# Remote Tomography and Entanglement Swapping Via von Neumann-Arthurs-Kelly Interaction

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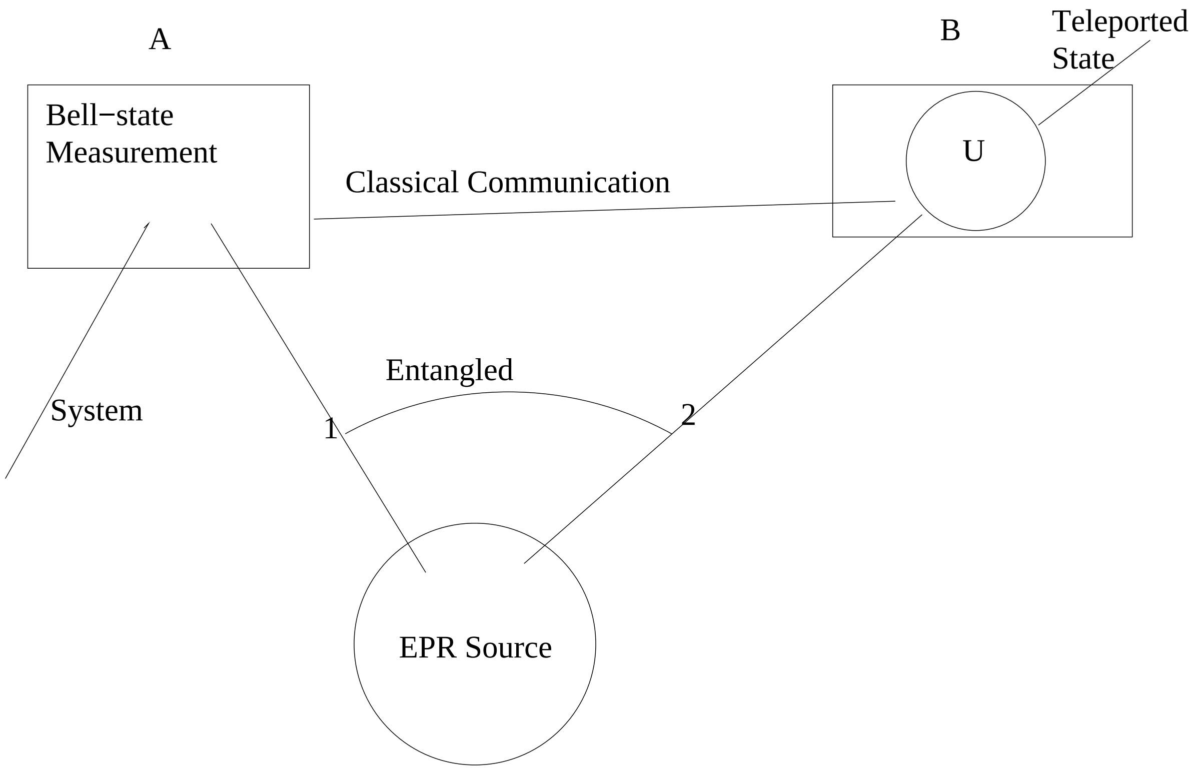
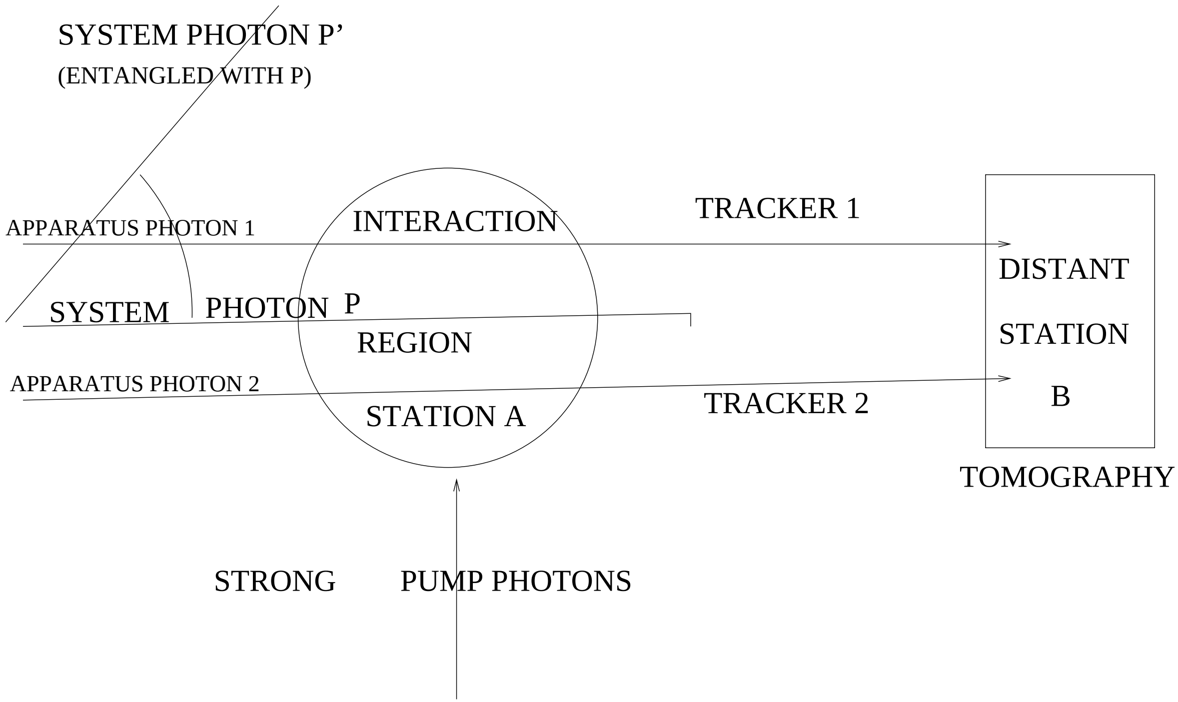
**Abstract**.

Teleportation usually involves entangled particles 1,2 shared by Alice and Bob, Bell-state measurement on particle 1 and system particle by Alice, classical communication to Bob, and unitary transformation by Bob on particle 2.




Usual Bennett et al [5]Protocol For Teleportation.

We propose a novel method: interaction-based remote tomography and entanglement swapping.




Remote Tomography and Entanglement Swapping via Von Neumann-Arthurs-Kelly interaction between system photon $P$ and tracker photons.If the photon $P'$ is EPR-entangled with $P$, the tracker photons become entangled with $P'$ .

Alice arranges an entanglement generating von Neumann-Arthurs-Kelly interaction between the system particle $P$ and two apparatus particles, and then transports the latter to Bob. Bob reconstructs the unknown initial state of the system not received by him by quadrature measurements only on the final off-diagonal reduced density matrix of the apparatus particles. Further, if the system particle $P$ is initially entangled with another system particle $P'$, the apparatus particles received by Bob will also be entangled with $P'$. These results follow from an exact solution of the Schrödinger equation with von Neumann-Arthurs-Kelly interaction between a system particle and two apparatus particles for generalized initial conditions.

**Introduction**. The idea of 'quantum tracking' of a single system observable by an apparatus observable first occurred in the measurement theory of Von Neumann $^{citevonN}$, and generalized to two canonically conjugate observables by Arthurs and Kelly Jr.[citeAK]. Suppose the initial state of the system-apparatus combine is factorized . If after interaction, the apparatus observable $X$ has the same expectation value in the final state as the system observable $A$ in the initial state, for arbitrary initial state of the system, then $X$ is said to track $A$. This nomenclature was probably used first by Arthurs and Goodman[citeAG] who , as well as, Gudder, Hagler, and Stulpe [citeAG] proved the joint measurement uncertainty relation. The Arthurs-Kelly interaction can also enable exact measurements of some quantum correlations between position and momentum $^{citeSMR}$.

We shall be concerned here not with joint measurements but with the completely different ideas of 'remote quantum tomography' and 'entanglement swapping' for continuous variables. These are akin to 'quantum teleportation' or the replication of an unknown quantum state of a particle at a distant location without physically transporting that particle. Teleportation, as first proposed by Bennett, Brassard, Crépeau, Jozsa, Peres and Wootters $^{citeBennett}$ and generalized to continuous variables by Vaidman $^{citeVaidman}$, usually involves four different technologies.(i) An EPR-pair $E_1, E_2$ is shared by observers $A$ (Alice) and $B$ (Bob) at distant locations. (ii) The system particle $P$ with unknown state is received by $A$ who makes a Bell-state measurement on the joint state of that particle and $E_1$ and (iii) communicates the result via a classical channel to $B$ , (iv) $B$ then makes a unitary transformation depending on the classical information on $E_2$ to replicate the unknown system state. Teleportation has been experimentally realized, e.g.

by Bouwmeester et al $^{citeBouwmeester}$, and the methods and uses extensively reviewed, e.g. by Braunstein et al $^{citeBraunstein}$. In particular the density matrix of the system particle can be constructed by quadrature measurements on $E_2$ (remote tomography).

**Interaction-based Remote Tomography and Teleportation of EPR-Entanglement**. We report here a completely new method which replaces the above four technologies by the single step of an interaction between the system particle and two apparatus particles. At Alice's location A, a system particle $P$ with unknown state interacts via an Arthurs-Kelly interaction with two apparatus particles $A_1, A_2$ in a known state. When the particles are photons, the interaction can easily be generated (see e.g. Stenholm $^{citeAK}$). The particles $A_1, A_2$ are then sent to a distant observer Bob ($B$). $B$ makes quantum tomographic measurements on them (quadrature measurements in the case of photons) and reconstructs the exact initial density matrix of the system particle without ever having received that particle. Further, if another particle $P'$ in Alice's hands is EPR-entangled with $P$, it will be EPR-entangled with the distant pair $A_1, A_2$. (See Fig.2). Practical implementation will require a quantum channel to send the two apparatus particles from location $A$ to the distant location of $B$ followed by tomographic measurements by $B$: for photons, a generalization of single photon *Optical Homodyne Tomography* (see e.g. [citeVogel], [citeBraunstein−Leonhardt] and [citeYuen] ) to two photons , which seems feasible and worthwhile .

From the 'application point of view' why is it practically useful to transport the apparatus particles with the system state imprinted on it ? Why can't Alice directly send the system particle to Bob ? There can be several reasons. E.g. the system particle might be unstable; or in the case of a photon, it might have a frequency unsuitable for optical fibre transmission. The apparatus photons can be chosen to have frequency in the telecom windows around 1300 nm or 1550 nm where optical fibres have very low absorption facilitating long distance transmission.

The scheme we propose exploits the entanglement between the system photon and the apparatus photons generated by the three-particle Arthurs-Kelly interaction. Multiparticle interactions to generate entanglement have previously been exploited for quantum enhanced metrology $^{citeRoy-Braunstein}$. We proceed now to put the new method on a rigorous footing.

**A Symmetry Property**. We shall use the Arthurs-Kelly system-apparatus interaction Hamiltonian , which is invariant under a class of simultaneous transformations on the system and apparatus specified below,

$$H = K(\widehat{q}\widehat{p}_1 + \widehat{p}\widehat{p}_2) = K(\widehat{q}_\theta \widehat{p}_{1,\theta} + \widehat{p}_\theta \widehat{p}_{2,\theta}) \quad (1)$$

where $K$ is a coupling constant , $\widehat{q}, \widehat{p}$ are position and momentum operators of the system, $\widehat{x}_1, \widehat{x}_2$ are two commuting position operators of the apparatus (e.g. two photons), with conjugate momenta $\widehat{p}_1, \widehat{p}_2$ which are coupled to $\widehat{q}$ and $\widehat{p}$ respectively.The rotated quadrature operators with subscript $\theta$ are defined using the rotation matrix $R$,

$$\begin{pmatrix}\widehat{q}_\theta\\ \widehat{p}_\theta\end{pmatrix} = R\begin{pmatrix}\widehat{q}\\ \widehat{p}\end{pmatrix}, \begin{pmatrix}\widehat{p}_{1,\theta}\\ \widehat{p}_{2,\theta}\end{pmatrix} = R\begin{pmatrix}\widehat{p}_1\\ \widehat{p}_2\end{pmatrix}, R = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & cos\theta\end{pmatrix}. \tag{2}$$

The operators $\widehat{p}_{j,\theta}$ are seen to be just the commuting momentum operators of the apparatus particles corresponding to rotated co-ordinates $x_{j,\theta}$, for $j = 1, 2$,

$$x_{1,\theta}+ix_{2,\theta} = exp(-i\theta)(x_1+ix_2), \widehat{p}_{j,\theta} = -i\partial/\partial x_{j,\theta}. \tag{3}$$

We also define,

$$\widehat{x}_{1,\theta} + i\widehat{x}_{2,\theta} = exp(-i\theta)(\widehat{x}_1 + i\widehat{x}_2). \tag{4}$$

Then, in the case of the apparatus being two photons with annihilation operators $a_i$ ,$i = 1, 2$,

$$\widehat{x}_{i,\theta} = a_i \operatorname{exp}(-i\theta)/\sqrt{2} + h.c., \ \widehat{p}_{i,\theta} = \widehat{x}_{i,\theta+\pi/2}. \tag{5}$$

**Exact Solution of the Schrödinger equation with generalized initial conditions**. We assume the constant $K$ to be so large that the free Hamiltonians of the system and the apparatus are negligible compared to $H$

during interaction time $T$. We start from an initial factorized state ,

$$\langle q|\langle x_1, x_2|\psi(t=0)\rangle\rangle = \langle q|\phi\rangle\chi(x_1, x_2), \quad (6)$$

where $\langle q|\phi\rangle$ is the unknown system wave fuction, and the apparatus wave function is chosen to be a product of two Gaussians, $\chi(x_1, x_2) = \chi_1(x_1)\chi_2(x_2)$,

$$\begin{aligned} \chi_1(x_1) &= \pi^{-1/4} b_1^{-1/2} \exp\left[-x_1^2/(2b_1^2)\right] \\ \chi_2(x_2) &= \pi^{-1/4} (2b_2)^{1/2} \exp\left[-2b_2^2 x_2^2\right]. \quad (7) \end{aligned}$$

Arthurs and Kelly chose $b_2 = b_1 = b$. We solve the Schrödinger equation with arbitrary

$b_1, b_2$ ; we need $b_1 \neq b_2$ to utilise the symmetry of the Hamiltonian.

The commutator of the two terms in $H$ in fact commutes with each of the terms. Hence,

$$\exp(-iHt) = \exp(-iKt\hat{q}\hat{p}_1)\exp(-iKt\hat{p}\hat{p}_2) \exp(iK^2t^2\hat{p}_1\hat{p}_2/2)\,. \quad (8)$$

If we work in the $q, x_1, p_2$ representation, the three exponentials on the right-hand side successively translate $x_1, q, x_1$ acting on the initial wavefunction. Hence the exact solution of the Schrödinger equation is,

$$\langle q, x_1, p_2|\psi(t)\rangle = \chi_1(x_1 - qKt + (1/2)p_2K^2t^2) \tilde{\chi}_2(p_2)\phi(q - p_2Kt), \quad (9)$$

where $\tilde{\chi}_2$ denotes a Fourier transform of $\chi_2$. The co-ordinate space wave function is given by a Fourier transform. Choosing $KT = 1$ we obtain,

$$\psi(q, x_1, x_2) = \int \psi(q, x_1, x_2, \xi) d\xi \ , \quad (10)$$

where,

$$\psi(q, x_1, x_2, \xi) = \phi(\xi) \exp\left(i(q-\xi)x_2\right)/(2\pi\sqrt{b_1b_2}) \exp\left(-\frac{(2x_1 - q - \xi)^2}{8b_1^2} - \frac{(q-\xi)^2}{8b_2^2}\right). \quad (11)$$

Tracing the system-apparatus density matrix over the system co-ordinate we obtain the apparatus density matrix at time T,

$$\langle x_1, x_2|\rho_{APP}(T)|x_1' x_2'\rangle = \int \psi(q, x_1, x_2, \xi)$$
$$\psi^*(q, x_1', x_2', \xi') dq d\xi d\xi' \, . \qquad (12)$$

The probability densities $P_1(x_1)$ and $P_2(x_2)$ for $x_1$ and $x_2$ are obtained by integrating the diagonal elements of this density operator over $x_2$ and $x_1$ respectively.In fact $P_1(x_1)$ and $P_2(x_2)$ can be obtained from the Arthurs-Kelly expressions by $b^2 \rightarrow (b_1^2 + b_2^2)/2$ and

$b^{-2} \to (b_1^{-2} + b_2^{-2})/2$ respectively. The resulting expectation values of $x_1, x_2$ equal those of $q, p$ respectively, but the dispersions are higher, $(\Delta x_1)^2 = (\Delta q)^2 + (b_1^2 + b_2^2)/2, (\Delta x_2)^2 = (\Delta p)^2 + (b_1^2 + b_2^2)/(8 b_1^2 b_2^2)$.

Our key new results require $b_1 \neq b_2$. First, integrating the off-diagonal elements of the apparatus density matrix over $x_2, x_2'$ ,

$$\int \langle x_1, x_2 | \rho_{APP}(T) | x_1' x_2' \rangle dx_2 dx_2' = \frac{1}{b_1 b_2} \int |\phi(q)|^2 \exp\left(-\frac{(x_1 - q)^2 + (x_1' - q)^2}{2b_1^2}\right) dq \, . \tag{13}$$

This shows that we can extract the exact initial system position probability density from the final apparatus density matrix as the expectation value of an apparatus observable.

$$\Big|\langle q = x_1|\phi\rangle\Big|^2 = lim_{b_1\to 0}\frac{b_2}{\sqrt{\pi}}\int dx_2 dx_2' \langle x_1, x_2|\rho_{APP}(T)|x_1 x_2'\rangle = lim_{b_1\to 0} Tr\rho_{APP}(T)Y(x_1), \quad (14)$$

where $Y(x_1)$ is the apparatus observable,

$$Y(x_1) = \frac{b_2}{\sqrt{\pi}}|x_1\rangle\langle x_1|\int |x_2'\rangle\langle x_2''|dx_2' dx_2'' = 2b_2\sqrt{\pi}|x_1\rangle\langle x_1||\widehat{p}_2 = 0\rangle\langle\widehat{p}_2 = 0| \quad (15)$$

Similarly, the exact initial system momentum probability density is an expectation value of an apparatus observable in the final apparatus density matrix,

$$\Big|\langle p = x_2|\phi\rangle\Big|^2 = lim_{b_2\rightarrow\infty}\frac{1}{2b_1\sqrt{\pi}}\int dx_1 dx_1' \langle x_1, x_2|\rho_{APP}(T)|x_1' x_2\rangle = lim_{b_2\rightarrow\infty} Tr\rho_{APP}(T)Z(x_2), \quad (16)$$

where $Z(x_2)$ is the apparatus observable,

$$Z(x_2) = \frac{\sqrt{\pi}}{b_1}|x_2\rangle\langle x_2|\widehat{p}_1 = 0\rangle\langle\widehat{p}_1 = 0|. \quad (17)$$

In the limit, $b_1 \rightarrow 0$, $b_2 \rightarrow \infty$, we have faithful tracking of both system position and system momentum, since $Y(x_1)$ tracks the position projectors $|\widehat{q} = x_1 >< \widehat{q} = x_1|$ for all $x_1$ and $Z(x_2)$ tracks the system momentum projectors $|\widehat{p} = x_2 >< \widehat{p} = x_2|$ for all $x_2$.

Further, the Wigner function of the initial system state can be calculated exactly from the final apparatus density matrix,

$$W(x_1, x_2) = lim_{b_1 \to 0, b_2 \to \infty} \frac{b_2}{2\pi b_1} \int dx_1' dx_2' \langle x_1, x_2 | \rho_{APP}(T) | x_1' x_2' \rangle. \qquad (18)$$

We now show that we can indeed measure a continuous infinity of apparatus observables on the final state to obtain the initial Wigner function of the system particle.

**Rotated quadratures and Quantum Tomography**. In order to harness the symmetry property mentioned above, we need a corresponding symmetry property of the initial apparatus state, $\chi(x_1, x_2) = \chi(x_{1,\theta}, x_{2,\theta})$. Therefore we are forced to use initial apparatus states very different from Arthurs and

Kelly. We need,

$$2b_1 b_2 = 1;\ \chi(x_1, x_2) = \chi(x_{1,\theta}, x_{2,\theta})$$
$$= \pi^{-1/2} b_1^{-1} \exp\left[-(x_1^2 + x_2^2)/(2b_1^2)\right]. \quad (19)$$

For this choice , the system-apparatus initial state can be rewritten for arbitrary $\theta$ as,

$$\langle \widehat{q}_\theta = q_\theta | \langle \widehat{x}_{1,\theta} = x_{1,\theta}, \widehat{x}_{2,\theta} = x_{2,\theta} | \psi(t=0)\rangle$$
$$= \langle \widehat{q}_\theta = q_\theta |\, \phi\rangle \chi(x_{1,\theta}, x_{2,\theta}), (20)$$

with the obvious notation $(\widehat{q}_\theta - q_\theta)|\widehat{q}_\theta = q_\theta\rangle = 0$. Since the Hamiltonian $H$ and the initial apparatus states have exactly the same form in terms of the rotated variables as in terms of the original variables, we can repeat the previous calculations with $\widehat{q}_\theta, \widehat{p}_\theta, q_\theta, p_\theta, x_{1,\theta}, x_{2,\theta}$ replacing $\widehat{q}, \widehat{p}, q, p, x_1, x_2$ respectively. Hence the matrix elements of $\rho_{APP.}$ are obtained by replacing in the previously obtained expressions

$$q, p, x_1, x_2, x'_1, x'_2 \to q_\theta, p_\theta, x_{1,\theta}, x_{2,\theta}, x'_{1,\theta}, x'_{2,\theta}.$$

Thus, we obtain for arbitrary $\theta$ ,

$$\left|\langle \widehat{q}_\theta = u|\phi\rangle\right|^2 = lim_{b_1\to 0} Tr \rho_{APP}(T) Y_\theta(u), \tag{21}$$

$$Y_\theta(u) \equiv \frac{\sqrt{\pi}}{b_1} |\widehat{x}_{1,\theta} = u\rangle\langle\widehat{x}_{1,\theta} = u|$$
$$|\widehat{p}_{2,\theta} = 0\rangle\langle\widehat{p}_{2,\theta} = 0|. \qquad (22)$$

Since, $\widehat{p}_\theta = \widehat{q}_{\theta+\pi/2}$ the initial system probability densities for it are obtained from above just by replacing $\theta \to \theta + \pi/2$.

We have proved that in the limit,

$$b_1 \to 0, b_2 = 1/(2b_1) \to \infty, \qquad (23)$$

we can recover exactly the initial system probability densities of arbitrary Hermitian linear combinations $\widehat{q}_\theta$,

$$\langle \widehat{q}_\theta = u|\, \rho_S |\widehat{q}_\theta = u\rangle = \left|\langle \widehat{q}_\theta = u|\phi\rangle\right|^2 \tag{24}$$

and hence the initial Wigner function, by measuring expectation values of Hermitian operators in the same final state of the apparatus after interaction.

**Reconstruction of the initial Density Matrix of the System from the final Apparatus Density Matrix**. Quantum tomography is completed by calculating the Wigner function $W(q,p)$ as an inverse Radon transform,

$$W(q,p) = (2\pi)^{-2} \int_0^{\infty} \eta d\eta \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} du$$

$$\exp\left(i\eta(u - (q\cos\theta + p\sin\theta))\right)\langle \widehat{q}_\theta = u|\, \rho_S|\widehat{q}_\theta = u\rangle, \quad (25)$$

and from that the density operator,

$$\langle q|\rho_S|q'\rangle = (2\pi)^{-1} \int_0^{\pi} |q - q'| d\theta (\sin\theta)^{-2}$$

$$\exp\left((-i(q^2 - q'^2)\cot\theta)/2\right) \int_{-\infty}^{\infty} du$$

$$\exp\left(iu(q - q')/\sin\theta\right)\langle \widehat{q}_\theta = u|\, \rho_S|\widehat{q}_\theta = u\rangle. \quad (26)$$

**Accounting for time evolution of the ap-**

**paratus photons during transit time $\tau$ to distant location B**. Note that

$$Tr\rho_{APP}(T)Y_\theta(u) = Tr\rho_{APP}(T+\tau) \times exp(-iH_0\tau)Y_\theta(u)exp(iH_0\tau),$$

where the Hamiltonian

$$H_0 = \omega(a_1 \dagger a_1 + a_2 \dagger a_2 + 1)$$

, if the photons have the same frequency $\omega$. Hence the $\langle \widehat{q}_\theta = u| \rho_S |\widehat{q}_\theta = u\rangle$ are equivalently given by replacing

$$\rho_{APP}(T), \widehat{x}_{1,\theta}, \widehat{p}_{2,\theta}$$

by

$$\rho_{APP}(T+\tau), cos(\omega\tau)\widehat{x}_{1,\theta} - sin(\omega\tau)\widehat{p}_{1,\theta},$$
$$cos(\omega\tau)\widehat{p}_{2,\theta} + sin(\omega\tau)\widehat{x}_{2,\theta}$$

respectively.We just have to measure different quadratures for the apparatus photons depending on the transit time $\tau$.

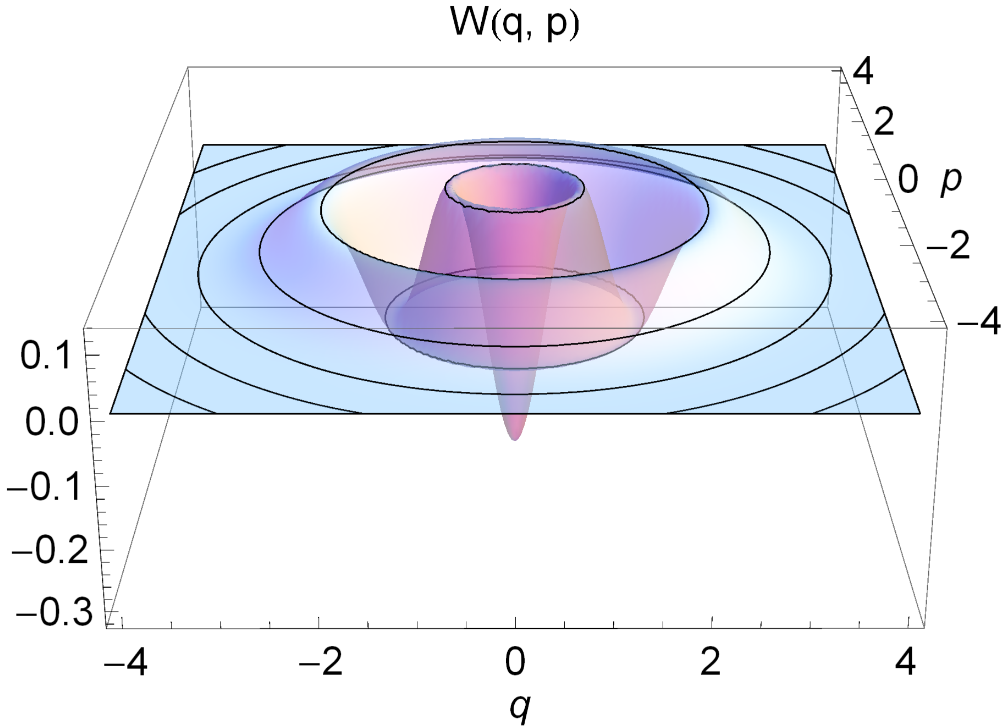
**Quantitative comparisons for the third excited state of the oscillator .**

Our exact theorems are for the limit $b_1 \to 0$. The purpose here is to estimate how small this parameter has to be for reasonably accurate reconstruction of the initial state which ,in this example, is chosen to be the highly non-classical third excited of the oscillator. The wave function in the position basis is

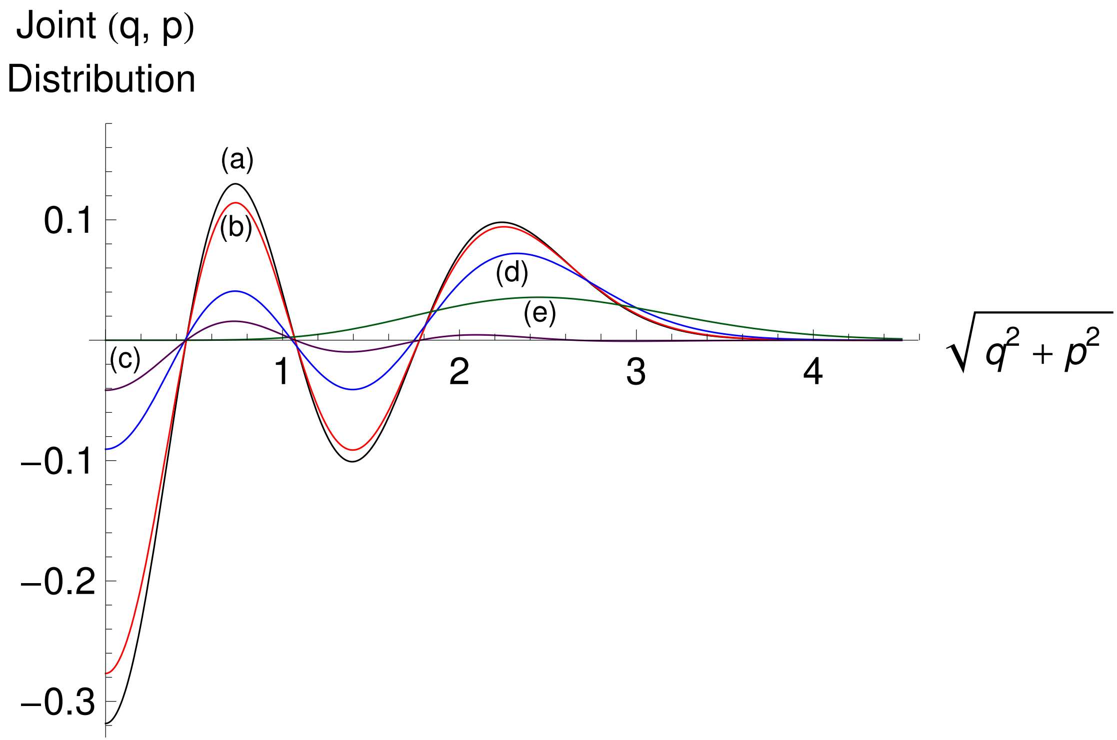
$$\phi(q) = (2q^3 - 3q)exp\left(-\frac{q^2}{2}\right)/(\sqrt{3}\ \pi^{1/4}). \tag{27}$$

The Wigner function is a function of $q^2 + p^2 \equiv d$

$$W(d) = exp(-d)[4d^3 - 18d^2 + 18d - 3]/(3\pi). \tag{28}$$

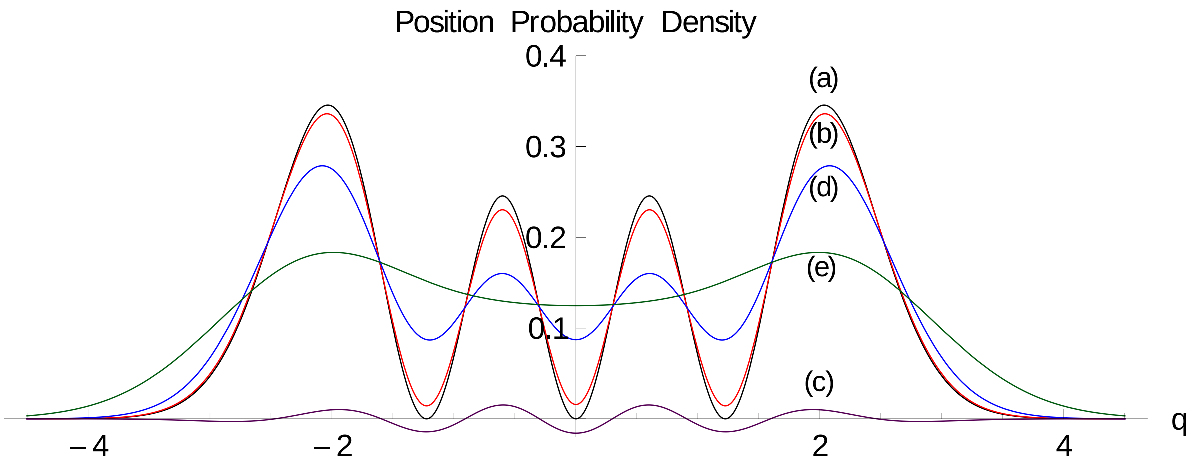



The Wigner function for the $3^{rd}$ excited state of the harmonic oscillator

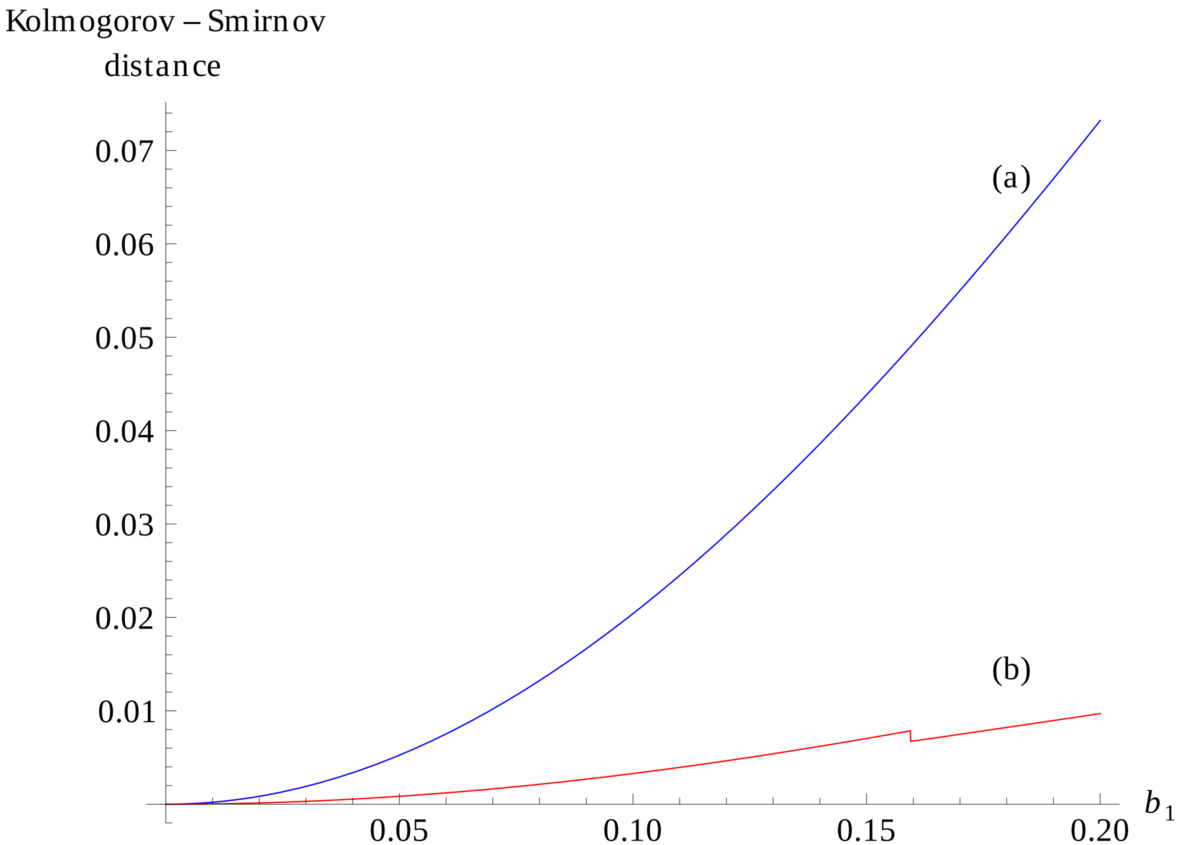



Joint distributions in $(q, p)$ for the third excited state of the oscillator as a function of $\sqrt{q^2 + p^2}$ (a): Wigner function (b):

Reconstructed Wigner function with $b_1 = 0.1$. (c): Difference between curves (a) and (b). (d): Reconstructed Wigner function with $b_1 = 0.3$. (e): Arthurs-Kelly probability distribution .




Position probability densities in for the third excited state. (a): Quantum probability density of the state. (b): Obtained from reconstructed Wigner function with $b_1 = 0.1$. (c): Difference between curves (a) and (b). (d): Obtained from reconstructed Wigner function with $b_1 = 0.3$. (e): Obtained from Arthurs-Kelly probability distribution .




Plots for the Kolmogorov-Smirnov (K-S)

distance between: a) The Wigner function and the reconstructed Wigner function, b) The position probability density and the reconstructed density , versus $b_1$. Even when $b_1$ is as large as 0.2, the K-S distance in case a) reaches a value of only 0.072. The agreement is even better in case b), ( the small discontinuity in the K-S distance at $b_1 = 0.16$ is due to the shifting of the position where the maximum K-S distance is reached).

In the figures we make quantitative comparisons between the Wigner function, our reconstructed Wigner function with $2b_1b_2 = 1$ (for $b_1 = \{0.1, 0.3\}$) and the Arthurs-Kelly Probability distribution .It is worth noting that for $b_1 = \frac{1}{\sqrt{2}}$, the reconstructed Wigner function is equal to the Arthurs-Kelly distribution which differs greatly from the true Wigner function. Towards practical utility, note that for $b_1 = .1$ the reconstructed Wigner function and the position probability derived from it are already very close to the actual.

A well-known measure of the distance between two probability distributions is given by the Kolmogorov-Smirnov distance, $D(K-S) = max_x|F_1(x) - F_2(x)|$, where $F_i(x)$ is the cumulative probability for the variable $X \leq x$ for the $i$-th probability distribution. This distance between the pseudo-probabilities given by the Wigner function and the reconstructed Wigner function, as well as for the corresponding position probabilities derived from them are plotted in Fig. 4. The distance (especially for the position probability) is very small even upto $b_1 = 0.2$ though the theorem of exact equality is only in the limit $b_1 \to 0$.

**Teleportation of Entanglement**. If the photon $P$ with co-ordinate $q$ is EPR-entangled with another photon $P'$ with co-ordinate $q'$ with initial wave function $\phi(q, q')$, the density matrix for particles $1, 2, P'$ after interaction can be shown to obey $^{citeSMR1}$ analogues of Eqs. (14), (15) with $\langle q = x_1|\phi\rangle$ replaced by $\langle q = x_1, q'|\phi\rangle$, and $Y(x_1)$ replaced by $Y(x_1)|q'\rangle\langle q'|$,

$$\left|\langle q = x_1, q'|\phi\rangle\right|^2 = lim_{b_1\to 0} Tr\rho_{APP}(T)Y(x_1)|q'\rangle\langle q'|.$$

Thus the apparatus photons after interaction with $P$ become entangled with $P'$ achieving interaction-based teleportation of EPR-entanglement. The exact initial probability densities for $q, q'$ (and similarly for $p, p'$), i.e. the exact EPR-correlations can be retrieved from this final entangled state.

**Conclusions and Outlook**. (i) We have shown that the Arthurs-Kelly interaction between an unknown state of a photon $P$ and chosen initial state of two apparatus photons enables a one-step remote tomographic reconstruction of the unknown initial state of

$P$, as well as teleportation of its entanglement with another photon $P'$, instead of the usual four step process. It is practically feasible because apparatus photon frequencies can be chosen in the telecom windows, and the technology of generating the Arthurs-Kelly interaction quantum optically is well established.

(ii) Remote Tomography requires the measurement of the two photon observable $Y_\theta(u)$ which is just a product of two commuting quadrature operators for the apparatus photons, each of the kind usually measured for a single photon. This generalization of optical homodyning to the two teleported photons will by itself be a stimulating development.

(iii) The Arthurs-Goodman result on impossibility of simultaneous accurate tracking of position and momentum by commuting observables of the apparatus is not violated. The secret is that the apparatus observables tracking position and momentum do not commute, $[Y(x_1), Z(x_2)] \neq 0$. This is not a problem since we are interested in tomography , not in the simultaneous measurement of position and momentum.

(iv) The final density operator of the system can also be exactly calculated and it can be seen that $< q >_T = < q >_0$ , $\Delta q_T^2 = \Delta q_0^2 + 2b_2^2$; since the final system state is different from the initial state, and depends on the initial states of both the system and the apparatus, the no-cloning [citeWootters] and no-hiding theorems [citeBraunstein−Pati] are respected.

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