Local Unitary Invariants from entanglement detecting maps, and some applications.

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Outline

- LU invariants and Review of a paper.
- PT, Realignment and new link transformations. Some proofs
- Characterization of 3-qubit pure states with a new measure.
- Some application to three coupled and chaotic rotors
- Summary

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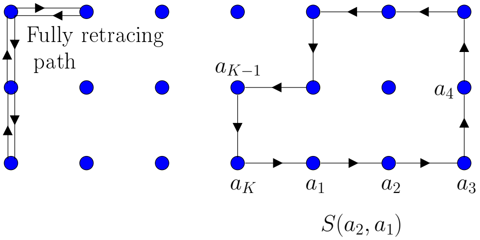
LU invariants

Properties of the state that are unchanged on LU operations:
$\rho_{12\cdots N} \mapsto U_1 \otimes U_2 \otimes \cdots \otimes U_N(\rho_{12\cdots N})U_1^\dagger \otimes U_2^\dagger \otimes \cdots \otimes U_N^\dagger$.

Review of a paper

Inspired by lattice gauge theory *M. S. Williamson, M. Ericsson, M. Johansson, E. Sjoqvist, A. Sudbery, V. Vedral, and W. K. Wootters*, Phys. Rev.A83, 062308 (2011) suggested the following method of generating LU for a multiqubit state

Consider closed path $\{a\}_K \equiv (a_1, a_2, \ldots, a_K)$



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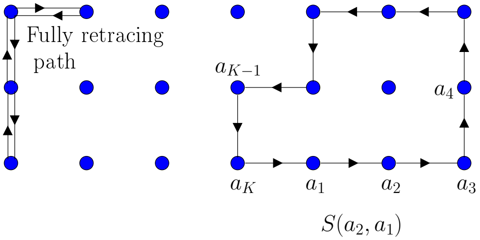

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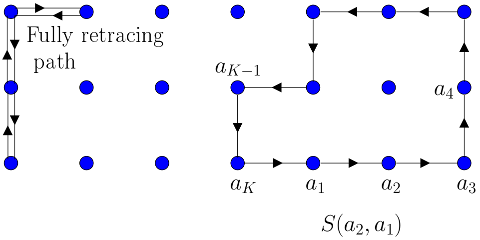

Link Transformation

$S(a_2, a_1)$ connecting two qubits:

$$S(a_2, a_1)_{nm} = \frac{1}{2}\text{tr}[\rho_{12}\, \sigma_m^{a_1} \otimes \sigma_n^{a_2}] \text{ where } m, n = 0, 1, 2, 3.$$

$m = 0$ corresponds to identity matrix and $m = 1, 2, 3$ corresponds to Pauli matrices.

The LU invariant

for closed path $\{a\}_K \equiv (a_1, a_2, \ldots, a_K)$ is then given as:

$$\text{tr}[S(a_1, a_K) \cdots S(a_3, a_2) S(a_2, a_1)].$$

Calculated 5 (excluding trace) invariants of three qubits ($N = 3$) in a pure state.

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Enter: Two entanglement detectors

Partial Transpose

$$\langle \mathbf{i}|\langle\beta|\rho_{\mathbf{12}}^{\mathbf{T_2}}|\mathbf{j}\rangle|\alpha\rangle = \langle\mathbf{i}|\langle\alpha|\rho_{\mathbf{12}}|\mathbf{j}\rangle|\beta\rangle. \text{ Entangled if } \rho_{12}^{T_2} < 0$$

Realignment

$$\langle \mathbf{i}|\langle\mathbf{j}|\mathcal{R}(\rho_{\mathbf{12}})|\alpha\rangle|\beta\rangle = \langle\mathbf{i}|\langle\alpha|\rho_{\mathbf{12}}|\mathbf{j}\rangle|\beta\rangle. \text{ Entangled if } \|\mathcal{R}(\rho_{12})\|_1 > 1$$

Partial Transpose followed by Realignment

$$\langle \mathbf{i}|\langle\mathbf{j}|\mathcal{R}(\rho_{\mathbf{12}}^{\mathbf{T_2}})|\beta\rangle|\alpha\rangle = \langle\mathbf{i}|\langle\alpha|\rho_{\mathbf{12}}|\mathbf{j}\rangle|\beta\rangle$$

Density operator reshaped:

$$d_1 d_2 \times d_1 d_2 \rightarrow d_1^2 \times d_2^2.$$

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Results: Use $\mathcal{R}(\rho_{ab}^{T_b})$ as Link Transformations

Lemma

Observation of **Left and Right** actions:

$$\mathcal{R}(\rho_{21}^{T_1}) = (U_2 \otimes U_2^*)^\dagger \mathcal{R}(\tilde{\rho}_{21}^{T_1})(U_1 \otimes U_1^*).$$

Proof.

Only local unitary U_2: $\rho_{21} = (U_2^\dagger \otimes I_1)\tilde{\rho}_{21}(U_2 \otimes I_1)$. It follows that:

$$(\rho_{21})_{i\alpha;j\beta} = (U_2^\dagger)_{i,i'}(\tilde{\rho}_{21})_{i'\alpha;j'\beta}(U_2)_{j',j}.$$

Using the definitions of the realignment and partial transpose:

$$\begin{aligned} \left(\mathcal{R}(\rho_{21}^{T_1})\right)_{ij;\beta\alpha} &= (U_2^\dagger)_{i,i'} \left(\mathcal{R}(\tilde{\rho}_{21}^{T_1})\right)_{i'j';\beta\alpha} (U_2)_{j',j} && (1) \\ &= (U_2^\dagger)_{i,i'}(U_2^*)^\dagger_{j,j'} \left(\mathcal{R}(\tilde{\rho}_{21}^{T_1})\right)_{i'j';\beta\alpha}. && (2) \end{aligned}$$

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Proposition

Under local unitary operations, U_i let the transformed two-body density matrices be $\tilde{\rho}_{ij} = U_i \otimes U_j \rho_{ij} U_i^\dagger \otimes U_j^\dagger$. If $\{a\}_K \equiv (a_1, a_2, \ldots, a_K)$ is a closed path, then

$$\mathcal{P}(\{a\}_K) \equiv \mathcal{R}(\rho_{1K}^{T_K}) \cdots \mathcal{R}(\rho_{32}^{T_2})\mathcal{R}(\rho_{21}^{T_1}) =$$
$$(U_1 \otimes U_1^*)^\dagger \mathcal{R}(\tilde{\rho}_{1K}^{T_K}) \cdots \mathcal{R}(\tilde{\rho}_{32}^{T_2})\mathcal{R}(\tilde{\rho}_{21}^{T_1})(U_1 \otimes U_1^*).$$

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$\square$

Slew of LU Invariants from closed paths

Eigenvalues of $\mathcal{P}\{a\}_K = \mathcal{R}(\rho_{1K}^{T_K}) \cdots \mathcal{R}(\rho_{32}^{T_2})\mathcal{R}(\rho_{21}^{T_1})$ provide a set of d_1^2 LU invariants.

Proposition

If the closed path $\{a\}_K$ is **fully retracing** then $\mathcal{P}(\{a\}_K)$ is a **positive operator**, else it has eigenvalues are either real or appear as complex conjugate pairs.

Proof

If S_i is swap on **copies** $\mathcal{H}_{d_1} \otimes \mathcal{H}_{d_1}$ then
$\mathcal{S}_2\mathcal{R}(\rho_{21}^{T_1})\mathcal{S}_1 = \mathcal{R}(\rho_{21}^{T_1})^* \implies \mathcal{S}_1\mathcal{P}(\{a\}_K)\mathcal{S}_1 = \mathcal{P}(\{a\}_K)^*$
$\mathcal{R}(\rho_{21}^{T_1}) = \mathcal{R}(\rho_{12}^{T_2})^\dagger \implies$ Retracing path $\mathcal{P}(\{a\}_K)$ is positive.

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Connection to the work of Williamson et. al.

Case of all $d_i = 2$:

$$S(a_2, a_1) = U^\dagger \mathcal{R}(\rho_{21}^{T_1}) U.$$

The matrix U written explicitly in the standard basis is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -i & 0 \\ 0 & 1 & i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}.$$

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Examples

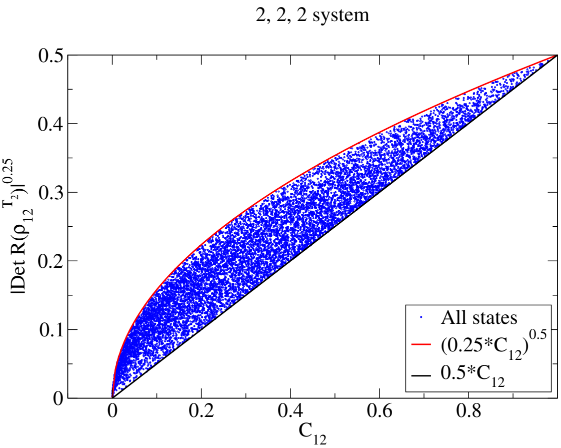
Two qubit rank-1 states. Pure states

$4\det[\mathcal{R}(\rho_{12}^{T_2})\mathcal{R}(\rho_{21}^{T_1})]^{1/4} = \tau$ τ: 2-tangle. or $2|\det \mathcal{R}(\rho_{12}^{T_2})|^{1/4} = C$

Examples

Two qubit rank-2 states/ Pure 3-qubit states

$$C_{12} \leq 2|\det \mathcal{R}(\rho_{12}^{T_2})|^{1/4} \leq \sqrt{C_{12}}$$



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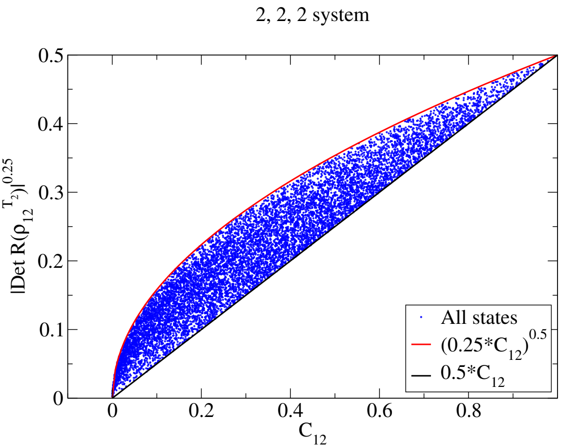
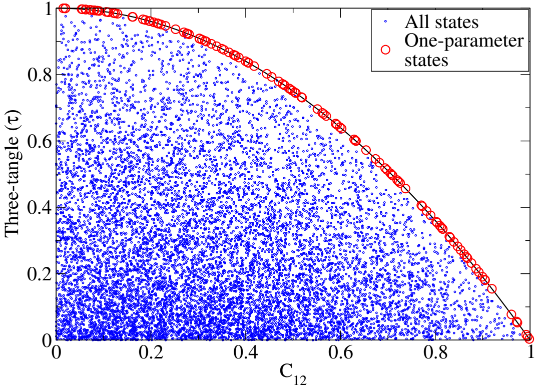


Border states

Lower boundary: $C_{12} = 2|\det \mathcal{R}(\rho_{12}^{T_2})|^{1/4}$: Three tangle $\tau = 0$: W states

Upper boundary: $\sqrt{C_{12}} = 2|\det \mathcal{R}(\rho_{12}^{T_2})|^{1/4}$ Maximize three tangle, given C_{12}. $\frac{1}{\sqrt{2}}\left(|000\rangle + C_{12}|110\rangle + \sqrt{1-C_{12}^2}|111\rangle\right)$



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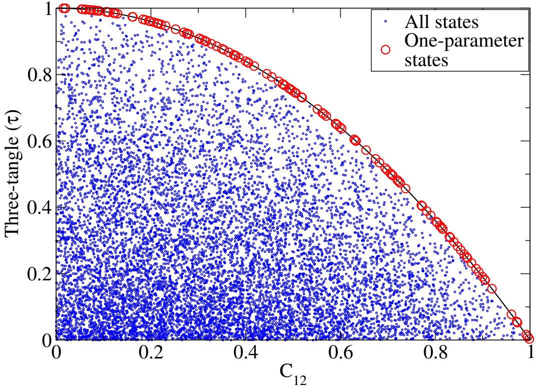

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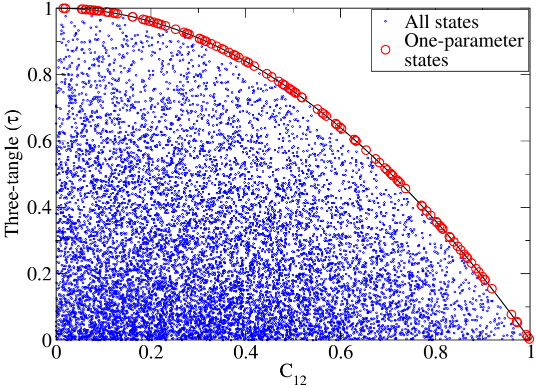

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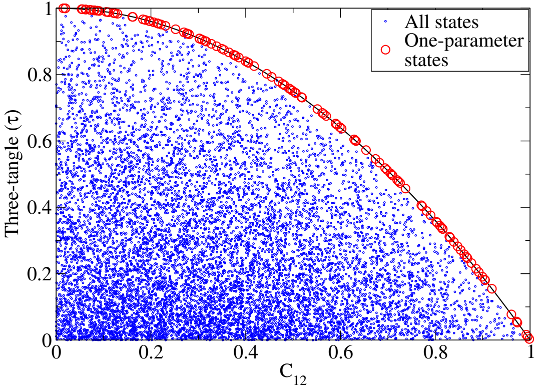

Larger Tripartite Pure States

Entanglement in tripartite qudit states

Kempe invariant

can be written as

$$\mathrm{tr}(\mathcal{P}(\{1,2,3\})) = \mathrm{tr}(\mathcal{R}(\rho_{13}^{T_3})\mathcal{R}(\rho_{32}^{T_2})\mathcal{R}(\rho_{21}^{T_1}))$$

.

$$\text{For pure states} = \mathrm{tr}\left(\rho_{12}^{T_2}\right)^3 = \mathrm{tr}\left(\rho_{23}^{T_3}\right)^3 = \mathrm{tr}\left(\rho_{31}^{T_1}\right)^3$$

RMT average Kempe invariant

$$\langle \mathrm{tr}\left(\rho_{12}^{T_2}\right)^3 \rangle = \frac{N_1^2 + N_2^2 + N_3^2 + 3N_1N_2N_3}{(N_1N_2N_3+1)(N_1N_2N_3+2)}$$

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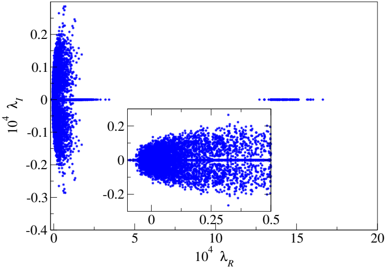
$$\langle \mathrm{tr}\left(\rho_{12}^{T_2}\right)^3 \rangle = \frac{N_1^2 + N_2^2 + N_3^2 + 3N_1N_2N_3}{(N_1N_2N_3+1)(N_1N_2N_3+2)}$$

Some LU invariant spectra: $10 \times 10 \times 10$ random pure

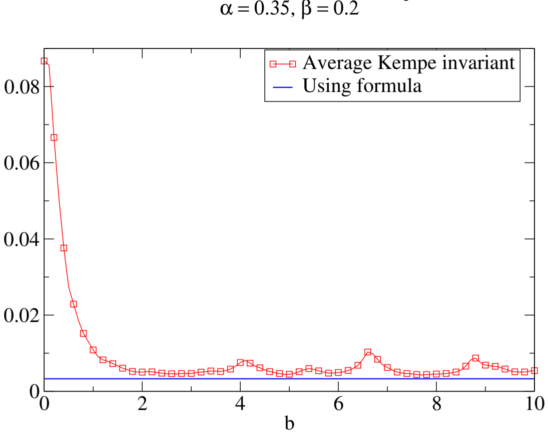


Real part (λ_R) and imaginary part (λ_I) of the eigenvalues of the matrix corresponding to $\mathcal{P}(\{1,2,3\})$ for 100 random tripartite pure states with subsystem dimensions $d_1 = d_2 = d_3 = 10$. The states are sampled according to the uniform Haar measure.

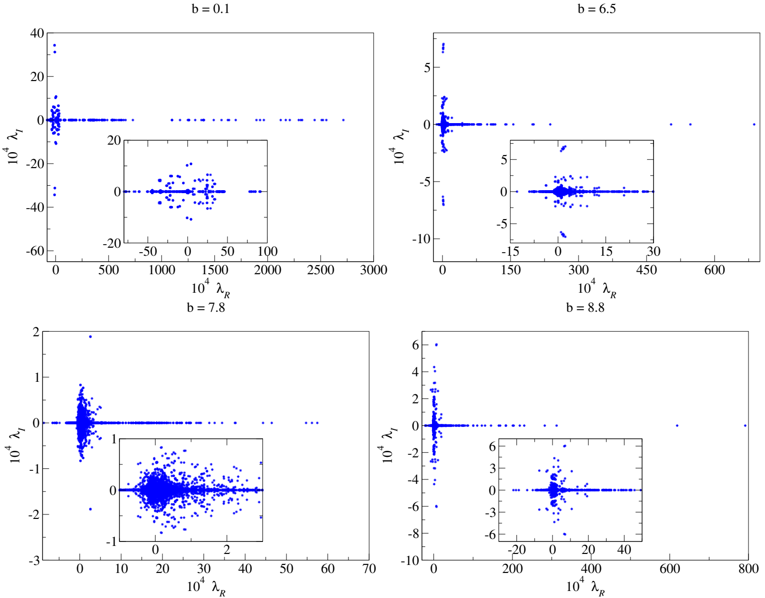
LU invariants in Eigenfunctions of complex systems

Three coupled kicked rotors on T^6:

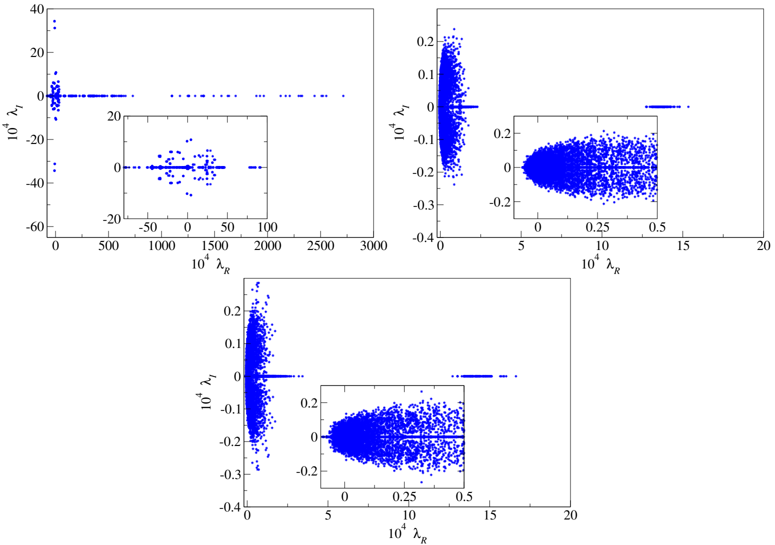
$$H = \sum_{i=1}^{3} \left(\frac{p_i^2}{2} + \frac{K_i}{2\pi} \cos 2\pi\theta_i \ \delta_t \right) + \frac{1}{4\pi^2} \sum_{i<j} b_{ij} \cos 2\pi(\theta_i - \theta_j) \ \delta_t,$$



Sensitivity of the LU invariants



LU invariants and integrability



(a) $K_i = 0,\ b_i = b = 0.1$. (b) $K_i \approx 6,\ b_i \approx 6$. (c) Random

Summary

- Showed that the operations of PT and realignment, hitherto used to detect entanglement, when used in combination are natural link transformations to generate local unitary invariants.
- Saw some applications to 3qubit states and to eigenfunction of three coupled rotors where these differentiate not only near integrable vs chaotic regimes but are sensitive to subtle features that are that not prominent in trace invariants.

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