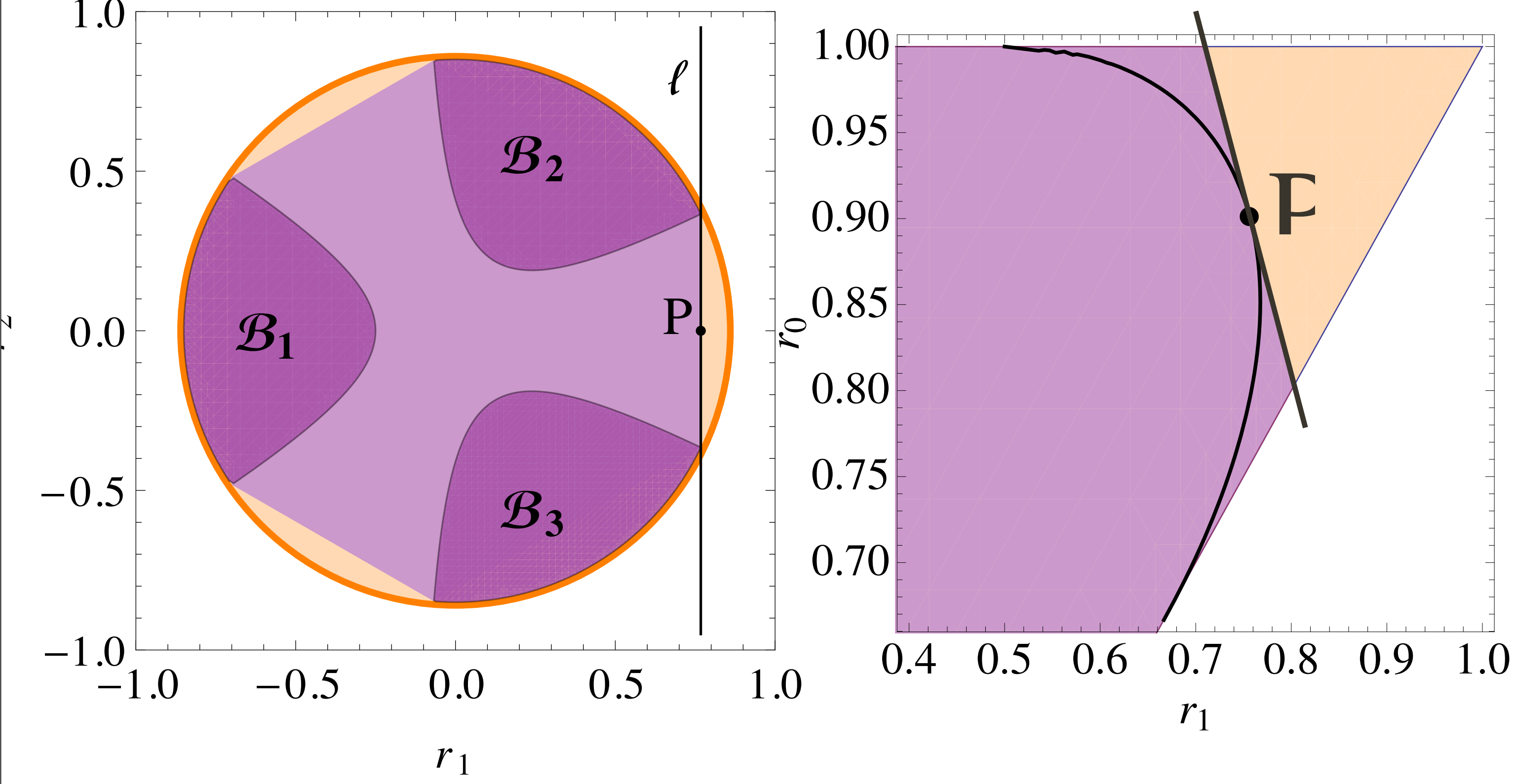


Stasinka et al. 2013

Optimal witness for real rotationally invariant ground states

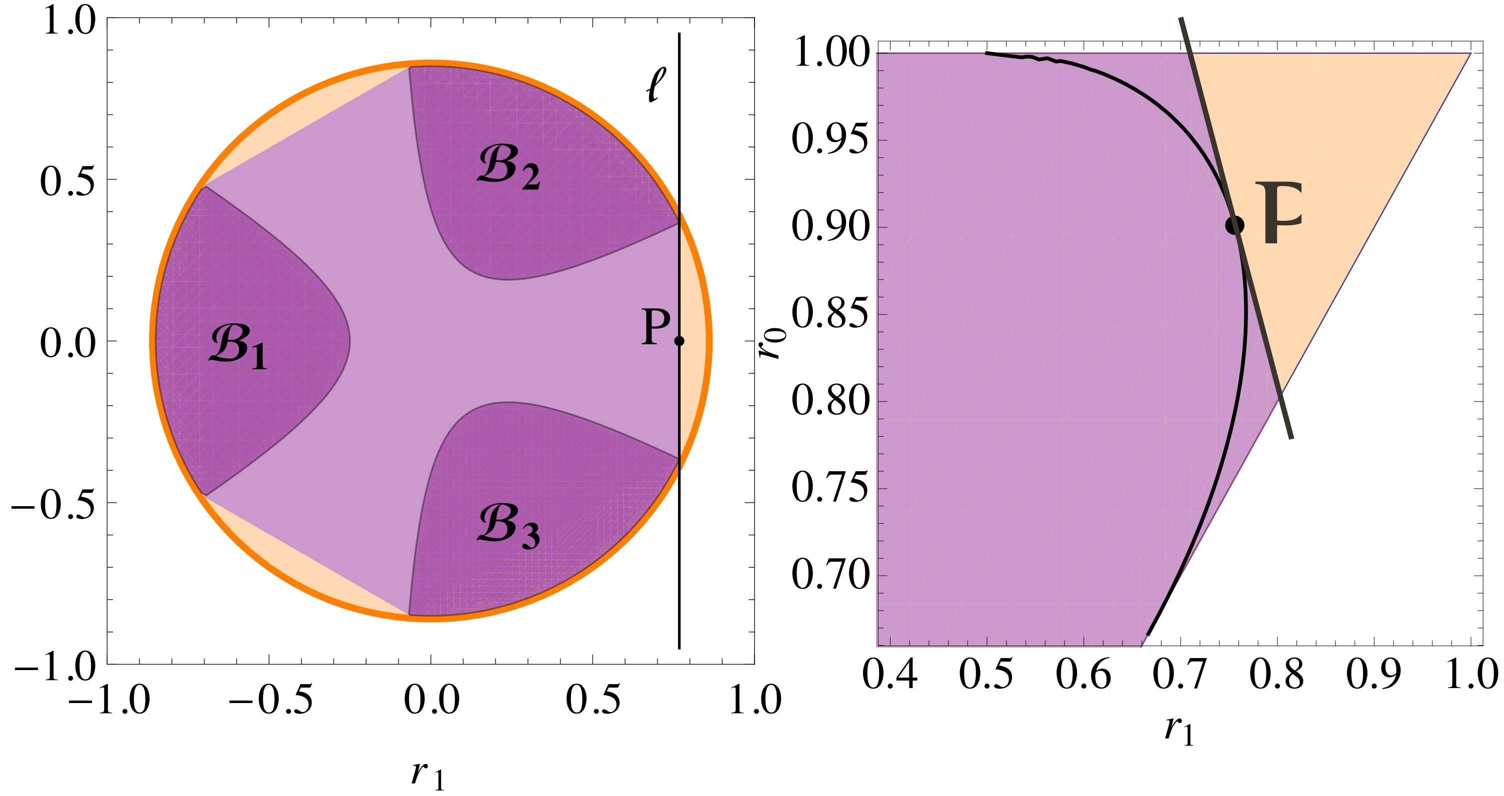


$$\exists\, \rho \in \mathcal{T} : \quad Tr(W\rho) < 0$$

$$Tr(W\rho) \geq 0 \;\; \forall \rho \in \mathcal{B}$$

This is equivalent to a twirling map and gives a sufficient but not necessary condition for 3-partite entanglement for non-rotationally invariant states !

Optimal witness for real rotationally invariant ground states



$$\exists\, \rho \in \mathcal{T} : \quad Tr(W\rho) < 0$$

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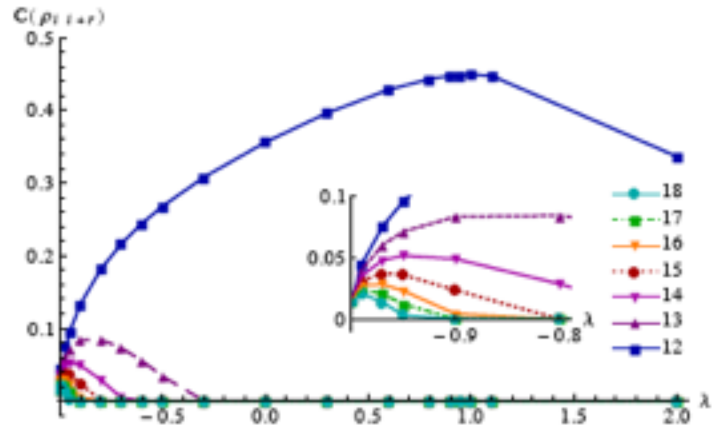
We take an SO(3) invariant W:

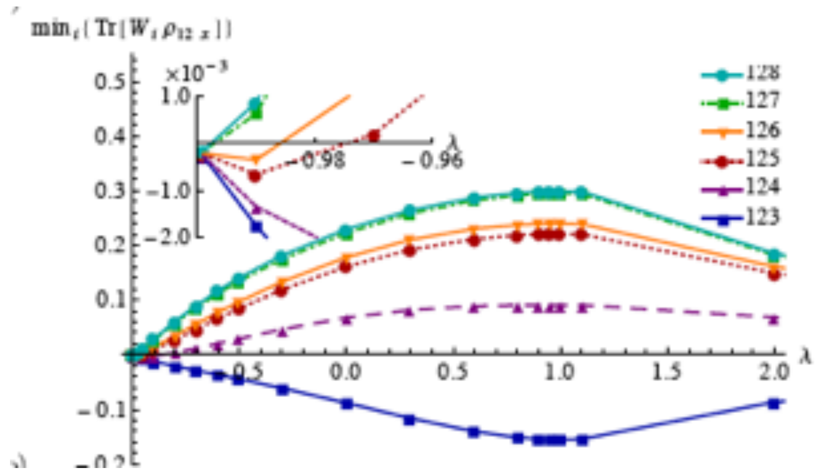
$$W = \sum_i c_i R_i$$

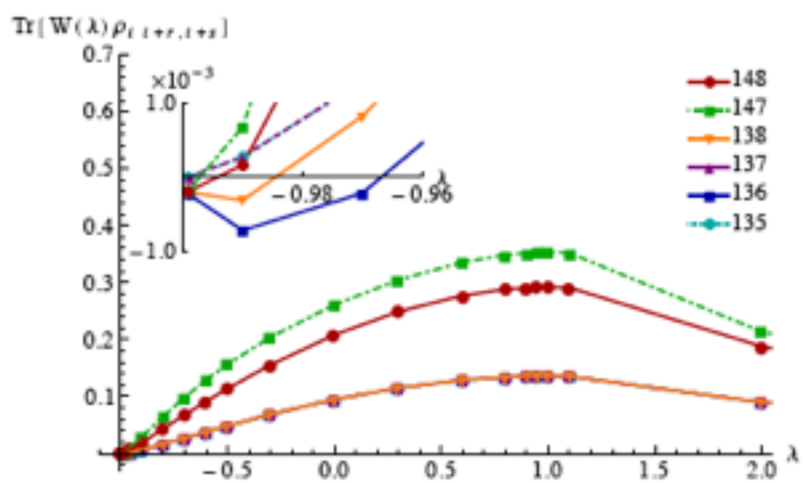
This is equivalent to a twirling map and gives a sufficient but not necessary condition for 3-partite entanglement for non-rotationally invariant states !

Strategy : Investigate QFT in spin-1/2 models

$\lambda \to -1^-$







<u>Bipartite concurrence near</u> $\lambda \to -1^+$
is not sensitive to SU(2) breaking
<u>always symmetric</u>

<u>Multipartite entanglement shows</u>:
<u>long range (distant multipartite)</u>
<u>signals the breaking of SU(2)</u>

J. Stasinska, B. Rogers, G. DeChiara, M. Paternostro, A. Sanpera (submitted)

Thanks for your attention