Weak Measurements for State Estimation in Quantum Mechanics

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QIP/FQM at IIER Mohali

- Arvind (Physics, Theory)
- Kavita Dorai (Physics NMR Expt)
- Mandip Singh (Physics Single photons BEC)
- Lingraj Sahu (Mathematics)
- Ritabrata Sengupta (Phd)
- Shruti Dogra (Phd)
- **Debmalaya Das (Phd)**
- Harpreet Singh (Phd)
- Jeebarathinam (Phd)
- Vikesh (MS-graduated)
- Abhishek (MS-graduated)
- Rajeev Nehra (MS)
- Aakash (MS)
- Mayank Mishra (MS)
- Himanshu (MS)
- Atul Mantri (MS)

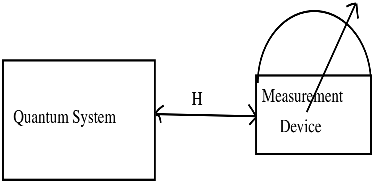
Plan of Talk

- Quantum measurement problem
- von Neumann's model of projective measurement
- Weak Measurement and Weak Values
- Applications
- Quantum State Tomography
- Quantum Tomography of a qubit with projective measurements.
- Why Weak Measurement for state tomography?
- Algorithm
- Simulation
- Results

Quantum Measurement Problem

- Emergence of pointer positions from quantum states
- Probabilities from wave function
- Classical form quantum
- Information extraction from quantum systems

von Neumann's Model



Interaction Hamiltonian:

$$H = g\delta(t - t_0) A \otimes p \qquad (1)$$

g =coupling Strength
A =system Observable
p =momentum conjugate to the pointer co-ordinate q

System state:

$$|\Psi_{in}\rangle = \sum_i C_i \, |A = a_i\rangle \qquad (2)$$

Pointer state:

$$|\Phi\rangle = (\frac{\epsilon}{2\pi})^{\frac{1}{4}} \int dq \exp(-\frac{\epsilon q^2}{4}) \, |q\rangle \qquad (3)$$

$\epsilon = (\frac{1}{\Delta q})^2$
Joint unitary evolution:

$$|\Psi_{in}\rangle \otimes |\Phi\rangle \longrightarrow \exp(-i \int H dt) \, |\Psi_{in}\rangle \otimes |\Phi\rangle = (\frac{\epsilon}{2\pi})^{\frac{1}{4}} \sum_i C_i \int dq \exp(-\frac{\epsilon(q - g a_i)^2}{4}) \, |a_i\rangle \, |q\rangle \qquad (4)$$

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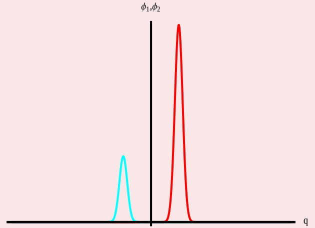
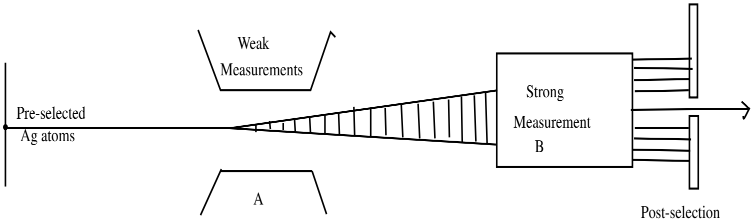
Weak Measurement in Quantum Mechanics

Strong vs Weak



$g << 1$ or $\epsilon << 1$ limit with postselection of a pure state $|\Psi_f\rangle$ of the system.



Weak Value in Quantum Mechanics

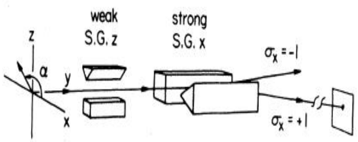
Pointer state after post-selection:

$$\left|\Phi_f\right\rangle = \left\langle\Psi_f\right| \exp -igA \otimes p \left|\Psi_{in}\right\rangle \otimes \left|\Phi_{in}\right\rangle \simeq \left(\frac{\epsilon}{2\pi}\right)^{\frac{1}{4}} \left\langle\Psi_f\right|\left.\Psi_{in}\right\rangle \int dq \exp\left(-\frac{\epsilon(q-gA_w)^2}{4}\right) |q\rangle \quad (5)$$

Weak value of A:

$$A_w = \frac{\left\langle\Psi_f\right| A \left|\Psi_{in}\right\rangle}{\left\langle\Psi_f\right|\left.\Psi_{in}\right\rangle} \quad (6)$$

If $\left|\Psi_f\right\rangle$ is nearly orthogonal to $|\Psi_{in}\rangle$ then A_w assumes a large value and can even go outside the eigenvalue spectrum !



$$(\sigma_z)_w \propto \tan\frac{\alpha}{2} \quad (7)$$

In the weak measurement limit, $\epsilon \longrightarrow 0$ which makes $(\sigma_z)_w$ assume a large value if the post-selected state is nearly orthogonal to the preselected one ($\alpha \approx \pi$).

More on Weak Values

- Weak values can be complex.
- The real and imaginary parts of a weak value can be related to the initial and the final expectation values of position and momentum coordinates of the apparatus as follows:

If $A_w = a + \iota b$ then

$$\langle q \rangle_f = \langle q \rangle_{in} + ga(m\frac{d}{dt}Var_q) \tag{8}$$

and

$$\langle p \rangle_f = \langle p \rangle_{in} + 2gbVar_p \tag{9}$$

where $\langle q \rangle_{in} = \langle \Phi_{in} | q | \Phi_{in} \rangle$ and $\langle q \rangle_f = \left\langle \Phi_f \right| q \left| \Phi_f \right\rangle$.

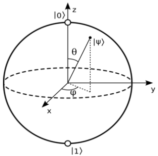
Weak Measurements: Some Applications

- Amplification of small signals.
- Testing Bell Inequalities and Legget-Garg Inequalities.
- Resolving Quantum Paradoxes: Wave-particle duality in double-slit experiment, Schroedinger's cat paradox, Hardy's paradox, etc.
- Direct measurement of photon wave function.
- Studying Super Quantum Discord.
- **Quantum State Estimation and Tomography.**

Quantum State Tomography

Any general quantum state is represented by

$$\rho = \frac{1}{2}(I + x\sigma_x + y\sigma_y + z\sigma_z) \tag{10}$$



where

$$\begin{aligned} x &= \langle \sigma_x \rangle \\ y &= \langle \sigma_y \rangle \\ z &= \langle \sigma_z \rangle \end{aligned} \tag{11}$$

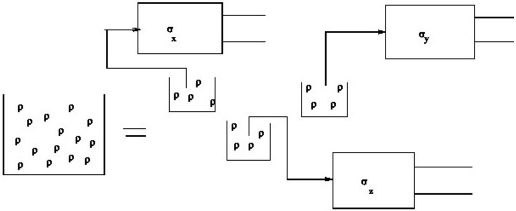
$(x, y, z) \equiv$ any general point inside the Bloch sphere.

Objective

Estimate (x, y, z) to estimate ρ.

Quantum State Tomography of a Qubit with Projective Measurements

The original ensemble is divided into three subensembles and upon the members of each, only one type of projective measurement is performed. If a measurement is made on any member then it cannot be reused for another measurement as its original state is destroyed.



The problem

- For ideal state estimation using projective measurements one needs an ensemble of infinite size which is not available in practice.
- If to start of we get an ensemble of very small size the estimation is poor.

Question:

Is there some other method using which we can do state tomography better than projective measurement in case of a small ensemble?

Why Weak Measurement?

Consider an initial pure spin state

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\frac{\alpha}{2} + \sin\frac{\alpha}{2} \\ \cos\frac{\alpha}{2} - \sin\frac{\alpha}{2} \end{pmatrix}$$

or

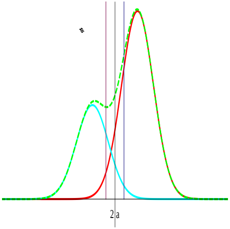
$$\rho_{in} = \frac{1}{2} \begin{pmatrix} 1 + \sin\alpha & \cos\alpha \\ \cos\alpha & 1 - \sin\alpha \end{pmatrix} \tag{12}$$

State after weak measurement with $g = 1$ and $\epsilon << 1$, considering terms which are till second-order in ϵ:

$$\rho_f = \frac{1}{2} \begin{pmatrix} 1 + \sin\alpha & (1 - \frac{\epsilon}{8})\cos\alpha \\ (1 - \frac{\epsilon}{8})\cos\alpha & 1 - \sin\alpha \end{pmatrix} \tag{13}$$

The disturbance induced to the system by a weak measurement is very small. This opens up the possibility of using a particular member of the ensemble more than once.

Measuring "weakly"



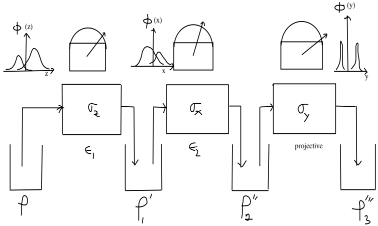
- In projective measurement of a qubit, the meter needle indicates one or the other result. This is possible as the distribution of the meter needle positions is not wide.
- In weak measurement case, ϵ being small, the distribution is wide and some needle positions may indicate any of the two outcomes. Such meter readings are ambiguous.

- Any meter reading falling in this region is discarded.
- The experimental expectation value of the observable is calculated by taking the the readings lying outside this region.

Algorithm

Procedure of State Estimation with weak measurements

We begin with the first box, represented by an ensemble of states ρ. One by one, a member is taken and σ_z measurement is performed. This measurement is of a strength, defined by the parameter ϵ_1. The outcomes are collected and put into another box which represents the ensemble of states given by ρ_1'. This is treated as the input for the second measurement of σ_x, performed one by one on the members of the second box. This measurement strength is defined by the parameter ϵ_2. The third box contains the outputs given by ρ_2'' and the input for the third measurement of σ_y which is projective. The final ensemble of states is represented by the box marked ρ_3'''.

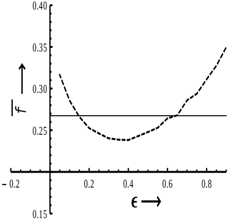


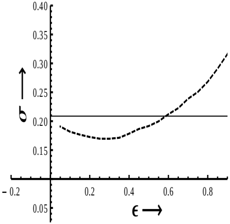
Simulation

- The simulation is performed in Wolfram Mathematica 9 and a fidelity or a distance measure given by
$$f = (x - x_{est})^2 + (y - y_{est})^2 + (z - z_{est})^2 \tag{14}$$
is calculated. Every experiment is performed 10000 times and the mean of these fidelities $\overline{f}$ is plotted with ϵ.
- States $\rho_1 = \begin{pmatrix} 0.699354 & -0.192359 + 0.0211906\iota \\ -0.192359 - 0.0211906\iota & 0.300646 \end{pmatrix}$

  and $\rho_2 = \begin{pmatrix} 0.527348 & -0.300716 + 0.198788\iota \\ -0.300716 - 0.198788\iota & 0.472652 \end{pmatrix}$ are used.
- In another study, the same procedure is repeated for 1000 randomly generated states and average of these mean fidelities are calculated.
- The corresponding fidelities for projective measurement method are also calculated.

Comparison of Projective and Weak measurement results

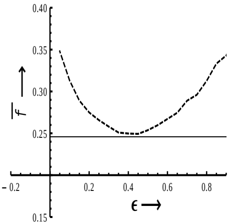


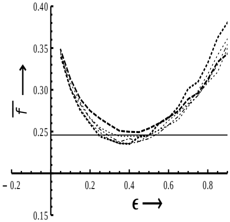


Average fidelity $\overline{f}$ plotted against ϵ (left) and standard-deviation in fidelity σ against ϵ (right) for weak measurement (dashed line) compared to the corresponding quantities in case of projective measurement (continuous line), where $\epsilon_1 = \epsilon_2 = \epsilon$, for the state ρ_1 and an ensemble size of 30. Weak measurement gives a better estimate of this state compared to projective measurement for a range of ϵ values.

Usefulness of discarding

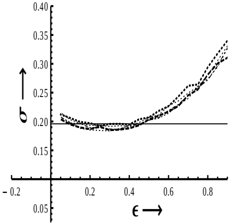
Plot of average fidelity $\overline{f}$ vs ϵ (top left). Weak measurement (dashed thick) fails to perform better than projective measurement (continuous) for the state ρ_2.





Why Discard?

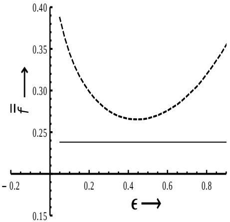
But if discards of $a = 0.2$ (dotted), 0.4 (dotdashed), 0.6 (dotted) and 0.8 (dotted thick) are used then it dips below the projective measuremnet mark (top right plot).
Plot of standard-deviation in fidelity σ against ϵ (right)

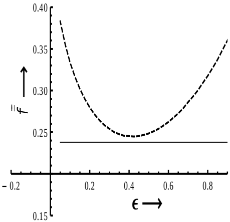


Performance on the average

1000 random states

Plots of mean of average fidelities $\overline{\overline{f}}$ against ϵ without any discard (left) and with a discard of $a = 0.6$ (right). In both cases the performance of projective measurement (black) is better than weak measurement (black dashed). However, while in the former case only 471 states out of 1000 are better estimated, 524 states are better estimated in the latter case.





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Work In progress

- NMR as weak measurement
- Weak measurement as a way to extract information from quantum systems and quantification of the cost.
- Weak measurements, Bell's inequalities and Quantum Entanglement
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Thank you !