Improving the fidelity of teleportation through noisy channels using weak measurement

Tanumoy Pramanik
Extended Senior Research Fellow

S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata 700 098, India

Phys. Lett. A **377**, 3209 (2013)
(arXiv:1301.0281)

Teleportation

Motivation

Here, one can send the information about unknown state of a qubit by sending two cbits of classical information.

Requirements

- Shared entanglement between the sender (Alice) and the receiver (Bob).
- Communication of two cbits of classical information from Alice to Bob.

Shared entanglement

- Alice prepares two qubits in one of the four maximally entangled states.
- She sends the 2nd qubit to Bob over the environment.

Decoherence

Environmental interaction

- In practice, at the time of transit, the 2nd qubit interacts with the environment.
- As a result entanglement decreases and for certain condition the entanglement can vanish.
- As well as, teleportation fidelity also decreases.

Amplitude damping channel (ADC)

$$|0\rangle_2|0\rangle_E \to |0\rangle_2|0\rangle_E$$
$$|1\rangle_2|0\rangle_E \to \sqrt{1-D_2}\ |1\rangle_2|0\rangle_E + \sqrt{D_2}\ |0\rangle_2|1\rangle_E$$

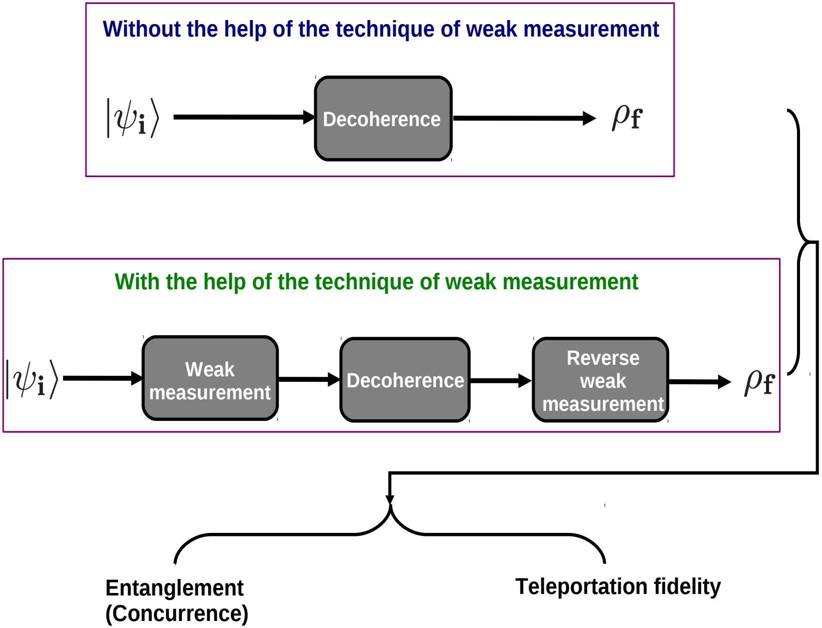
Using the technique of weak measurement, one can protect the entanglement when the environmental interaction is modeled by ADC.

Y-S. Kim, J.-C. Lee, O. Kwon, and Y-H. Kim, Nature Phys. **8**, 117 (2012); Z.-X. Man, Y.-J. Xia, N.B. An, Phys. Rev. A **86** 052322 (2012).

Motivation

Is the protection of entanglement equivalent to the protection of teleportation fidelity ?

We try to address this question using the technique of weak measurement.


Without the help of the technique of weak measurement
$|\psi_i\rangle$
Decoherence
ρ_f
With the help of the technique of weak measurement
$|\psi_i\rangle$
Weak measurement
Decoherence
Reverse weak measurement
ρ_f
Entanglement (Concurrence)
Teleportation fidelity

Prepared entangled state

Alice prepares two qubits in one of the following entangled states :

$$|\psi^{\pm}\rangle = \frac{|00\rangle_{12} \pm |11\rangle_{12}}{\sqrt{2}}$$

$$|\phi^{\pm}\rangle = \frac{|01\rangle_{12} \pm |10\rangle_{12}}{\sqrt{2}}$$

How the entanglement and the teleportation fidelity are affected by the interaction of the 2nd qubit with the environment ?

Decoherence

Alice sends the 2nd qubit to Bob over the environment.

ADC channel : Operator representation

$$\sigma \longrightarrow W_{2,0}\ \sigma\ W_{2,0}^{\dagger} + W_{2,1}\ \sigma\ W_{2,1}^{\dagger},$$

$$W_{2,0} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_2} \end{pmatrix}; \qquad W_{2,1} = \begin{pmatrix} 0 & \sqrt{D_2} \\ 0 & 0 \end{pmatrix}$$

After environmental interaction, the state ρ_W becomes :

$$\rho_D = (I \otimes W_{2,0})\ \rho\ (I \otimes W_{2,0}^{\dagger}) + (I \otimes W_{2,1})\ \rho\ (I \otimes W_{2,1}^{\dagger}),$$

where $\rho \in \{|\psi^{\pm}\rangle, |\phi^{\pm}\rangle\}$

Concurrence and teleportation fidelity of ρ_D

Concurrence

$$C(\rho_D) = 1 - D_2$$

Teleportation fidelity

$$F(\rho_D) = \frac{2f(\rho_D)+1}{3} = \frac{1}{6}[4 - D_2 + 2\sqrt{1 - D_2}]$$

Singlet fraction

$$f(\rho_D) = \max[\langle\psi^+|\rho_D|\psi^+\rangle, \langle\psi^-|\rho_D|\psi^-\rangle, \langle\phi^+|\rho_D|\phi^+\rangle, \langle\phi^-|\rho_D|\phi^-\rangle]$$

The effect of the technique of weak measurement in the protection of entanglement and teleportation fidelity.

Weak measurement

Befor sending the 2nd qubit, Alice makes a weak measurement with strength p_2.

Weak Measurement

Weak measurement is achieved by reducing the sensitivity of the detector, i.e., the detector detects the input qubit with probability p_2 if it is in the state $|1\rangle_2$ and never detects if the input qubit is in the state $|0\rangle_2$.

$$M_{2,0} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p_2} \end{pmatrix}; \qquad M_{2,1} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p_2} \end{pmatrix}$$

$$\rho_W = (I \otimes M_{2,0})\ \rho\ (I \otimes M_{2,0}^{\dagger})$$

Success probability

Due to post selection (depending on the detection by the detector) the success probability associated with the weak measurement is $Tr[(I \otimes M_{2,0})\ \rho\ (I \otimes M_{2,0}^{\dagger})]$.

Decoherence

Alice sends the 2nd qubit to Bob over the environment.

ADC channel : Operator representation

$$\sigma \longrightarrow W_{2,0}\ \sigma\ W_{2,0}^{\dagger} + W_{2,1}\ \sigma\ W_{2,1}^{\dagger},$$

$$W_{2,0} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_2} \end{pmatrix}; \qquad W_{2,1} = \begin{pmatrix} 0 & \sqrt{D_2} \\ 0 & 0 \end{pmatrix}$$

After environmental interaction, the state ρ_W becomes :

$$\rho_D = (I \otimes W_{2,0})\ \rho_W\ (I \otimes W_{2,0}^{\dagger}) + (I \otimes W_{2,1})\ \rho_W\ (I \otimes W_{2,1}^{\dagger})$$

Reverse weak measurement

After receiving the 2nd qubit, Bob makes a reverse weak measurement which is a reverse operation of Alice's weak measurement.

$$N_{2,0} = \begin{pmatrix} \sqrt{1-q_2} & 0 \\ 0 & 1 \end{pmatrix}; \qquad N_{2,1} = \begin{pmatrix} \sqrt{q_2} & 0 \\ 0 & 0 \end{pmatrix}$$

Shared State

$$\rho_R = \frac{(I \otimes N_{2,0}) \; \rho_D \; (I \otimes N_{2,0}^{\dagger})}{Tr[(I \otimes N_{2,0}) \; \rho_D \; (I \otimes N_{2,0}^{\dagger})]}$$

Success Probability

$$P_{Success} = Tr[(I \otimes N_{2,0}) \; \rho_D \; (I \otimes N_{2,0}^{\dagger})]$$

Optimal concurrence of ρ_R

Optimal concurrence

Optimal concurrence is obtained by maximizing $C(\rho_R)$ with respect to q_2 and corresponding reverse weak measurement strength $q_2^O|_E$ is called optimal reverse weak measurement strength for the protection of entanglement.

$$C^O(\rho_R) = \frac{1}{1+D_2-D_2 p_2}$$

$$q_2^O|_E = \frac{2D_2(1-p_2)+p_2}{1+D_2(1-p_2)}$$

Success Probability

$$P^1_{Success}|_E = (1-D_2)(1-p_2).$$

Comparison of Concurrences

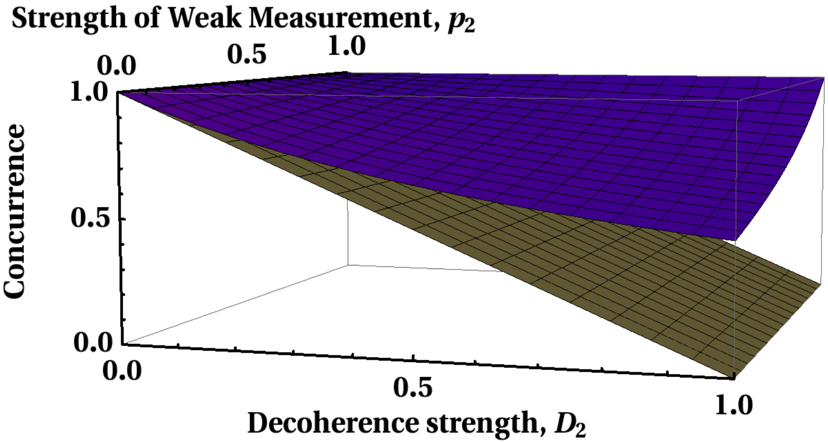


Figure : The brown colored surface represents the concurrence when only decoherence affects on the 2nd qubit. The purple colored surface represents the concurrence when the technique of weak measurement is applied.

Optimal teleportation fidelity of ρ_R

Optimal teleportation fidelity

Optimal teleportation fidelity is obtained by maximizing $F(\rho_R)$ with respect to q_2 and corresponding reverse weak measurement strength $q_2^O|_F$ is called optimal reverse weak measurement strength for the protection of teleportation fidelity.

$$F^O(\rho_R) = \frac{1}{3}\frac{3+2D_2(1-p_2)}{1+D_2(1-p_2)}$$

$$q_2^O|_F = \frac{3D_2(1-p_2)+D_2^2(1-p_2)^2+p_2}{(1+D_2(1-p_2))^2} \neq q_2^O|_E = \frac{2D_2(1-p_2)+p_2}{1+D_2(1-p_2)}$$

Success Probability

$$P^1_{Success}|_F = \frac{(1-D_2)\ (1-p_2)\ (2+D_2(1-p_2))}{2+2D_2(1-p_2)}.$$

Comparison of teleportation fidelity

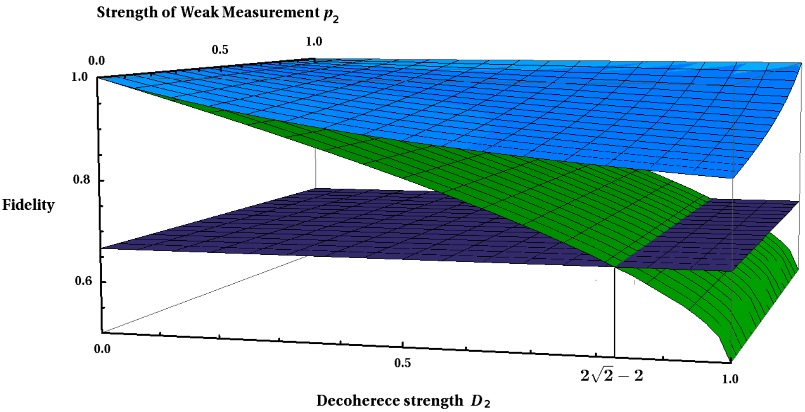


Figure : The flat plane represents the average classical fidelity $\frac{2}{3}$. The green colored surface represents the fidelity when only decoherence affects on the 2nd qubit. The blue colored surface represents the fidelity when the technique of weak measurement is applied.

T. Pramanik and A. S. Majumdar, Phys. Lett. A. 377 3209 (2013)

Teleportation fidelity when both qubits are affected by decoherence

Prepared state : $|\psi^{\pm}\rangle = \frac{|00\rangle_{12} \pm |11\rangle_{12}}{\sqrt{2}}$

In this case, the teleportation fidelity can be protected with the help of the technique of weak measurement only for the prepared states $|\psi^{\pm}\rangle$.

Assumptions

- $D_1 = D_2 = D$.
- $p_1 = p_2 = p$.
- $q_1 = q_2 = q$.

Without using the technique of weak measurement, i.e., $p = q = 0$

$$F(\rho_D) = \frac{1}{3}[D^2 - 2D + 3]$$

Using the technique of weak measurement

$$F^O(\rho_R) = \frac{1}{3} \times \frac{2+2D^2(1-p)^2+(1+D(1-p))\sqrt{1+D^2(1-p)^2}}{1+D^2(1-p)^2+D(1-p)\sqrt{1+D^2(1-p)^2}}$$

$$q_F^O = \frac{1+D^2(1-p)^2-\sqrt{(1-D)^2(1-p)^2(1+D^2(1-p)^2)}}{1+D^2(1-p)^2} \neq q_E^O = p + D(1-p)$$

Y-S. Kim, J.-C. Lee, O. Kwon, and Y-H. Kim, Nature Phys. **8**, 117 (2012).

Success Probability

$$P^2_{Success}|_F = \frac{1}{1+D^2(1-p)^2}\left((1-D)^2(1-p)^2\left(1+D(1-p)\sqrt{1+D^2(1-p)^2}+D^2(1-p)^2\right)\right).$$

Comparison of teleportation fidelity

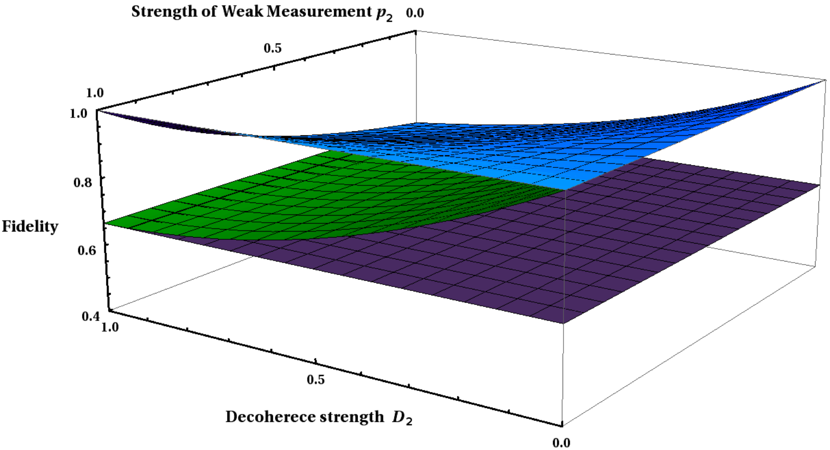


Figure : The flat plane represents the average classical fidelity $\frac{2}{3}$. The green colored surface represents the fidelity when the technique of weak measurement is not performed. The blue colored surface represents the fidelity $F^O(\rho_R)$.

T. Pramanik and A. S. Majumdar, Phys. Lett. A. 377 3209 (2013)

Comparison of success probability $P^2_{Success}|_F$ with the success probability $P^2_{Success}|_F$

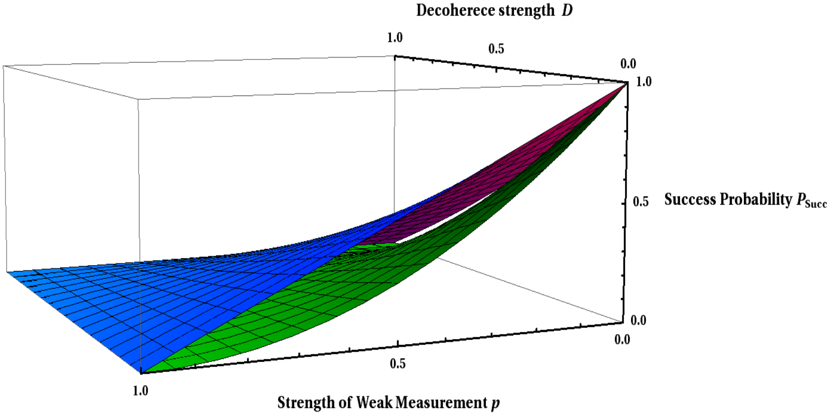


Figure : The blue colored surface represents the success probability $P^1_{Success}$ when the weak measurement technique is applied in single qubit. The green colored surface represents the success probability $P^2_{Success}$ when the weak measurement technique is applied on both qubits.

Prepared state : $|\phi^{\pm}\rangle = \frac{|01\rangle_{12} \pm |10\rangle_{12}}{\sqrt{2}}$

For the prepared state $|\phi^{\pm}\rangle$ with the assumption $p_1 = p_2 = p$, the prepared state remains unaffected when detectors do not detect the qubits.

Result

Hence, the technique of weak measurement is not useful to protect both entanglement and teleportation fidelity for the prepared state $|\phi^{\pm}\rangle$ under the assumption $p_1 = p_2 = p$.

Conclusion

- The technique of weak measurement can reduce the effect of decoherence.
- The protection of entanglement is not equivalent to the protection of teleportation fidelity as the strength of reverse weak measurement to protect the entanglement is not same the strength corresponding to the teleportation fidelity.
- As the weak measurement technique is associated with the post-selection, the protection of both entanglement and teleportation fidelity are associated with the success probability.
- The success probability when weak measurement is performed on single qubit is larger than when it is performed on two qubits.

Phys. Lett. A **377**, 3209 (2013) (arXiv:1301.0281)

Concurrence of ρ

$$C(\rho) = \max[0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}],$$

where λ_is are eigenvalues (in descending order) of $\rho\tilde{\rho}$. Here $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

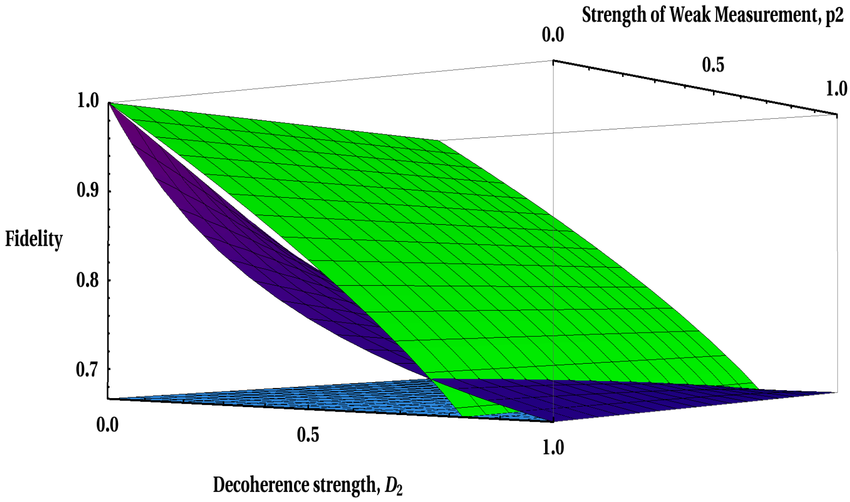


Figure : The upper surface represents the fidelity $\overline{F}_1$ when the technique of weak measurement is not applied. The middle surface represents the success probability $F_1^{\text{Av}} = F^O(\rho_R)P^1_{Success}|_F + \frac{2}{3}(1 - P^1_{Success}|_F)$. The flat surface represents the classical fidelity 2/3.