Pre-Verification of Quantum Processes

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Outline

1. Introduction
2. Pre-Verification
3. Chernoff & Neighbors
4. Simulation & Experiment
5. Conclusions

pre-verification

$\equiv$

classical testing quantum data

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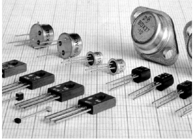
Introduction

- Data and device “fidelity”.



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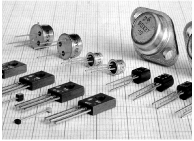
- Data "fidelity" checked via error control theory.



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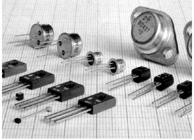

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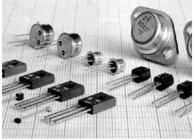

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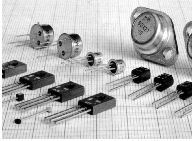


- Data and device "fidelity".
- Data "fidelity" checked via error control theory.
- Device "fidelity" checked via hypothesis testing theory.
- **Error Detection** ☑ & Error Correction ☒.
- "Naive": Check all possible I/O combinations.

Quantum Case: States and Processes

States

- Data transmission $\rightarrow$ State transmission.
- Density matrices have $\mathrm{d_s^2} - 1$ elements.

Processes

- Devices $\rightarrow$ Processes: $\mathcal{H}_\mathcal{S} \rightarrow \mathcal{H}_\mathcal{S}$.
- Density matrices characterized by $\mathrm{d_s^2} - 1$ elements.
- Processes characterized by $\mathrm{d_s^4} - \mathrm{d_s^2}$ elements.
- Costly when $\mathrm{d_s} = 2^\mathrm{m}$, for m qubits[a].

[a] pseudo-spins are different

Process Tomography-I

Basics

- Basis ϱ_k.
- Any density matrix $\rho = \sum_{k=1}^{\mathrm{d_s^2}} \mathrm{c_k} \varrho_\mathrm{k}$
- For any process Φ, measure $\tilde{\varrho}_k = \Phi(\varrho_k)$.
- From linearity, $\Phi(\rho) = \sum_{k=1}^{\mathrm{d_s^2}} \mathrm{c_k} \Phi(\varrho_\mathrm{k}) = \sum_{\mathrm{k}=1}^{\mathrm{d_s^2}} \mathrm{c_k} \tilde{\varrho}_\mathrm{k}$.
- Count again : $\mathrm{d_s^2} - 1$ for each of the $\mathrm{d_s^2}$ density matrices.
- Total $\boxed{\Rightarrow \mathrm{d_s^4} - \mathrm{d_s^2}}$.

$\mathrm{d_s}$	$\mathrm{d_s^4} - \mathrm{d_s^2}$
2	12
3	72
4	240
5	600
6	1260
7	2352
8	4032
9	6480
10	9900

Process Tomography-II: EAPT

- Entanglement Assisted Process Tomography (EAPT)[1].
- Choi-Jamiołkowski: $\Phi \Leftrightarrow \varrho_{\Phi}$, where $\varrho_{\Phi} := [\Phi \otimes \mathrm{I}]\left(|\phi\rangle\langle\phi|\right)$.
- $|\phi\rangle = \frac{1}{\sqrt{\mathrm{d_s}}} \sum_{j} |j\rangle|j\rangle$ is maximally entangled state in dilated space.
- Employ $|\phi\rangle$ and use state tomography to find ρ_{Φ}.
- Caveat: Production of highly entangled state $|\phi\rangle$! As many measurements!

[1] Altepeter, Joseph B., et al. Physical Review Letters 90.19 (2003): 193601.

Process Tomography-III: AAPT

- Ancilla Assisted Process Tomography (AAPT)[2].
- $\mathrm{R} = \sum_{\mathrm{k}}^{\mathrm{d_s^2}} \lambda_{\mathrm{k}} \mathrm{A_k} \otimes \mathrm{B_k}$, $\lambda_k > 0$.
- maximal rank state R faithful in mapping Φ:

$$[\Phi \otimes \mathrm{I}]\,(\mathrm{R}) = \sum_{\mathrm{k}}^{\mathrm{d_s^2}} \lambda_{\mathrm{k}}\; \Phi(\mathrm{A_k}) \otimes \mathrm{B_k}.$$

- max rank states are “easy” to produce.
- Caveat: As many measurements!

[2] Altepeter, Joseph B., et al. Physical Review Letters 90.19 (2003): 193601.

Compressive Sensing

- Compressive Sensing: Prelude to CQPT[3].
- Res.(Mbps$\rightarrow$Gbps) $\times$ Mult. Freq. $\times$ (# of Sensors) $\Rightarrow$ "Data Deluge" $|x\rangle$
- $rank(|x\rangle) = K \ll N$: Nyquist/Shannon + Throwaway + Transmit.
- Access compressible inf. $K \ll N$ directly via $M = \mathcal{O}\left(K \log(N)\right)$ meas.
- Randomized measurements are universal.
- Take away: $rank(|x\rangle) = K \Rightarrow \mathcal{O}\left(K \log(N)\right)$ measurements.

[3] Candes, E. J.,et.al., Communications on pure and applied mathematics 59.8 (2006): 1207-1223.

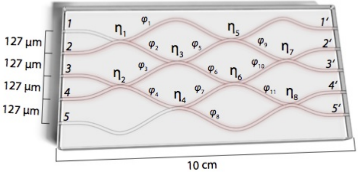
Process Tomography-IV: CQPT

- Compressive Quantum Process Tomography(CQPT)[4].
- Take away: It takes $\mathcal{O}\left(K \log(N)\right)$ measurements.
- Assume that the Process Matrix ϱ_{Φ} is sparse.
- Small (logarithmically) number of measurements suffice.
- Caveat: Not $\forall$ processes, measure zero applicability!

[4] Shabani, A., et al. Physical Review Letters 106.10 (2011): 100401.

Construct, Convince & Measure

a)



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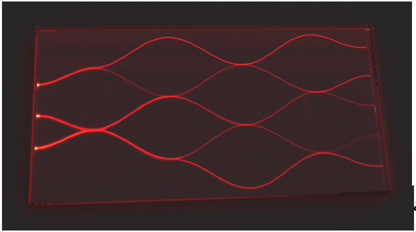
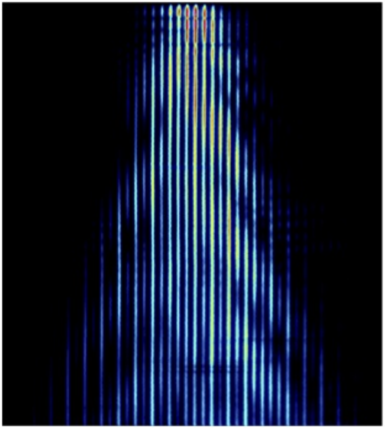
b)





[a]

[a] Tillmann, Max, et al.Nature Photonics (2013), Driesow *et.al.,PRL* **105**, *143902(10)*

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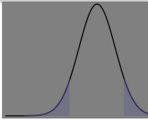
Hoeffding's Inequality



- $Z_1, \ldots Z_M$, independent random variables.

[a] Hoeffding, Wassily. Journal of the American statistical association 58.301 (1963): 13-30.

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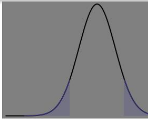

- $Z_1, \ldots Z_M$, independent random variables.
- Assume that $a_i \leq Z_i \leq b_i \; \forall \; i \in [1, M]$

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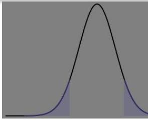

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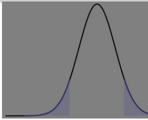

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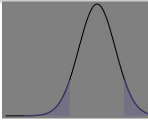

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- If $\{Z_1 \ldots Z_M\}$ from the correct hypothesis, $\bar{Z} \approx \mathbb{E}(\bar{Z})$[a].

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Application to Pre-Verification

- Assume $Z_j = \mathrm{tr}(\mathcal{E}_j \Phi(\rho_j)) + \sqrt{\eta_j} d\mathbb{W}_j$.

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- Distinguishing neighbors follows common sense.

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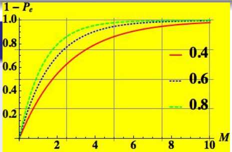
Chernoff Bound for Processes



Asymptotic Error Probability

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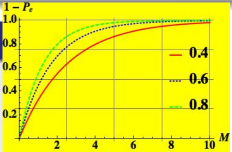

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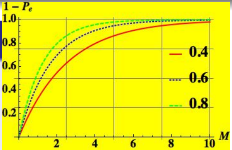

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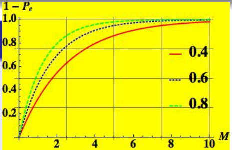

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- Spectral decomposition $\varrho_\Phi = \sum_i \lambda_i |i\rangle\langle i|$

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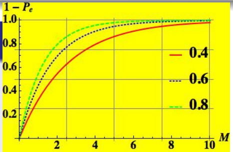

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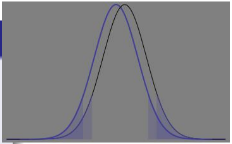
- Spectral decomposition $\varrho_\Phi = \sum_i \lambda_i |i\rangle\langle i|$
- $\xi_{qcb} = \frac{\varepsilon}{2}\sum_{i,j}\frac{\left|\langle i|d\rho_\Phi|j\rangle\right|^2}{(\sqrt{\lambda_i}+\sqrt{\lambda_j})^2} = \frac{\varepsilon}{2}d\xi.$

Distinguishing Neighbors Via Q. Chernoff & Hoeffding



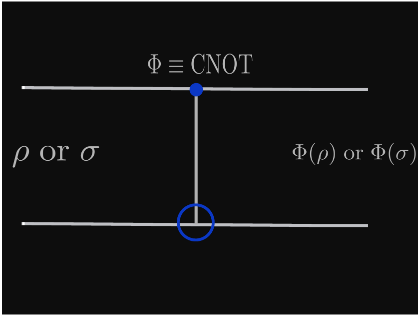
Chenoff

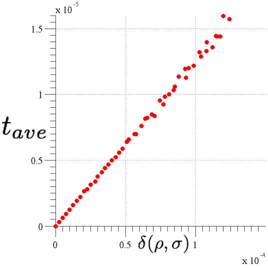
- $\mathbb{P}_e = exp(-\frac{M\varepsilon}{2} d\xi)$
- As $\varepsilon \to 0$ for a given M, $\mathbb{P}_e \to 1$.
- $M = -\frac{2}{\varepsilon d\xi} \log(\mathbb{P}_e)$ Scales badly.

Hoeffding

- $\mathbb{E}(\bar{Z}) \to \mathbb{E}(\bar{Z}) + \varepsilon \mathbb{E}(d\bar{Z})$
- As if $\mathbb{P}(|\bar{Z} - \mathbb{E}(\bar{Z})| \geq t) \leq 2e^{-\frac{Mt^2}{\sigma^2}} \to \mathbb{P}(|\bar{Z} - \mathbb{E}(\bar{Z})| \geq t) \leq 2e^{-\frac{M(t-\varepsilon\mathbb{E}(d\bar{Z}))^2}{\sigma^2}}$
- $M = -\frac{\sigma^2}{(\varepsilon \mathbb{E}(d\bar{Z}))^2} \log\left(\frac{\mathbb{P}_e}{2}\right)$ Scales badly as well.

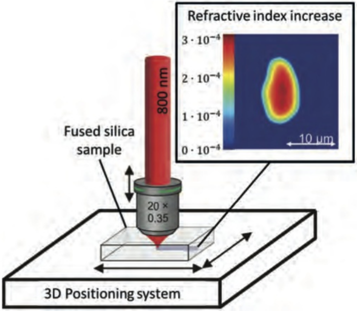
Simulation

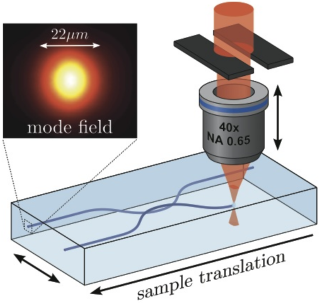




- Choose 10 random measurements from 240.
- 10^4 random neighbors of ρ with average δ.
- Exponent increases.

Experiment(“Construct”)



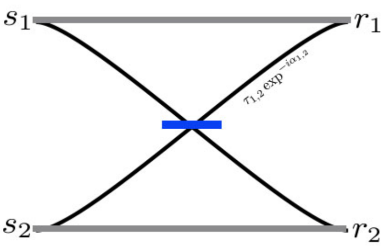


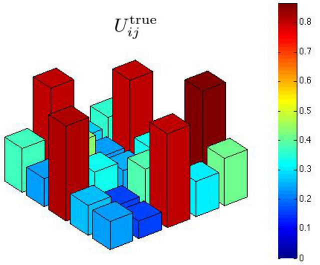
- Femtosecond Lasers used to etch waveguides.[5]
- Velocity→ n. Evanescent coupling.
- 1% errors.

[5] Szameit A. and Nolte, J. Phys. B 43, 163001 (2010)

Experiment("Convince")

$$P_2 = \begin{pmatrix} \sqrt{r_j} & 0 \\ 0 & \sqrt{r_g} \end{pmatrix} * \begin{pmatrix} e^{i\alpha_{j,k}}\tau_{j,k} & e^{i\alpha_{j,h}}\tau_{j,h} \\ e^{i\alpha_{g,k}}\tau_{g,k} & e^{i\alpha_{g,h}}\tau_{g,h} \end{pmatrix} * \begin{pmatrix} \sqrt{s_j} & 0 \\ 0 & \sqrt{s_g} \end{pmatrix}$$





$$Q = |\text{PerP}|^2$$
$$C = \text{Per}|\text{P}|^2$$
$$V = (C - Q)/C$$

Measure these

"Super Stable Tomography"[a]

[a] Laing, Anthony, and Jeremy L. O'Brien 1208.2868

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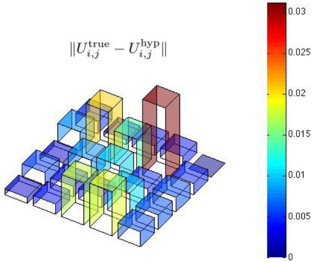
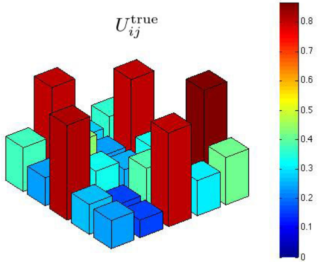
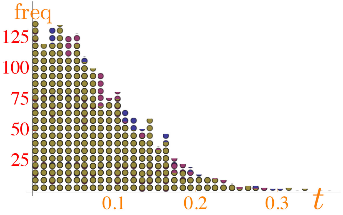
Convince-II: Pre-Verify







- min 2% dev. 14 times the min 1% value.

Conclusions

- Mesoscopic devices are state of the art...quantum data deluge!
- Bypass "fluorescence" + "imaging" type techniques.
- Replace QPT (when possible) with confidence test.
- No requirements: sparsity , linearity...
- Can be used "alongside" QPT.
- Doesn't violate canon on undecidability[6]...

[6] Rosgen, Bill. *Th. Q. Comp., Comm., and Crypt*. Springer Berlin Heidelberg, 2011. 63-76.

The End