Decoherence in Multiqubit systems

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Quantum Information

- Science: A search for truth about the world around us.
- Quantum Mechanics: In our endevour to understand nature, we reached the quantum limit some 100 years back.
- The concept of entanglement was coined by Einstein during 1940's.
- In 1980's Feynmann suggested that there is ample space at the bottom for information storage, i.e., there is lot more scope of storage of information at the quantum level.
- In 1990's few applications of which most significant were, Shor's algorithm Quantum Teleportation revolutionised the study in the field of quantum information processing. Efforts began to harness the information contained in superposition of quantum states and strength of quantum entanglement.

Quantum Information contd..

- Henceforth came the problem of controlling, quantifying and rectifying the information contained in quantum systems and quantum entanglement.
- It resulted in an urgent need to understand the decoherence in quantum systems. Our work is a small effort in that direction.

What is Decoherence?

Deoherence is decay of a system when it interacts with the environment. At the quantum level, particles interact with the environment and get entangled with it. The entanglement of the system with the environment leads to leaking of the information contained in system to the environment in an irreversible manner.

Why is study of decoherence significant?

It is significant because in order to use quantum systems for any useful operations, we need to preserve the information contained in the system. So, in order to preserve the information contained in the system, there is a need to understand how it entangles with the environment and how the information leaks to the environment.

Our motivation behind the work: Looking at the problem from broader perspective

A lot of work has been done till date to comprehend decoherence in quantum system and how rate of decoherence increases with the increasing number of qubits. For this purpose various types of states have been considered.
In our paper, we have looked at the problem from two aspects:

- We have studied the rate of decoherence of general states with respect to increase in number of qubits of the state. For this purpose, for a system of the given size, we have sampled a very large part of Hilbert space by generating random states.
- We have studied the effect of different channels on the state.

Our System : Multiqubit system

We have considered three types of pure states:

- General states $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ where $\alpha, \beta, \gamma, \delta$ are complex numbers s.t. $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$
- General GHZ states $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$ where α, β are complex numbers s.t. $|\alpha|^2 + |\beta|^2 = 1$
- General W-states $|\psi\rangle = \alpha|01\rangle + \beta|10\rangle$ where α, β are complex numbers s.t. $|\alpha|^2 + |\beta|^2 = 1$

We have defined the general GHZ state and general W state by the expression given above.

Decoherence channels

The effect of decoherence has been formulated using 4 types of channels which are as follows

- Depolarizing channel: The effect of the depolarising channel on the states is that it adds equal degree of mixedness to all the states.
- Dephasing channel: The effect of this channel is reduction in the coherence of the states.
- System interacting with zero temperature bath: Zero temperature bath implies the environment with which the system is interacting is in its ground state. So what it will do to the system will be two fold. It will decrease the coherence of the system as well as bring the system from excited state to the lower state.
- Collective dephasing channel: It is a special kind of a channel. As the name implies, it acts as a dephasing channel on two qubits simultaneously.

Measure of decoherence

As our system is a pure state, we have used Fidelity as the measure of decoherence. The general expression for fidelity is as follows:

$$F = \langle\psi|\,\rho\,|\psi\rangle$$

where $|\psi\rangle$ is the initial pure state and ρ is final mixed state.

Models used for decoherence channels

We have used two approaches to model our channels:

- Kraus operator approach
- Master equation approach

We have used two different approaches to model the two channels only for the sake of simplicity in calculating the final state.

Kraus operator approach

We have used Kraus operator approach to model two of the four channels. The channels are collective dephasing channel and depolarising channel. The general form of Kraus operator is given by

$$\rho(t) = E(\rho) = \sum_{\nu=1}^{n} K_\nu^\dagger \rho(0) K_\nu$$

where K_ν and $K_\nu^\dagger$ obeys the condition

$$\sum_\nu K_\nu^\dagger K_\nu = 1,$$

Master equation approach

We have used Lindblad form of Master equation for the dephasing channel and zero temperature bath channel. The Lindblad form of Master equation is as follows:

$$\frac{d\rho}{dt} = \sum_{k=1}^{n} (I \otimes ...... \otimes I \otimes L_k \otimes I ...... \otimes I)\rho$$

where ρ is reduced density matrix of the system and Lindblad operator is given by expression

$$L_k \rho = \sum_i \frac{\gamma_i}{2} (2c_i \rho c_i^\dagger - c_i^\dagger c_i \rho - \rho c_i^\dagger c_i)$$

where c_i and γ_i describes the system environment interaction operator and its strength respectively. For dephasing channel $c_i = \sigma_- \sigma_+$ and for zero temperature bath the value of $c_i = \sigma_-$ where σ_- and σ_+ are lowering and raising operator.

Methodology

- We have considered a general state
- The operation of the channel is performed on the state.
- Evaluate the expression of the final state.
- Computed the expression of fidelity between the final and initial state.
- The above mentioned procedure is performed for all the four channels. i.e., we have evaluated the expressions of fidelities of states upto eight qubits for all four channels.

Methodology contd...

- After all these computations, we start with numerical simulations.
- We generated large number of random states and fed them to the channels.
- We have plotted the fidelity distributions for various channels.
- We did a comparitive study of decoherence of three types of states under different channels.

General expression of final state under depolarising channel

Depolarising Channel: Expression of depolarising channel

$$E(\rho) = (1-p)\rho + \frac{p}{4}(\sigma_x \rho \sigma_x + \sigma_y \rho \sigma_y + \sigma_z \rho \sigma_z)$$

The Kraus operators used are the Identity Matrices plus the Pauli spin matrices $\sigma_x, \sigma_y, \sigma_z$ and p is the probability with which state moves towards the maximally mixed state.

General expression of state under dephasing channel

Dephasing Channel: The general expression of the state under the effect of dephasing channel is given as under:

$$\rho(t) = \begin{pmatrix} \rho_{11}(0) & e^{-t\gamma/2}\rho_{12}(0) \\ e^{-t\gamma/2}\rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$$

where γ corresponds to the coupling strength of the state with the environment.

General expression of state under collective dephasing channel

The general expression of the final state under the effect of collective dephasing channel is considered for two qubits as it is the smallest system subjected to the channel.

$$\rho(t) = \begin{pmatrix} (\gamma^2+\omega_1^2)\rho_{11} & \gamma\rho_{12} & \gamma\rho_{13} & (\gamma^2+\omega_1\omega_2)\rho_{14} \\ \gamma\rho_{21} & \rho_{22} & \rho_{23} & \gamma\rho_{24} \\ \gamma\rho_{31} & \rho_{32} & \rho_{33} & \gamma\rho_{34} \\ (\gamma^2+\omega_1\omega_2)\rho_{41} & \gamma\rho_{42} & \gamma\rho_{43} & (\gamma^2+\omega_2^2+\omega_3^2)\rho_{44} \end{pmatrix}$$

where

$$\begin{aligned} \omega_1(t) &= \sqrt{1-e^{\frac{-t}{T}}} \\ \omega_2(t) &= -e^{-t/T}\sqrt{1-e^{\frac{-t}{T}}} \\ \omega_3(t) &= \sqrt{(1-e^{-t/T})(1-e^{-2t/T})} \end{aligned}$$

$$\gamma(t) = \exp\left(\frac{-t}{2T}\right)$$

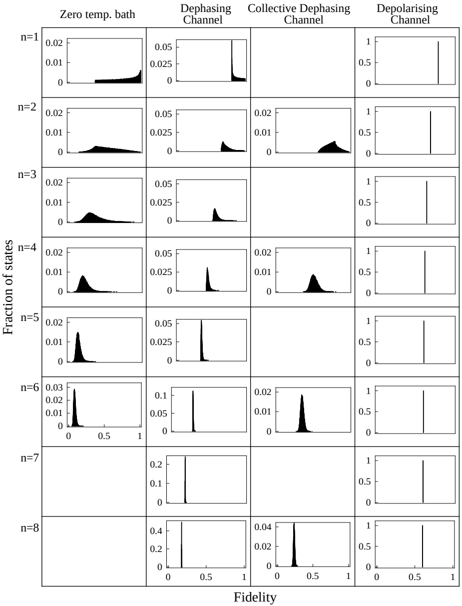
General expression of state under effect of zero temperature bath

The general expression of a state under the effect of zero temperature bath is given by the following expression

$$\rho(t) = \begin{pmatrix} e^{-t\gamma}\rho_{11}(0) & e^{-t\gamma/2}\rho_{12}(0) \\ e^{-t\gamma/2}\rho_{21}(0) & (1-e^{-t\gamma})\rho_{11}(0)+\rho_{22}(0) \end{pmatrix}$$

where γ corresponds to the coupling strength of the system with the environment.

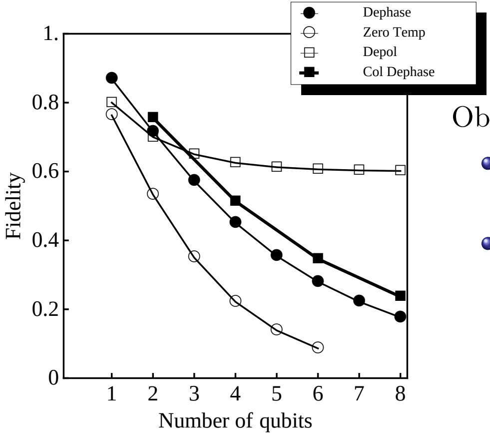
Fidelity distribution for general states



Observations made:

- All the states under the effect of depolarising channel are getting destroyed in exactly same way.
- Variance in the fidelity distribution is more in case of zero temperature bath in comparison to other channels.

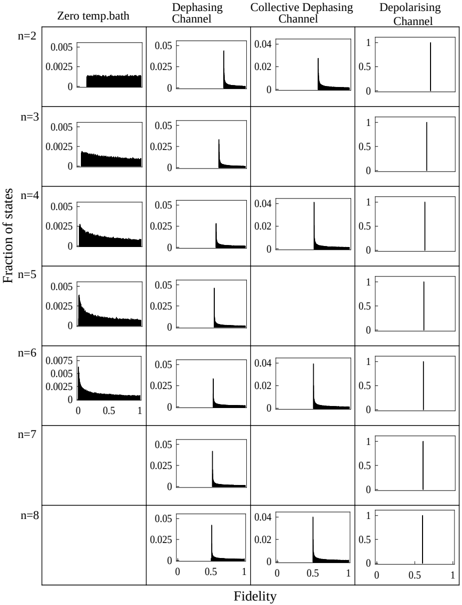
Relative effect of all four channels on general state



Observations made:

- Impact of depolarising channel is minimum.
- The state decays under all other channels in a similar way.

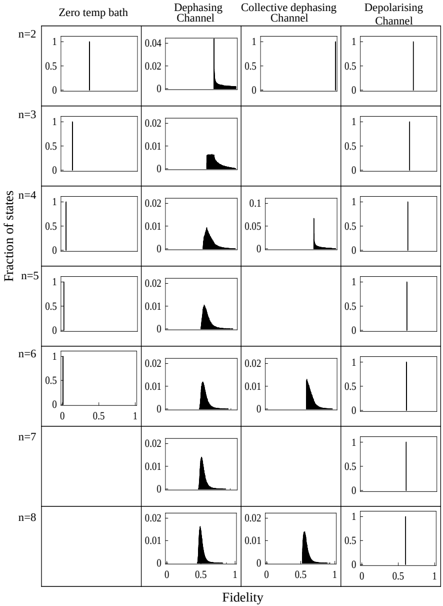
Fidelity distribution for general GHZ states



Observations made:

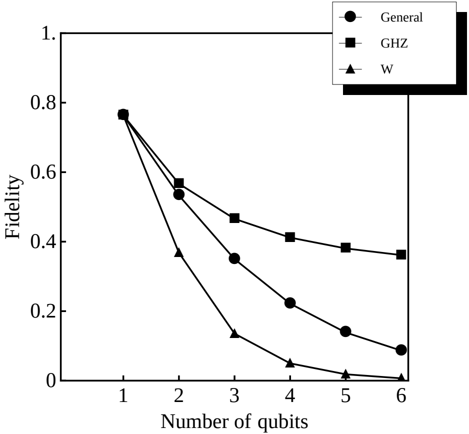
- Fidelity distribution for depolarsing channel is same as earlier.
- Fidelity distribution for dephasing and collective dephasing is completely similar.
- Fidelity distributions is even more scattered than in general states.

Fidelity distribution for general W-states



Observations made:

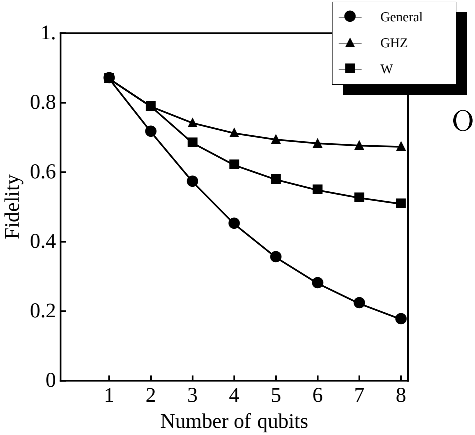
- fidelity distributions of w states for zero temperature bath is peculiar, it is completely opposite to its impact on general GHZ states.
- Effect of all other channels is similar to the other states.



Observations made:

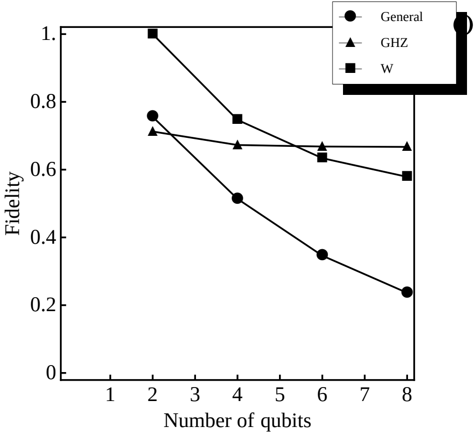
- In this case we have considered the states upto 6 qubits.
- The decay of general W state is fastest.
- The decay of general GHZ state is least.
- Comparing the the number of occupied states and phases involved, two states don't seem to be much different.

Dephasing channel



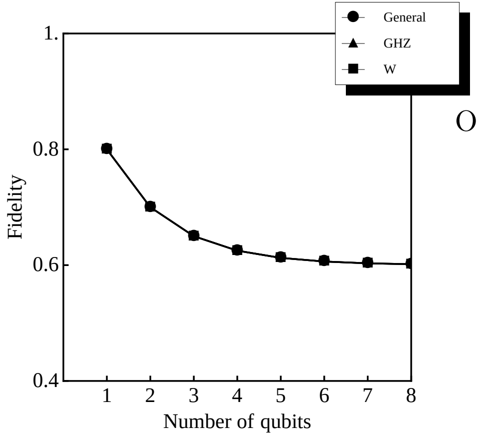
Observations made:

- The effect of dephasing channel is minimum in case of general GHZ state and maximum in case of general W state



Observations made:

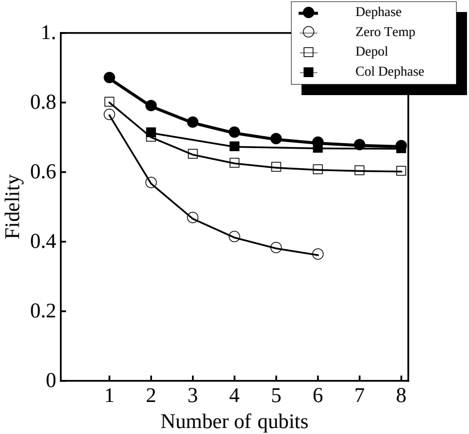
- The effect of collective dephasing channel on general GHZ state is minimum.
- Effect is maximum on the general state.
- Though the fidelity distribution is similar in all three states, decay rate is different.



Observations made:

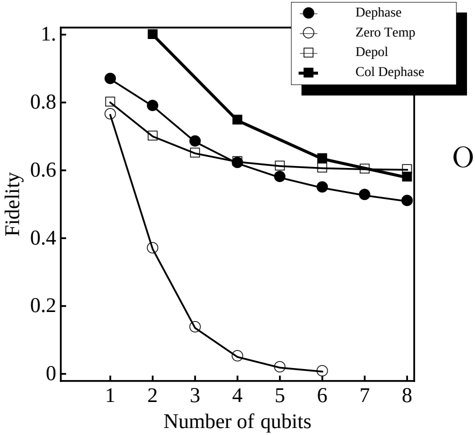
- Three types of states are getting destroyed in the same way. Thus it is independent of the initial state, which is as expected.

Relative effect of different channels on general GHZ state



Observations made:

- Effect of dephasing and collective dephasing channels is almost similar, rather the impact becomes completely similar when we increase the number of qubits.
- Less impact of the above mentioned two channels on the system can be attributed to lesser number of phases involved in the state



Observations made:

- The decay of general W state is similar to the decay of other two types of states.

References

1. Andre R. R. Carvalho, Florian Mintert, and Andreas Buchleitner, PRL 93, 230501 (2004)
2. J. M. Cai, Z. W. Zhou and G. C. Guo, Phys. Rev. A, 72, 022312(2005)