ENTANGLEMENT IN CLASSICAL OPTICS

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1 Introduction

Hilbert spaces occur not only in quantum mechanics but also in areas of mathematics and physics that have nothing to do with quantum mechanics. For example, classical electrodynamics has a Hilbert space structure. Once this is recognized, it becomes at once clear that entanglement is possible in classical electrodynamics without implying the special features associated with quantum Mechanics and nonlocality.

R. J. C. Spreeuw, (1998), (2001); P. Ghose & M. K. Samal (2001).

PG & A. Mukherjee, Rev. in Theoret. Sc (2013); arXiv (2013)

2 Hilbert Spaces

EXAMPLES:

THE SPACE $\mathbb{C}^n$ OF N-TUPLES OF COMPLEX NUMBERS $z = (z_1, z_2, ..., z_n)$ AND $z' = (z'_1, z'_2, ..., z'_n)$ IS A HILBERT SPACE UNDER THE INNER PRODUCT DEFINED BY $\langle z, z' \rangle = \sum_{k=1}^{n} z_k \bar{z}'_k$.

THE SQUARE INTEGRABLE FUNCTIONS ON L^2 ALSO FORM A HILBERT SPACE. FOR ANY f AND g IN L^2 ONE DEFINES THE INNER PRODUCT $\langle f, g \rangle = \int_\Omega f(x)\overline{g(x)}dx$. HENCE, $\langle f, f \rangle < \infty$.

Superposition principle

THE LINEARITY OF HILBERT SPACES ALLOWS VECTORS TO BE SUPERPOSED TO FORM OTHER VECTORS IN THE SAME SPACE. AN EXAMPLE IS THE INTERFERENCE OF TWO COHERENT CLASSICAL WAVES AT A POINT x. LET $F_1(x,t) = ae^{i(\phi_1(x)-\omega t)}$ AND $F_2(x,t) = be^{i(\phi_2(x)-\omega t)}$ BE TWO WAVES AT x AT TIME t. THEN, THE SUM OF THE TWO WAVES IS $G(x,t) = F_1(x,t) + F_2(x,t)$ AND THE INTENSITY AT x IS GIVEN BY $I(x) = \langle G(x,t), G(x,t) \rangle = \int G(x,t)\overline{G(x,t)}dt = a^2 + b^2 + 2ab\cos(\phi_1 - \phi_2)$. THE SECOND TERM GIVES RISE TO THE FAMILIAR INTERFERENCE PATTERN.

Projection operators

One can define projection operators $\pi_i = |x_i)(x_i|$ for every basis vector $|x_i)$ which are hermitian ($\pi_i^\dagger = \pi_i$) and idempotent ($\pi_i.\pi_i = \pi_i$). They have the important property that $\pi_i(1 - \pi_i) = 0$, i.e. a projector and its complement are orthogonal. Thus, $\pi_i|X) = \sum_j c_j|x_i)(x_i||x_j) = c_i|x_i)$, and hence $c_i = (x_i|\pi_i|X)$ and $|c_i|^2 = (X|\pi_i|X)$. Thus, $|c_i|$ is a measure of the membership of $|x_i)$ in $|X)$.

Linear transformations and choice of basis

An important characteristic of vector spaces is the complete freedom to choose any set of basis vectors related by unitary transformations. Thus, for example, one can express $|X) = \sum_i c_i|x_i)$ or as $|X) = \sum_i c_i'|x_i')$ provided $|x_i') = \sum_j S_{ij}|x_j)$ with S a unitary matrix, i.e. $S^\dagger S = SS^\dagger = 1$ so that $S^\dagger = S^{-1}$. Operators $\mathcal{O}$ (represented by $n \times n$ matrices in the old basis) acting on $\mathcal{H}$ then transform to the new basis as $\mathcal{O}' = S\mathcal{O}S^{-1}$ which is a similarity transformation.

Operators on Hilbert space and groups

BOUNDED OPERATORS ON A HILBERT SPACE CAN FORM NON-ABELIAN LIE GROUPS AND GENERATE THEIR ALGEBRAS. A SIMPLE EXAMPLE IS THE ROTATION GROUP $SO(3)$ IN EUCLIDEAN SPACE $\mathbb{R}^3$. $SO(3)$ HAS A UNIVERSAL COVERING GROUP $SU(2)$, THE GROUP OF ALL 2×2 UNITARY MATRICES WITH COMPLEX ELEMENTS AND DETERMINANT 1. ALTHOUGH THE GROUP $SU(2)$ IS USED IN QUANTUM PHYSICS TO DESCRIBE PARTICLES WITH SPIN $\frac{1}{2}\hbar$ (FERMIONS), THE GROUP ITSELF IS INDEPENDENT OF QUANTUM MECHANICS, AND CAN BE USED TO DESCRIBE EVEN A CLASSICAL BEAM SPLITTER.

Tensor products

APART FROM THE DIRECT SUM $\mathcal{H}_1 \oplus \mathcal{H}_2$ OF TWO HILBERT SPACES, ONE CAN ALSO FORM TENSOR PRODUCTS $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ WHOSE DIMENSION IS $(n_1 \times n_2)$.

There is a mathematical theorem which states that every pair of vector spaces has a tensor product.

Hence, nonseparable states in such spaces are inevitable.

3 Hilbert Spaces in Quantum Mechanics

Certain additional restrictions are imposed on Hilbert spaces to get quantum mechanics.

1. First, in order to have a probabilistic interpretation, quantum state vectors are all normalised (i.e. of unit norm). Hence, the set of all pure states corresponds to the unit sphere in Hilbert space, with the additional requirement that all vectors that differ only by a complex scalar factor (a phase factor) are identified with the same state. Thus, quantum mechanics operates on coset spaces and Grassmanian manifolds.

2. Second, a linear and unitary equation of motion, the Schrödinger equation, is postulated that specifies the time evolution of states in Hilbert space.

3. Third, the results of observations are obtained by projective measurements $M_i = |\psi_i\rangle\langle\psi_i|$ acting on the state $|\Psi\rangle : \hat{\rho} = \sum_i M_i \rho M_i^\dagger$. This projection is additional to the linear and unitary time evolution. This is the measurement postulate. Its ad hoc and non-unitary character has spawned a plethora of interpretations of quantum mechanics, some nonlocal (e.g. de Broglie-Bohm), some local (QBism).

4. Fourth, every pair of canonical dynamical variables (p_i, q_j) is postulated to be represented by hermitian operators $(\hat{p}_i, \hat{q}_j)$ with the commutation rules $[\hat{p}_i, \hat{q}_j] = -i\hbar\delta_{ij}$ resulting in the famous Heisenberg uncertainty relations $\Delta q \, \Delta p \geq \hbar/2$. This is the canonical quantization postulate. These commutation relations vanish in the formal limit $\hbar \to 0$, and hence do not arise from a non-abelian character of the operators and are not intrinsic to Hilbert spaces.

5. Fifth, entangled states in quantum mechanics are extremely fragile, i.e. the entanglement disappears very rapidly and the system decoheres and breaks into pieces when exposed to an environment.

4 Hilbert Space and Classical Polarization Optics

The Hilbert space structure of classical electrodynamics was first explicilty used by Spreeuw (1998, 2001) and independently demonstrated by Ghose and Samal in 2001.

The complete description of an ordinary state in classical electrodynamics involves the direct product of two disjoint Hilbert spaces, namely a space $\mathcal{H}_{path}$ of square integrable functions and a two-dimensional space of polarization states $\mathcal{H}_{pol}$. Hence, a state of unit intensity can be written as

$$\frac{1}{\sqrt{|A|^2}}|A) \otimes |\lambda) \in \mathcal{H}_{path} \otimes \mathcal{H}_{pol} \tag{1}$$

where $A(\mathbf{r}, t)$ are solutions of the scalar wave equation and $|\lambda) \in \mathcal{H}_{pol}$ is the vector

$$|\lambda) = e^{i\phi} \begin{pmatrix} \cos\theta \\ e^{i\chi} \sin\theta \end{pmatrix} \tag{2}$$

of the transverse polarizations λ_1 and λ_2. This can also be written as the Jones vector

$$|J) = \frac{1}{\sqrt{(J|J)}} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

with $E_x = A_0 \hat{e}_x \exp(\mathrm{i}\phi_\mathrm{x})$ and $E_y = A_0 \hat{e}_y \exp(\mathrm{i}\phi_\mathrm{y})$ the complex transverse electric fields, $\hat{e}_x$ and $\hat{e}_y$ the unit polarization vectors, and $(J|J) = |E_x|^2 + |E_y|^2 = A_0^2$ the intensity I_0.

Entanglement and Classical Polarization States

That classical optical fields like thermal light are necessarily path-polarization entangled has been recently shown by Xiao-Feng Qian and Eberly (2011, 2013). They have shown that the degree of polarization of a thermal light field corresponds to the degree of separability of these two spaces. This leads to a natural measure of the degree of polarization applicable to any optical field, whether beamlike or not. It turns out that only homogeneously polarized light beams correspond to fully separable or factored states. Interestingly, an ideal thermal light field is shown to be a Bell state that violates a Bell inequality, showing that typical Bell correlations are not exclusive to quantum states:

$$|e\rangle = \kappa_1 |e_1\rangle \otimes |f_1\rangle + \kappa_2 |e_2\rangle \otimes |f_2\rangle \tag{3}$$

where $\langle u_j|u_k\rangle = \langle f_j|f_k\rangle = \delta_{jk}$ and κ_1 and κ_2 are normalization constants, both equal to $1/\sqrt{2}$ in the exact thermal case. For an arbitrary field state, such a decomposition is guaranteed by the Schmidt theorem.

"Our results thus clarify two long-standing and widely held misimpressions: that indeterministic entanglement is unique to quantum mechanics, and that quantum mechanics is unique to violate Bell inequalities."

Cylindrically Polarized Light

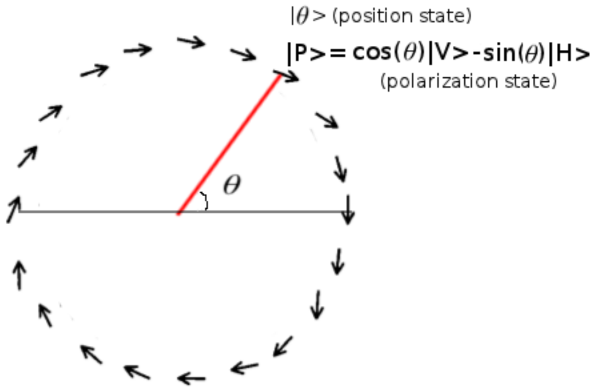

Figure 1: Variation of polarization with space of an emerging azimuthally polarized beam.

Nonseparable cylindrically polarized laser beams have been extensively studied and used since 1993

R. Oron, S. Blit, N. Davidson & A. A. Friesem (2000).

S. C. Tidwell, G. H. Kim & W. D. Kimura (1993)

Y. Kozawa & S. Sato (2005).

A. Holleczek, A. Aiello1, C. Gabriel1, C. Marquardt, G. Leuchs, Dec 2010.

C. Gabriel, A. Aiello, W. Zhong, T. G. Euser, N. Y. Joly, P. Banzer, M. Fortsch, D. Elser, U. L. Andersen, C. Marquardt, P. S. J. Russell & G. Leuchs, (2011).

The scalar field distributions of the $TEM_{01(x)}$ and $TEM_{01(y)}$ Laguerre-Gaussian modes are given by

$$E_x(r,\theta) = E_0\sqrt{\rho}\exp(-\rho/2)\cos\theta, \quad (4)$$
$$E_y(r,\theta) = E_0\sqrt{\rho}\exp(-\rho/2)\sin\theta, \quad (5)$$

where r and θ are the cylindrical coordinates, E_0 the magnitude of the electric field, $\rho = 2r^2/w^2$ with w as the waist of the Gaussian beam. A coherent summation of such modes with orthogonal polarizations leads to either azimuthally or radially polarized modes

$$E_\theta(r,\theta) = \hat{y}E_x(r,\theta) - \hat{x}E_y(r,\theta) = \hat{\theta}E_0\sqrt{\rho}\exp(-\rho/2), \quad (6)$$
$$E_r(r,\theta) = \hat{x}E_x(r,\theta) + \hat{y}E_y(r,\theta) = \hat{r}E_0\sqrt{\rho}\exp(-\rho/2). \quad (7)$$

These states are evidently polarization-position entangled and robust. Fig. 1 is a representation of azimuthally polarized light

Mueller Matrices and Entanglement

B. N. Simon, S. Simon, F. Gori, M. Santarsiero, R. Borghi, N. Mukunda & R. Simon (2010).

Recently, it has been shown that non-quantum entanglement plays an essential role in resolving the basic issue of the choice of the appropriate subset of 4×4 real matrices that should be accepted as Mueller matrices in classical optics. While Jones matrices are adequate to describe fully polarized light, Mueller matrices are required to describe the transformation of all forms of light passing optical elements. Any light beam, polarized or unpolarized, can be described by the Stokes vector $\vec{S}$. After it passes a linear optical element, it is transformed by the Mueller matrix $M : \vec{S}' = M\vec{S}$ where M is a 4×4 real matrix. Let Ω^{pol} be the state space of all real vectors $\vec{S} \in \mathbb{R}^4$. It turns out that although all real 4×4 matrices map Ω^{pol} into itself, not all of them are physical Mueller matrices. To be considered as physical Mueller matrices, they must satisfy a stronger positivity criterion that follows from nonseparability or entanglement of path and polarization of inhomogeneous light fields.

Bell-like Inequality Criterion for Entangled Classical Light

C. V. S. Borges, M. Hor-Meyll, J. A. O. Huguenin & A. Z. Khoury, (2010).

Borges et al have proposed an inequality criterion in analogy with Bell's inequality for the separability of the spin and orbit degrees of freedom of a laser beam and shown that this inequality is violated by classical optical states for which the spin-orbit degrees are nonseparable.

Classical optical coherence and optical signal processing

K. H. Kagalwala, G. Di Giuseppe, A. F. Abouraddy and B. E. A. Saleh, Nature Photonics (2013)

These authors have used Bell's measure as a quantitative tool in classical optics to delineate incoherence associated with statistical fluctuations from entanglement based incoherence . These results demonstrate the applicability of quantum information processing concepts to the study of classical optical coherence and optical signal processing.

Classical Vortex Beams with Topological Singularities

P. Chowdhury, A. S. Majumdar & G. S. Agarwal (2013)

That light can be twisted like a corkscrew around its axis of travel was first pointed out by Nye and Berry. This twisting results in the cancellation of light amplitudes on the axis. Such light is called an optical vortex. When incident on a flat surface, an optical vortex looks like a ring of light with a dark hole in the centre. This has led to the discovery of classical optical beams with topological singularities and charges (the number of twists in one wave length) that have diverse applications such as in optical tweezers, and to the study of the coherence properties of vortex beams. Such beams have Schmidt decomposition and share many properties with quantum optical systems. Using the Wigner function, Chowdhury et al have shown that the coherence properties of such beams imply violations of Bell inequalities for continuous variables.

NOTWITHSTANDING ALL THIS, IT MUST BE POINTED OUT THAT CLASSICAL AND QUANTUM ENTANGLEMENT HAVE SIGNIFICANTLY DIFFERENT **IMPLICATIONS**, AND HENCE THE APPROPRIATE NOMENCLATURE TO BE USED IN THE CASE OF CLASSICAL LIGHT HAS BEEN A MATTER OF SOME DEBATE. SOME PREFER TO USE 'NONSEPARABILITY', SOME 'STRUCTURAL INSEPARABILITY' AND SOME 'NON-QUANTUM ENTANGLEMENT' FOR CLASSICAL LIGHT. WE PREFER TO USE THE TERM 'CLASSICAL ENTANGLEMENT'. THIS IS TO EMPHASIZE THE FACT THAT NON-FACTORIZABILITY IS NOT AN EXCLUSIVE FEATURE OF QUANTUM MECHANICS; IT IS INTRINSIC TO CLASSICAL OPTICS AS WELL.

Implications of Entanglement/Nonseparability in Classical Optics

- Entanglement and BI violation are no longer exclusive signatures of quantumness
- Classical entanglement can be used to simulate the manipulations necessary for quantum information processing except those depending on quantum nonlocality. This was first shown by Spreeuw.

- The fact that classical entanglement is robust is an added advantage.

- Another very important aspect of classical entanglement is its bearing on the concept of noncontextuality and realism. It has recently been shown that the polarization and spatial modes of classically entangled light are **CONTEXTUAL** variables, which is a real surprise in classical physics.

5 Bell-like States in Classical Optics and Noncontextuality

PG & A. Mukherjee (2013)

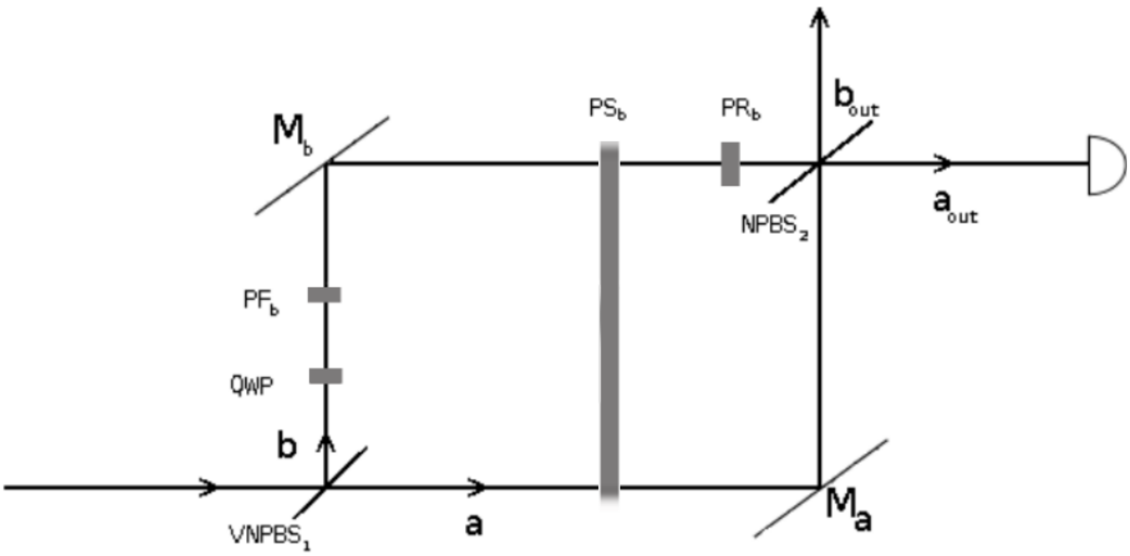

Figure 2: Schematic diagram: A V polarized classical light beam passes through a Mach-Zehnder interferometer. PF_b is a polarization flipper that converts V to H, PS_b is a phase-shifter, and PR_b is a polarization rotator.

It can be easily shown that the action of the interferometer shown in Figure 2 is to produce the final path-polarization entangled state

$$|\Phi^+) = \frac{A}{\sqrt{2I_0}}[|a) \otimes |V) + |b) \otimes |H)] \tag{8}$$

Our aim is to find out whether these classical states are consistent with the notion of **NONCONTEXTUALITY**, namely the innocuous classical notion that a physical property must be independent of the context in which it is measured. What this implies is that physical systems have properties with predetermined values that are not affected by how the value is measured, i.e. not affected by previous or simultaneous measurement of any other compatible or co-measureable observable.

Whether this holds in classical optics can be tested by making a joint measurement of the path and polarization of a single beam. We start by defining a correlation

$$E(\theta, \phi) = (\Psi|\sigma_\theta.\sigma_\phi|\Psi) \tag{9}$$

where $|\Psi)$ is an arbitrary normalized classical optical state and

$$\sigma_\theta = (e^{-i\theta}|V)(H| + e^{i\theta}|H)(V|) \otimes I_{path}, \tag{10}$$

$$\sigma_\phi = I_{pol} \otimes (e^{-i\phi}|a)(b| + e^{i\phi}|b)(a|). \tag{11}$$

These are projection operators $(\sigma_\theta^2 = 1, \sigma_\phi^2 = \mathbb{I})$ that represent polarization and path measurements. Since σ_θ and σ_ϕ act upon disjoint Hilbert spaces, they commute.

Let us now consider a general normalized product state

$$|\Psi) = |\psi_{pol})|\psi_{path}) = (\cos\alpha|V) + e^{i\beta}\sin\alpha|H))(\cos\gamma|a) + e^{i\delta}\sin\gamma|b)) \tag{12}$$

where $\alpha, \beta, \gamma, \delta$ are arbitrary parameters.

The correlation for such a factorizable state is

$$\begin{aligned} E(\theta, \phi) &= (\psi_{pol}|\sigma_\theta|\psi_{pol})(\psi_{path}|\sigma_\phi|\psi_{path}) \\ &= E_{pol}(\theta)E_{path}(\phi), \end{aligned} \tag{13}$$

with

$$E_{pol}(\theta) = \sin\alpha\cos(\beta - \theta), \tag{14}$$

$$E_{path}(\phi) = \sin\gamma\cos(\delta - \phi). \tag{15}$$

$$-1 \leqslant E(\theta, \phi) \leqslant 1. \tag{16}$$

Define a quantity S by

$$S(\theta_1, \phi_1; \theta_2, \phi_2) \; = \; E(\theta_1, \phi_1) + E(\theta_1, \phi_2) - E(\theta_2, \phi_1) + E(\theta_2, \phi_2) \tag{17}$$

Clearly,

$$|S| \leq 2 \tag{18}$$

This result depends only on the fact that **the correlations lie between** -1 **and** $+1$ and **no discreteness assumption is necessary**. Hence, this is a new and non-trivial result for classical optics, analogous to CHSH-Bell inequalities in particle mechanics.

Let us now calculate the correlation for the normalized state

$$|\Phi^+) = \frac{A}{\sqrt{2I_0}} \left[|a) \otimes |V) + |b) \otimes |H) \right] \tag{19}$$

Then,

$$\begin{aligned} E(\theta, \phi) \; &= \; (\Phi^+|\sigma_\theta \cdot \sigma_\phi|\Phi^+) \\ &= (\Phi^+| \, [(+)\sigma_{\theta,0} + (-)\sigma_{\theta,\pi}].[(+)\sigma_{\phi,0} + (-)\sigma_{\phi,\pi}]|\Phi^+). \end{aligned}$$

$$\begin{aligned} E(\theta, \phi) \; &= \; \frac{I(\theta,\phi) + I(\theta+\pi,\phi+\pi) - I(\theta+\pi,\phi) - I(\theta,\phi+\pi)}{I(\theta,\phi) + I(\theta+\pi,\phi+\pi) + I(\theta+\pi,\phi) + I(\theta,\phi+\pi)} \\ &= \; \cos(\theta+\phi). \end{aligned} \tag{20}$$

Notice that this is not in a product or factorizable form. It follows from this that the noncontextuality bound $|S| \leq 2$ is violated by the state $|\Phi^+)$ for the set $\theta_1 = 0, \theta_2 = \pi/2, \phi_1 = \pi/4, \phi_2 = -\pi/4$ for which $S = 2\sqrt{2}$.

There is no nonlocality in this result because the path and polarization changes are made on the same state in path b. This result shows that the path and polarization of classical light in entangled states like $|\Phi^+\rangle$ are **contextual**, i.e. the measurement of path/polarization depends on the measurement of polarization/path although they are compatible properties..

Hence innocuous classical realism does not hold in classical optics, the paradigm of realist classical physics.

6 Information Processing with Classical Light

We will now discuss to what extent classical polarization optics can be used to simulate quantum information processing tasks. This is very important from a practical point of view because coherence and entanglement are robust in classical optics, thus circumventing the problem of decoherence in quantum information processing. However, as already emphasized, there are certain fundamental differences between classical optical coherence and entanglement and their quantum counterparts, namely (a) there is no nonlocality in classical optics, and (b) there is no 'no-cloning theorem' in classical optics. As a result, error correction protocols have to be specially designed. The exponential resources problem with classical optics also needs further investigation. We will now show how some simple quantum logic gates and networks can be realized with classical optics.

6.1 Gates analogous to single qubit gates

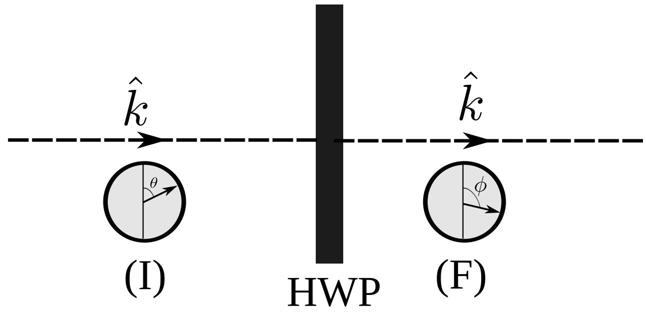

Figure 3: The action of a half-wave plate HWP with its fast axis inclined at$(\frac{\phi-\theta}{2})$ to the vertical on a linearly polarized light beam. (I) and (F) represent the initial and final states of polarization respectively.

Let us first consider the case of a linearly polarized classical light beam passing through a half-wave plate HWP with its fast axis inclined at$(\frac{\phi-\theta}{2})$ to the vertical (Fig. 3). The transition from the initial state θ to the final state ϕ of polarization is given by

$$\begin{aligned} J\left((\phi-\theta)/2\right)|\theta\rangle &= |\phi\rangle, \\ J((\phi-\theta)/2) &= \begin{pmatrix} \cos(\phi-\theta) & \sin(\phi-\theta) \\ \sin(\phi-\theta) & -\cos(\phi-\theta) \end{pmatrix} \\ |\theta\rangle &= \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \end{aligned}$$

where J is a Jones matrix. More generally, the Jones vector $|\theta\rangle$ can also be written in the form

$$|\theta\rangle = c_0|0\rangle + c_1|1\rangle \tag{21}$$

with c_0 and c_1 classical complex amplitudes and $|0\rangle$ and $|1\rangle$ denoting the basis states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{22}$$

so that

$$|\theta\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}. \tag{23}$$

The coefficients $|c_0|^2$ and $|c_1|^2$ are the intensities measured by photodetectors. Then (21) is the classical analog of a qubit, and may be called a polarization *cebit*. A similar relation holds for path cebits with $|0\rangle$ and $|1\rangle$ denoting two orthogonal paths. In the classical optical case, though the path and polarization cebits are locally specified, they are independent degrees of freedom, being elements of disjoint Hilbert spaces. They can therefore be used as control or target cebits.

Gates analogous to two qubit unitary gates

The CNOT Gate

Let us first consider the propagation mode or the path as the control cebit and the polarization as the target cebit. After the beam splitter NPBS (Fig. 4) the state of an incident H beam is

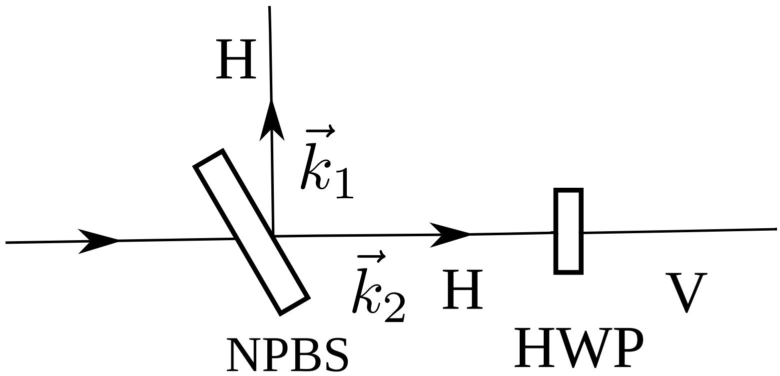

Figure 4: A CNOT gate with path as the control cebit. The fast axis of the half-wave plate HWP is oriented at 45^0 to the vertical.

$$|\Psi\rangle = (|\vec{k}_1\rangle + |\vec{k}_2\rangle)|H\rangle \tag{24}$$

If a half-wave plate HWP is placed in the path $\vec{k}_2$, it flips the H polarization state to the V polarization state. Therefore the final state is

$$|\Psi\rangle = |\vec{k}_1\rangle|H\rangle + |\vec{k}_2\rangle|V\rangle \tag{25}$$

This is a CNOT gate as it flips the polarization of the target cebit only if the control cebit is the path $\vec{k}_2$ and not $\vec{k}_1$. Since the intensity remains the same, we have a succesful realization of CNOT.

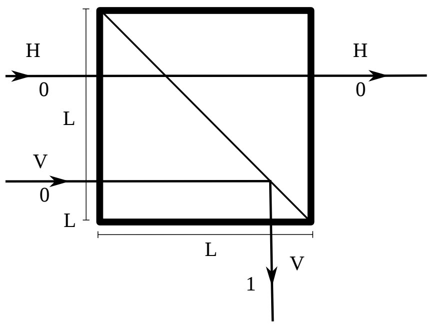

Figure 5: A CNOT gate with polarization as the control bit. PBS is a polarizing beam splitter .

One can also have a CNOT gate with polarization as the control cebit to fix the path as shown in Fig. 5. A light beam with electric field parallel to the plane of the paper (H) gets transmitted, and one with electric field perpendicular to the plane of the paper (V) gets reflected. Hence, it is a successful realization of a CNOT gate. The truth table for this CNOT gate is

Control Bit	Target Bit	Output
H	0	0
H	1	1
V	0	1
V	1	0

One can an also construct Toffoli Gates and carry out teleportation between the disjoint path and polarization Hilbert spaces.