Review of the Firewall Paradox

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Based on work by

Mathur;

AMPS(S) (Ahmheiri, Marolf, Sully, Polchinski (+Stanford));

Harlow, Hayden; Maldacena, Susskind; $\cdots$

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Some conventions and background material

We shall use natural units:

$$\hbar = 1, \quad c = 1, \quad G_N = 1, \quad k_{Boltzman} = 1$$

Unit of length $\sim 10^{-33}$cm, unit of mass $\sim 10^{-5}$gm

Unit of time $\sim 10^{-43}$ seconds

Mass of the sun $\sim 10^{38}$

All space-times will be represented by their Penrose diagrams.

Change coordinates such that

1. Asymptotic regions to which light can travel come within a finite range of coordinates

2. The metric is proportional to the Minkowski metric.

Example: Take Minkowski space-time

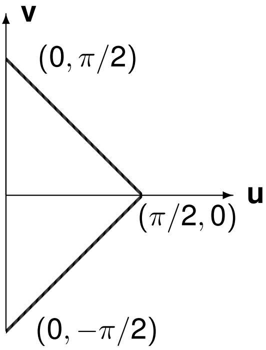
$$\mathbf{ds^2 - -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

Ignore the angular coordinates θ, ϕ since they span a finite range anyway.

Change coordinates:

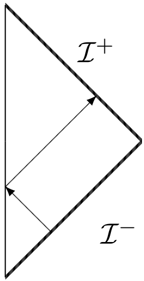
$$\tan(u \pm v) = r \pm t$$

$$(0 \leq r < \infty, -\infty < t < \infty) \quad \Rightarrow \quad u > 0,\ -\frac{\pi}{2} < u \pm v < \frac{\pi}{2}$$



$$-dt^2 + dr^2 = (dr + dt)(dr - dt) \propto (-dv^2 + du^2)$$

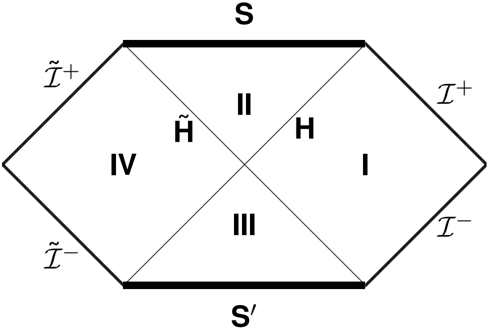
$$\mathbf{ds^2} \propto (-\mathbf{dv^2} + \mathbf{du^2})$$



Note: Light rays travel at 45° angle with the vertical, beginning at $\mathcal{I}^-$ and ending at $\mathcal{I}^+$.

Massive particle trajectories travel upwards, with tangent making $< 45^\circ$ angle with the vertical.

Schwarzschild black hole: A classical solution to general theory of relativity with the following Penrose diagram:



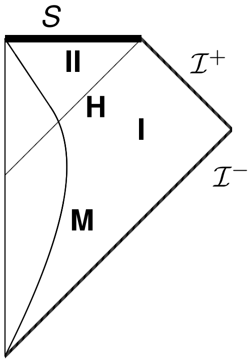
S, S′: singularities, H, $\widetilde{\mathrm{H}}$: Horizons

Note 1: region II cannot send signal to region I.

Any particle in region II must end up in the singularity.

Note 2: Two asymptotic regions contained in I and IV

The Penrose diagram of a realistic black hole formed out of collapse of matter M:



Note: This has only one asymptotic region.

Result of the 70's (Bekenstein, Hawking)

In quantum theory, a black hole behaves as a thermal object with finite temperature, entropy etc.

$$T \propto 1/M, \quad S_{BH} = A_H/4 \propto M^2$$

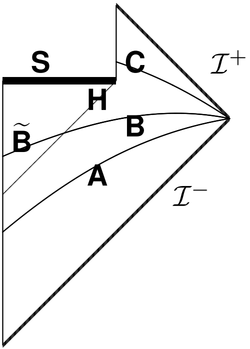
As the black hole evaporates by thermal radiation its mass goes down.

$\Rightarrow$ the temperature goes up.

It evaporates in a finite time $\sim M^3$.

Big question: How is the information about the wave-function of the matter before collapse encoded in the radiation that comes out of the black hole?

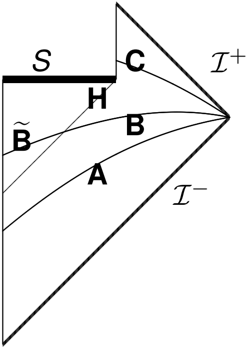
Penrose diagram of evaporating black hole



A, B $+$ $\widetilde{\text{B}}$ and C are three space-like slices on which we can define the wave-function of the state.

We expect unitary evolution from A $\Rightarrow$ B $+$ $\widetilde{\text{B}}$ $\Rightarrow$ C.

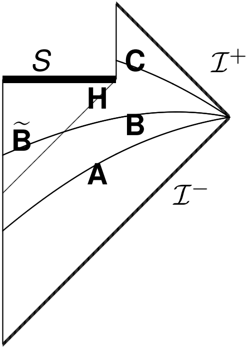
Question: How can any information from $\widetilde{\text{B}}$ propagate to C?



How can information from $\widetilde{B}$ propagate to C?

One possible resolution: There is no information in $\widetilde{B}$, i.e. the Hilbert space on $\widetilde{B}$ is one dimensional.

Then it is possible to have unitary evolution A $\Rightarrow$ B $\Rightarrow$ C.



Can the Hilbert space on $\widetilde{\mathbf{B}}$ be one dimensional?

$\Rightarrow$ for an infalling observer all information will suddenly disappear as she crosses the horizon.

– violation of equivalence principle which says that the horizon has weak gravity and so there will only be mild effect on an infalling observer.

Proposed resolution: Black hole complementarity

’t Hooft; Susskind; Susskind, Thorlacius, Uglam

The quantum mechanical description of an outside observer need not agree with that of an inside observer.

Even if the inside observer sees a regular Hilbert space consistent with her local physics, as long she cannot send the information outside, there is no contradiction.

In the description of quantum mechanics of an outside observer, the information about the quantum state of the black hole always resides at the (stretched) horizon.

Note: Even though this is an apparently consistent resolution, there has not been a concrete microscopic realization of this idea yet

(but hear Suvrat Raju’s talk!)

Recent developments

To follow these developments we have to go back a few steps and understand the origin of Hawking radiation.

$\mathbf{a}^\dagger, \mathbf{a}$: particle creation/annihilation operators from infalling observer's viewpoint

$\mathbf{b}^\dagger, \mathbf{b}$: creation/annihilation operators from a stationary outside observer's viewpoint

$\widetilde{\mathbf{b}}^\dagger, \widetilde{\mathbf{b}}$: creation/annihilation operators for modes inside the horizon.

One finds that a's are linear combinations of b, $\mathbf{b}^\dagger$, $\widetilde{\mathbf{b}}$, $\widetilde{\mathbf{b}}^\dagger$.

$\mathbf{a}|\Omega\rangle = \mathbf{0} \quad \Rightarrow \quad (\mathbf{b} - \lambda\widetilde{\mathbf{b}}^\dagger)|\Omega\rangle = 0$ **and** $(\widetilde{\mathbf{b}} - \lambda\mathbf{b}^\dagger)|\Omega\rangle = 0$
for some constant λ.

$\Rightarrow |\Omega\rangle = \exp(\lambda\mathbf{b}^\dagger\widetilde{\mathbf{b}}^\dagger)|\mathbf{0}\rangle, \quad \mathbf{b}|\mathbf{0}\rangle = \widetilde{\mathbf{b}}|\mathbf{0}\rangle = \mathbf{0}$

$$|\Omega\rangle = \mathbf{exp}(\lambda \mathbf{b}^\dagger \widetilde{\mathbf{b}}^\dagger)|\mathbf{0}\rangle$$

Thus $|\Omega\rangle$ appears as a superposition of multiparticle states from the point of view of the stationary observer outside.

Two important points:

1. If we do not observe the interior modes created by $\widetilde{\mathbf{b}}^\dagger$ then the state outside is described by a density matrix.

2. The entanglement between the modes created by $\mathbf{b}^\dagger$ and $\widetilde{\mathbf{b}}^\dagger$ is maximal.

Thus every time the black hole emits a Hawking quantum the entanglement between the state of the black hole and the outside radiation increases.

The entanglement between the black hole and the outside radiation increases with every emission of a Hawking quantum.

But the black hole has finite entropy S_{BH} and hence a finite dimensional Hilbert space.

The entanglement entropy between the black hole and the outside radiation cannot exceed S_{BH}.

This apparently happens roughly at a time by which the black hole has radiated away half its entropy.

$\rightarrow$ page time $t_{page} \sim M^3$

After the page time the entanglement entropy between the black hole and the outside radiation must decrease.

Page: For a generic state of the black hole this means that the Hawking quanta emitted after the page time must be nearly maximally entangled with the radiation emitted before the page time.

On the other hand Hawking's analysis says that the emitted Hawking quanta must be (nearly) maximally entangled with the interior mode of the black hole.

– impossible to satisfy both conditions.

This has been made more precise by applying strong subadditivity theorem of entanglement entropy on the combined system of

E: Total Hawking radiation emitted before a time $t_0 >$ page time

B: A Hawking quantum emitted after t_0 (assumed to be a qubit)

$\widetilde{B}$: Hawking partner of B inside the horizon (also a qubit)

Strong subadditivity: $S_{EB} + S_{B\widetilde{B}} \geq S_B + S_{EB\widetilde{B}}$

Decrease in entanglement $\Rightarrow S_{EB} < S_E$,

Near maximal entanglement between B and $\widetilde{B}$

$\Rightarrow S_{B\widetilde{B}} \simeq 0$ and $S_{EB\widetilde{B}} \simeq S_E$

$\Rightarrow S_E > S_{EB} \geq S_B + S_E$

$\Rightarrow$ requires $S_B \simeq 0$ but this is in conflict with B being maximally entangled with $\widetilde{B}$, giving $S_B = \ln 2$

What about complementarity?

The above argument requires treating E, B and $\widetilde{\mathrm{B}}$ to be independent system.

Outside observer does not have access to $\widetilde{\mathrm{B}}$.

Thus we could try to avoid the contradiction by postulating that the interior mode $\widetilde{\mathrm{B}}$ is not an independent system but is embedded inside the big Hilbert space E of early Hawking radiation.

$$\widetilde{\mathrm{B}} \subset \mathrm{E}$$

Then B can be maximally entangled with $\widetilde{\mathrm{B}}$ and also E.

However a late infalling observer in principle has access to E, B and $\widetilde{\mathrm{B}}$.

$\Rightarrow$ **could discover violation of strong subadditivity.**

AMPS argument leading to a contradiction of $\widetilde{B} \subset E$:

The experiment involves an observer who jumps into the black hole later than the page time.

Assume that the observer knows the full dynamics describing formation and evaporation of the black hole.

She knows how exactly $\widetilde{B}$ is embedded in E.

Now suppose she collects all of E, applies an appropriate unitary operator (quantum computer) so that the information about $\widetilde{B}$ is contained in a single qubit.

She then collects the qubit and jumps into the black hole to access $\widetilde{B}$.

She now has a clone of $\widetilde{B}$ in conflict with quantum no cloning theorem.

Counterargument

Harlow, Hayden

Quantum computation takes time.

They argue that the time taken for extracting the qubit from E takes exponentially large time in M, and hence is much larger that the life of the black hole $\sim$ M^3.

$\Rightarrow$ **such an experiment cannot be performed.**

AMPSS refutal

Refutal 1. After time t_0

a) surround the black hole by a perfectly reflecting mirror to stop evaporation,

b) do the quantum computation to extract $\widetilde{B}$ in a single qubit,

c) remove the mirror,

d) jump in carrying the qubit.

We now again have a conflict with quantum no cloning theorem.

Refutal 2. Suppose $\widetilde{B} \subset E$.

Now manipulate a single qubit in E (apply some operator $\mathcal{O}$ on E).

In general since $\widetilde{b}, \widetilde{b}^\dagger$ are operators in E,

$$[\mathcal{O}, \widetilde{\mathbf{b}}] \sim \mathbf{1}, \quad [\mathcal{O}, \widetilde{\mathbf{b}}^\dagger] \sim \mathbf{1}$$

Since $\widetilde{b}$ is a linear combination of $a^\dagger$ and a – the creation / annihilation operator in the frame of the infalling observer – we have generically

$$[\mathcal{O}, \mathbf{a}] \sim \mathbf{1}$$

If the system was initially in the vacuum state of the infalling observer, after applying $\mathcal{O}$ it will not remain in the vacuum.

As this will be true for all modes seen by the infalling observer, she will encounter a

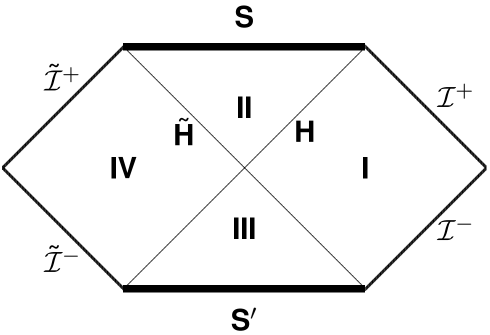
Firewall

Thus by manipulating a single qubit in E, we can create a firewall across the horizon for an infalling observer.

From this AMPSS goes on to argue that since an operation on a faraway system should not decide the fate of an infalling observer, the firewall must be there whether or not the operator $\mathcal{O}$ was applied on the system E.

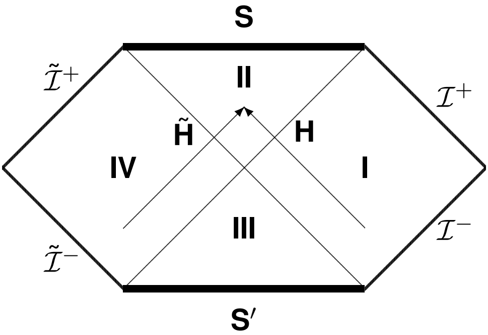
Counterexample: How an operation on a faraway system could change the state of black hole interior

Maldacena, Susskind



Earlier we had considered this as an unphysical solution with two asymptotic spaces, but we could identify the two asymptotic spaces.

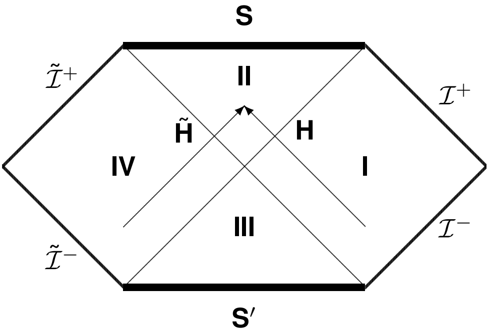
New interpretation: Two far away black holes in the same space-time whose interiors are connected by a wormhole (Einstein-Rosen bridge)



We cannot use a wormhole to communicate between two observers by staying outside the black hole horizons.

However two observers falling through the horizon of these black holes could meet and communicate (and eventually hit the singularity).

Thought experiment: Suppose we collect all the early Hawking quanta and make them form a new black hole far away.



Suppose that the interior of this far away black hole is connected to that of the original black hole via a wormhole.

Then by throwing in something into the far away black hole we could create a firewall for an observer falling through the horizon of the original black hole.

– explains how operations performed far away could affect the interior of the original black hole and yet there need not be any firewall if such operations had not been performed.

Note: This argument does not explain how a wormhole could be formed between the two black holes or what happens if we do not make the early radiation form a black hole.

Intuitive picture: Each Hawking quantum may be connected to the black hole by a microscopic wormhole.

When they are made to collapse and form a big black hole, then the microscopic wormholes join to form a big wormhole connecting the two black holes.

– promoted to a general principle

$$\textbf{EPR} = \textbf{ER}$$

(Macroscopic) entanglement

$$\|$$

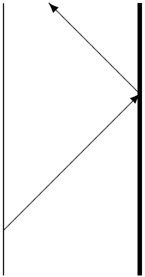
connection via Einstein-Rosen bridge

Shifting the battleground

Even though there is very little agreement on various issues among the opposing camps, there is one point on which most practitioners seem to agree.

A better place to fight out the battle is in anti-de Sitter (AdS) space.

Penrose diagram of global AdS



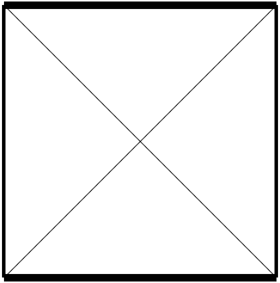
Left boundary: Like the r=0 origin of Minkowski space.

Reflection from there is fake.

The right boundary is the genuine boundary of AdS.

Note: Light rays keep getting reflected from this boundary.

Penrose diagram of AdS black hole



As in the case of Minkowski space, for black holes formed out of collapse the left hand side and bottom of the diagram will be replaced by that of global AdS.

Note that Hawking radiation gets reflected from the boundary and falls into the black hole.

Thus the black hole never evaporates.

Why AdS?

1. AdS/CFT correspondence relates quantum gravity theory in AdS to an ordinary quantum field theory (CFT) sitting at the boundary of AdS.

Thus there is no violation of the laws of quantum mechanics from the point of the asymptotic observer who sits at the boundary.

2. In AdS the black hole represents the field theory in equilibrium at a finite temperature.

3. Since the black hole never evaporates the AMPS argument does not apply directly.

But there is a more controlled version of the AMPS argument that works.

Given an AdS black hole, we can place an external absorber at the boundary which absorbs Hawking radiation.

– corresponds to coupling the CFT to another system which can absorb energy from the CFT and reduce its temperature.

Switch on the interaction at time 0, and switch it off at a time t_0 by which more that half of the entropy of the black hole (CFT at finite temperature) has been absorbed by the absorber.

At this stage the role of early Hawking radiation E is played by the quantum state of the absorber.

We can now run the same argument as AMPS(S) to arrive at a contradiction.

In this form the question of whether $\widetilde{B} \subset E$ reduces to whether we can construct the operators $\widetilde{b}$, $\widetilde{b}^\dagger$ of the interior modes in terms of operators of the CFT at the boundary.

– topic of Suvrat's talk.