Quantum Information: From the Perspective of Quantum Optics

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Overview of Talk

- Information is represented and communicated as quantum states.
- Any physical system (process) is subjected to the effects of its environment, responsible for the phenomenon of decoherence, dissipation and it is essential to understand the functioning of the process in the presence of these.
- After a brief discussion of open quantum systems and some measures of quantum information, I will talk about the dynamics of quantum correlations and holonomic quantum computation in a number of models motivated from quantum optics.
- In the end, diverting somewhat from the main theme, I will briefly talk about geometric phase in a frustrated spin system which also brings out an interesting connection with Parrondo's effect.

Quantum Information: Fundamental Goals

- Identify elementary classes of static resources in quantum mechanics: example...qubit. Another example is a Bell state shared between two distant parties (entanglement).
- Identify elementary classes of dynamic resources in quantum mechanics: example...quantum information transmission between two parties and process of protecting quantum information processing against the effects of noise, a natural consequence of open systems.
- Quantify resource tradeoffs incurred performing elementary dynamical processes: example: what are minimal resources required to reliably transfer quantum information between two parties using a noisy communications channel?

Open Quantum Systems: A Brief Preview and Motivation

- The theory of open quantum systems addresses the problems of damping and dephasing in quantum systems by the assertion that all real systems of interest are 'open' systems, surrounded by their environments (U. Weiss: (1999); H. -P. Breuer and F. Petruccione: (2002)).
- Quantum optics provided one of the first testing grounds for the application of the formalism of open quantum systems (W. H. Louisell: (1973)). Application to other areas was intensified by the works of (Caldeira and Leggett: (1983)) and (Zurek: (1993)), among others.
- The recent upsurge of interest in the problem of open quantum systems is because of the spectacular progress in manipulation of quantum states of matter, encoding, transmission and processing of quantum information, for all of which understanding and control of the environmental impact are essential (Turchette *et al.*: (2000); Myatt *et al.*: (2000); Haroche group: (1996)). This increases the relevance of open system ideas to quantum computation and quantum information.

Open Quantum Systems: continued...

- Hamiltonian of the total (closed system):

$$H = H_S + H_R + H_{SR}.$$

- S- system, R- reservoir (bath), $S-R$-interaction between them.
- System-reservoir complex evolves unitarily by:

$$\rho(t) = e^{-\frac{i}{\hbar}Ht}\rho(0)e^{\frac{i}{\hbar}Ht}.$$

- We are interested in the reduced dynamics of the system S, taking into account the influence of its environment. This is done by taking a trace over the reservoir degrees of freedom, making the reduced dynamics *non-unitary*:

$$\rho^s(t) = \mathrm{Tr_R}(\rho(\mathrm{t})) = \mathrm{Tr_R}\left[\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\mathrm{Ht}}\rho(0)\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\mathrm{Ht}}\right].$$

Open Quantum Systems: continued...

- Open quantum systems can be broadly classified into two categories:
  (A). Quantum non-demolition (QND), where $[H_S, H_{SR}] = 0$ resulting in decoherence without any dissipation (Braginsky *et al.*: (1975), (1980); Caves *et al.*: (1980); G. Gangopadhyay, S. M. Kumar and S. Duttagupta: (2001); SB and R. Ghosh: (2007)) and
  (B). Quantum dissipative systems, where $[H_S, H_{SR}] \neq 0$ resulting in decoherence with dissipation (Caldeira and Leggett: (1983); H. Grabert, P. Schramm and G-L. Ingold: (1988); SB and R. Ghosh: (2003), (2007)).
- Open system ideas have been applied extensively in quantum optics (W. H. Louisell: (1973); F. Haake: (1973); G. S. Agarwal: (1974)).
- These ideas have been used in quantum information theoretic processes (SB and R. Srikanth: (2007)).
- Ideas developed by R. Landauer: (1961) and C. H. Bennett: (1988), established a deep connection between information and thermodynamics.
- In the parlance of quantum information theory, the noise generated by a QND open system would be a "phase damping channel", while that generated by a dissipative (Lindblad) evolution would be a "(generalized) amplitude damping channel".

Connection to quantum noise processes

- Interpret our results in terms of familiar noisy channels. How these environmental effects can affect quantum computing.
  In operator-sum representation, action of superoperator $\mathcal{E}$ due to environmental interaction

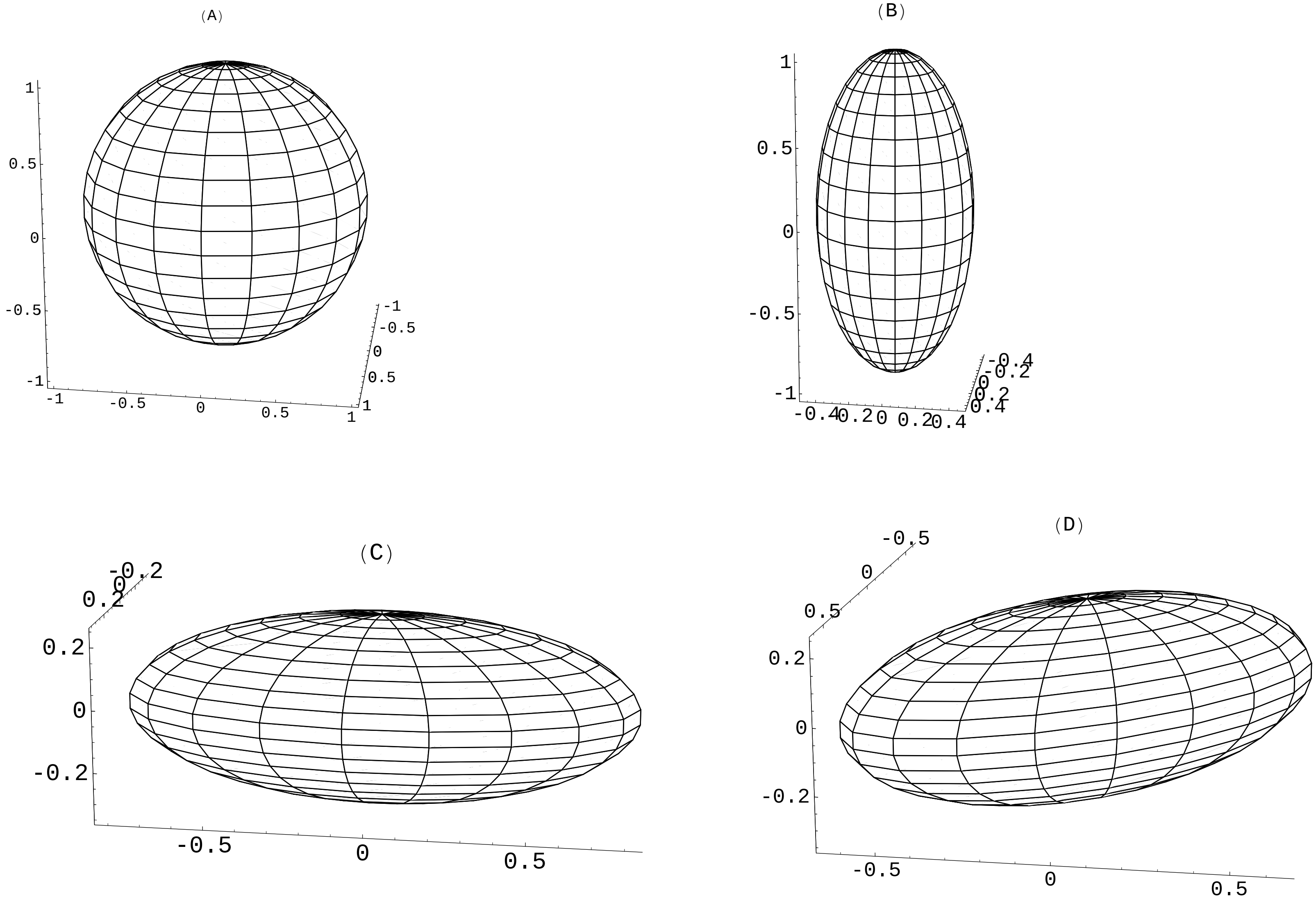
$$\rho \longrightarrow \mathcal{E}(\rho) = \sum_k \langle e_k | U(\rho \otimes |f_0\rangle\langle f_0|) U^\dagger | e_k \rangle = \sum_j E_j \rho E_j^\dagger,$$

  unitary operator U represents free evolution of system, environment, as well as the interaction between the two; $|f_0\rangle$: environment's initial state; $\{|e_k\rangle\}$ a basis for the environment.
- environment-system assumed to start in a separable state.
- $E_j \equiv \langle e_k | U | f_0 \rangle$ are the Kraus operators; partition of unity: $\sum_j E_j^\dagger E_j = \mathcal{I}$. Any transformation representatable as operator-sum is a completely positive (CP) map.

Connection to quantum noise processes continued ...

- Some examples of noisy channels useful in quantum informatiom are:
- **quantum phase damping channel**: uniquely non-classical quantum mechanical noise process, describing the loss of quantum information without the loss of energy. (SB and R. Ghosh: (2007))
- **squeezed generalized amplitude damping channel**: allows for dissipation along with decoherence and accounts for finite bath squeezing. (R. Srikanth and SB: (2007))

Bloch representation of Noisy Channels



continued ...

Fig.1 : Effect of QND and dissipative interactions on the Bloch sphere: (A) the full Bloch sphere; (B) the Bloch sphere after time $t = 20$, with $\gamma_0 = 0.2$, $T = 0$, $\omega = 1$, $\omega_c = 40\omega$ and the environmental squeezing parameter $r = a = 0.5$, evolved under a QND interaction ; (C) and (D) the effect of the Born-Markov type of dissipative interaction with $\gamma_0 = 0.6$ and temperature $T = 5$, on the Bloch sphere – the x and y axes are interchanged to present the effect of squeezing more clearly. (C) corresponds to $r = 0.4$, $\Phi = 0$ and $t = 0.15$ while (D) corresponds to $r = 0.4$, $\Phi = 1.5$ and $t = 0.15$.
(SB and R. Ghosh: (2007))

Quantum Correlations

Entanglement

- What is entanglement and what is its use?
- Separability versus entanglement: that which is not separable is entangled.
- A pure state is separable if it can be expressed as a tensor product of subsystem states: $|\psi\rangle = |a\rangle \otimes |b\rangle$.
- Examples for pure states:
  (a). separable states: $|00\rangle$, $|11\rangle$
  (b). entangled states: $|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$; $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$: Bell states.

Entanglement continued...

- A mixed state is separable if it can be represented as a mixture of product states: $\rho = \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i|$. Correlations between different subsystems due to incomplete knowledge of quantum states completely characterized by classical probabilities p_i.
- Examples for mixed states:
  (a). separable state: $\rho = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$
  (b). entangled state: $\rho_W = (1-p)\frac{1}{4}I + p|\Phi_+\rangle\langle\Phi_+|$, where $1/3 < p \leq 1$: Werner state.

Entanglement continued...

- Entanglement can be used to perform tasks not possible classically. E.g.: Using entanglement it is possible to teleport a qubit in state $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ using a shared entangled state $|\Phi_+\rangle$.
- Thus entanglement is a resource in quantum communication and information.

Concurrence

- For a pair of qubits there exists a general formula for the entanglement of formation: E_f: based on the quantity "CONCURRENCE". (W. K. Wootters: (1998))
- Consider pure state $|\Phi\rangle$ of a pair of qubits. Concurrence

$$C(\Phi) = |\langle\Phi|\tilde{\Phi}\rangle|$$

, where $|\tilde{\Phi}\rangle = (\sigma_y \otimes \sigma_y)|\Phi^*\rangle$, σ_y is the Pauli operator, $|\Phi^*\rangle$ is the complex conjugate of $|\Phi\rangle$.

- Spin flip operation, via σ_y, when applied to a pure product state, takes the state of each qubit to the orthogonal state, i.e., state diametrically opposite on the Bloch sphere resulting in zero concurrence. A completely entangled state is left invariant by a spin flip, resulting in C taking the maximum value 1.

Concurrence continued...

- $\mathcal{E}(C)$ is monotonically increasing for $0 \leq C \leq 1$ implying that concurrence can be regarded as a measure of entanglement in its own right.
- Concurrence of a mixed state of two qubits is: (W. K. Wootters: (1998))
  $C(\rho) = max\Big\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\Big\}$,
  where λ_i are the square roots of the eigenvalues of $\rho\tilde{\rho}$ in descending order and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

Quantum Correlations

Discord

- Correlation between two random variables X and Y is: `Mutual Information' $J(X:Y) = H(X) - H(X|Y)$.
- Here $H(X|Y)$ is the conditional entropy of X given that Y has already occured and $H(X)$ is the Shannon entropy of the random variable X.
- $H(X|Y) = H(X,Y) - H(Y)$: an alternative expression for mutual information $I(X:Y) = H(X) + H(Y) - H(X,Y)$.
- Classically: no ambiguity between these two expressions of mutual information and they are same.

Discord continued...

- Situation different in quantum regime (H. Ollivier and W. H. Zurek:(2001); L. Henderson and V. Vedral:(2001); S. Luo: (2008)).
- Consider a bipartite state ρ_{XY}: where ρ_X and ρ_Y are the states of the individual subsystems.
- Shannon entropies $H(X), H(Y)$ are replaced by von-Neumann entropies (e.g: $H(X) = S(\rho_X) = -Tr_X \rho_X Log(\rho_X)$).
- Conditional entropy $S(X|Y)$ requires a specification of the state of X given the state of Y.
- Such a statement in quantum theory is ambiguous until the to-be-measured set of states of Y are selected.
- Focus on perfect measurements of Y defined by a set of one dimensional projectors $\{\pi_j^Y\}$. The subscript j is used for indexing different outcomes of this measurement.

Discord continued...

- The state of X, after the measurement is given by

$$\rho_{X|\pi_j^Y} = \frac{\pi_j^Y \rho_{XY} \pi_j^Y}{Tr(\pi_j^Y \rho_{XY})},$$

 with probability $p_j = Tr(\pi_j^Y \rho_{XY})$.

- $S(\rho_{X|\pi_j^Y})$ is the von-Neumann entropy of the system in the state ρ_X, given that projective measurement is carried out on system Y.
- The conditional entropy of X, given the complete set of measurements $\{\pi_j^Y\}$ on Y is: $S(X|\{\pi_j^Y\}) = \sum_j p_j S(\rho_{X|\pi_j^Y})$.
- The quantum analogue of $J(X:Y)$ is thus

$$J(X:Y) = S(X) - S(X|\{\pi_j^Y\}),$$

 where a supremum is taken over all $\{\pi_j^Y\}$.

Discord continued...

- $I(X:Y)$ is similar to its classical counterpart

$$I(X:Y) = S(X) + S(Y) - S(X,Y).$$

- It is clearly evident that these two expressions are not identical in quantum theory. Quantum discord is the difference between these two generalizations of classical mutual information,

$$D(X:Y) = I(X:Y) - J(X:Y).$$

- Quantum discord aims to quantify the amount of quantum correlation that remains in the system and also points out that classicality and separability are not synonymous. In other words, it actually reveals the quantum advantage over the classical correlation.

Quantum Correlations

Bell's Inequality

- Bell's inequality: one of the first tools used to detect entanglement. (J. Bell: (1965;1971)): It is not possible for a local, realistic theory to reproduce all the statistical predictions of quantum mechanics.
- An important step in this direction is the Clauser-Horne-Shimony-Holt inequality (J. F. Clauser and A. Shimony: (1978)), derived on the premises of a local realistic theory.
- Interestingly, it can be seen that in standard quantum theory, it is always possible to design experiments for which this inequality gets violated (A. Aspect, P. Grangier and G. Roger: (1981)). This shows that quantum physics can violate local realism.

- One can express the most general form of Bell-CHSH inequality for the two-qubit mixed state
  $\rho = \frac{1}{4}[I \otimes I + (r.\sigma) \otimes I + I \otimes (s.\sigma) + \sum_{n,m=1}^{3} t_{mn}(\sigma_m \otimes \sigma_n)]$
  as $M(\rho) < 1$,
  where $M(\rho) = max(u_i + u_j)$, and u_i, u_j are the eigenvalues of the matrix $T^\dagger T$ (where the elements of the correlation matrix T is given by,
  $t_{mn} = Tr[\rho(\sigma_m \otimes \sigma_n)]$) (Horodecki's: (1995)).
- Violation of Bell's inequality for a given quantum state indicates that the state is entangled. But at the same time, there are certain entangled states which do not violate Bell's inequality.

Quantum Correlations

Teleportation Fidelity

- In addition to all these measures of quantum correlation one could also attempt to quantify them in terms of an application, for e.g., fidelity of teleportation (C. H. Bennett et al.: (1993)).
- The basic idea is to use a pair of particles in a singlet state shared by sender (Alice) and receiver (Bob). Pairs in a mixed state could be still useful for (imperfect) teleportation (S. Popescu: (1994)).
- The general mixed state of a two-qubit system :

$$\begin{aligned} \rho &= \frac{1}{4}[I \otimes I + (r.\sigma) \otimes I \\ &+ I \otimes (s.\sigma) + \sum_{n,m=1}^{3} t_{mn}(\sigma_m \otimes \sigma_n)]. \end{aligned}$$

- The quantities $t_{nm} = Tr[\rho(\sigma_n \otimes \sigma_m)]$ are the coefficients of a real matrix denoted by T. This representation is most convenient when one talks about the inseparability of mixed states. In fact, all the parameters fall into two different classes: those that describe the local behaviour of the state, i.e., (r and s), and those responsible for correlations (T matrix).

Teleportation Fidelity *continued...*

- In the standard teleportation scheme a mixed state ρ acts as a quantum channel.
- The maximum teleportation F was shown by (Horodecki : (1996)) to be

$$\begin{aligned} F_{max} &= \frac{1}{2}(1+\frac{1}{3}N(\rho)) \\ &= \frac{1}{2}(1+\frac{1}{3}[\sqrt{u_1}+\sqrt{u_2}+\sqrt{u_3}]). \end{aligned}$$

  Here u_i and u_j are the eigenvalues of $U = T^\dagger(\rho)T(\rho)$, where $T(\rho) = [T_{ij}], T_{ij} = Tr[\rho(\sigma_i \otimes \sigma_j)]$ and $T^\dagger$ implies the Hermitian conjugate of T. The classicial fidelity of teleportation in the absence of entanglement is obtained as $\frac{2}{3}$. Thus whenever $F_{max} > \frac{2}{3}(N(\rho) > 1)$, teleportation is possible.
- At this point it is interesting to note that there is a non-trivial interplay between Bell's inequality and teleportation fidelity. This is because both $M(\rho), N(\rho)$ are dependent on the correlation matrix T. The relationship between these two quantities is the inequality $N(\rho) > M(\rho)$. Hence, it is clear that states which do violate Bell's inequality are always useful for teleportation. However, this does not rule out the possibility of existence of entangled states that do not violate Bell's inequality, but can still be useful for teleportation.

Quantum Correlations

Measurement Induced Disturbance (MID)

- Given a bipartite state ρ living in the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, and the reduced density matrices being denoted by ρ_A and ρ_B, a reasonable measure of total correlations between systems A and B is the mutual information:

$$I(\rho) = S(\rho_A) + S(\rho_B) - S(\rho),$$

where $S(\cdot)$ denotes von Neumann entropy.

MID continued...

- If $\rho_A = \sum_i p_A^i \Pi_A^i$ and $\rho_B = \sum_i p_B^i \Pi_B^i$, then the measurement induced by the spectral resolution of the reduced states is

$$\Pi(\rho) \equiv \sum_{j,k} \Pi_A^j \otimes \Pi_B^k \rho \Pi_A^j \otimes \Pi_B^k.$$

- The state $\Pi(\rho)$ may be considered classical in the sense that there is a (unique) local measurement strategy, namely Π, that leaves $\Pi(\rho)$ unchanged. This strategy is special in that it produces a classical state in ρ while keeping the reduced states invariant.
- If we accept that $I[\Pi(\rho)]$ is a good measure of *classical* correlations in ρ, then one may consider MID, given by

$$Q(\rho) = I(\rho) - I[\Pi(\rho)],$$

as a reasonable measure of quantum correlation (S Luo: (2008)).

Dynamics of the Reduced Density Matrix for two-qubit Dissipative system

(SB, V. Ravishankar and R. Srikanth: (2009))

- Hamiltonian, describing the dissipative, position dependent, interaction of two qubits with bath (modelled as a 3-D electromagnetic field (EMF)) via dipole interaction as:

$$
\begin{aligned}
H &= H_S + H_R + H_{SR} \\
&= \sum_{n=1}^{N=2} \hbar\omega_n S_n^z + \sum_{\vec{k}s} \hbar\omega_k (b^\dagger_{\vec{k}s} b_{\vec{k}s} + 1/2) - i\hbar \sum_{\vec{k}s} \sum_{n=1}^{N} [\vec{\mu}_n . \vec{g}_{\vec{k}s}(\vec{r}_n)(S_n^+ + S_n^-) b_{\vec{k}s} - h.c.].
\end{aligned}
$$

$\vec{\mu}_n$: transition dipole moments, dependent on the different atomic positions $\vec{r}_n$

$$S_n^+ = |e_n\rangle\langle g_n|, \; S_n^- = |g_n\rangle\langle e_n| :$$

dipole raising and lowering operators satisfying the usual commutation relations

$$S_n^z = \frac{1}{2}(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|) :$$

energy operator of the nth atom

Dynamics of the Reduced Density Matrix continued...

$b^{\dagger}_{\vec{k}s}$, $b_{\vec{k}s}$: creation and annihilation operators of the field mode (bath) $\vec{k}s$ with the wave vector $\vec{k}$, frequency ω_k and polarization index $s = 1, 2$

- System-Reservoir (S-R) coupling constant:

$$\vec{g}_{\vec{k}s}(\vec{r}_n) = (\frac{\omega_k}{2\varepsilon\hbar V})^{1/2}\vec{e}_{\vec{k}s}e^{i\vec{k}.r_n}.$$

  V: the normalization volume and $\vec{e}_{\vec{k}s}$: unit polarization vector of the field.

- S-R coupling constant: dependent on the atomic position r_n.
  This leads to a number of interesting dynamical aspects.

Dynamics of the Reduced Density Matrix continued...

- Assuming separable initial conditions, and taking a trace over the bath the reduced density matrix of the qubit system in the interaction picture and in the usual Born-Markov, rotating wave approximation (RWA) is obtained as

$$
\begin{aligned}
\frac{d\rho}{dt} &= -\frac{i}{\hbar}[H_{\tilde{S}}, \rho] - \frac{1}{2}\sum_{i,j=1}^{2} \Gamma_{ij}[1+\tilde{N}](\rho S_i^+ S_j^- + S_i^+ S_j^- \rho - 2S_j^- \rho S_i^+) \\
&- \frac{1}{2}\sum_{i,j=1}^{2} \Gamma_{ij}\tilde{N}(\rho S_i^- S_j^+ + S_i^- S_j^+ \rho - 2S_j^+ \rho S_i^-) \\
&+ \frac{1}{2}\sum_{i,j=1}^{2} \Gamma_{ij}\tilde{M}(\rho S_i^+ S_j^+ + S_i^+ S_j^+ \rho - 2S_j^+ \rho S_i^+) \\
&+ \frac{1}{2}\sum_{i,j=1}^{2} \Gamma_{ij}\tilde{M}^*(\rho S_i^- S_j^- + S_i^- S_j^- \rho - 2S_j^- \rho S_i^-).
\end{aligned}
$$

Dynamics of the Reduced Density Matrix continued...

$$\tilde{N} = N_{\mathrm{th}}(\cosh^2(r) + \sinh^2(r)) + \sinh^2(r),$$

$$\tilde{M} = -\frac{1}{2}\sinh(2r)e^{i\Phi}(2N_{\mathrm{th}} + 1) \equiv Re^{i\Phi(\omega_0)},$$

with

$$\omega_0 = \frac{\omega_1 + \omega_2}{2},$$

and

$$N_{\mathrm{th}} = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}.$$

- Here N_{th} is the Planck distribution giving the number of thermal photons at the frequency ω and r, Φ are squeezing parameters. The analogous case of a thermal bath without squeezing can be obtained from the above expressions by setting these squeezing parameters to zero, while setting the temperature (T) to zero one recovers the case of the vacuum bath.

Dynamics of the Reduced Density Matrix continued...

$$H_{\tilde{S}} = \hbar \sum_{n=1}^{2} \omega_n S_n^z + \hbar \sum_{\substack{i,j \\ (i \neq j)}}^{2} \Omega_{ij} S_i^+ S_j^- ,$$

where

$$\begin{aligned}
\Omega_{ij} &= \frac{3}{4}\sqrt{\Gamma_i \Gamma_j} \left[-[1 - (\hat{\mu}.\hat{r}_{ij})^2] \frac{\cos(k_0 r_{ij})}{k_0 r_{ij}} + [1 - 3(\hat{\mu}.\hat{r}_{ij})^2] \right. \\
&\times \left. [\frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^2} + \frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^3}] \right] .
\end{aligned}$$

$\hat{\mu} = \hat{\mu}_1 = \hat{\mu}_2$ and $\hat{r}_{ij}$ are unit vectors along the atomic transition dipole moments and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, respectively.
$k_0 = \omega_0/c$, $r_{ij} = |\vec{r}_{ij}|$.

Dynamics of the Reduced Density Matrix continued...

- Wavevector $k_0 = 2\pi/\lambda_0$, λ_0 being the resonant wavelength, occuring in the term $k_0 r_{ij}$ sets up a length scale into the problem depending upon the ratio r_{ij}/λ_0. This is thus the ratio between the interatomic distance and the resonant wavelength, allowing for a discussion of the dynamics in two regimes:
  (a). localized decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \geq 1$
  and
  (b). collective decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \rightarrow 0$.
- Collective decoherence would arise when the qubits are close enough for them to feel the bath collectively or when the bath has a long correlation length (set by the resonant wavelength λ_0) in comparison to the interqubit separation r_{ij}.

Dynamics of the Reduced Density Matrix continued...

- Ω_{ij}: a collective coherent effect due to the multi-qubit interaction and is mediated via the bath through the terms

$$\Gamma_i = \frac{\omega_i^3 \mu_i^2}{3\pi\varepsilon\hbar c^3}.$$

- The term Γ_i is present even in the case of single-qubit dissipative system bath interaction and is the spontaneous emission rate, while

$$\Gamma_{ij} = \Gamma_{ji} = \sqrt{\Gamma_i \Gamma_j} F(k_0 r_{ij}),$$

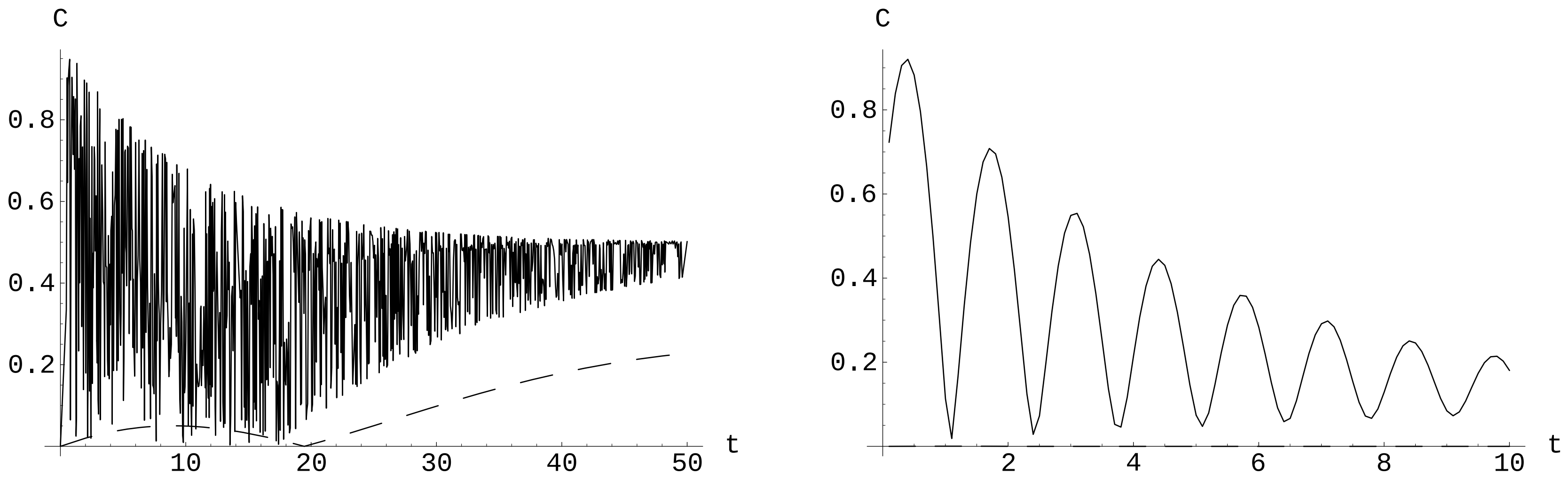
where $i \neq j$ with

$$\begin{aligned} F(k_0 r_{ij}) &= \frac{3}{2}\Bigg[[1-(\hat{\mu}.\hat{r}_{ij})^2]\frac{\sin(k_0 r_{ij})}{k_0 r_{ij}} + [1-3(\hat{\mu}.\hat{r}_{ij})^2] \\ &\times [\frac{\cos(k_0 r_{ij})}{(k_0 r_{ij})^2} - \frac{\sin(k_0 r_{ij})}{(k_0 r_{ij})^3}]\Bigg]. \end{aligned}$$

- Γ_{ij}: collective incoherent effect due to the dissipative multi-qubit interaction with the bath. For the case of identical qubits, as considered here, $\Omega_{12} = \Omega_{21}$, $\Gamma_{12} = \Gamma_{21}$ and $\Gamma_1 = \Gamma_2 = \Gamma$.

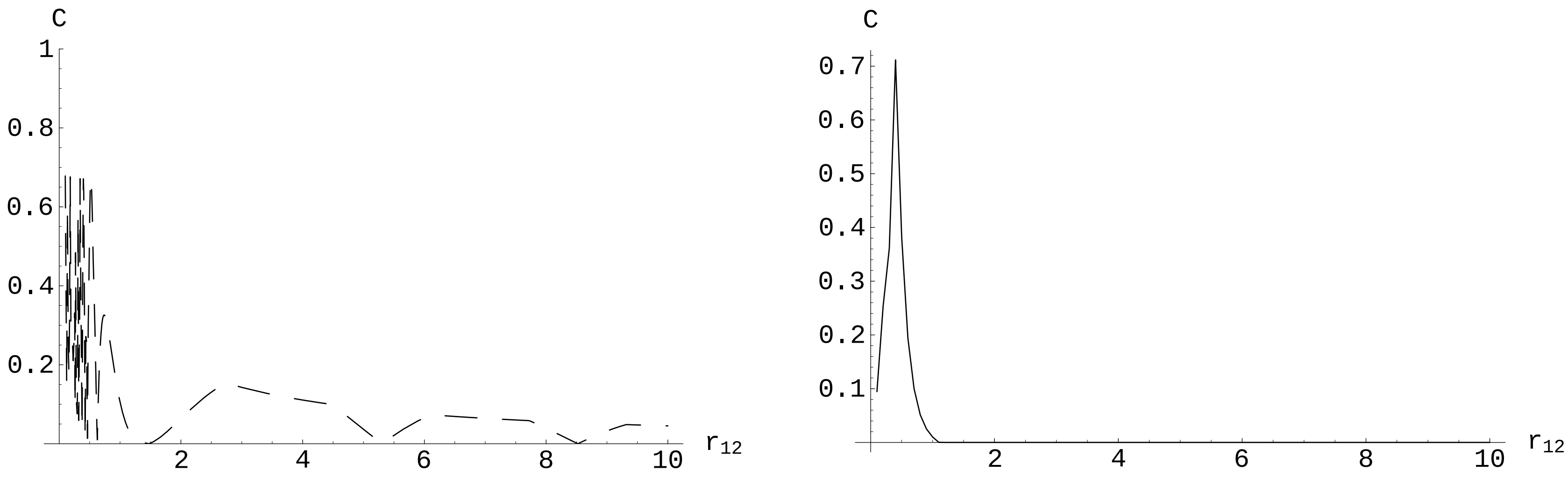
Dynamics of Entanglement

Dissipative Evolution



Figs. 2 & 3 : Concurrence $\mathcal{C}$ as a function of time of evolution t. Figure (2) deals with the case of vacuum bath ($T = r = 0$), while figure (3) considers concurrence in the two-qubit system interacting with a squeezed thermal bath, for a temperature $T = 1$ and and bath squeezing parameter r equal to 0.1. In both the figures the bold curve depicts the collective decoherence model ($kr_{12} = 0.05$), while the dashed curve represents the independent decoherence model ($kr_{12} = 1.1$). In figure (3) for the given settings, the concurrence for the independent decoherence model is negligible and is thus not seen.

Dissipative Evolution

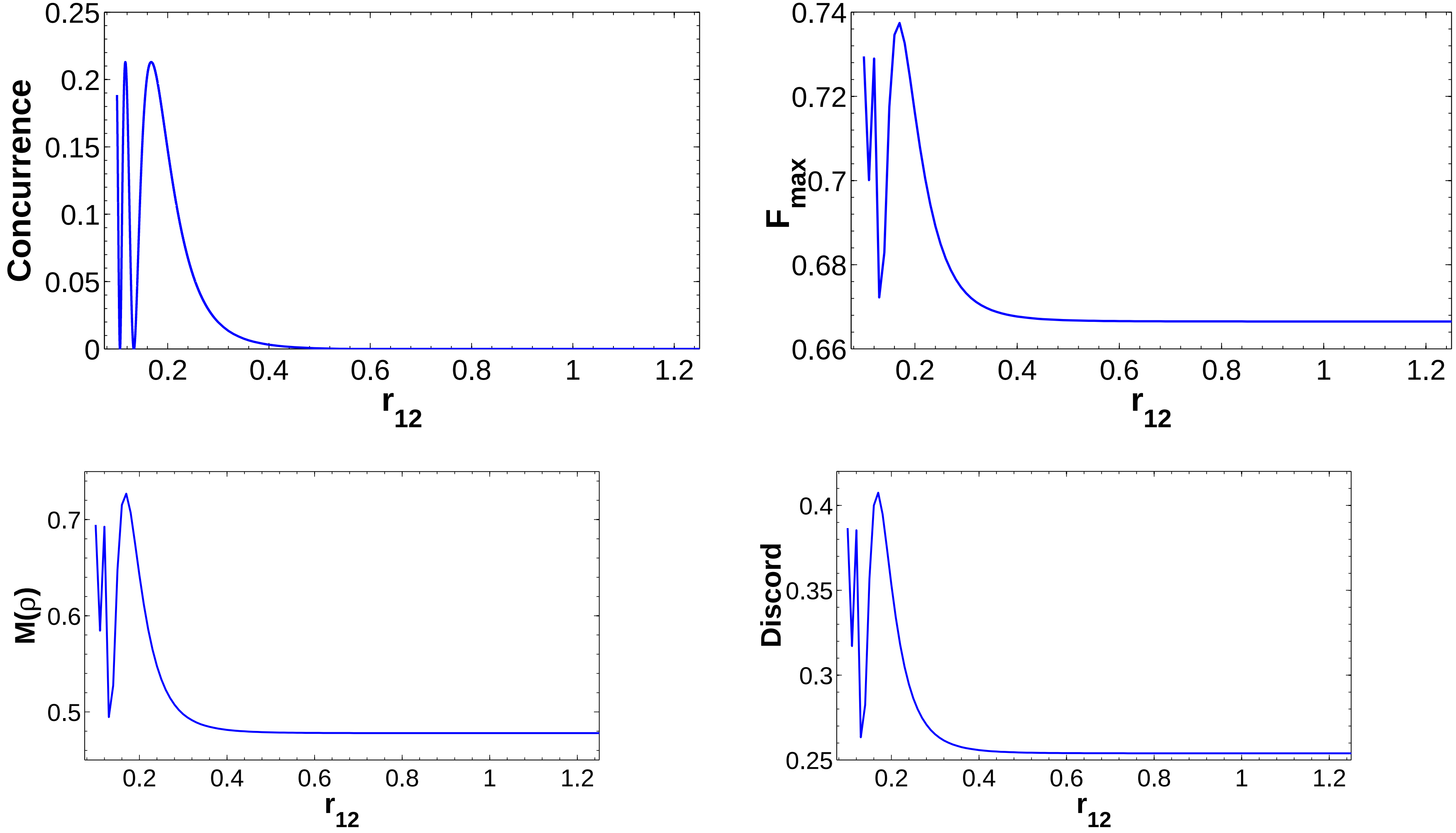


Figs. 4 & 5 : Concurrence $\mathcal{C}$ with respect to inter-qubit distance r_{12}. Figure (4) deals with the case of vacuum bath ($T = r = 0$), while figure (5) considers concurrence in the two-qubit system interacting with a squeezed thermal bath, for $T = 1$, evolution time $t = 1$ and bath squeezing parameter r equal to 0.1. In figure (4) the oscillatory behavior of concurrence is stronger in the collective decoherence regime, in comparison with the independent decoherence regime ($kr_{12} \geq 1$). In figure (5), the effect of finite bath squeezing and T has the effect of diminishing the concurrence to a great extent in comparison to the vacuum bath case. Here the concurrence for the localized decoherence regime is negligible, in agreement with the previous figure.

Dynamics of Quantum Correlations

- We made a comparative study, on states generated by our open system two-qubit models: (SB, V. Ravishankar and R. Srikanth: (2009), (2010)), of various features of quantum correlations like teleportation fidelity (F_{max}), violation of Bell's inequality $M(\rho)$ (violation takes place for $M(\rho) \geq 1$), concurrence $C(\rho)$ and discord with respect to various experimental parameters like, bath squeezing parameter r, inter-qubit spacing r_{12}, temperature T and time of evolution t (I. Chakrabarty, SB, N. Siddharth: (2011)).
- A basic motivation of this work is to have realistic open system models that generate entangled states which can be useful for teleportation, but at the same time, not violate Bell's inequality. We provide below some examples of such states. Interestingly, we also find examples of states with positive discord, but zero entanglement, reiterating the fact that entanglement is a subset of quantum correlations.

Dynamics of Quantum Correlations: Dissipative



Dynamics of Quantum Correlations: Dissipative continued...

Fig. 6 : Quantum correlations in a two-qubit system undergoing a dissipative evolution. The Figs. (a), (b), (c) and (d) represent the evolution of concurrence, maximum teleportation fidelity F_{max}, test of Bell's inequality $M(\rho)$, discord as a function of inter-qubit distance r_{12}. Here temperature $T = 300$, evolution time t is 0.1 and bath squeezing parameter $r = -1$. From Fig. (6 (a)), we find that the two qubit density matrix is entangled with a positive concurrence except at the point 0.133 (approx) and for $r_{12} \geq 0.4$. Figure 65 (b)) illustrates that $F_{max} > \frac{2}{3}$, for all values of r_{12} except where there is no entanglement. However, from Fig. (6 (c)) we find that $M(\rho) < 1$ for all values of r_{12}, clearly demonstrating that the states can be useful for teleportation despite the fact that they satisfy Bell's inequality. Moreover, from Fig. (6 (d)), a positive discord is seen for the complete range of r_{12}, even in the range where there is no entanglement. As a function of the inter-qubit distance, the various correlation measures exhibit oscillatory behavior, in the collective regime of the model, but flatten out subsequently to attain almost constant values in the independent regime of the model. This oscillatory behavior is due to the strong collective behavior exhibited by the dynamics due to the relatively close proximity of the qubits in the collective regime.

Dynamics of Quantum Correlations: Quantum Optical Model

- The use of generic quantum optical models, such as semiclassical three-level atomic systems, that can be experimentally implemented and observed, can serve as an important tool to generate, investigate, verify and control nonclassical correlations and their features (H. Dhar, SB, A. Chatterjee, R. Ghosh (2013)).
- We study the nonclassical correlation properties of the photon states emitted from a three-level atomic system interacting with two classical driving fields. The interactions generate two-mode single photon states, arising from two controlled coherent transitions connecting the three levels, under the single photon approximation (SPA) (B. R. Mollow (1975)).
- The system can be set up in three different configurations, Ξ, Λ and V. We establish a qualitative relation between the two different theoretical classes of correlation measures, entanglement and the measurement-based correlations such as MID, QD and WD and Dynamics of Quantum Correlations: Quantum Optically. The control parameters in the system enable us to define specific regimes where certain correlations are enhanced based on the nature of the output photon states.

Dynamics of Quantum Correlations: Quantum Optical continued...

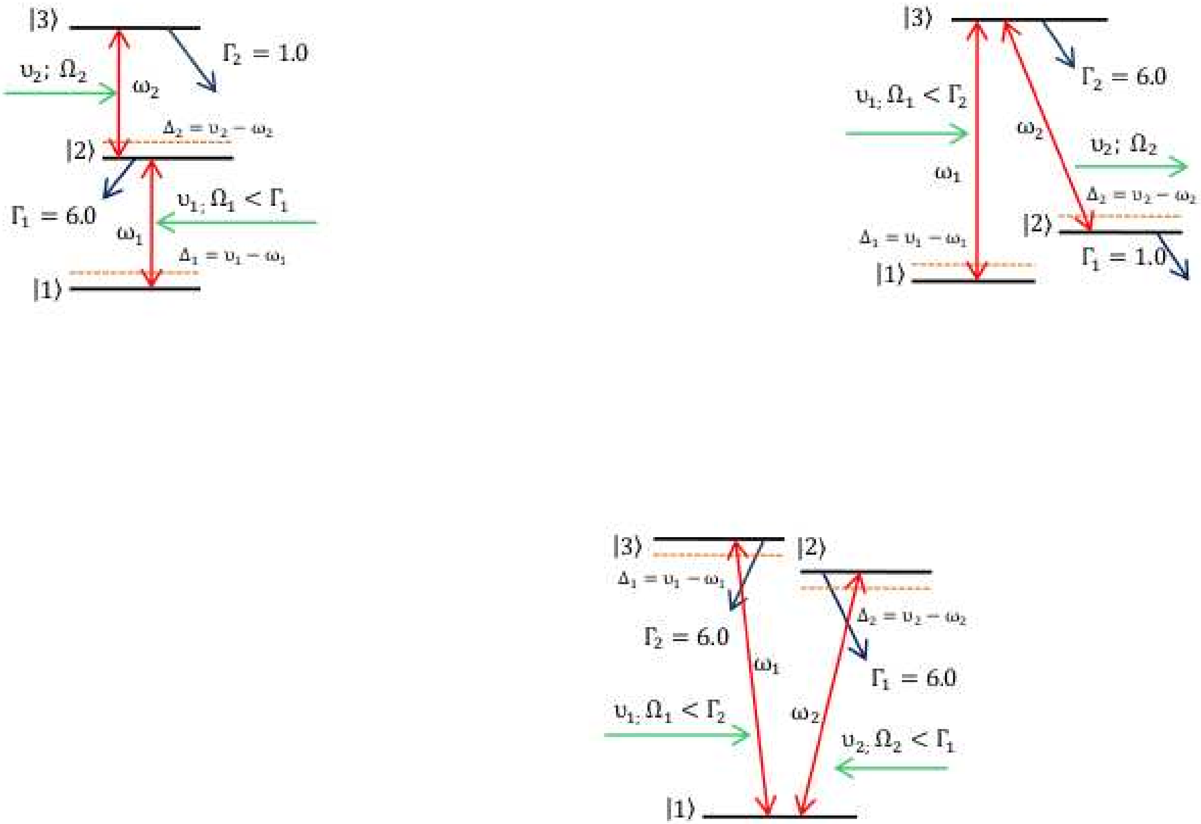


Fig. 7 : A three-level atom in the (a) Ξ, (b) Λ, and (c) V configuration. Γ_1 and Γ_2 are the decay constants (numbers shown in units of MHz) of the levels $|2\rangle$ and $|3\rangle$. ν_1, ν_2, and Ω_1, Ω_2 are the optical frequencies and the Rabi frequencies of the two near-resonant driving fields. ω_1 and ω_2 are the two atomic transition frequencies. Δ_1 and Δ_2 are the field detunings, set to zero here.

Dynamics of Quantum Correlations: Quantum Optical continued...

- A three-level atom can be used in three different configurations, namely, Ξ, Λ and V, for example, we focus on a gas of rubidium (Rb) atoms (J. G-Banacloche et al. (1995)).
- The energy levels $5S_{1/2}$, $5P_{3/2}$ and $5D_{5/2}$ of Rb can be suitably used to generate each of the three configurations.
- Level $5S_{1/2}$ is the ground state and does not decay. Level $5D_{5/2}$ is metastable, with a decay rate of about 1.0 MHz. while $5P_{3/2}$ has a decay rate of $\approx$ 6.0 MHz.

Dynamics of Quantum Correlations: Quantum Optical continued...

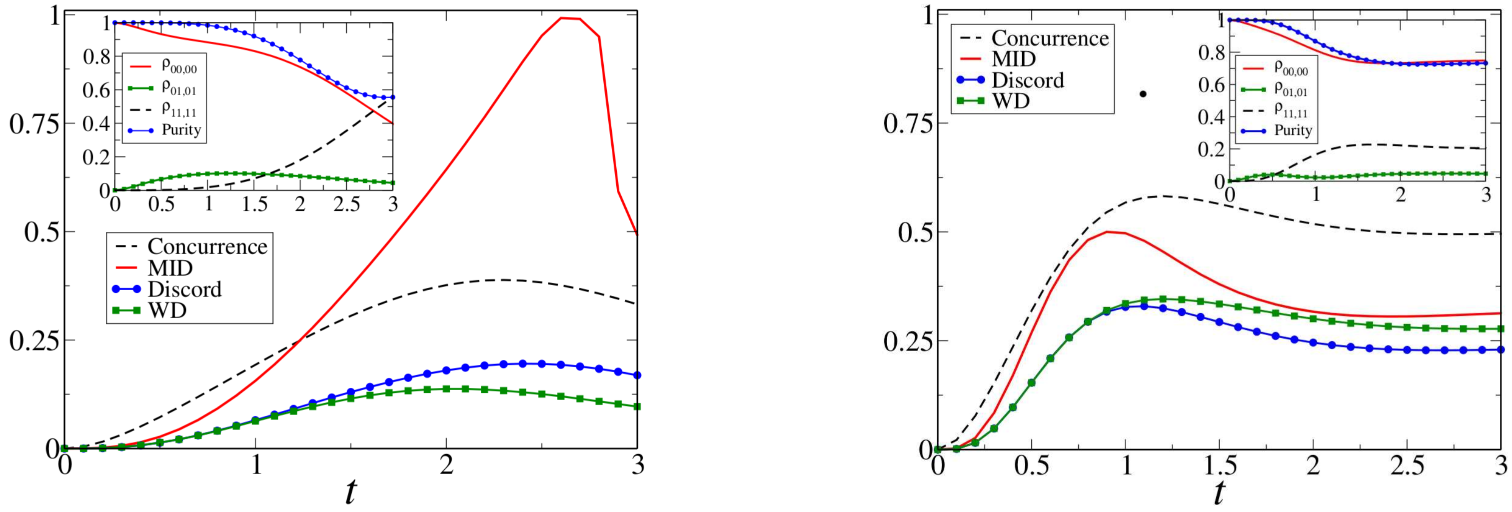
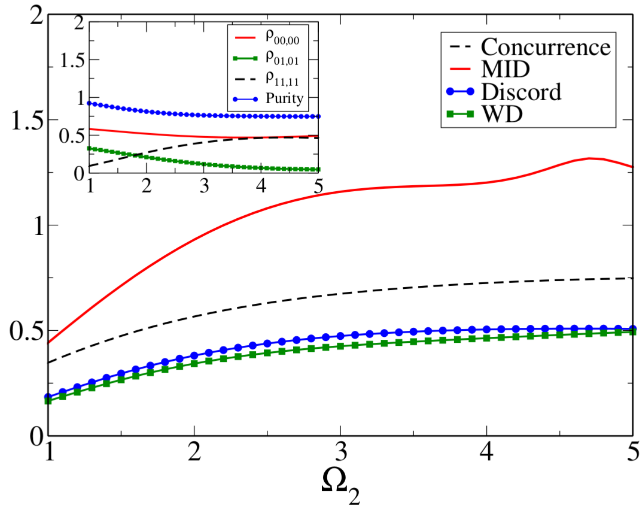


Fig. 8 & 9 : The time evolution for correlation measures MID (red continuous), discord (blue circles) and work deficit (green squares) along with the entanglement measure concurrence (black dashed) for the cascade (Ξ) configuration. The field detunings are $\Delta_1 = \Delta_2 = 0$, and the phases of the Rabi frequencies are $\phi_1 = \phi_2 = 0$. The level decay rates are $\Gamma_1 = 6.0$, $\Gamma_2 = 1.0$. SPA for this configuration requires that $\Omega_1 < \Gamma_1$. The driving field strengths are $\Omega_1 = 2.0$, $\Omega_2 = 1.0$, and $\Omega_1 = 2.0$, $\Omega_2 = 5.0$, in Figures (8) and (9), respectively. The inset shows the evolution of population elements of the two-photon density matrix and its purity.

Dynamics of Quantum Correlations: Quantum Optical continued...



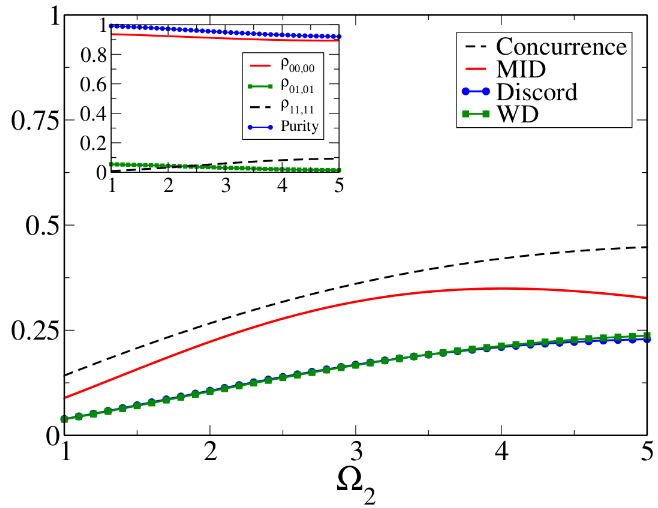


Fig. 10 & 11 : Fixed time (t = 1.0) MID (red continuous), discord (blue circles), work deficit (green squares), and concurrence (black dashed) in the Ξ configuration as a function of the driving field strength Ω_2. The field detunings are $\Delta_1 = \Delta_2 = 0$, and the phases of the Rabi frequencies are $\phi_1 = \phi_2 = 0$. The level decay rates re $\Gamma_1 = 6.0$, $\Gamma_2 = 1.0$. SPA for this configuration requires that $\Omega_1 < \Gamma_1$. One driving field strength Ω_1 is fixed at 1.5, and 3.5, in Figures (10) and (11), respectively. The inset shows the variation of population elements of the two-photon density matrix and its purity.

Dynamics of Quantum Correlations: Quantum Optical continued...

- Some general observations can be made that are consistent with known results: MID always serves as an upper bound on the other measurement based correlations, such as, QD and WD (Girolami, Paternostro and Adesso (2010); B R Rao, R Srikanth, C M Chandrashekar and SB (2011)).
- In Fig. (8), for $\Omega_1 > \Omega_2$, MID forms a non-monotonic upper bound on concurrence at times $t > 1.0$. For the field regime $\Omega_1 < \Omega_2$ $(\Omega_1 = 2.0, \Omega_2 = 5.0)$, Fig. (9), concurrence forms a monotonic upper bound on the measurement-based correlations. We observe that the behavior of the correlations is closely related to the dynamics of the populations (inset of Figures). The non-monotonic behavior of MID is associated with the population difference in the two photon modes $|00\rangle$ and $|11\rangle$. It is clear from the plots that the sudden increase in MID occurs when the populations of the modes $|00\rangle$ and $|11\rangle$ are nearly equal. This could be due to the fact that the non-optimization of the correlation measure in MID is skewed in these regions. For cases where MID is monotonic with the other measurement based measures, the population is distinctly unequal. Observing the purity in these regimes, one can state that the monotonicity is observed at higher levels of purity.

Dynamics of Quantum Correlations: Quantum Optical continued...

- A similar dichotomy in behavior can also be observed for fixed time dynamics of the system if the interaction is allowed to vary across driving field strengths. In Figures. (10) and (11), keeping the evolution time fixed and varying the two classical driving field strengths, a similar behavior of the correlations is observed. MID is greater than concurrence and non-monotonic at times where the population levels are equal with significantly lower purity as compared to regimes with unequal populations and higher purity where the measurement based correlations are monotonic to concurrence. Hence, we observe that the fixed time dynamics allows us to manipulate the correlation hierarchy by changing the ground-state driving field strength, Ω_1. The generation of monotonic correlations can be controlled by using parameter regions that allow higher purity in the output photon state.

Dynamics of Quantum Correlations: Quantum Optical continued...

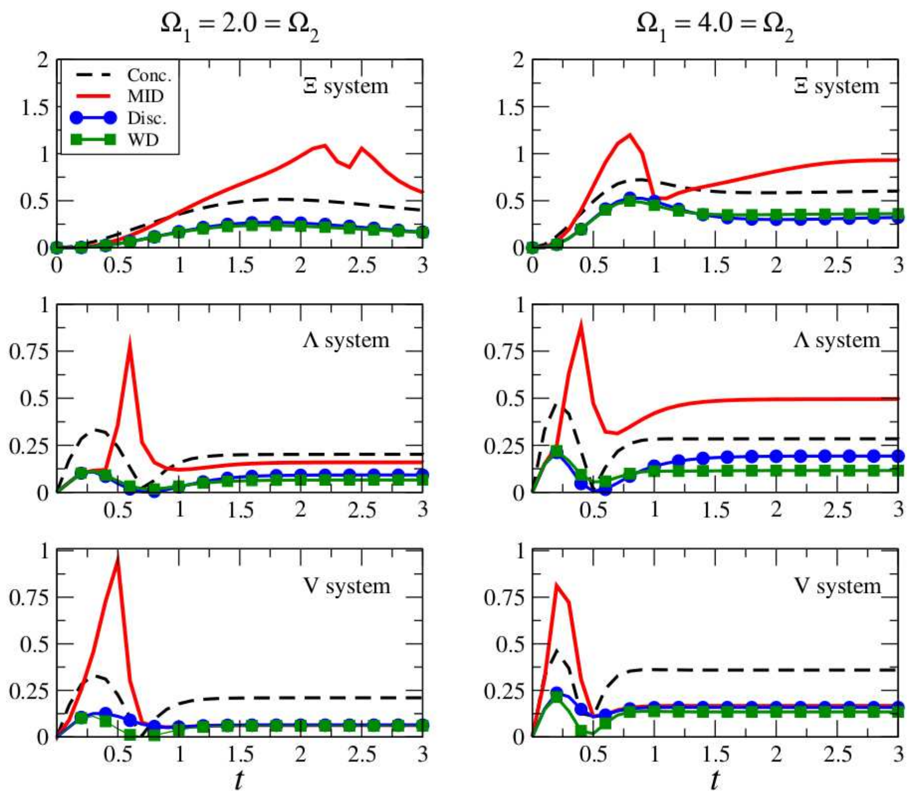


Fig. 12 : Time evolutions of MID (red continuous), discord (blue circles), work deficit (green squares) and concurrence (black dashed) for all the three configurations, Ξ (top), Λ (middle) and V (bottom). The field detunings are Δ_1 = Δ_2 = 0, and the phases of the Rabi frequencies are ϕ_1 = ϕ_2 = 0. The chosen driving field strengths of $\Omega_1 = \Omega_2$ = 2.0 (left panel) and $\Omega_1 = \Omega_2$ = 4.0 (right panel) satisfy the SPA for all three configurations.

Dynamics of Quantum Correlations: Quantum Optical continued...

- The correlation behavior is observed to be configuration dependent. The manipulation of control parameters in different configurations leads to variations in the dynamic evolution of the correlations.
- The Ξ configuration produces photon states with relatively high correlation even at low driving fields. The Ξ system can be suitably controlled using the driving fields to generate correlations dominated both by MID or concurrence and is ideally suited to experimentally study the temporal evolution of the two measures with respect to the evolution of the system in the Hilbert space.
- Λ and V systems are better suited for generating steady monotonic correlations in both low and high strength driving field regimes. The Λ system can be suitably tuned to generate steady correlations with either entanglement or MID as an upper bound.
- V systems, however, can generate ideally pure correlated photons bounded by concurrence at all field strength regimes. The absence of a metastable state in the V system allows production of pure correlated output photon states. The measurement-based correlations are all equal at steady values. However, in the Λ and V systems, significant correlation is generated only for high driving field strengths.

Dynamics of Quantum Correlations: Quantum Optical continued...

- Hence, specific regimes and configurations can be used to generate and manipulate the correlations in the output two-photon state as desired. These results may be useful in practical implementations with such interacting photon states.

Geometric Phase (GP) in Open Quantum Systems

Brief history of GP

- Pancharatnam defined a phase characterizing the intereference of classical light in distinct states of polarization (1956).
- Berry (1984) discovered that under cyclic adiabatic evolution, system acquires extra phase over dynamical phase.
- Simon (1983) establised geometric nature of GP, linked to notion of parallel transport, depends only on area covered by motion, independent of how motion is executed.
  (consequence of the holonomy in a line bundle over parameter space)
- Generalization of GP to non-adiabatic evolution (Aharonov and Anandan 1987) to non-cyclic evolution (Samuel and Bhandari 1988)
- GP as a consequence of quantum kinematics (Mukunda & Simon 1993).
- GP defined for nondegenerate density opertors undergoing unitary evolution (Sjöqvist *et al.* 2000)
- Extended by Singh *et al.* (2003) to the case of degenerate density operators.
- Kinematic approach to define GP in mixed states undergoing nonunitary evolution Tong *et al.* (2004) – which we use.

Motivation

- The geometric nature of GP implies an inherent fault tolerance and would be useful for quantum computers (Duan, Cirac, Zoller: (2001)).
- There have been proposals to observe GP in superconducting nanocircuits (Falci, Fazio, Palma, *et al.*: (2000)). Here the effect of the environment is never negligible (Nakamura, Pashkin, Tsai: (1999)).
- The above points provide a strong motivation for studying GP in the context of Open Quantum Systems. Work along these lines was initiated by (Whitney, Gefen: (2003); Whitney, Makhlin, Shnirman, Gefen: (2005)).

A. Geometric Phase (GP) in Two-Level Open System

(SB and R. Srikanth: (2007))

- System Hamiltonian

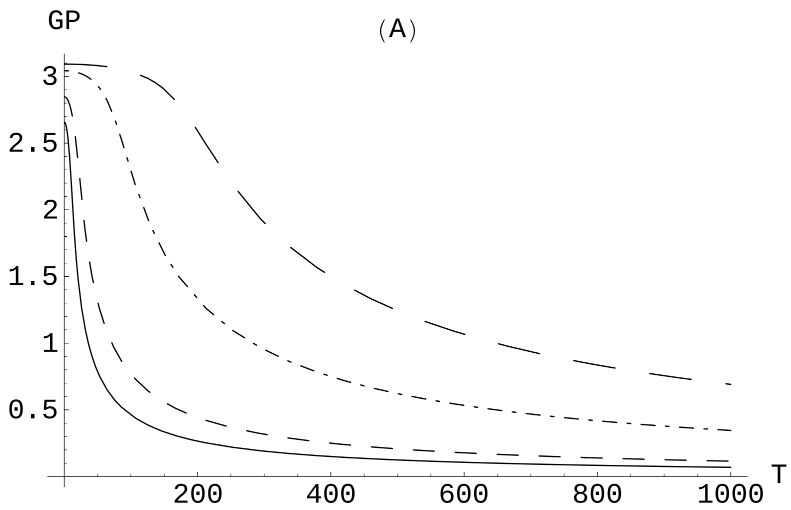
$$H_S = \frac{\hbar\omega}{2}\sigma_z .$$

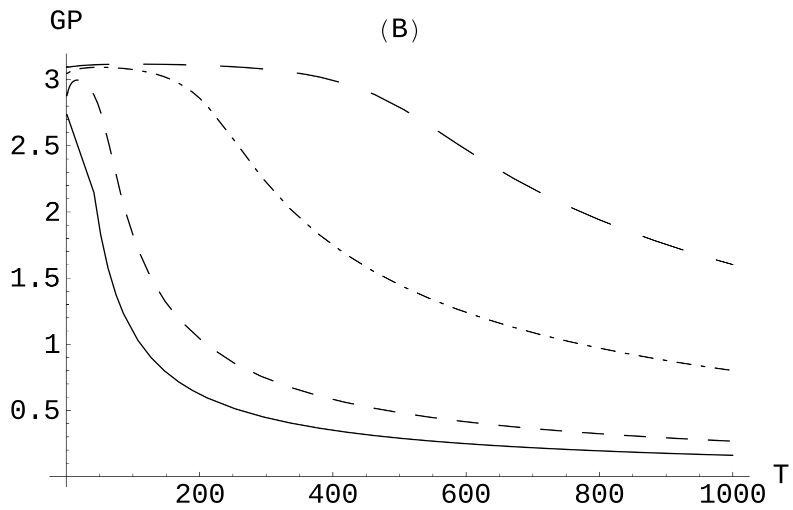
- System interacts with a squeezed-thermal bath via a QND or a dissipative interaction.
- Use prescription of Tong *et al.* (2004)

$$\Phi_{\rm GP} = \arg\left(\sum_{k=1}^{N}\sqrt{\lambda_k(0)\lambda_k(\tau)}\langle\Psi_k(0)|\Psi_k(\tau)\rangle e^{-\int_0^\tau dt\langle\Psi_k(t)|\dot{\Psi}_k(t)\rangle}\right).$$

- $\lambda_k(\tau)$, $\Psi_k(\tau)$: eigenvalues, eigenvectors of reduced density matrix.

Geometric Phase (GP)... continued...





Figs. 13 (A) & (B) : GP (in radians) as function of temperature (T) for dissipative interaction with a bath of harmonic oscillators. Here $\omega = 1$, $\theta_0 = \pi/2$, the large-dashed, dot-dashed, small-dashed and solid curves, represent, $\gamma_0 = 0.005, 0.01, 0.03$ and 0.05, respectively. Fig. 13(A) zero squeezing, Fig. 13(B): squeezing non–vanishing, with $r = 0.4$ and $\Phi = 0$. GP falls with T. Effect of squeezing: GP varies more slowly with T, by broadening peaks and flattening tails. Counteractive action of squeezing on influence of T on GP useful for practical implementation of GP phase gates.

B. Geometric Phase (GP) in Three-Level Quantum Optical System

(Sandhya and SB: (2011))

- Holonomic quantum computation requires the evolution of qubits in parameter space: by the control of parameters which are physically feasible.
- Atom photon interaction provides a rich ground for exploring geometric phase with the added advantage of the existence of control parameters for manipulating the photon states.
- We study the geometric phase of the two-photon state corresponding to the two modes emitted by the two dipole transitions of three level cascade system interacting with two driving fields. The two photon state is in general a mixed state.
- The evolution in the state space is made by varying the control parameters namely, the driving field strength and detuning.

B. Geometric Phase (GP) in Three-Level... continued...

Model

- The scheme considered here corresponds to a three level cascade system interacting with two coherent fields which address the only two allowed dipole transitions $|i\rangle \leftrightarrow |i+1\rangle, i = 1, 2$ with energy separation given by ω_i.
- Two counter propagating (Doppler free geometry) driving fields of nearly equal frequencies ω_{L1} and ω_{L2} and respective strengths Ω_1 and Ω_2 are resonant with these two transitions.
- The decay constants of the energy levels $|3\rangle$ and $|2\rangle$ are indicated by Γ_3 and Γ_2, respectively. The parameters $\Delta_1,\ \Delta_2$ refer to the detunings of the driving fields.
- This scheme may be realized for example in ^{87}Rb vapor with the corresponding energy levels $5S_{1/2}, 5P_{3/2}$ and $5D_{5/2}$ and has been used by (Banacloche *et al.*: (1995)).
- We use the prescription given in (Tong *et al.* (2004)) to determine the geometric phase of the two-photon mixed states emitted by the three level system.

B. Geometric Phase (GP) in Three-Level... continued...

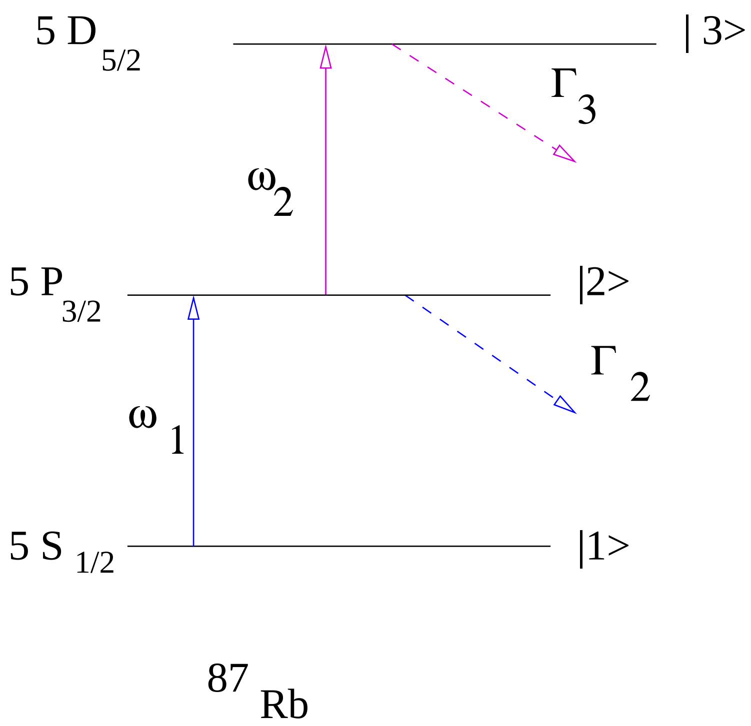
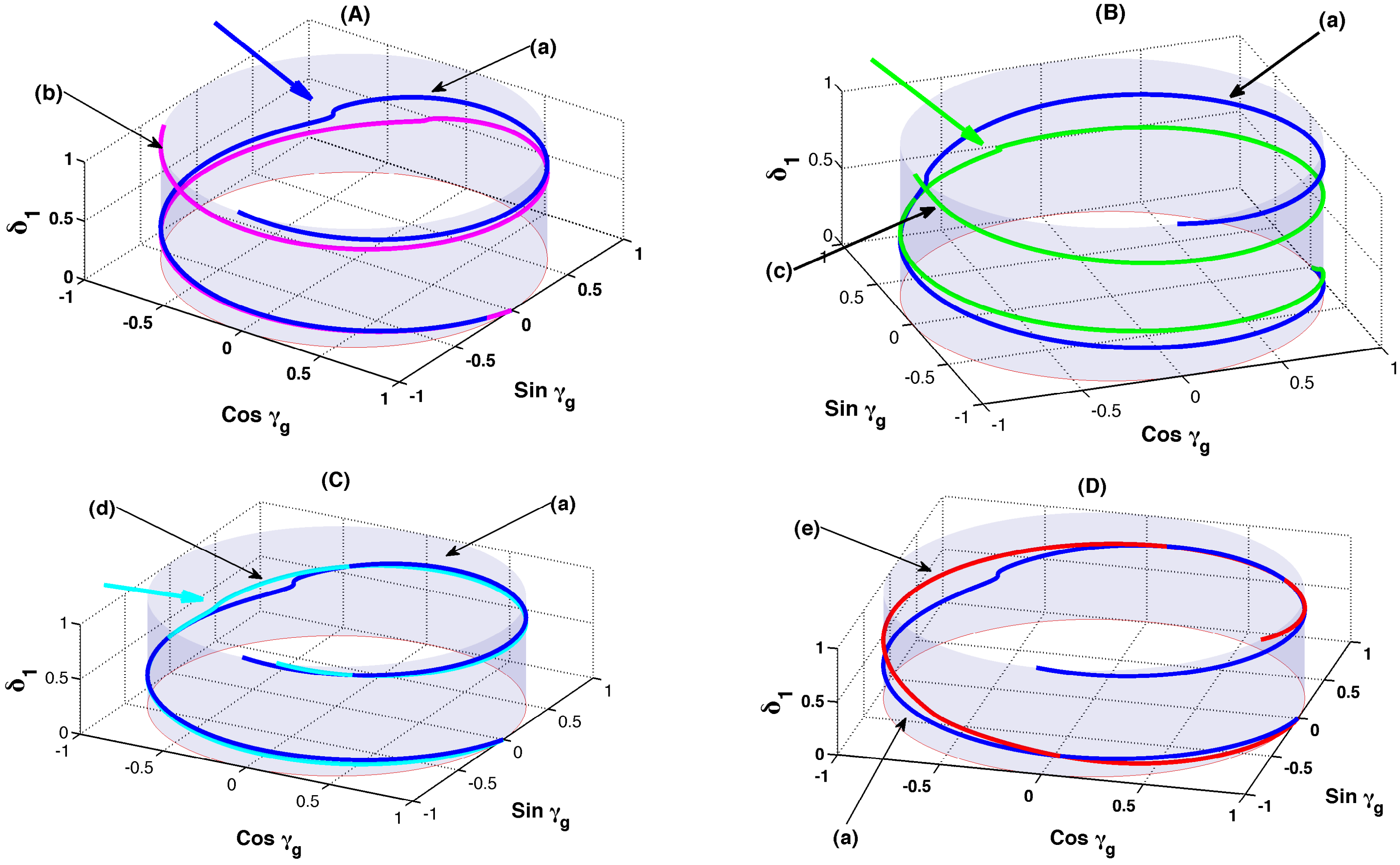


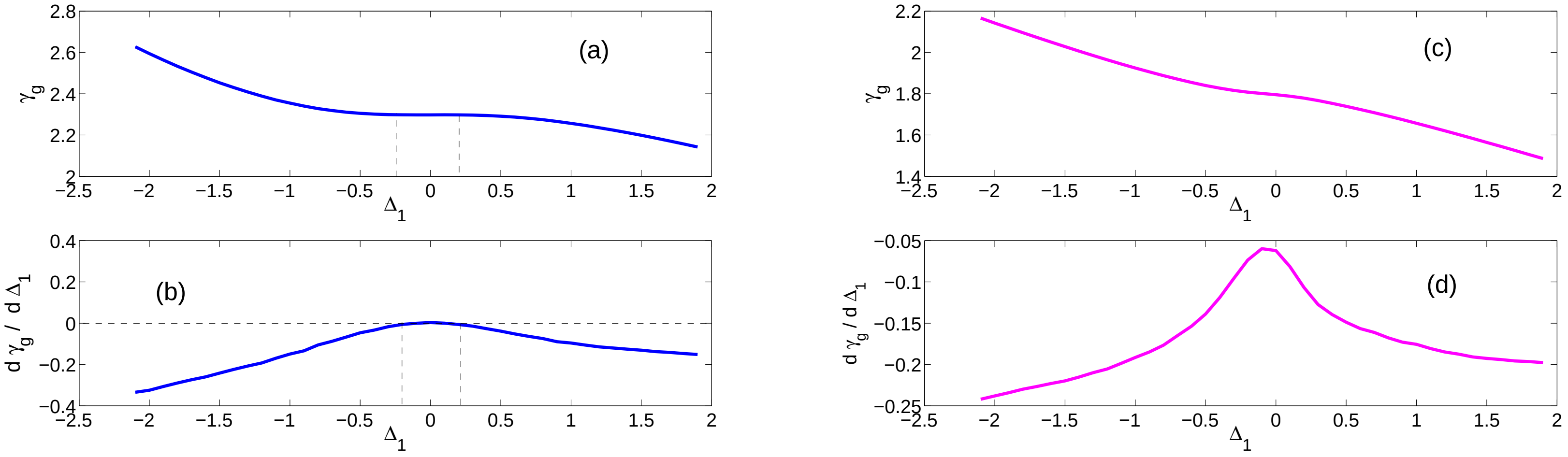
Fig. 14 : Three level cascade system corresponding to the ^{87}Rb atoms driven by two fields ω_1 and ω_2.

B. Geometric Phase (GP) in Three-Level... continued...



Figs. 15 : Variation of the geometric phase with the rescaled detuning parameter $\delta_1 = (\Delta_1 + 10.0)/20.0$.

B. Geometric Phase (GP) in Three-Level... continued...



Figs. 16 : Variation of the geometric phase and its derivative with the detuning parameter Δ_1. Parameter values of figures 16 (a) and (b) are those corresponding to the curve (a) of Figs. 15; figures 16 (c) and (d) correspond to curve (b) of Figs. 15. The parameter values of the curves are: (a) $\Omega_1 = \Omega_2 = 6.0, \Delta_2 = 0$, (b) $\Omega_1 = 3.0, \Omega_2 = 6.0, \Delta_2 = 0$, (c) $\Omega_1 = 6.0, \Omega_2 = 3.0, \Delta_2 = 0$, (d) $\Omega_{1,2} = 6.0, \Delta_2 = 3.0$ and (e) $\Omega_{1,2} = 6.0, \Delta_2 = 6.0$.

B. Geometric Phase (GP) in Three-Level... continued...

Model

- In Figs. 15, for the case of curve (a) γ_g does not change in the neighborhood of $\delta_1 = 0.5$ which results in a smaller sweep while the variation of the angle is uniformly slow in the case of curve (e).
- In Figs. 16, the details of the variation of the curves (a) and (b) in the neighborhood of $\Delta_1 = 0$ are presented.
- Figs. 16 (a), (b) show the variation of γ_g and the rate of change of γ_g, respectively with Δ_1 near $\Delta_1 = 0$ of the curve (a) of Figs. 15. γ_g is constant in the region $-0.25 < \Delta_1 < 0.25$. This is substantiated by the vanishing of the derivative in this region thus indicating *that the geometric phase in this case is stable under small perturbations in the neighborhood of* $\Delta_1 = 0$.
- In contrast, in the case of Figs. 16 (c), (d) γ_g is never a constant even though the rate of change of γ_g slows down.

C. Geometric Phase (GP) in a Frustrated Spin System

(SB, C. M. Chandrashekar and A. K. Pati: (2013))

- A practical implementation of GP would involve a qubit interacting with its environment, resulting in its inhibition. This calls for the need to have settings where the inhibition of GP, due to the ubiquitous environment, could be arrested.
- Quantum frustration of decoherence (QFD), would be a potential candidate for achieving this.
- QFD is the term ascribed to the general phenomena when a quantum system coupled to two independent environments by canonically conjugate operators results in an enhancement of quantum fluctuations, that is, decoherence gets suppressed (E. Novais et al.: (2008)). The reason for this is attributable to the non-commuting nature of the conjugate coupling operators that prevents the selection of an appropriate pointer basis to which the quantum system could settle down to.

C. Geometric Phase (GP) in a Frustrated... continued...

Model

- It has been studied in various guises, such as an extension of the dissipative two-level system problem where the two non-commuting spin operators of the central spin system were coupled to independent harmonic oscillator baths, or a harmonic oscillator, modelling a large spin impurity in a ferromagnet, coupled to two independent oscillator baths via its position and momentum operators (H. Kohler and F. Sols: (2008)).
- In each case, irrespective of the system of interest or the coupling operators, QFD was observed.

C. Geometric Phase (GP) in a Frustrated... continued...

Model

- We study the influence of QFD on GP by taking up a simple model involving a central spin, or a qubit which would be our system of interest, interacting with two independent spin baths via two non-commuting spin operators

$$\begin{aligned} H &= H_S + H_{SR} \\ &= \omega \frac{\sigma_z}{2} + \alpha_1 \frac{\sigma_x}{2} \otimes \Sigma_{k=1}^N I_x^k + \alpha_2 \frac{\sigma_y}{2} \otimes \Sigma_{l=1}^N J_y^l, \end{aligned} \tag{1}$$

  where H_S is the system (single qubit) Hamiltonian and H_{SR} is the system-reservoir interaction Hamiltonian. Here σ_i, $i = x, y, z$ are the three Pauli matrices for the central spin, I_x^k and J_y^l are the bath spin operators. Also, α_1, α_2 are the two spin-bath coupling constants and ω comes from the basic system Hamiltonian, representing the initial magnetic field.

- Such a model could be envisaged in solid state spin systems with dominant spin-environment interactions, such as quantum dots.

C. Geometric Phase (GP) in a Frustrated... continued...

- The full form of the initial density matrix with an unpolarized initial bath state is $\rho_{SR}(0) = \frac{1}{2^{2N}}\rho_S(0) \otimes \mathcal{I}_{2^N} \otimes \mathcal{I}_{2^N}$, where N is the total number of spins present in each bath.
- Here

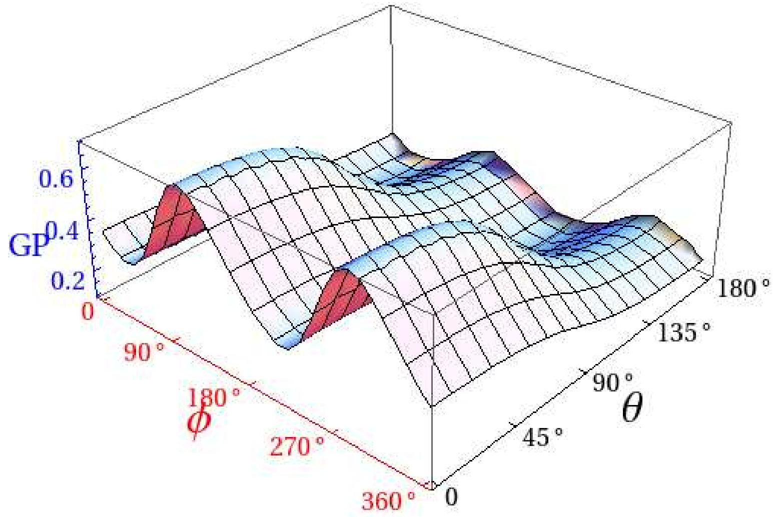
$$\begin{aligned} \rho_S(0) &= \cos^2\left(\frac{\theta}{2}\right)|\downarrow\rangle\langle\downarrow| + \sin^2\left(\frac{\theta}{2}\right)|\uparrow\rangle\langle\uparrow| \\ &+ \frac{i}{2}\sin(\theta)e^{i\phi}\left[|\uparrow\rangle\langle\downarrow| - e^{-i2\phi}|\downarrow\rangle\langle\uparrow|\right]. \end{aligned} \quad (2)$$

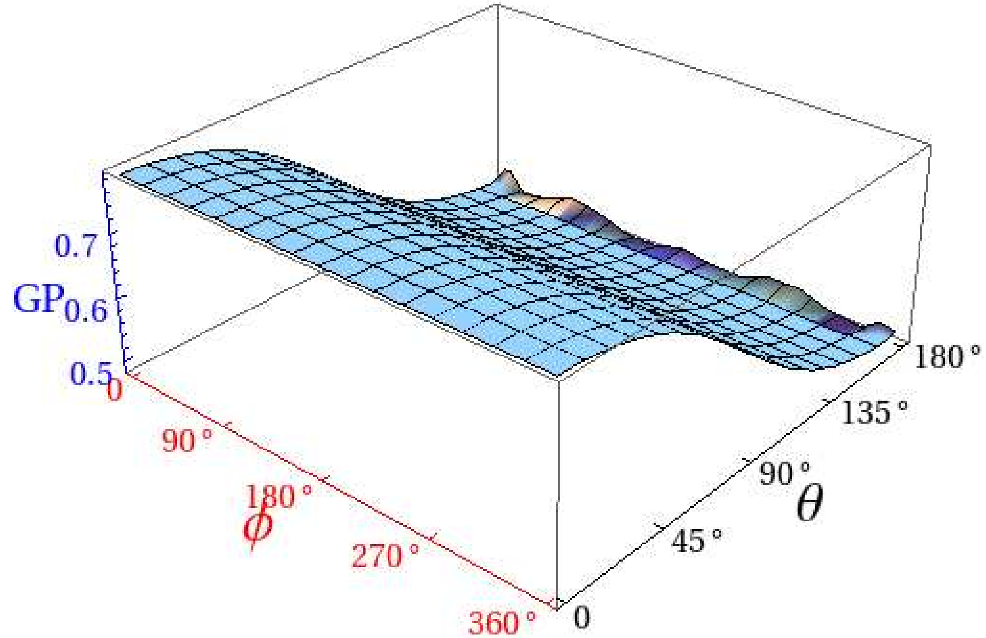
  Also, $\theta \in \{0, \pi\}$ and $\phi \in \{0, 2\pi\}$ are the polar and azimuthal angles, respectively.
- After interaction, the reduced state of the central spin, in the Bloch vector representation is

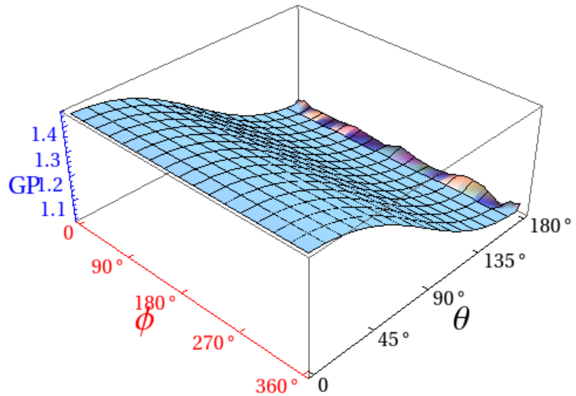
$$\rho_S(t) = \frac{1}{2}\begin{pmatrix} 1 + \langle\sigma_z(t)\rangle & \langle\sigma_x(t)\rangle - i\langle\sigma_y(t)\rangle \\ \langle\sigma_x(t)\rangle + i\langle\sigma_y(t)\rangle & 1 - \langle\sigma_z(t)\rangle \end{pmatrix}, \quad (3)$$

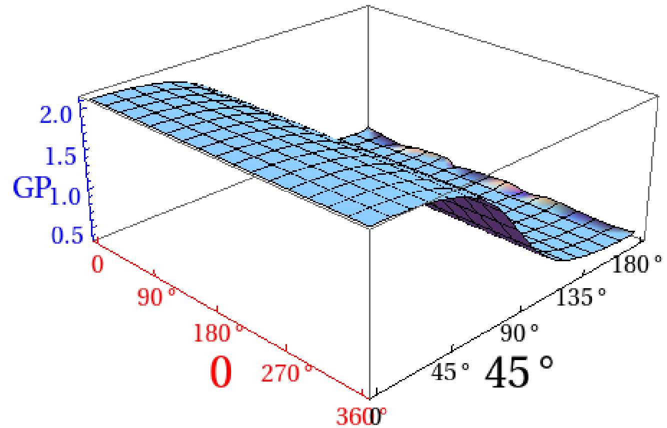
  where $\langle\sigma_i(t)\rangle$ are the average polarizations of the central spin. (SB et al.: (2013))
- This is then used in the Tong et al. prescription to study the GP.

C. Geometric Phase (GP) in a Frustrated... continued...









continued ...

Fig. 17 : GP ($\gamma_g(\tau)$) with respect to θ and ϕ (a) when $\alpha_1 = 1$ and $\alpha_2 = 0$, that is, for the case of a single bath, (b) when $\alpha_1 = \alpha_2 = \frac{1}{\sqrt{2}}$, (c) when $\alpha_1 = \frac{\sqrt{3}}{2}$ and $\alpha_2 = \frac{1}{2}$ (d) when $\alpha_1 = \alpha_2 = \frac{1}{4}$. Here $\omega = 2$, and time $t = 50$. A comparison between (a), (b), (c), and (d) reiterates the point that the decay of GP gets frustrated when both the baths are acting and one of the best strategies is seen to be the case where $\alpha_1 = \alpha_2 = \frac{1}{4}$.
(SB et al.: (2013))

C. Geometric Phase (GP) in a Frustrated... continued...

Analogy to Parrondos Paradox

- The effect of frustration on GP could be thought of as a Parrondo's game: each game on its own is "a single qubit interacting with its bath; one with σ_x, and with another σ_y"; this would result in decoherence and dissipation leading to inhibition of GP. This would be the situation where each player looses his game.
- However, when the two games are played in a synchronized fashion; corresponding, here, to the case of "the qubit interacting with two independent baths via non-commuting operators with coupling strengths α_1 and α_2", then the decoherence and dissipation can get frustrated leading to improvement in GP over some range of parameters.

Conclusion

- Here we have discussed some aspects of Quantum Information with an interface from Quantum Optics.
- We have made use of a number of aspects of Open Quantum Systems in our analysis.

Thank you!