Quantifying the Quantum

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"I think I can safely say that nobody understands Quantum Mechanics"-Richard Feynman, *The Character of Physical Law, 1965.*



Wave function hypothesis

???



- ["] R.P.Feynman, R.Leighton and M.Sands, *The Feyman Lectures on Physics* Vol. **3**, 1963.
- C.Cohen-Tannoudji, B.Diu and F.Laloe, *Quantum Mechanics I*, 2005.
- " R.Shankar, *Principles of Quantum Mechanics*, 1994.



Feynmancs path integral formalism



(,) [,]

An integration over all possible paths that can be taken by the particle in going from A to B.



Apply Path integral formalism to the slit problem, instead of two slits now consider the triple slit problemõ



All possible pathsõ .not only straight paths from source to detector through any slit (classical path) but also looped (non-classical) paths.



Modification of the wave function hypothesis which now becomes:

$$\psi_{AB} = \psi_A + \psi_B + \psi_L$$

 ψ is the contribution due to the looped i.e. non-classical paths.

We have quantified the effect of such non-classical paths in interference experiments for the first time. Proposed simple slit-based experiments which could be used to & e+such non-classical paths in table top experimentsõ.

Whirling waves in interference experiments, J.Samuel, R.Sawant, A.Sinha, S.Sinha and <u>U.Sinha,</u> Under review, 2013, arXiv: 1308.2022



A triple slit experiment provides a simple way to express the failure of the wave function hypothesis in terms of directly measurable quantities. The triple slit (path) set up has been used as a test bed for testing fundamental aspects of quantum mechanics over the last few yearsõ.

- 1. %Quantum Mechanics as quantum measure theory", R.D.Sorkin, Modern Physics Letters A 9, 3119 (1994).
- 2. %*Ruling out multi-order interference in quantum* mechanics+, <u>U.Sinha</u>, C.Couteau, T.Jennewein, R.Laflamme and G.Weihs, *Science* **329**, 418-421 (2010).
- 3. *%Three slit experiments and the structure of quantum theory+*, C.Ududec, H.Barnum and J.Emerson, *Foundations of Physics* **41**, 396-405 (2011).
- 4. *%Testing Born's rule in quantum mechanics for three mutually exclusive* events+, I. Soellner, B. Gschoesser, P.Mai, B.Pressl, Z.Voros and G.Weihs, *Foundations of Physics* **42**, 742-751 (2012).
- 5. %Three path interference using nuclear magnetic resonance: a test of the consistency of Born's rule+, D.K.Park, O.Moussa and R.Laflamme, New Journal of Physics **14**, 113025 (2012).
- 6. *%Three-slit experiments and quantum non-locality+*, G.Niestegge, *Foundations of Physics* **43**, 805-812 (2013).







The Quantum World: A place where there is no penalty for interference - Raymond Laflamme, IQC.







Interference and Sum Rule

Rafael D. Sorkin, *Quantum Mechanics as Quantum Measure Theory,* Modern Physics Letters A **9**, 3119-3127 (1994).



Interference describes the deviation from the classical additivity of the probabilities of mutually exclusive events.

If additivity holds, we call that a sum rule.

A sum rule says that an interference term I = 0

Define a hierarchy of interference terms:



Sum Rules

I(A) = P(A) $I(A, B) = P(A \sqcup B) - P(A) - P(B)$

 $I(A, B, C) = P(A \sqcup B \sqcup C) - P(A \sqcup B) - P(A \sqcup C) - P(B \sqcup C) + P(A) + P(B) + P(C)$

The zeroeth sum rule needs to be violated $(I(A) \neq 0)$ for a non-trivial measure.

If the first sum rule holds (I(A,B) = 0) one gets regular probability theory, e.g. for classical stochastic processes.

Violation of the first sum $(I(A,B) \neq 0)$ rule consistent with quantum mechanics.

A sum rule always entails that the higher ones in the hierarchy hold.

As far as we know, the second sum rule holds in known physics:

I(A,B,C) = 0

i.e. triadditivity of mutually exclusive possibilities is always true!

Can we test this?



Three Slits

Particles encounter three slits, that are assumed to be **mutually exclusive possibilities** / paths (no loops)



$$P(A, B, C) = |\psi_A + \psi_B + \psi_C|^2 =$$

$$= |\psi_A|^2 + |\psi_B|^2 + |\psi_C|^2 +$$

$$+ \psi_A^* \psi_B + \psi_B^* \psi_A + \psi_A^* \psi_C + \psi_C^* \psi_A + \psi_B^* \psi_C + \psi_C^* \psi_B =$$

$$= P(A) + P(B) + P(C) + I(A, B) + I(A, C) + I(B, C)$$

$$= P(A) + P(B) + P(C) +$$

$$+ P(A, B) - P(A) - P(B) +$$

$$+ P(A, C) - P(A) - P(C) +$$

$$+ P(B, C) - P(B) - P(C) =$$

$$= P(A, B) + P(A, C) + P(B, C) - P(A) - P(B) - P(C)$$

 $I(A, B, C) := P(A, B, C) - P(A, B) - P(A, C) - P(B, C) + P(A) + P(B) + P(C) \equiv 0$



Testing the 2nd Sum Rule



 $\epsilon := P(A, B, C) - P(A, B) - P(A, C) - P(B, C) + P(A) + P(B) + P(C) - P(0)$

 $\frac{1}{\delta}$

 δ is a suitable normalization factor.

U.Sinha, C.Couteau, T.Jennewein, R.Laflamme and G.Weihs in *Science*, Vol. **329**, No. 5990, pp 418-421, 23rd July 2010.



 $\kappa = \frac{\epsilon}{\delta}$

Many experimental boundsõ.

- All experiments reported in literature which measure this quantity, implicitly assume that only classical paths contribute to the interference.
- ^{π} Numerical simulations of classical Maxwellos equations using FDTD analysis have shown that due to difference in boundary conditions, the classical κ can be non-zero^{*}.
- "What is the effect of ‰on-classical+paths on κ ???
- [%] A non-zero contribution to κ by taking into account all possible paths in the path integral formalism
 - => Wave function hypothesis not strictly true.
 - => Non zero κ does not necessarily imply falsification of Born rule.
 - =>

First theoretical bound on \mathcal{K} .





Path integrals in the lab: The green line demonstrates a representative classical path. The purple line demonstrates a representative non classical path.

Observation of a non-zero κ which is expected from the proposed modification to the wavefunction hypothesis would in fact serve as an experimental validation of the full scope of the Feynman path integral formalism.



The calculation õ

Assumptions:

- We use the free particle propagator in our calculations. For a particle in free space and away from the slits, this is a reasonable approximation. We account for the slits by simply removing from the integral all paths that pass through the opaque metal.
- We use a steady state description. We assume that the detectors integrate over a duration of time which is much longer than any other time scale in the problem like for instance the travel time across the apparatus which justifies the use of the steady state approximation. We go on to use the time independent Feynman path integral.
- We also suppose that the wavelength of the incident source is much smaller than any other length scale in the problem, the sizes and separations of the slits and the distance to the source and the detector.
- We will present results which are applicable to the Fraunhofer regime.



The normalized energy space propagator K for a free particle with wave number k from a position \mathbf{r} to \mathbf{r} :

$$K(\vec{r},\vec{r}') = \frac{k}{2\pi i} \frac{1}{|\vec{r}-\vec{r}'|} e^{ik|\vec{r}-\vec{r}'|} .$$
(1)



- 1. Paths which cross the slit plane exactly once pertaining to a probability amplitude K_c (green)
- 2. Paths which cross the slit plane more than once pertaining to a probability amplitude K_{nc} (purple)

$$\therefore K = K_c + K_{nc}$$

(2)

*We should point out that there are corrections to the propagator due to closed loops in momentum space from quantum field theory considerations. We have explicitly estimated that the effects of such corrections will be negligibly small.



We wish to estimate K_{nc} relative to K_c to test the domain of validity of the wave function hypothesis.

A representative K_c in our problem is the probability to go from the source (-L,0,0) to the detector (D,y_D,0) through slit A which we call $K_c^A(S,D,k)$. This makes use of the decomposition theorem, i.e. a path in FPI can be broken into many sub-paths and the propagator is the product of the individual propagators. Thus:

$$K_c^A = -\left(\frac{k}{2\pi}\right)^2 \int_{d-\frac{w}{2}}^{d+\frac{w}{2}} \int_{-h}^{h} dy \, dz \, \frac{e^{ik(l_1+l_2)}}{l_1 l_2} \tag{3}$$

In the Fraunhofer regime this becomes:

$$K_{c}^{A} = -\gamma \left(\frac{k}{2\pi}\right)^{2} \int_{d-\frac{w}{2}}^{d+\frac{w}{2}} \int_{-h}^{h} dy \, dz \, e^{ik\left[\frac{(y)^{2}+z^{2}}{2L}+\frac{(y_{D}-y)^{2}+z^{2}}{2D}\right]}$$
(4)

These are Fresnel integrals and have been solved using Mathematica.

A representative K_{nc} in our problem is the probability to go from the source (-L,0,0) to the detector (D,y_D,0) following a path where the particle goes from the source to

the first slit, then loops around the second and third slits before proceeding to the detector. We call this $K_{nc}{}^{A}(S,D,k)$.

$$K_{nc}^A = i(rac{k}{2\pi})^3 \int dy_1 dy_2 dz_1 dz_2 rac{e^{ik(l_1+l_2+l_3)}}{l_1 l_2 l_3}$$

 y_1 integral runs over slit A, y_2 integral runs over slits B and C.

$$d = \text{inter-slit distance}$$

$$w = \text{slit width}$$

$$h = \text{slit height}$$

$$l_1 \approx L + \frac{y^2 + z^2}{2L}$$

$$l_2 \approx D + \frac{(y_D - y)^2 + z^2}{2D}$$

$$\gamma = \frac{1}{LD} e^{ik(L+D)}$$

$$\begin{bmatrix} l_1^2 = (y_1 - y_S)^2 + L^2 + z_1^2 \\ l_2^2 = (y_2 - y_1)^2 + (z_2 - z_1)^2 \\ l_3^2 = (y_D - y_2)^2 + D^2 + z_2^2. \end{bmatrix}$$
(5)



Taking approximations appropriate to the Fraunhofer regime, using stationary phase approximation for the oscillatory integrals, eqn.(5) becomes:

$$K_{nc}^{A} = \gamma i^{3/2} \left(\frac{k}{2\pi}\right)^{5/2} \int dy_1 \, dy_2 \, dz_1 \, (y_2 - y_1)^{-1/2} e^{ik \left[\frac{y_1^2 + z_1^2}{2L} + (y_2 - y_1) + \frac{(y_D - y_2)^2 + z_1^2}{2D}\right]} \tag{6}$$

In terms of K_c and K_{nc} the propagator to go from the source to the detector when all three slits are open is:

$$K^{ABC} = K_c^A + K_c^B + K_c^C + K_{nc}^{ABC}$$
(7)

Similarly,

 K_{nc}^{ABC} nc terms when all slits open, K_{nc}^{AB} nc terms when A and B openõ.

(9)

$$K^{AB} = K_c^A + K_c^B + K_{nc}^{AB} \tag{8}$$

Thus, in terms of propagators:

And,
$$\epsilon = |K^{ABC}|^2 - |K^{AB}|^2 - |K^{AC}|^2 - |K^{BC}|^2 + |K^A|^2 + |K^B|^2 + |K^C|^2$$

 $\frac{\delta}{\delta}$

κ

where $\delta = |K^{ABC}(0)|^2$ and $|K^{ABC}(0)|^2$ is the value of $|K^{ABC}|^2$ at the central maximum of the triple slit interference pattern.





Normalized values of κ as a function of detector position (blue line).

 $I_n = |K^{ABC}(y)|^2 / |K^{ABC}(0)|^2 \text{ which is a plot of the triple slit interference pattern as a function of detector position (red line), it gives a clearer understanding of the modulation in the plot for <math>\kappa$. Slit width = 30µm, inter-slit distance = 100µm, distance between source and slits and slits and detector = 18cm and incident wavelength = 810nm (parameters taken from [*])

* U.Sinha et al, *Science*, Vol. **329**, No. 5990, pp 418-421, 23rd July 2010.





Normalized values of κ as a function of detector position (blue line).

 $I_n = |K^{ABC}(y)|^2 / |K^{ABC}(0)|^2$ which is a plot of the triple slit interference pattern as a function of detector position (red line), it gives a clearer understanding of the modulation in the plot for κ . Slit width = 62nm, inter-slit distance = 272nm, distance between source and slits = 30.5cm and slits and detector = 24cm and DeBroglie wavelength = 50pm (parameters taken from [**])

** R.Bach et al, New Journal of Physics 15, 033018 (2013)



"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong."

Richard Feynman



Triple slit interference pattern









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U.Sinha, C.Couteau, T.Jennewein, R.Laflamme and G.Weihs in *Science,* Vol. **329**, No. 5990, pp 418-421, 23rd July 2010.



HIGHER ORDER INTERFERENCE

- Improve throughput
- Shutter paths independently
- Access entire phase space



Institute for Quantum



I.Sollner, B.Gschosser, P.Mai, B.Pressl, Zoltan Voros and Gregor Weihs, Foundatons of Physics, Vol. 42, No.6, pp 742751, June 2012.





 $\kappa = 4 \times 10^{-5} \pm 4 \times 10^{-4}$



NMR Implementation



D.K.Park, O.Moussa and R.Laflamme, New Journal of Physics, Vol. 14, No.11, pp 114025, November 2012.



"The triple slit experiment that we had performed earlier could serve as the perfect table top experiment to test for the presence of non-classical pathsõ.

"The earlier version had systematic errors (dond) get me started!!) and we were limited to being only able to measure a value of κ upto 10^{-2} .

" future version of the experiment which is devoid of such systematic errors could definitely be used to quantify non-classical paths through a κ measurement.

But we could do betterõ $\,\tilde{o}\,\,\tilde{o}\,\,\tilde{o}$.



The non-zeroness of κ is very strongly dependent on certain experimental parameters and one can definitely find a parameter regime where κ would be even bigger, hence easier to observeo

"Keeping all other experimental parameters fixed, κ increases with an increase in wavelength.

[%] For instance, for an incident beam of wavelength 4cm (microwave regime) and slit width of 120cm and inter-slit distance of 400cm, a theoretical estimate for κ would be 10⁻³. This is an experiment which can be performed for instance in a radio-astronomy lab.

"We have been talking to our friends in radio-astronomy (RRI is a world leader in radio-astronomy research and you should definitely visit RRI to know more!) and they are very excited! We have been doing preliminary simulations for their set-up and the results are encouraging to say the least

[~] Such explicit demonstration of non-classical paths is very rare in literatureõ The other systems where such demos can happen include measuring an Aharonov-Bohm type phase using charged particles. These are very fiddly experiments and sometimes the results are very open to interpretationõ Our proposal is for a % imple+slit based interference experiment and letœ face it, the double slit experiment was voted the most beautiful experiment of all time by Physics World a few years ago!!



On going work

Non classical paths in a quantum walk

Aravind H.V., A.Sinha and U.Sinha



2 Random walk from the diffusion equation:

 $\textbf{Diffusion-equation} \leftrightarrow \textbf{Random-walk correspondence:}$

(Heat/Diffusion equation in 1 dimension): $\frac{\partial u(x,t)}{\partial t} = \alpha \frac{\partial^2 u(x,t)}{\partial x^2}$

$$\begin{aligned} u(x,t+\partial t) &= u(x,t) + \alpha \frac{\partial t}{(\partial x)^2} \left[u(x+\partial x,t) + u(x-\partial x,t) - 2 u(x,t) \right] \\ \text{Discretizing:} \quad u(x,t+\Delta t) &= \left[1 - 2 \frac{\Delta t}{(\Delta x)^2} \right] u(x,t) + \alpha \frac{\Delta t}{(\Delta x)^2} \left[u(x+\Delta x,t) + u(x-\Delta x,t) \right] \end{aligned}$$

If we now interpret u(x,t) as the probability of finding a particle in the neighborhood $\left[x - \frac{\Delta x}{2}, x + \frac{\Delta x}{2}\right]$ during the time interval $[t, t + \Delta t]$, it turns out that such a particle will be executing a random walk.

INTERPRETATION:

 $\begin{bmatrix} 1 - 2\frac{\Delta t}{(\Delta x)^2} \end{bmatrix} u(x,t) \Rightarrow \text{ probability of the particle remaining in the same region during the time interval } \Delta t$ $\alpha \frac{\Delta t}{(\Delta x)^2} u(x + \Delta x, t) \Rightarrow \text{ probability of the particle in the neighborhood } x + \Delta x \text{ "diffusing" into the neighborhood around } x \text{ in the time interval } \Delta t$ $\alpha \frac{\Delta t}{(\Delta x)^2} u(x - \Delta x, t) \Rightarrow \text{ probability of the particle in the neighborhood } x - \Delta x \text{ "diffusing" into the neighborhood } x - \Delta x \text{$

⇒ probability of the particle in the neighborhood $x - \Delta x$ "diffusing" into the neighborhood around x in the time interval Δt

Adopting the above method for the case of the Schrödinger equation will fail to provide us with a unitary evolution operator



Constructing quantum walk from 1-D Schrodinger equation

Consider the Schrodinger Equation:

$$\begin{split} i\hbar \frac{\partial \psi}{\partial t} &= \frac{-\hbar^2}{2m} \frac{(\partial^2 \psi)}{(\partial x)^2} \\ \frac{\partial \psi}{\partial t} &= \frac{i\hbar}{2m} \frac{(\partial^2 \psi)}{(\partial x)^2} \\ \psi_x(t+\partial t) &= \psi_t(x) + \frac{i\hbar}{2m} \frac{\partial t}{(\partial x)^2} \left[\psi_t(x+\partial x) + \psi_t(x-\partial x) - 2\psi_t(x)\right] \end{split}$$

Now we will consider the coordinates to be discrete and ψ_t to be a vector in the index x. Also define the second derivative to be the matrix operator:

So the equation now becomes

$$\psi_x(t+\partial t) = \psi_x(t) - \frac{i\hbar}{2m} \frac{\partial t}{(\partial x)^2} M_{x,\bar{x}} \psi_{\bar{x}}(t) \quad \text{(here, } \bar{x} \text{ index is summed over)}$$

define: $C := \frac{\hbar}{2m} \frac{\partial t}{(\partial x)^2}$

 $\vec{\psi}(t+\partial t) = e^{-iC\overline{M}}\vec{\psi}(t)$ (here we have dropped the index notation and used $\vec{\psi}$ and \overline{M} instead)

Now $e^{-iC\overline{M}}$ is a difficult term to evaluate so we try to find good approximations for it,



(\		(\		(\
	-1	2	-1	0	0	0	0				0	1	-1	0	0	0	0				-1	1	0	0	0	0	0	
	0	-1	2	-1	0	0	0				0	-1	1	0	0	0	0		\perp		0	0	1	-1	0	0	0	
	0	0	-1	2	-1	0	0		=		0	0	0	1	$^{-1}$	0	0		' I		0	0	-1	1	0	0	0	
	0	0	0	-1	2	-1	0				0	0	0	-1	1	0	0				0	0	0	0	1	-1	0	
	0	0	0	0	-1	2	-1				0	0	0	0	0	1	-1				0	0	0	0	-1	1	0	
\)		()		(/

Now the first matrix on the right hand side will be addressed as \overline{M}_0 and second matrix as \overline{M}_1 , so

 $\vec{\psi} (t + \partial t) = e^{-iC(\overline{M}_0 + \overline{M}_1)} \vec{\psi} (t)$ $\approx e^{-iC\overline{M}_0} e^{-iC\overline{M}_1} \vec{\psi} (t)$

(by doing this, we are neglecting terms of higher order in C, which is a small, but more importantly this method gives us a unitry evolution operator as opposed to the previous approximation)

Continuing this way, we get the expressions for the time evolution operators to be:

$$e^{-iC\overline{M}_{0}} = 1 - \frac{\overline{M}_{0}}{2} (1 - e^{-2iC})$$
$$e^{-iC\overline{M}_{1}} = 1 - \frac{\overline{M}_{1}}{2} (1 - e^{-2iC})$$

So, Define:

$$U_0 := 1 - \frac{M_0}{2} (1 - e^{-2iC})$$
$$U_1 := 1 - \frac{\bar{M}_1}{2} (1 - e^{-2iC})$$

The final evolution operator is a product of the two.

Similar method can also be adopted for the massless Dirac equation.



The Matrix,

This comes out to be

$$\bar{U}_{0} = \begin{pmatrix} \cos\left(\frac{\partial t}{\partial x}\right)\bar{I} & -i\sin\left(\frac{\partial t}{\partial x}\right)\bar{M}_{0} \\ i\sin\left(\frac{\partial t}{\partial x}\right)\bar{M}_{0} & \cos\left(\frac{\partial t}{\partial x}\right)\bar{I} \end{pmatrix} \qquad \bar{U}_{1} = \begin{pmatrix} \cos\left(\frac{\partial t}{\partial x}\right)\bar{I} & -i\sin\left(\frac{\partial t}{\partial x}\right)\bar{M}_{1} \\ i\sin\left(\frac{\partial t}{\partial x}\right)\bar{M}_{1} & \cos\left(\frac{\partial t}{\partial x}\right)\bar{I} \end{pmatrix}$$



Now, we consider how these operators act on the wavefunctions,

$$\bar{U}_{0} \vec{\psi} = \begin{pmatrix} \cos\left(\frac{\partial t}{\partial x}\right) \bar{I} & -i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{0} \\ i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{0} & \cos\left(\frac{\partial t}{\partial x}\right) \bar{I} \end{pmatrix} \begin{pmatrix} \psi_{\uparrow} \\ \psi_{\downarrow} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\partial t}{\partial x}\right) \psi_{\uparrow} - i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{0} \psi_{\downarrow} \\ \cos\left(\frac{\partial t}{\partial x}\right) \psi_{\downarrow} + i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{0} \psi_{\uparrow} \end{pmatrix}$$

$$\text{similarly:} \quad \bar{U}_{1} \vec{\psi} = \begin{pmatrix} \cos\left(\frac{\partial t}{\partial x}\right) \psi_{\uparrow} - i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{1} \psi_{\downarrow} \\ \cos\left(\frac{\partial t}{\partial x}\right) \psi_{\downarrow} + i\sin\left(\frac{\partial t}{\partial x}\right) \bar{M}_{1} \psi_{\uparrow} \end{pmatrix}$$

$$\text{with:} \quad \begin{pmatrix} \bar{M}_{0} \rangle_{x, \bar{x}} \psi_{\bar{x}} = (-1)^{(\bar{x})} \psi_{[\bar{x} + (-1)^{(\bar{x})}]} \\ (\bar{M}_{1})_{x, \bar{x}} \psi_{\bar{x}} = (-1)^{(\bar{x} + 1)} \psi_{[\bar{x} + (-1)^{(\bar{x} + 1)}]} \end{cases}$$

At this point we are done with the discretization of Dirac equation.

3 Some results for the case of Schrodinger's equation

Parameters used in the simulation:

Distance between source and slit : 10CM

Distance between slit and screen : $10 \,\mathrm{CM}$

Slit width : $5\times 10^{-4}{\rm M}$

Distance between slits : 50×10^{-4} M

Wavelength : 4×10^{-4} M



































Future plans

- "Better understanding of the non classical paths in a quantum walk using real experimental parameters.
- "Perform the triple slit experiment to See+non classical paths both in IR as well as microwave regime.
- ["] Looking at the connection between Feynman paths and Bohmian trajectories with Dipankar Home and Aninda Sinha.
- " Explore the concept of triple slit based % putrits+.













http://www.rri.res.in/downloads/JRF-QuIC-Announcement.pdf



11.8, 2010 Vine Cottage, High Street Lyncham, Oxfordshire OX7 6QL Tel/Fax: 01993 830 492 Dear Dr. Suika, 1 thought I would like to tell you of the great surprise and pleasure your paper in the July issue of Science has given me. I am the son of Max Born, and an greatly gratified that theoretical work he did in 1926 (when Iwas glive) should now receive experimental confirmation,



3/4 of a century later. I am a biomedical scientist, for from properly understanding your work - and my fathers litter, of course. But he and I were very close in every way, and so you can magne my gladhers when a physicist found of mine drew my attention to you, clearly excellent work. please lit me know of any Jollow-up papers on this. With best withes, your spicerely, Custav Born.