A mechanical switch for state transfer in cavity opto-mechanical systems.

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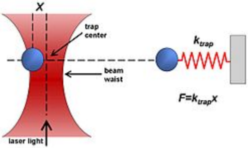
Outline

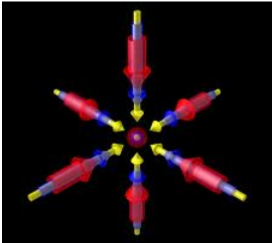
1. An introduction to cavity Opto-Mechanics (OM)
2. Production of quantum states in OM systems
3. Laser cooling of opto-mechanical resonator.
4. Transfer of quantum optical states to mechanical states
5. Quantum state transfer using Interference.
6. Dual cavity dual spring systems.
7. Conclusions.

Radiation Pressure Force.

- Radiation pressure force causes momentum transfer from light to matter. Optical tweezers trap biological samples using such a force.



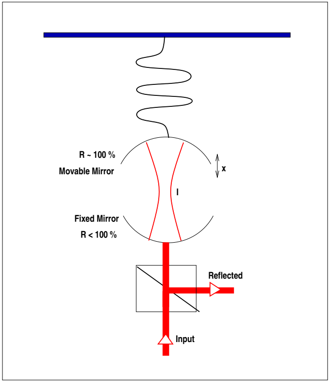
- Radiation pressure force is primarily responsible for laser cooling of neutral alkali atoms that led to their BEC.



Radiation pressure force on a movable cavity mirror.

- The displacement of an end mirror of an optical cavity caused by radiation pressure force of light inside the cavity creates a **non-linear interaction**.

Prototype opto–mechanical system.



$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hat{H}_{int}$$

$$\hat{H}_{int} = \hbar g_0 x_{zpf} \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b})$$

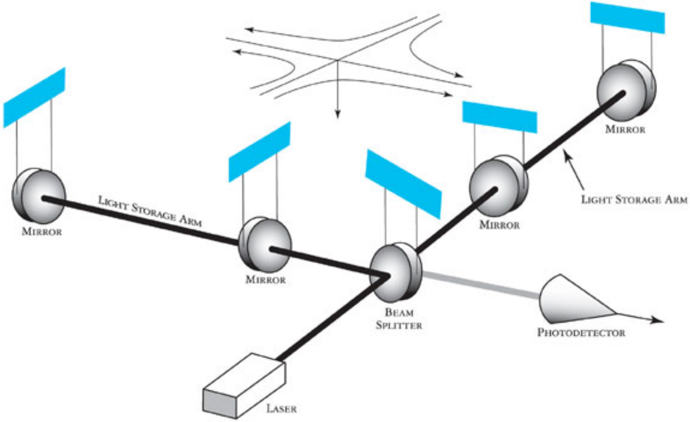
$$\hat{x} = x_{zpf} (\hat{b}^\dagger + \hat{b})$$

$$x_{zpf} = \sqrt{\frac{\hbar}{2\omega_m m}}$$

$$g_0 = \frac{d\omega_c}{dx}$$

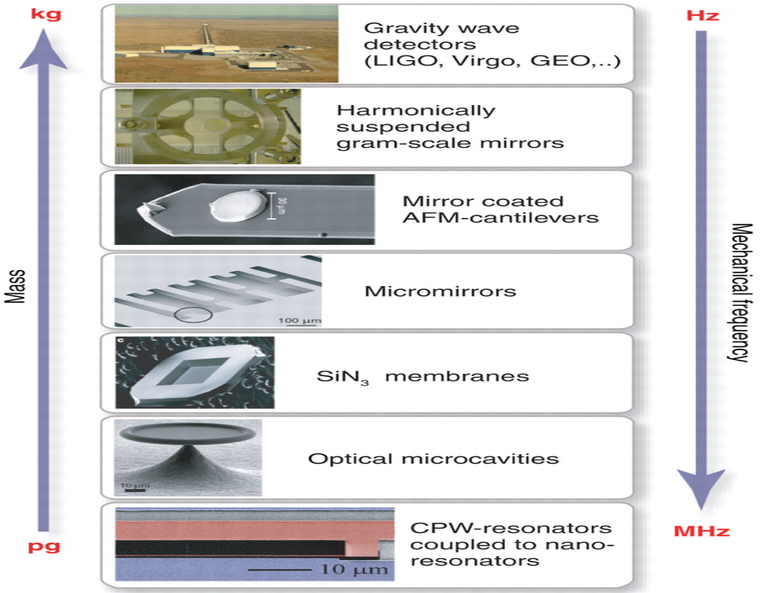
Radiation pressure force induced measurements.

- The displacement of an end mirror of an optical cavity caused by radiation pressure of an external signal (like a gravitational wave) can be measured to an accuracy of $10^{(-19)}$ m $/\sqrt{Hz}$! using optical interference.



Experimental range in opto-mechanical systems.

Science, **321**, 1172, (2008)



Quantum superposition states through time evolution.

S. Mancini et. al, Phys. Rev. A, **55**, 3042 (1997).

For the opto-mechanical Hamiltonian, the time evolution operator is

$$\hat{U}(t) = e^{iE(t)(\hat{a}^\dagger \hat{a})^2} e^{iF(t)(\hat{a}^\dagger \hat{a})\hat{x}(t)} (e^{-i\omega_c \hat{a}^\dagger \hat{a} \frac{t}{\omega_m}} e^{-i\hat{b}^\dagger \hat{b} t})$$

$$\hat{x}(t) = \hat{b} e^{i\frac{t}{2}} + \hat{b}^\dagger e^{-i\frac{t}{2}},\ E(t) = \beta^2(t - sin(t)),\ F(t) = 2\beta sin(t/2),$$
$$\beta = \frac{G}{\omega_m},\ G = g_0 x_{zpf}.$$

$$\rho_{th} = (1-z)\sum_n z^n |n\rangle\langle n|, z = e^{\frac{-\hbar\omega_m}{k_B T}}$$

$$\rho(0) = |\alpha_0\rangle\langle\alpha_0| \otimes \rho_{th}$$

$$\rho(t^*) = e^{iE(t^*)(\hat{a}^\dagger \hat{a})^2} |\alpha_0\rangle\langle\alpha_0| \otimes \rho_{th} e^{iE(t^*)(\hat{a}^\dagger \hat{a})^2} \quad with \quad t^* = 2\pi m_1$$

At t^*, $F(t^*) = 0$; The cavity field and the thermal field are disentangled.

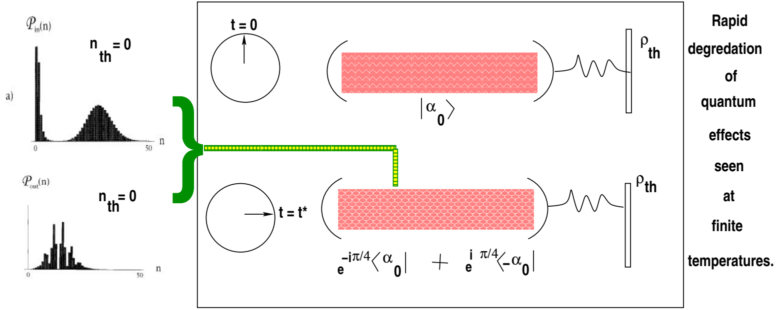
Quantum superposition states through time evolution.

S. Mancini et. al, Phys. Rev. A, **55**, 3042 (1997)

For production of superposition states of the cavity field $|\alpha_0\rangle$, if $E(t^*) = \beta^2 t^* = \frac{\pi}{2} + 2\pi m_2 \implies \beta^2 = \frac{1}{m_1}(\frac{1}{4} + m_2)$, then

$$\rho(t^*) = \frac{1}{2}(e^{-i\frac{\pi}{4}}|\alpha_0\rangle + e^{i\frac{\pi}{4}}|-\alpha_0\rangle) \times \left\{ \langle -\alpha_0|e^{-i\frac{\pi}{4}} + \langle \alpha_0|e^{i\frac{\pi}{4}} \otimes \rho_{th} \right\}$$

Production of macroscopic superposition states of the cavity field.



What about superposition states of the oscillating mirror? Phys. Rev. Lett. **91**, 130401, (2003)

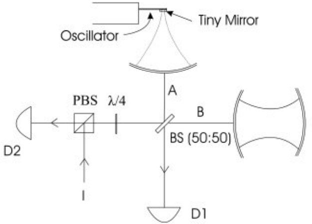


FIG. 1. The proposed setup: a Michelson interferometer for a single photon, where in each arm there is a high-finesse cavity. The cavity in arm A has a very small end mirror mounted on a micromechanical oscillator. The single photon comes in through I. If the photon is in arm A, the motion of the small mirror is affected by its radiation pressure. The photon later leaks out of either cavity and is detected at D1 or D2.

its ground state $|0\rangle_m$. Then the initial state is $|\psi(0)\rangle = (1/\sqrt{2})(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B)|0\rangle_m$. After a time t the state of the system will be given by [6,12]

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega_c t}[|0\rangle_A|1\rangle_B|0\rangle_m + e^{i\kappa^2(\omega_m t - \sin\omega_m t)}|1\rangle_A|0\rangle_B \times |\kappa(1 - e^{-i\omega_m t})\rangle_m], \quad (2)$$

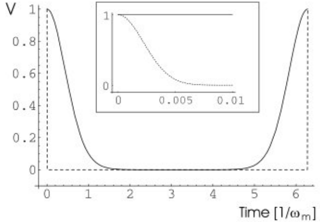
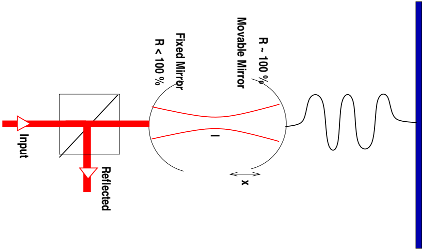


FIG. 2. Time evolution of the interference visibility V of the photon over one period of the mirror's motion for the case where the mirror has been optically cooled close to its ground state ($\bar{n} = 2$, solid line) and for $T = 2$ mK, which corresponds to $\bar{n} = 100\,000$ (dashed line—see also inset). The visibility decays after $t = 0$, but in the absence of decoherence there is a revival of the visibility after a full period. The width of the revival peak scales like $1/\sqrt{\bar{n}}$.

Quantifying optomechanical interaction:I



$$\frac{da}{dt} = i\Delta(x)a - \left(\frac{1}{2\tau}\right)a + i\sqrt{\frac{1}{\tau_{ex}}}s$$

$$\frac{d^2x}{dt^2} + \left(\frac{\omega_m}{2Q_m}\right)\frac{dx}{dt} + \omega_m^2 x = \frac{F_{RP}(t)}{m_{eff}} + \frac{F_L(t)}{m_{eff}}$$

$$F_{RP} \sim \frac{|a|^2}{T_{rt}}; \quad \Delta(x) = \omega - \omega_0(x) = \Delta(L) + \frac{\omega_0}{L}x$$

Quantifying optomechanical interaction: II

T. J. Kippenberg, K. J. Vahala, *Science*, **321**, 1172 (2008)

Solutions: The static response.

The mirror position becomes a function of applied power giving rise to bi-stable behaviour.

Dynamical response.(Adiabatic)

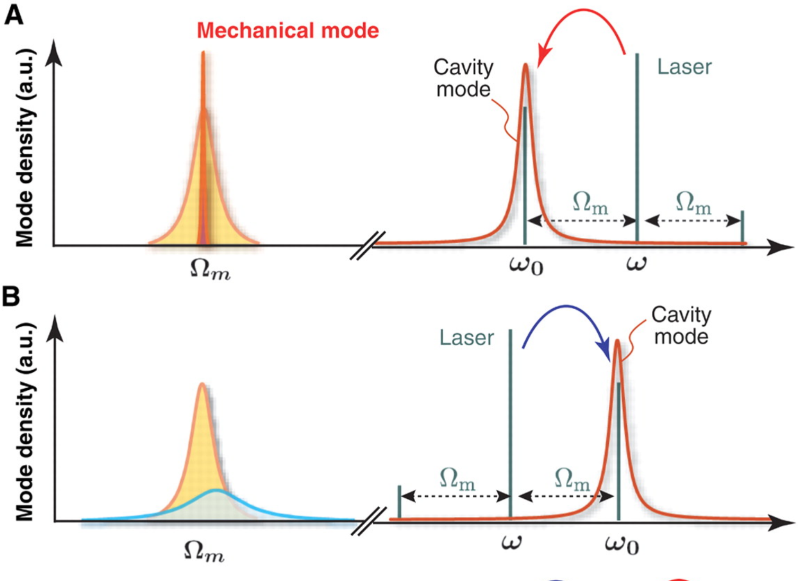
The x dependent contribution provides an optical contribution to the stiffness of the mechanical spring. This is called *Light Induced Rigidity*

Dynamical response.(Non-Adiabatic)

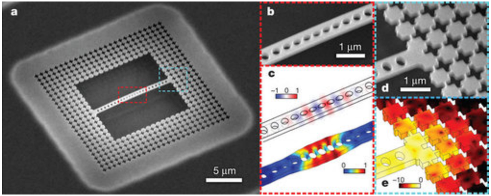
For $\kappa >> \omega_m$, the mechanical damping term becomes a function of Δ. Depending on whether $\Delta < 0$ or $\Delta > 0$, the radiation pressure force extracts energy from mechanical oscillator or dumps energy into it. For $\Delta < 0, \gamma_{OM} = \frac{-4g^2 n_c}{\kappa}$

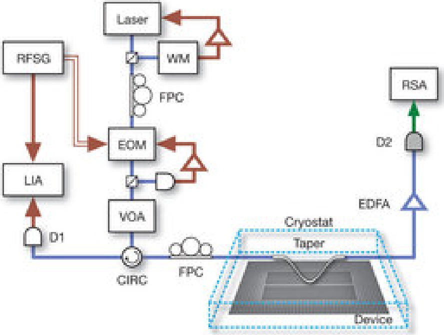
Thus cooling and amplification of the mechanical motion occurs due to dynamic backaction of radiation pressure force.

Opto-mechanical cooling.

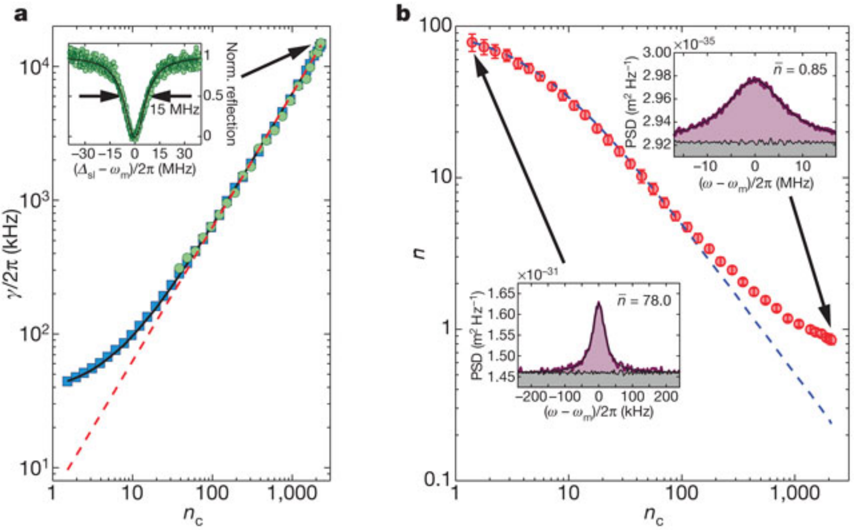


Laser cooling into the quantum ground state of a nano-mechanical oscillator. Chan et. al, *Nature*, **478**, 89, (2011)





Average phonon occupancy in ground state.

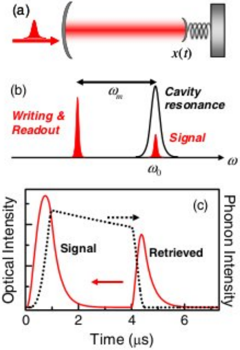


Summary so far.

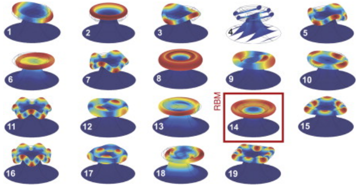
- The canonical mechanical oscillator (*coupled mass-spring*) is used in a variety of sensitive measurements including detection of weak forces and gravity wave detection.
- Coupling a single mode optical cavity to this mechanical oscillator, gives rise to quantum features both in states of mechanical motion and in states of light.
- To detect and use these quantum features requires cooling the mechanical oscillator to its quantum ground state.
- Laser cooling of mechanical oscillator has indeed experimentally achieved this quantum ground state.

Storing optical information as mechanical excitation.

Victor et. al, *Phys. Rev. Lett.*, **107**, 133601, (2011)



(a)



(b)

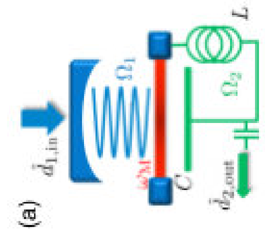
Why transfer to a mechanical mode?

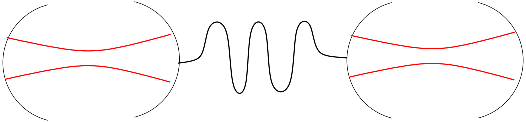
- Storing in mechanical modes facilitates transfer from one EM frequency to a vastly different EM frequency.
- It thus facilitates transfer of quantum optical states between very different quantum computing architectures–from circuit QED to optical computing.
- Requirements: Very low bath temperatures, High Q for both mechanical and optical modes.
- Can this situation be improved?

High fidelity quantum state transfer using interference.

Wang and Clerk, Phys. Rev. Lett., **108**, 153603, (2012)

Proposes a new cavity opto-mechanical coupling scheme which minimises mechanical dissipation.

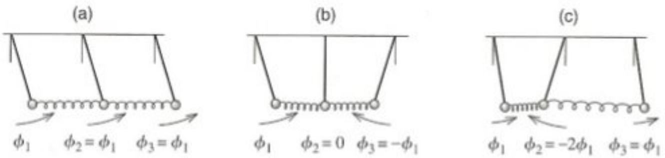




- Each segment of the opto-mechanical system can be thought of an oscillator in the *Resolved Sideband Limit*. This is a (three) coupled pendulum system.

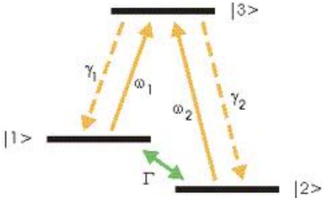
Normal modes

- The normal modes of the pendulum are



- In (b) the middle pendulum does not oscillate at all and the motion is **delocalized** on the edge pendula.
- Identifying the middle pendulum as the mechanical oscillator, we can effect transfer of states from optical cavity to microwave cavity without storing energy in the mechanical oscillator.
- Such a scheme for quantum state transfer is already in use in quantum atom optics and goes under the name of *Electro-magnetically Induced Transparency (EIT).*

Electromagnetically Induced Transparency

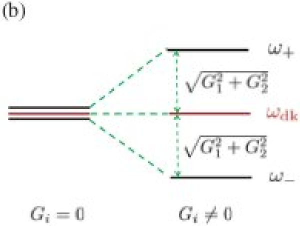


$$|DS\rangle = \frac{\Omega_2|1\rangle - \Omega_1|2\rangle}{\sqrt{\Omega_1^2 + \Omega_2^2}} \qquad (1)$$

- The dark state is an eigen state of the atom+field system.
- Once the atom gets into the dark state it is decoupled from transitions to the excited state.

Role of Interference (I)

- For the **Dark Mode** transfer both the optical and microwave cavity have to be simultaneously driven.
- The energy eigen values are



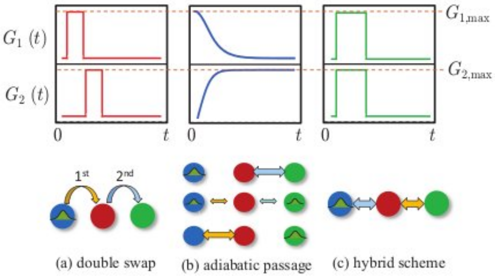
  - G_is represent coupling strengths of EM oscillators (1 and 2) with the mechanical oscillator.

- Two regimes of dark mode transfer.
- **Hybrid** $G = G_1 = G_2$ at all times.
- **Adiabatic Transfer** $G = \sqrt{G_1(t)^2 + G_2(t)^2}$.

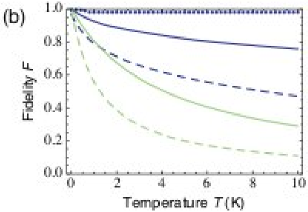
Role of Interference (II)

- The mechanical oscillator couples both the cavities *equally* but with opposite phase.
- A complete transfer through the dark mode is possible only for equal decay time of photons in each cavity.
- The AT protocol is an adiabatic protocol where the rate of change of G_is is made such that it minimises dissipation. It is very similar to the STIRAP protocol in quantum atom optics.
- The Hybrid transfer protocol is useful when the cavity decay is also non-negligible.

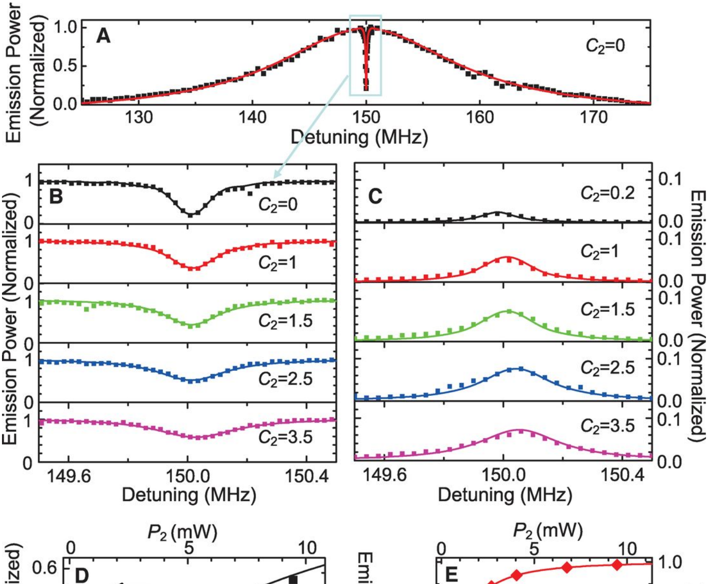
Transfer fidelities



- Transfer Fidelity for transfering a coherent state $|\alpha\rangle$.



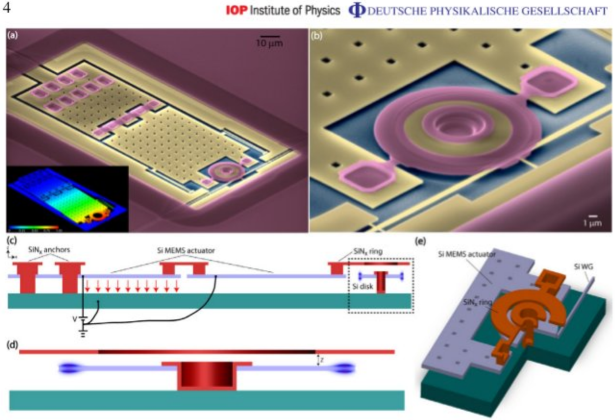
Experimental verification of transfer through a dark mode. C. Dong. et. al Science, **338**, 1609, (2012)



Presence of other mechanical modes in OM systems.

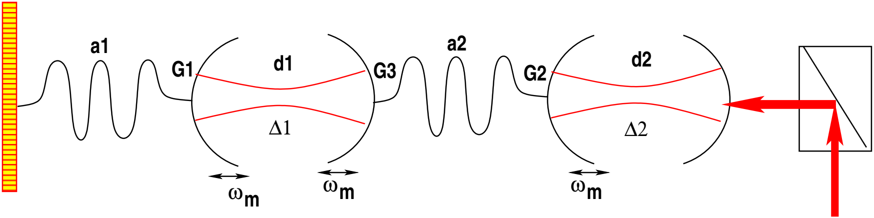
H. Miao, K Srinivasan and V Aksyuk, NJP, **14**, 075015 (2012)

Cascaded opto-mechanical systems are being now used for SQL limited sensing purposes.$4.6 * 10^{-15} \pm 0.6 * 10^{-15} \frac{m}{\sqrt{Hz}}$.



Dual Cavity dual spring system.

S. Sainadh U & A. Narayanan Phys. Rev. A **88**, 033802 (2013)



The Hamiltonian for this system is given by

$$\hat{H} = \sum_{i=1}^{2} \left(w_m \hat{a}_i^\dagger \hat{a}_i - \Delta_i \hat{d}_i^\dagger \hat{d}_i + G_i(\hat{a}_1^\dagger \hat{d}_i + \hat{a}_1 \hat{d}_i^\dagger) \right) + G_3(\hat{a}_2^\dagger \hat{d}_1 + \hat{a}_2 \hat{d}_1^\dagger) \qquad (2)$$

This Hamiltonian is written in a frame displaced by the strength ($|c_i|^2$) of the drive fields and in the interaction picture. The internal losses of the cavities are denoted by κ_is and that of the springs by γ_is. We take the good cavity limit with $G_i \gg \kappa_i$ and work in the resolved sideband regime with $\kappa_i \ll w_m$.

Hybrid Scheme for state transfer.

We consider $G_1 = G_2 =$G with $G_3 = pG$.
The Heisenberg equations of motion, in the absence of cavity dissipation κ_i and mechanical dissipation γ_i are given by

$$e^{iw_m t}\hat{d}_2(t) = \hat{d}_2(0)\left(\frac{\nu_- \cos\left(\frac{h_+ t}{4}\right) + \nu_+ \cos\left(\frac{h_- t}{4}\right)}{2}\right) +$$

$$\hat{a}_2(0)\frac{4ipG}{\sqrt{4+p^4}}\left(\frac{\sin\left(\frac{h_- t}{4}\right)}{h_-} - \frac{\sin\left(\frac{h_+ t}{4}\right)}{h_+}\right) + \hat{d}_1(0)\left(\frac{\cos\left(\frac{h_+ t}{4}\right) - \cos\left(\frac{h_- t}{4}\right)}{\sqrt{4+p^4}}\right)$$

$$-\hat{a}_1(0)(2iG)\left(\left(\nu_- + \frac{2}{\sqrt{4+p^4}}\right)\frac{\sin\left(\frac{h_+ t}{4}\right)}{h_+} + \left(\nu_+ - \frac{2}{\sqrt{4+p^4}}\right)\frac{\sin\left(\frac{h_- t}{4}\right)}{h_-}\right)$$

where $\nu_\pm = 1 \pm \frac{p^2}{\sqrt{4+p^4}}$, $h_\pm = \sqrt{8G^2(2 + p^2 \pm \sqrt{4+p^4})}$.

At all times, there is non-vanishing contribution of the mechanical modes a_i to the state of second cavity d_2. At $t = t_0 = 4\pi/h_+$, $\hat{d}_1(t_0) \approx \hat{d}_1(0)$ and $\hat{d}_2(t_0) \approx \hat{d}_2(0)$.

Effect of additional mechanical mode (I)

- Even in the absence of dissipation,the presence of even one additional mechanical mode drastically affects the quantum state transfer between the cavities.

A full Lindbladian time evolution incorporating the cavity (κ) and thermal (γ) losses and the presence of a bath at a finite temperature T has the following features.

- We consider a initial Gaussian state.
- The Fidelity is expressed using Wigner functions of initial and final states which in turn is written in terms of symmetrized covariance matrices.

$$F = \frac{1}{1+\overline{n_h}} e^{-\lambda^2/(1+\overline{n_h})} \tag{4}$$

$\overline{n_h}$ is the heating generated during tranfer generated by cavity noise, mechanical resonator noise and initial thermal population of mechanical state. λ is the amplitude decay.

Effect of additional mechanical mode (II)

For input squeezed state

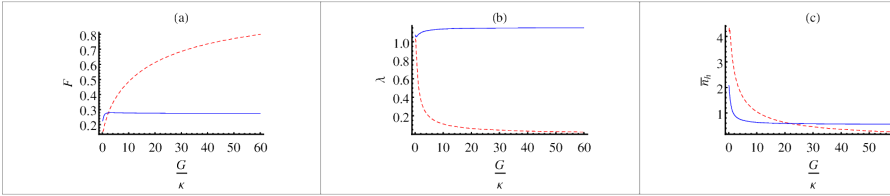


Figure: Graphs (a)-(c) show fidelity (F),amplitude decay (λ) and $\overline{n}_h$, as a function of coupling strength $(\frac{G}{\kappa})$. This is drawn for an initial squeezed state of squeezing parameter $r = 1$, with $|\alpha| = 1$,$\phi = \frac{\pi}{4}$ and with $p = 5$ (blue and thick). In all the three graphs a comparison plot is drawn for $p = 0$ (red and dashed) which is the case where the extra spring is absent. The graphs are plotted for experimentally realisable parameters.

Effect of additional mechanical mode (III)

For input coherent state

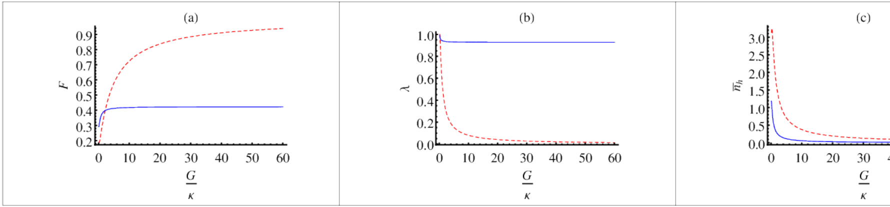


Figure: The above graphs show fidelity (F),λ and $\overline{n}_h$, as a function of coupling strength $(\frac{G}{\kappa})$, for a coherent state with $r = 0$, $\alpha = 1$ and for $p = 5$ (blue and thick). Shown also in each graph are plots for $p = 0$ (red and dashed) which signifies the absence of the extra spring. The graphs are plotted for experimentally realisable parameters.

Summary.

- Cavity opto-mechanics is a powerful emerging field which couples high Q electromagnetic and mechanical modes.
- Realising quantum features at *Macro* scales is very feasible in these systems.
- OM systems will find their best use as hybrid systems interfacing between disparate quantum architectures.
- Currently they are the best sensors for force measurements.
- Our main result highlights the need to isolate additional mechanical modes when the OM system is used as a sensor for sensing quantum features.