Einstein's Recoiling-Slit Experiment: Uncertainty and Complementarity

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BITS Pilani K K Birla Goa Campus

QIPA 2013, 4th Dec

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Center for Theoretical Physics
Jamia Millia Islamia, New Delhi

Outline

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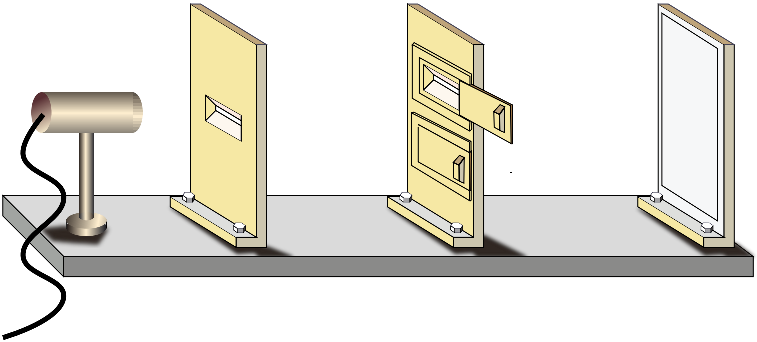
1. Two-Slit Experiment and Complementarity
   - Two Slit Experiment with Quantum Particles
   - Complementarity
   - Einstein's Recoiling Slit Experiment
   - ...and Bohr's Reply
2. Complementarity and Entanglement
   - von Neuman Measurements
   - Which-way Information and Interference
   - Path Distinguishability and Fringe Visibility
3. Complementarity and Uncertainty
   - Duality and Uncertainty
4. Conclusions

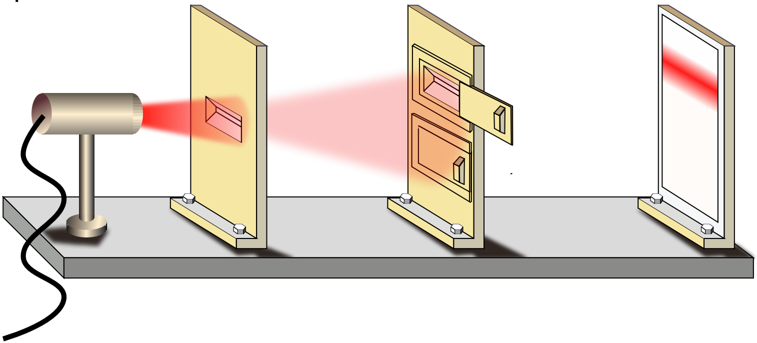
The Two-Slit Experiment with Quantum particles

Setup



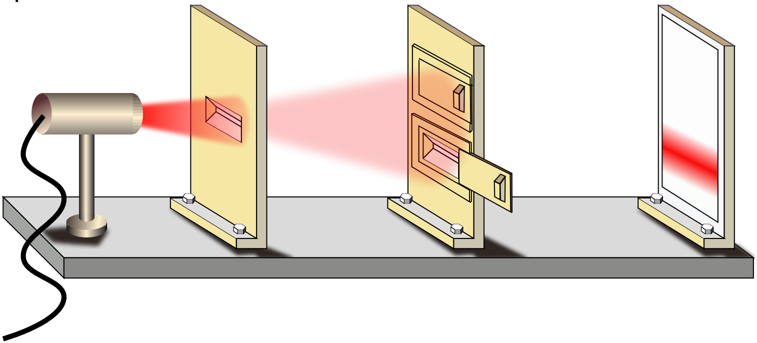
The Two-Slit Experiment with Quantum particles

Slit 1 open



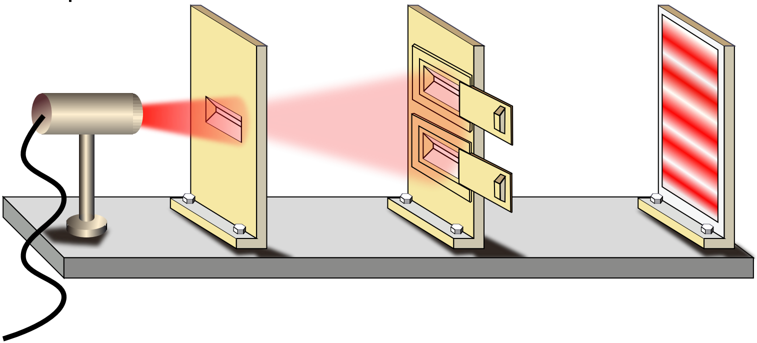
The Two-Slit Experiment with Quantum particles

Slit 2 open



The Two-Slit Experiment with Quantum particles

Both slits open



Two-slit experiment with electrons

Tonomura, Endo, Matsuda, Kawasaki, Ezawa, *Am. J. Phys.* **57**(2) (1989).

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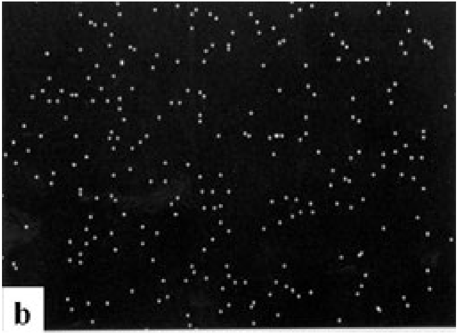

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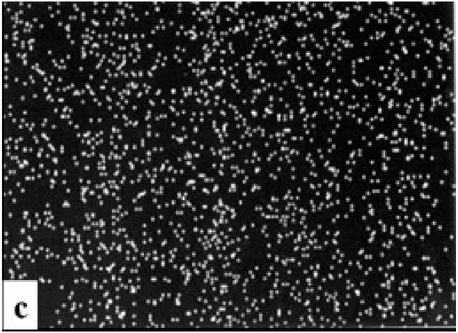

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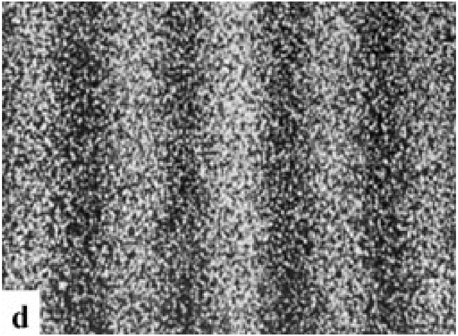
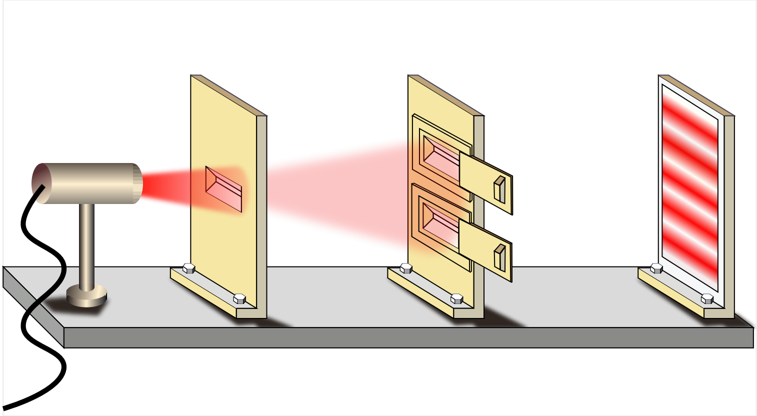


Outline

Which slit did the electron pass through?

Getting the “*Welcher-Weg*" (which-way) information



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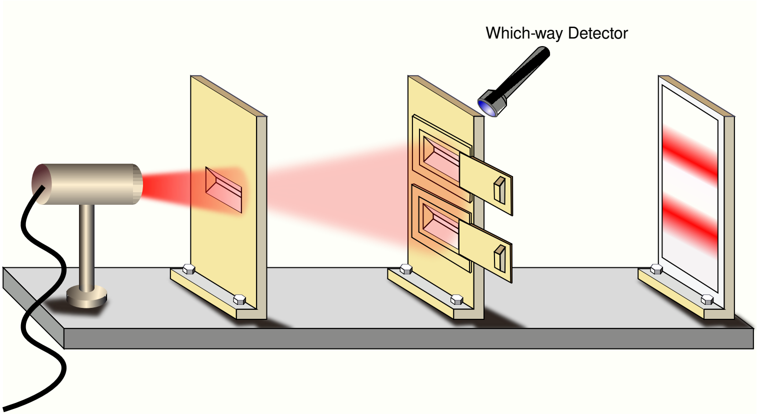

No Interference!

Bohr's Complementarity Principle

Niels Bohr in 1928

In describing the results of quantum mechanical experiments, certain physical concepts are complementary. If two concepts are complementary, an experiment that clearly illustrates one concept will obscure the other complementary one. . . .
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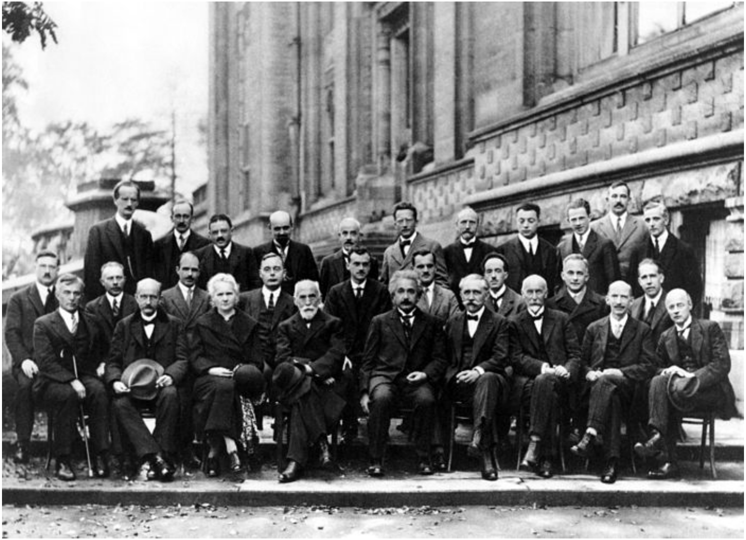
They can NEVER be observed at the same time, in the same experiment.

5th Solvay Conference (1927)



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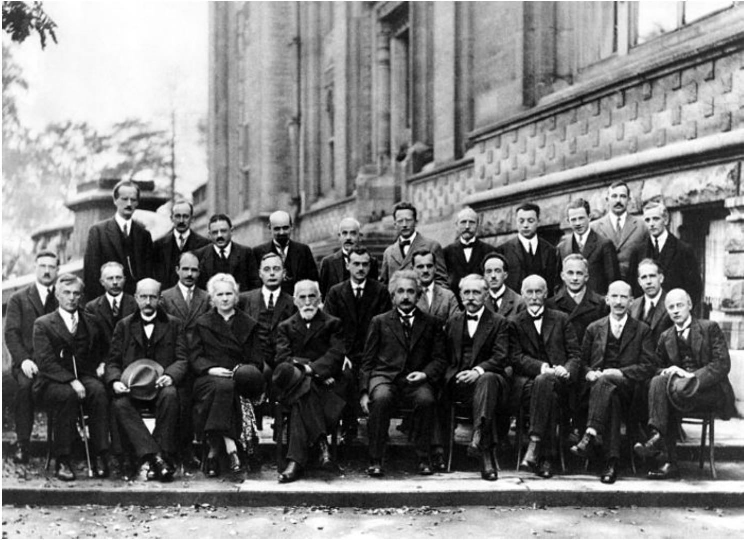

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Einstein's Recoiling-Slit *Gedanken* Experiment

... Einstein thought he had found a counterexample to the uncertainty principle. *"It was quite a shock for Bohr .... he did not see the solution at once. During the whole evening he was extremely unhappy, going from one to the other and trying to persuade them that it couldn't be true, that it would be the end of physics if Einstein were right; but he couldn't produce any refutation. I shall never forget the vision of the two antagonists leaving the club [of the Fondation Universitaire]: Einstein a tall majestic figure, walking quietly, with a somewhat ironical smile, and Bohr trotting near him, very excited ....*

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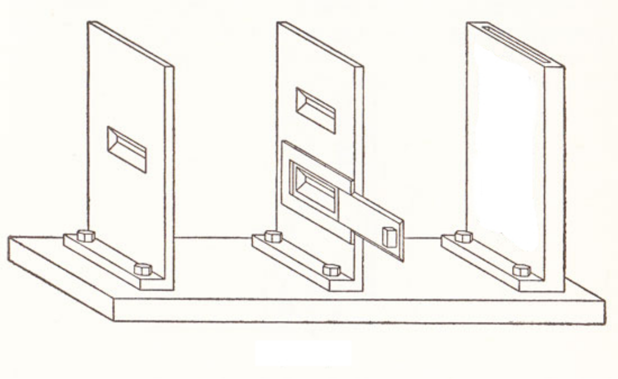
Replace the static source slit



Figures after Bohr

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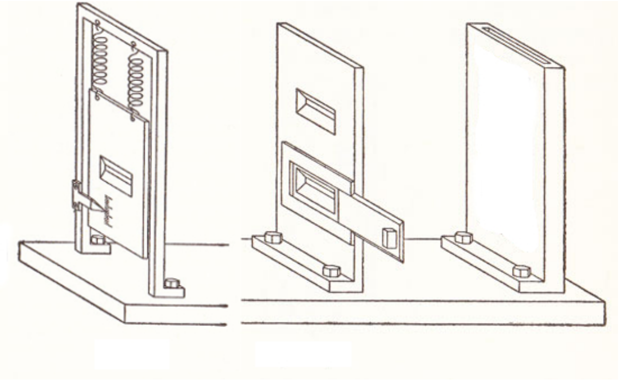

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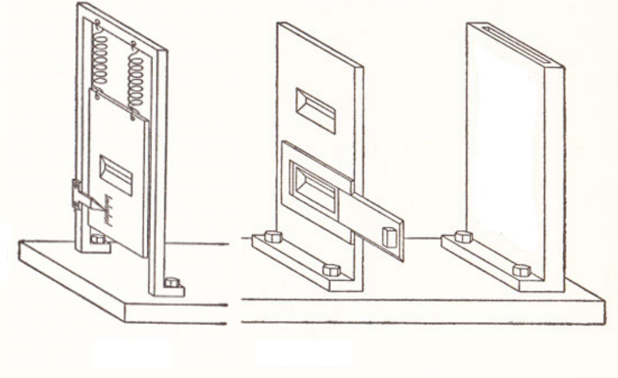
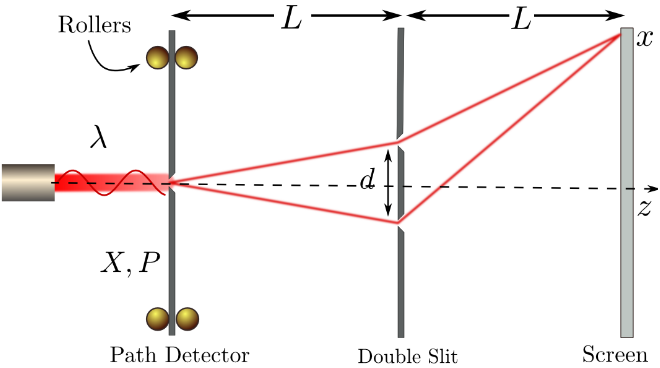


by a movable slit
to obtain which-way information without disturbing the particle

Figures after Bohr

Einstein's Recoiling-Slit *Gedanken* Experiment



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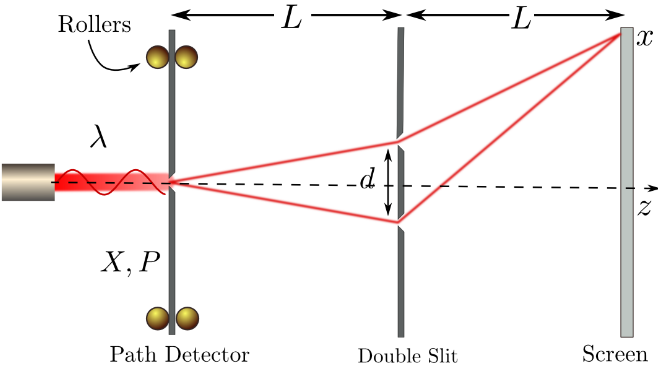

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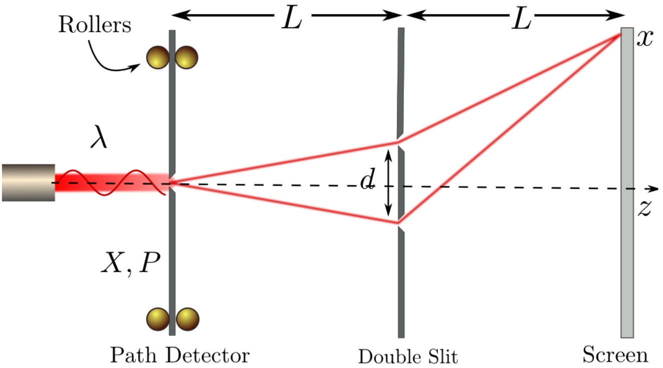

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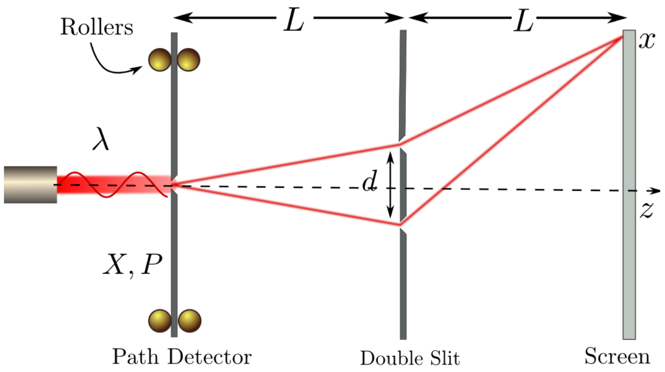

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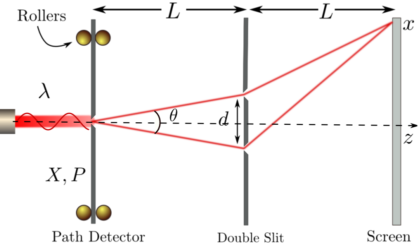
- Momentum of slit $\rightarrow$ which-way information

Outline

Bohr's reply



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$$\Delta p_x = 2p\sin(\theta/2)$$

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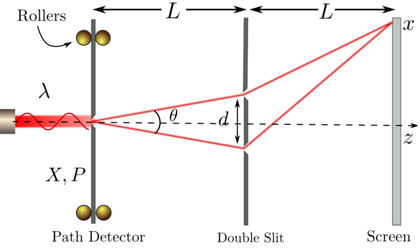

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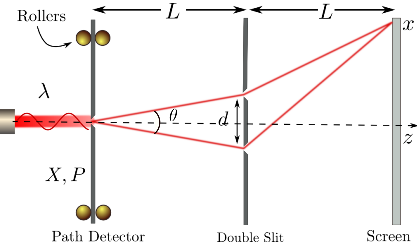

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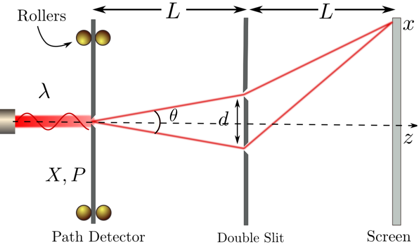

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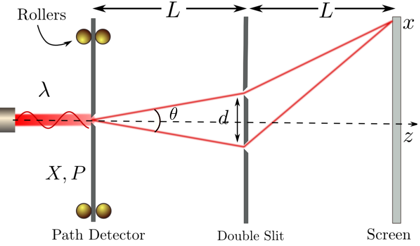

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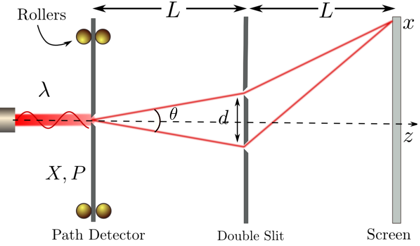

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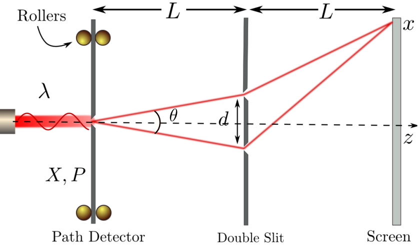

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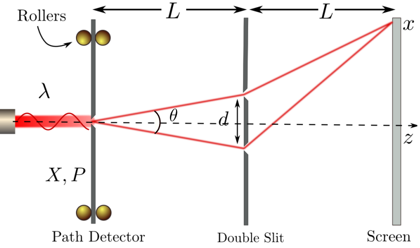

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- Fringe separation = $\frac{\lambda L}{d}$.
- Interference pattern is lost!

Implication of Bohr's resolution

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- This viewed as a restatement of Uncertainty Principle

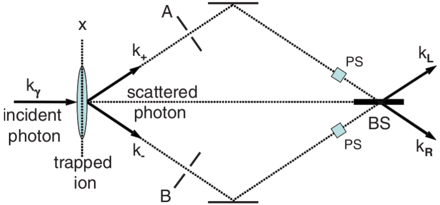
Realization of Recoiling-Slit Experiment

PHYSICAL REVIEW A **75**, 062105 (2007)

Trapped-ion realization of Einstein's recoiling-slit experiment

Robert S. Utter and James M. Feagin*
Department of Physics, California State University-Fullerton, Fullerton, California 92834, USA
(Received 10 July 2006; revised manuscript received 9 October 2006; published 13 June 2007)

We analyze photon scattering by a harmonically trapped ion using two-port interferometry of the scattered photon and coherent-state measurement of the ion's external recoil motion. We examine how the coherent-state measurement could be used to mimick both momentum and position ion measurements and thus a modern realization of Wootters and Zurek's pioneering analysis of Einstein's historic recoiling-slit gedanken experi-



Realization of Recoiling-Slit Experiment

Letters to Nature > Abstract

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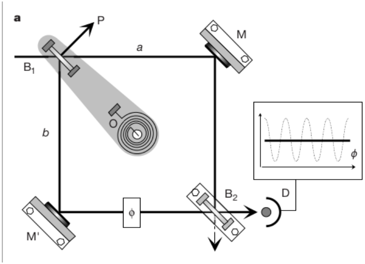
Letters to Nature

Nature **411**, 166-170 (10 May 2001) | doi:10.1038/35075517; Received 22 December 2000; Accepted 7 March 2001

A complementarity experiment with an interferometer at the quantum–classical boundary

P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond & S. Haroche ← Physics Nobel 2012

1. Laboratoire Kastler Brossel, Département de Physique, Ecole Normale Supérieure, 24 rue Lhomond, F-75231, Paris Cedex 05, France



Is uncertainty a requirement for Complementarity?

Now it turns out that the concept of Uncertainty is not necessary for explaining complementarity!

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Obtaining information about a quantum system is through Measurement, which yields classical result.

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Quantum measurement

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"The Measurement Problem".

Which-way Detection in Einstein's experiment

Using von Neumann's process 1

Two orthogonal states of the particle depending on the path:

slit 1: $|\psi_1\rangle$ slit 2: $|\psi_2\rangle$

Two momentum states of the recoiling slit: $|p_1\rangle$ and $|p_2\rangle$.

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(a) Final state of particle+slit: necessary entanglement :

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and is enough to rule out interference!

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Which-way Information and Interference

- **Without which-way information**
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Can this argument be made more quantitative?

Outline

Path-distinguishability and Interference

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Is there a relationship between them to capture complementarity?

Path-distinguishability and Interference

Gaussian Wave-packet Model

$t = 0$: particle emerges from the double-slit with amplitude

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After time t, traveling a distance L, amplitude for particle to arrive at x on screen:

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where $$A_t = \frac{1}{\sqrt{2}}[\sqrt{2\pi}(\epsilon + i\hbar t/2m\epsilon)]^{-1/2}$$

Probability of finding particle at point x on the screen

$$
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&\times \left(1 + |\langle d_1|d_2\rangle| \frac{\cos\left(\frac{xd\lambda L/2\pi}{4\epsilon^4+(\lambda L/2\pi)^2} + \theta\right)}{\cosh(xd/2\sigma_t^2)}\right)
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\langle d_1|d_2\rangle &= |\langle d_1|d_2\rangle| e^{i\theta} \\
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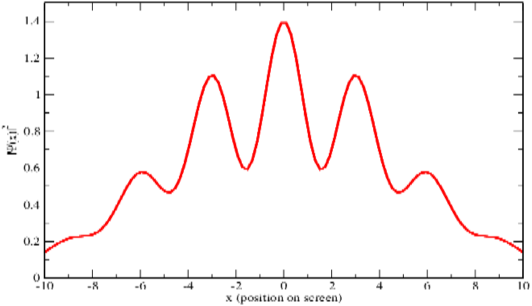

Fringe width =

$$\frac{\lambda L}{d} + \frac{16\pi^2\epsilon^4}{\lambda d L}.$$

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Englert-Greenberger-Yasin duality relation

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A *quantitative* statement of complementarity

Origin of Complementarity?

- Quantum correlations ?
  - D.M. Greenberger, A. Yasin, Phys. Lett. A 128, 391 (1988),
    “Simultaneous wave and particle knowledge in a neutron interferometer",
  - B-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996),
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  - H. Wiseman, *Phys. Lett. A* **311**, 285 (2003),

    “Directly observing momentum transfer in twin-slit which-way experiments"
- Does the particle really receive a “momentum kick"?
  - S. Durr, T. Nonn, G. Rempe, *Nature* **395**, 33 (1998), “Origin of quantum-mechanical complementarity probed by a which-way experiment in an atom interferometer."
  - C.S. Unnikrishnan, *Phys. Rev. A* **62**, 015601 (2000),

    “Origin of quantum-mechanical complementarity without momentum back action in atom-interferometry experiments".

Uncertainty principle and complementarity

Other work

- G. Bjork, J. Soderholm, A. Trifonov, T. Tsegaye, A. Karlsson, *Phys. Rev. A* **60**, 1874 (1999), “Complementarity and the uncertainty relations".
- K-P Marzlin, B.C. Sanders, P.L. Knight, *Phys. Rev. A* **78**, 062107 (2008), “Complementarity and uncertainty relations for matter-wave interferometry",
- J-H Huang, S-Y Zhu, *arXiv:1011.5273 [physics.optics]*, “Complementarity and uncertainty in a two-way interferometer".
- G.M. Bosyk, M. Portesi, F. Holik, A. Plastino, *arXiv:1206.2992 [quant-ph]* “On the connection between complementarity and uncertainty principles in the Mach-Zehnder interferometric setting".
- Paul Busch, Christopher R. Shilladay. *arXiv:quant-ph/0609048, Phys Rep* **435**, 1-31 (2006)

Outline

1. Two-Slit Experiment and Complementarity
   - Two Slit Experiment with Quantum Particles
   - Complementarity
   - Einstein's Recoiling Slit Experiment
   - ...and Bohr's Reply
2. Complementarity and Entanglement
   - von Neuman Measurements
   - Which-way Information and Interference
   - Path Distinguishability and Fringe Visibility
3. **Complementarity and Uncertainty**
   - **Duality and Uncertainty**
4. Conclusions

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Uncertainty and duality

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- The particle states can be correlated with these states:

$$\Psi(x) = \frac{c_1}{\sqrt{2}}[\psi_1(x) + \psi_2(x)]|q_1\rangle + \frac{c_2}{\sqrt{2}}[\psi_1(x) - \psi_2(x)]|q_2\rangle$$

Correlate the detected particles on the screen with the measured eigenstate of $\hat{Q}$ ($c_1 = c_2$ case)

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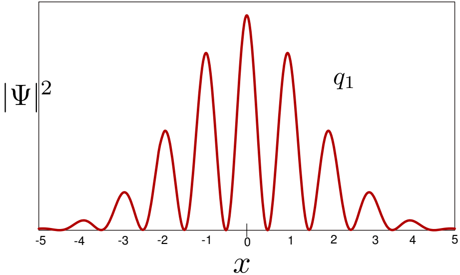

Two complementary interference patterns corresponding to $|q_1\rangle$ and $|q_2\rangle$.

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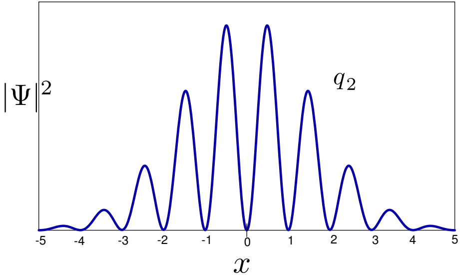

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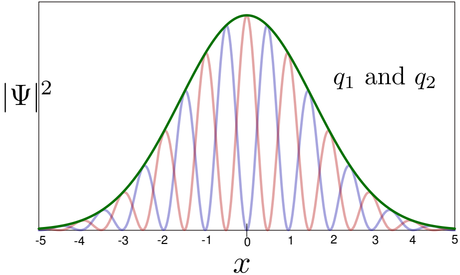

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For any $c_1,\ c_2$,

$$|\Psi(x)|^2 = \frac{|\psi_1(x)|^2 + |\psi_2(x)|^2}{2} + \frac{|c_1|^2 - |c_2|^2}{2}\left[\psi_1^*(x)\psi_2(x) + \psi_2^*(x)\psi_1(x)\right].$$

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Fringe visibility: $\mathcal{V}^2 \leq (|c_1|^2 - |c_2|^2)^2$.

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$$|\Psi(x)|^2 = \frac{|\psi_1(x)|^2 + |\psi_2(x)|^2}{2} + \frac{|c_1|^2 - |c_2|^2}{2}\left[\psi_1^*(x)\psi_2(x) + \psi_2^*(x)\psi_1(x)\right].$$

Fringe visibility: $\mathcal{V}^2 \leq (|c_1|^2 - |c_2|^2)^2$.

The uncertainty in $\hat{Q}$, in this *entangled* state:

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Combining with the earlier result $\mathcal{D}^2 = 1 - \Delta P^2$, we get

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 2 - [\Delta P^2 + \Delta Q^2].$$

Sum uncertainty relation for angular momenta [1]

$$\Delta L_x^2 + \Delta L_y^2 + \Delta L_z^2 \geq \ell$$

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The duality relation also emerges from the *sum* uncertainty relation.

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Conclusions

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- Momentum back-action of the recoiling slit on the particle plays no role in complementarity.

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THANK YOU!