The effect of w-term in visibility correlation and power spectrum estimation

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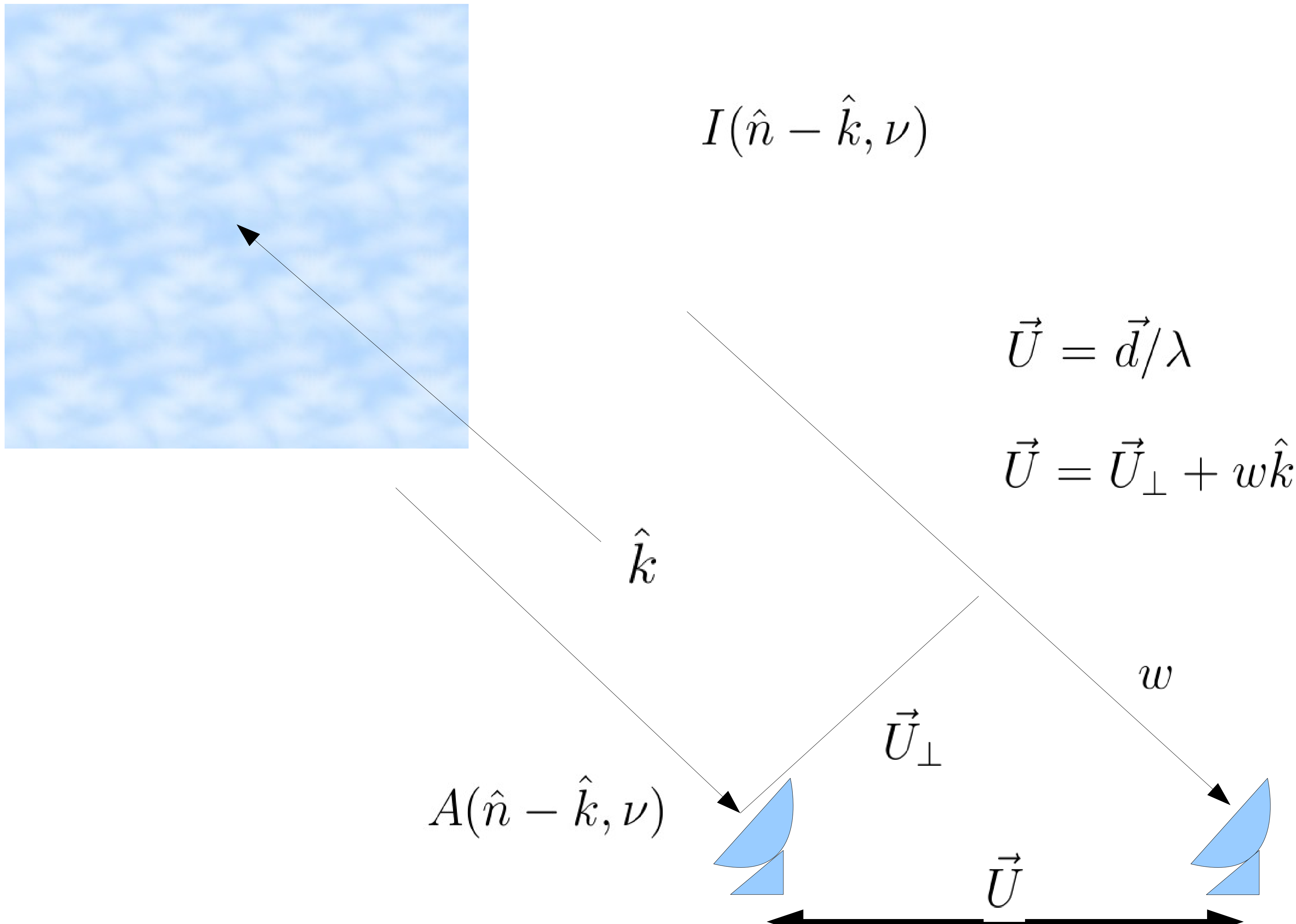
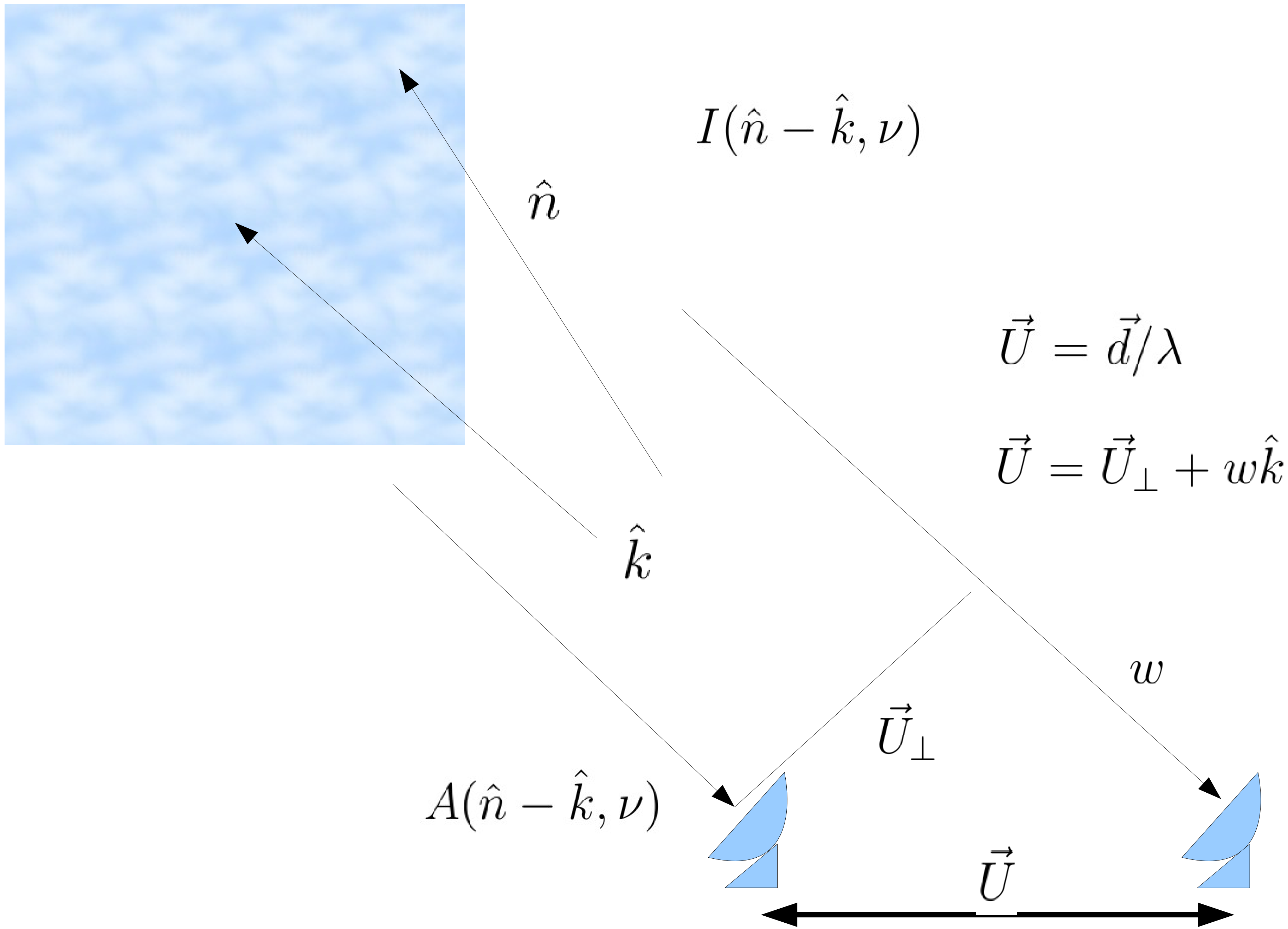


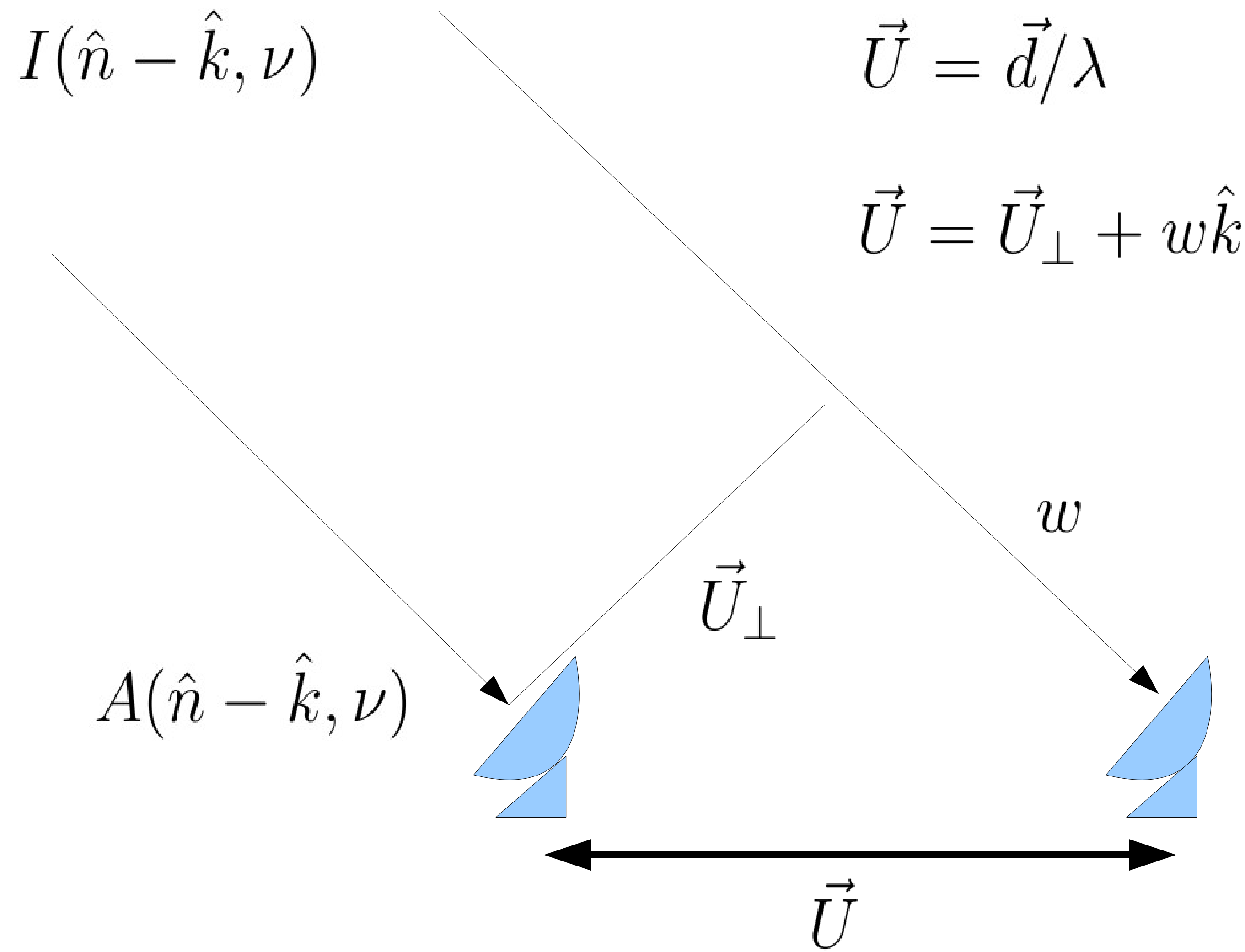
$I(\hat{n} - \hat{k}, \nu)$
$\vec{U} = \vec{d}/\lambda$
$\vec{U} = \vec{U}_\perp + w\hat{k}$
$\hat{k}$
w
$\vec{U}_\perp$
$A(\hat{n} - \hat{k}, \nu)$
$\vec{U}$



$I(\hat{n}-\hat{k},\nu)$
$\hat{n}$
$\vec{U}=\vec{d}/\lambda$
$\vec{U}=\vec{U}_\perp + w\hat{k}$
$\hat{k}$
w
$\vec{U}_\perp$
$A(\hat{n}-\hat{k},\nu)$
$\vec{U}$

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d\Omega_{\hat{n}} e^{2\pi \vec{U}\cdot(\hat{n}-\hat{k})} A(\hat{n}-\hat{k},\nu) I(\hat{n}-\hat{k},\nu)$$

$$\hat{n} - \hat{k} = \vec{\theta}$$

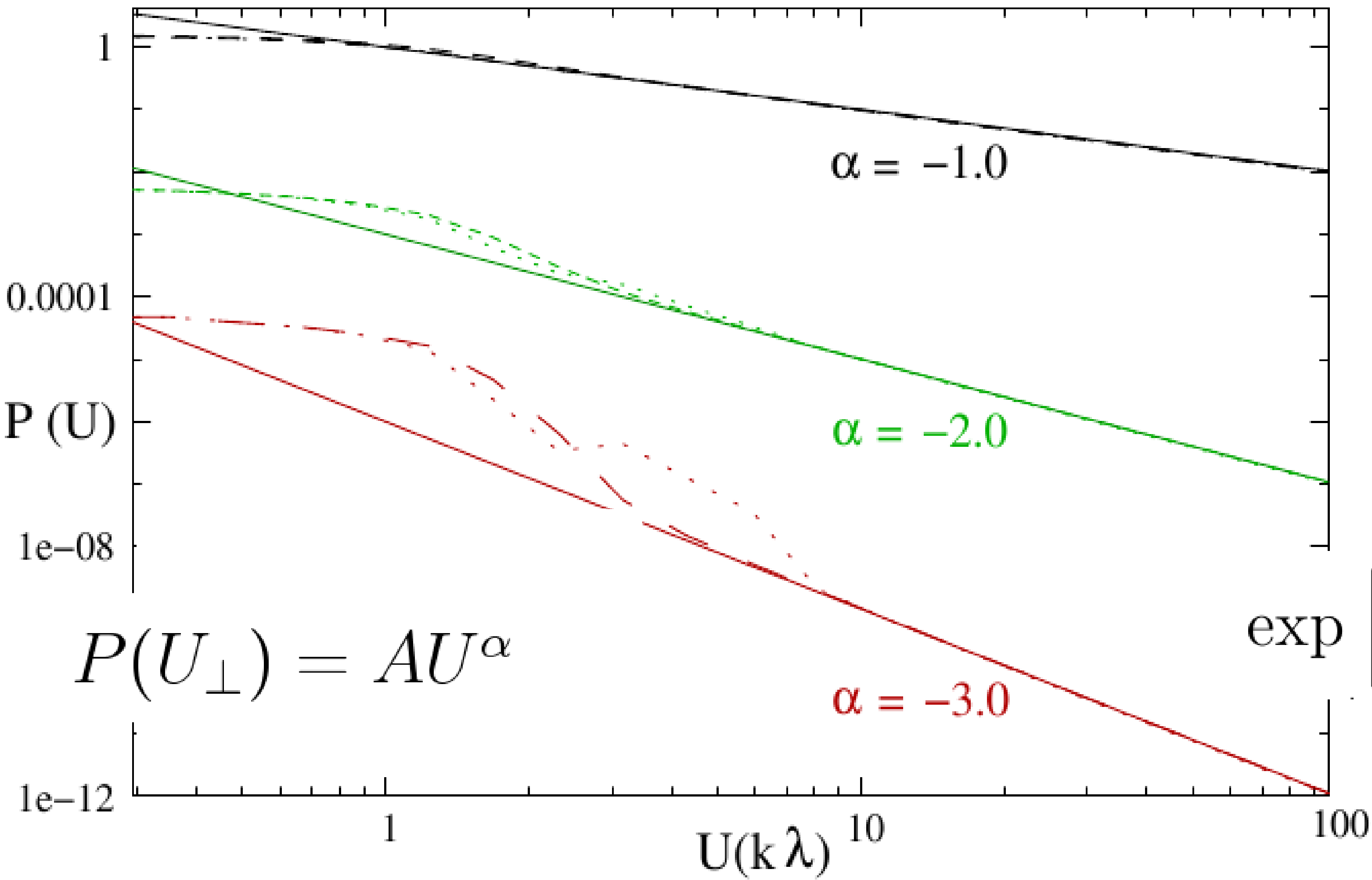


$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d\Omega_{\hat{n}} e^{2\pi \vec{U}\cdot(\hat{n}-\hat{k})} A(\hat{n}-\hat{k},\nu) I(\hat{n}-\hat{k},\nu)$$

$$\mathcal{V}^{2D}(\vec{U}_\perp,\nu) = \int d^2\vec{\theta}\, e^{2\pi i \vec{U}_\perp\cdot\vec{\theta}}\, A(\vec{\theta},\nu)\; I(\vec{\theta},\nu)$$

$$V_2^{3D}(\vec{U}_a,\vec{U}_b) = \langle \mathcal{V}^{3D}(\vec{U}_a)\; \mathcal{V}^{3D*}(\vec{U}_b)\rangle$$

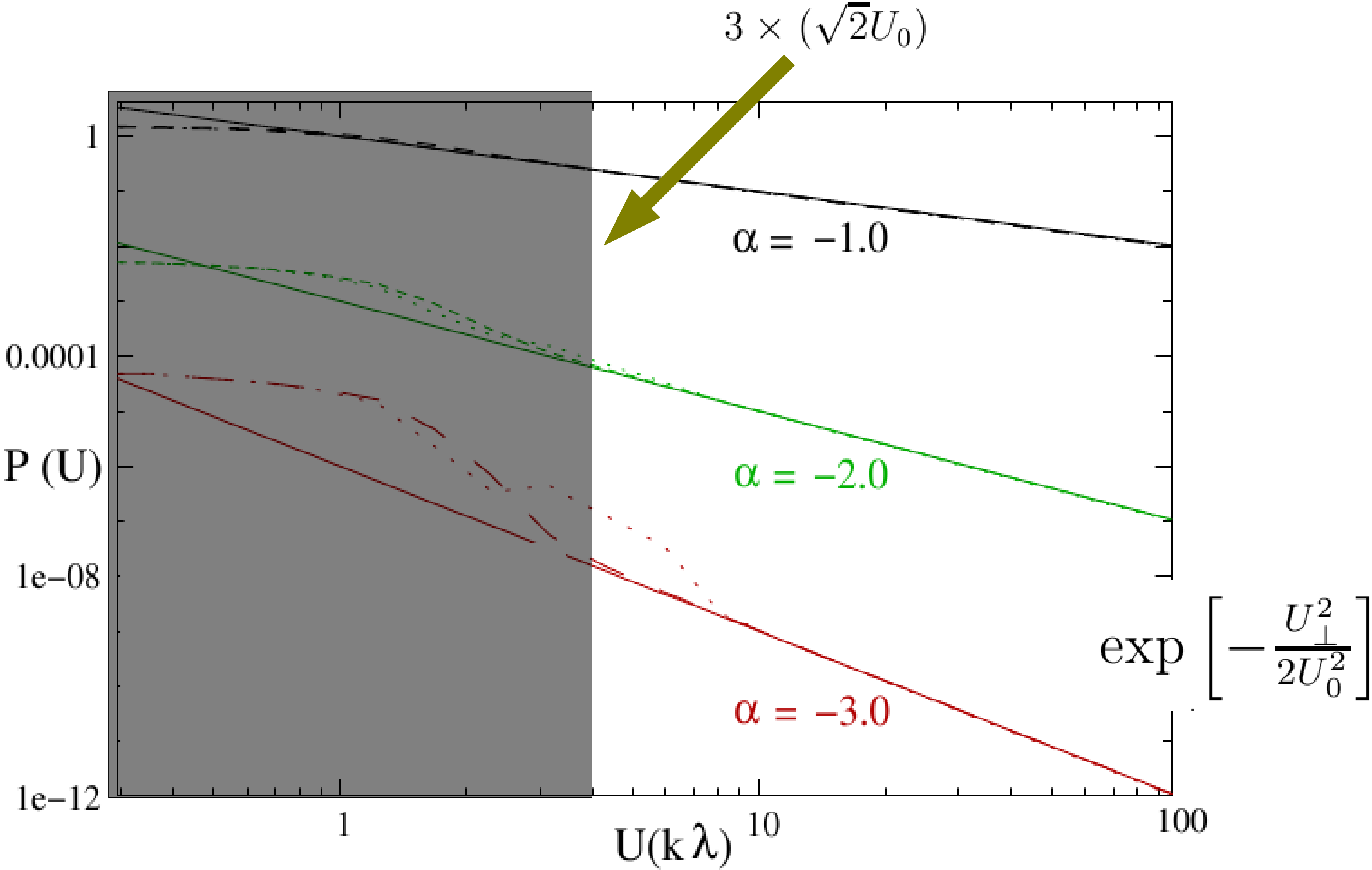
$$V_2^{2D}(U_\perp) = \int d^2\vec{U}'_\perp\, \left|\tilde{A}(\vec{U}_\perp - \vec{U}'_\perp)\right|^2 P(U'_\perp)$$



$$P(U_\perp) = AU^\alpha$$

$$\exp\left[-\frac{U_\perp^2}{2U_0^2}\right]$$

$$V_2^{2D}(U_\perp) = \int d^2\vec{U}'_\perp \left|\tilde{A}(\vec{U}_\perp - \vec{U}'_\perp)\right|^2 P(U'_\perp)$$



$$V_2^{2D}(U_\perp) = \int d^2\vec{U}'_\perp \left|\tilde{A}(\vec{U}_\perp - \vec{U}'_\perp)\right|^2 P(U'_\perp)$$

- General Wisdom: Smaller the frequency greater is the effect of the w-term.

- Usually when visibility correlation is done for the smaller frequency (150 MHz), correlation is done only for small w.

- Visibility correlation is used as an estimator of the power spectrum for the CMB observations.

Bharadwaj, S., Sethi, S. K., & Saini, T. D. 2009, PRD, 79, 083538

Bharadwaj, S., & Ali, S. S. 2005, MNRAS, 356, 1519

Hobson, M. P., & Maisinger, K. 2002, MNRAS, 334, 569

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d\Omega_{\hat{n}} e^{2\pi \vec{U}\cdot(\hat{n}-\hat{k})} A(\hat{n}-\hat{k},\nu) I(\hat{n}-\hat{k},\nu)$$

$$\mathcal{V}^{2D}(\vec{U}_\perp,\nu) = \int d^2\vec{\theta}\, e^{2\pi i \vec{U}_\perp\cdot\vec{\theta}}\, A(\vec{\theta},\nu)\, I(\vec{\theta},\nu)$$

$$I(\vec{\theta},\nu) = \bar{I}_\nu + \delta I(\vec{\theta},\nu)$$

$$\mathcal{V}^{2D}(\vec{U}_\perp,\nu) = \qquad \tilde{A}(\vec{U}_\perp,\nu) \otimes \tilde{\delta I}(\vec{U}_\perp,\nu)$$

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d\Omega_{\hat{n}} e^{2\pi \vec{U}\cdot(\hat{n}-\hat{k})} A(\hat{n}-\hat{k},\nu) I(\hat{n}-\hat{k},\nu)$$

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d^2\vec{U}'_\perp \, \tilde{\mathcal{A}}(\vec{U}-\vec{U}'_\perp) \, \tilde{\delta I}(\vec{U}'_\perp)$$

$$\mathcal{V}^{2D}(\vec{U}_\perp,\nu) = \qquad \tilde{A}(\vec{U}_\perp,\nu) \otimes \tilde{\delta I}(\vec{U}_\perp,\nu)$$

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d\Omega_{\hat{n}} e^{2\pi \vec{U}\cdot(\hat{n}-\hat{k})} A(\hat{n}-\hat{k},\nu) I(\hat{n}-\hat{k},\nu)$$

$$\mathcal{V}^{3D}(\vec{U},\nu) = \int d^2\vec{U}'_\perp \, \tilde{\mathcal{A}}(\vec{U}-\vec{U}'_\perp) \, \tilde{\delta I}(\vec{U}'_\perp)$$

$$\tilde{\mathcal{A}}(\vec{U},\nu) = 4\pi \int d^2\vec{U}'_\perp \, j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{A}(\vec{U}'_\perp,\nu)$$

EFFECT OF $w - term$ ON THE APERTURE FUNCTION

$\vec{U}_\perp$ $\quad w \quad \nu \quad$ $\tilde{A}(\vec{U}_\perp, \nu)$

GMRT

$\exp\left[-\frac{U_\perp^2}{2U_0^2}\right]$

$$\mathcal{V}^{3D}(\vec{U}, \nu) = \int d^2\vec{U}'_\perp \, \tilde{\mathcal{A}}(\vec{U} - \vec{U}'_\perp) \, \tilde{\delta I}(\vec{U}'_\perp)$$

$$\tilde{\mathcal{A}}(\vec{U}, \nu) = 4\pi \int d^2\vec{U}'_\perp \, j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{A}(\vec{U}'_\perp, \nu)$$

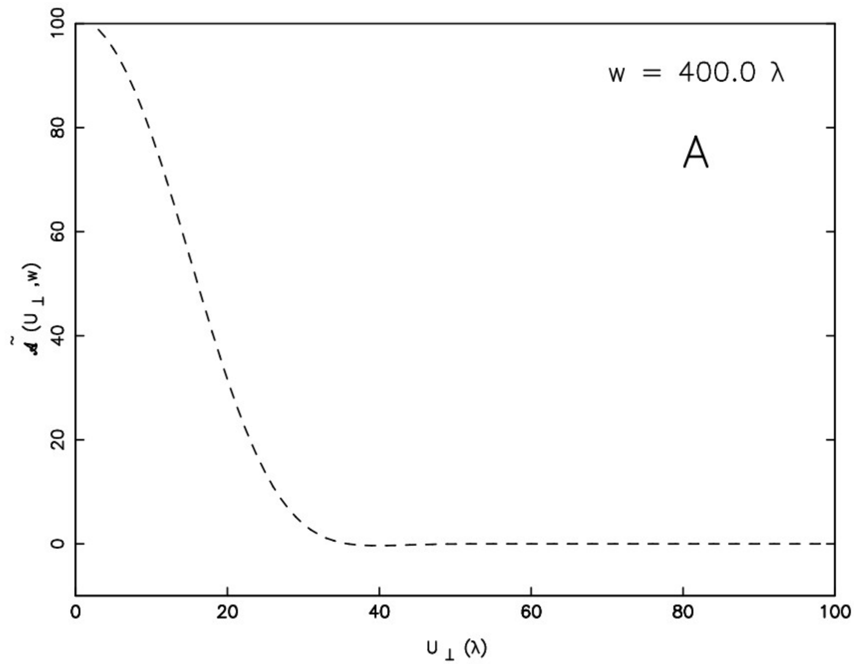


Figure 1. Modified aperture $\tilde{\mathcal{A}}(\vec{U}_\perp, w)$ plotted as a function of $\vec{U}_\perp$ for two different values of w, (A) $w = 400.\lambda$ and (B) $w = 12.$ kλ at $\nu = 150$ MHz. Solid line in B shows the gaussian envelope.

$$\tilde{\mathcal{A}}(\vec{U}, \nu) = 4\pi \int d^2\vec{U}'_\perp \; j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{A}(\vec{U}'_\perp, \nu)$$

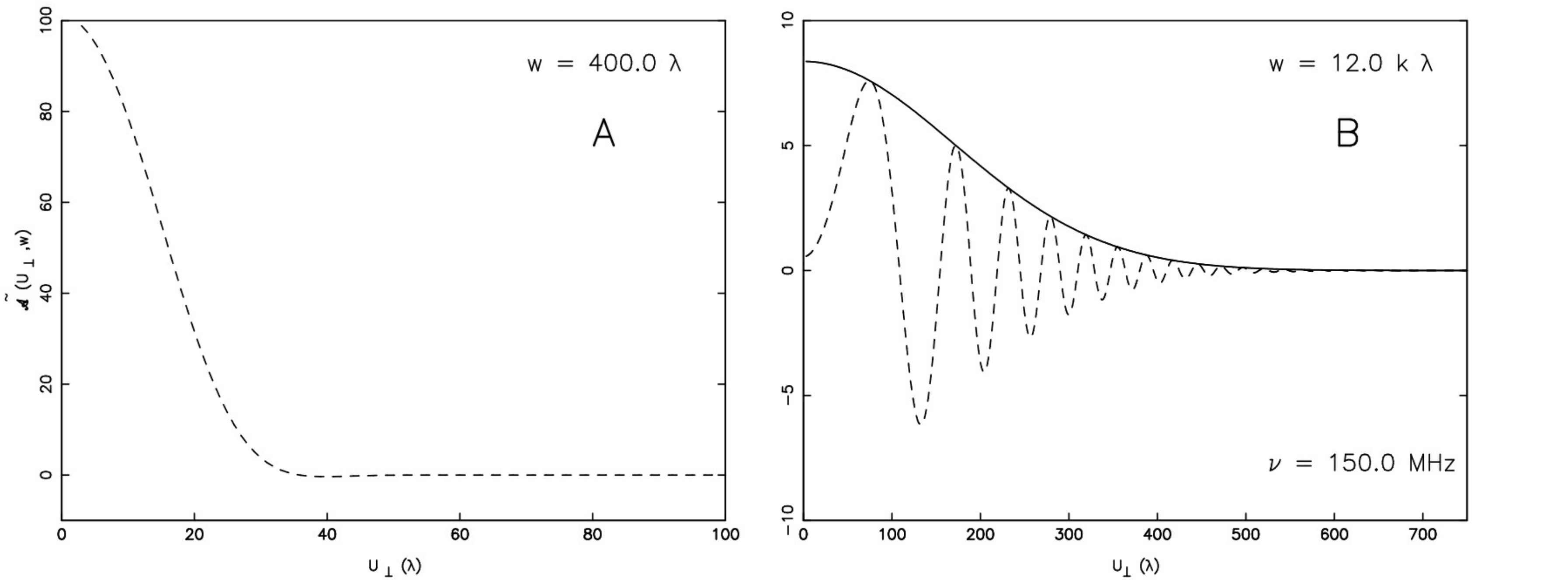


Figure 1. Modified aperture $\tilde{\mathcal{A}}(\vec{U}_\perp, w)$ plotted as a function of $\vec{U}_\perp$ for two different values of w, (A) $w = 400.\lambda$ and (B) $w = 12.$ kλ at $\nu = 150$ MHz. Solid line in B shows the gaussian envelope.

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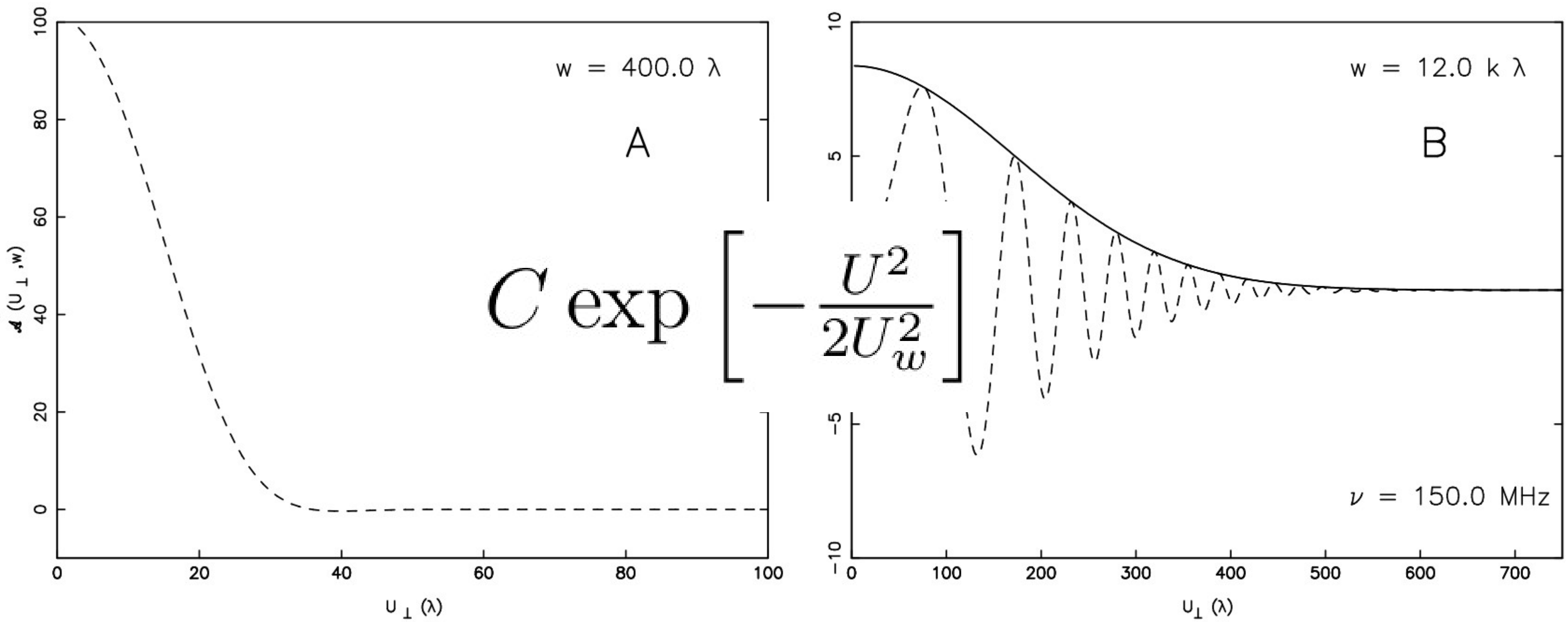
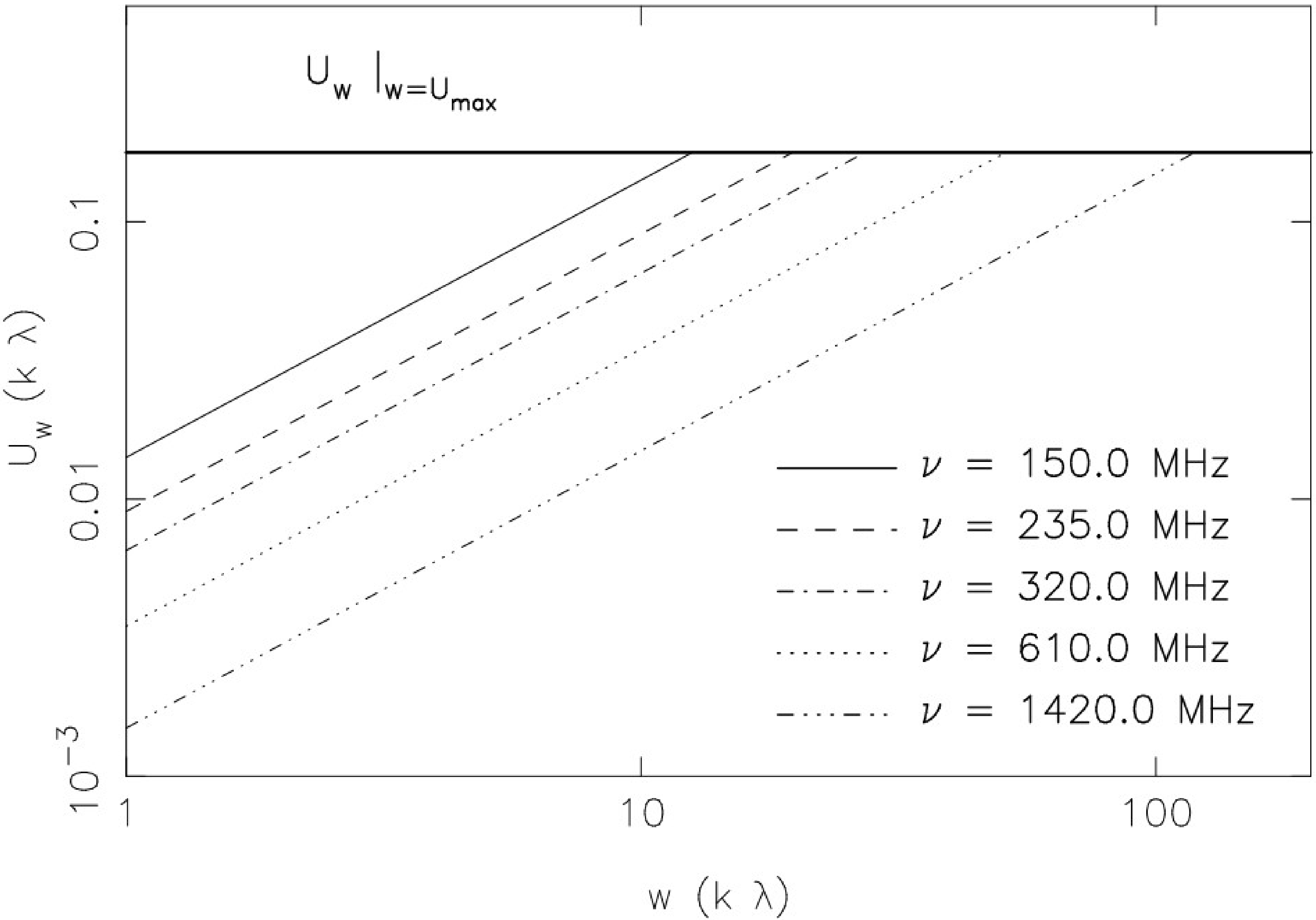


$$C \exp\left[-\frac{U^2}{2U_w^2}\right]$$

Figure 1. Modified aperture $\tilde{\mathcal{A}}(\vec{U}_\perp, w)$ plotted as a function of $\vec{U}_\perp$ for two different values of w, (A) $w = 400.\lambda$ and (B) $w = 12.$ kλ at $\nu = 150$ MHz. Solid line in B shows the gaussian envelope.

$$\tilde{\mathcal{A}}(\vec{U}, \nu) = 4\pi \int d^2\vec{U}'_\perp \; j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{A}(\vec{U}'_\perp, \nu)$$

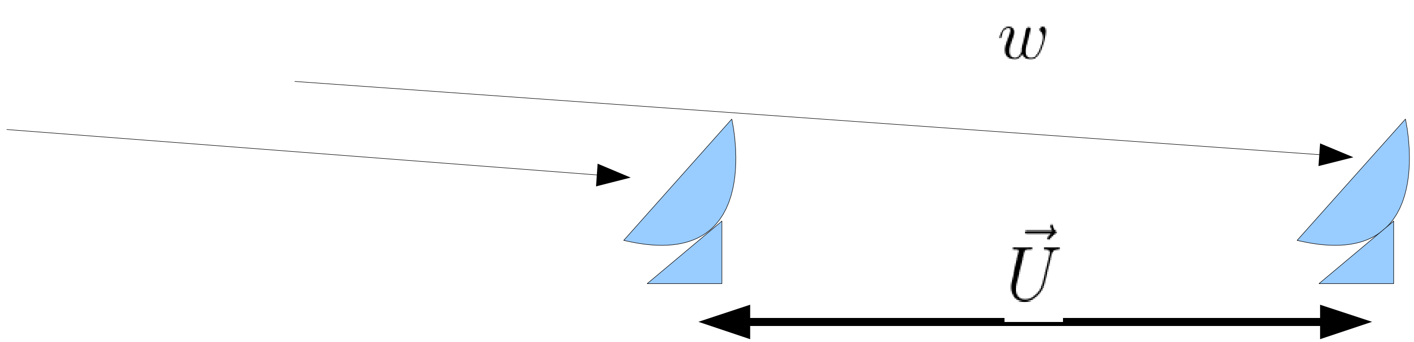


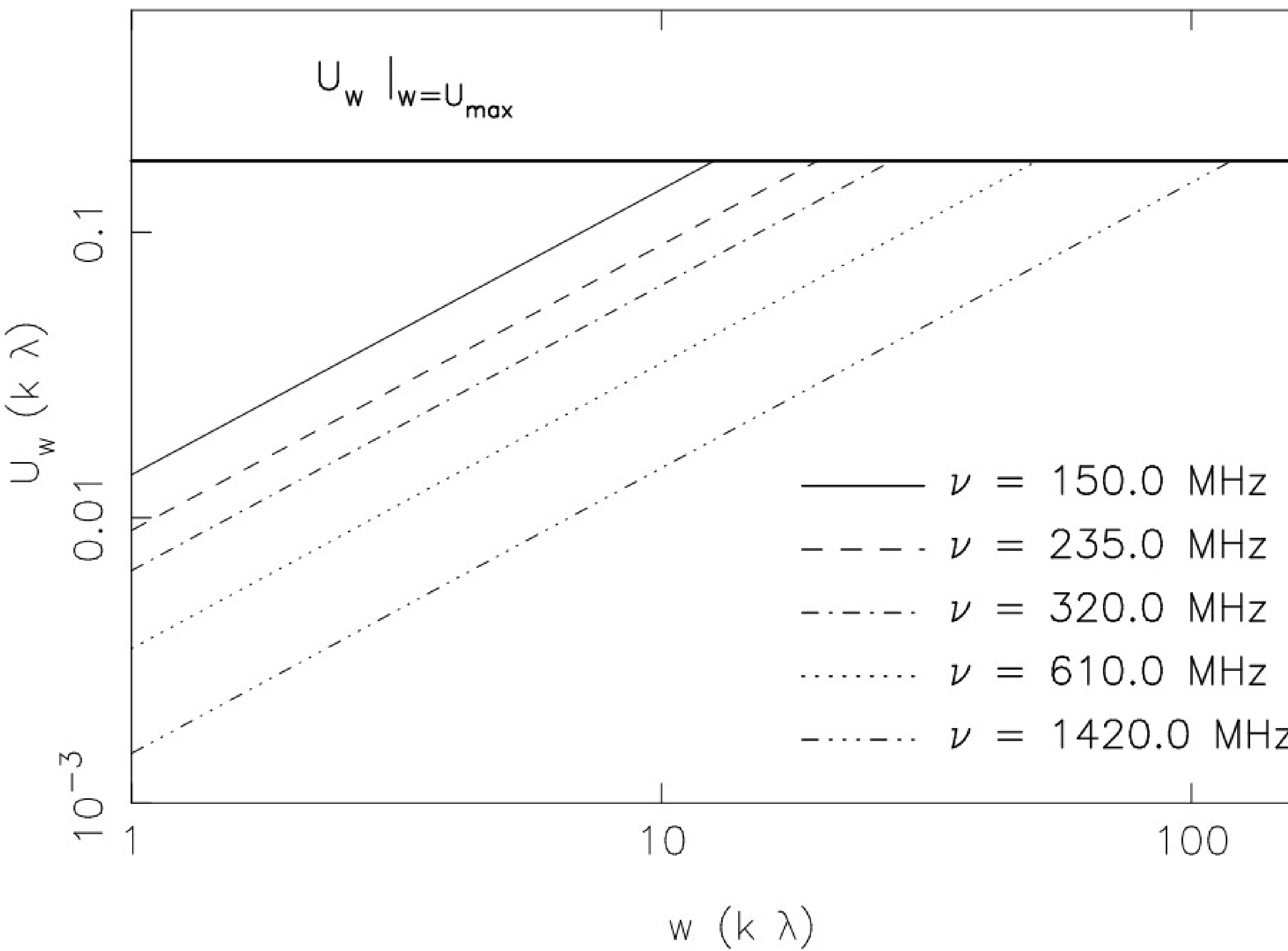
$U_w \; |_{w=U_{max}}$
0.1
0.01
10^{-3}
U_w (k λ)
1
10
100
w (k λ)
ν = 150.0 MHz
ν = 235.0 MHz
ν = 320.0 MHz
ν = 610.0 MHz
ν = 1420.0 MHz

$$I(\hat{n} - \hat{k}, \nu)$$

$$\vec{U} = \vec{d}/\lambda$$

$$\vec{U} = \vec{U}_\perp + w\hat{k}$$





$U_w \,|_{w=U_{max}}$
0.1
0.01
10^{-3}
1
10
100
U_w (k λ)
w (k λ)
ν = 150.0 MHz
ν = 235.0 MHz
ν = 320.0 MHz
ν = 610.0 MHz
ν = 1420.0 MHz

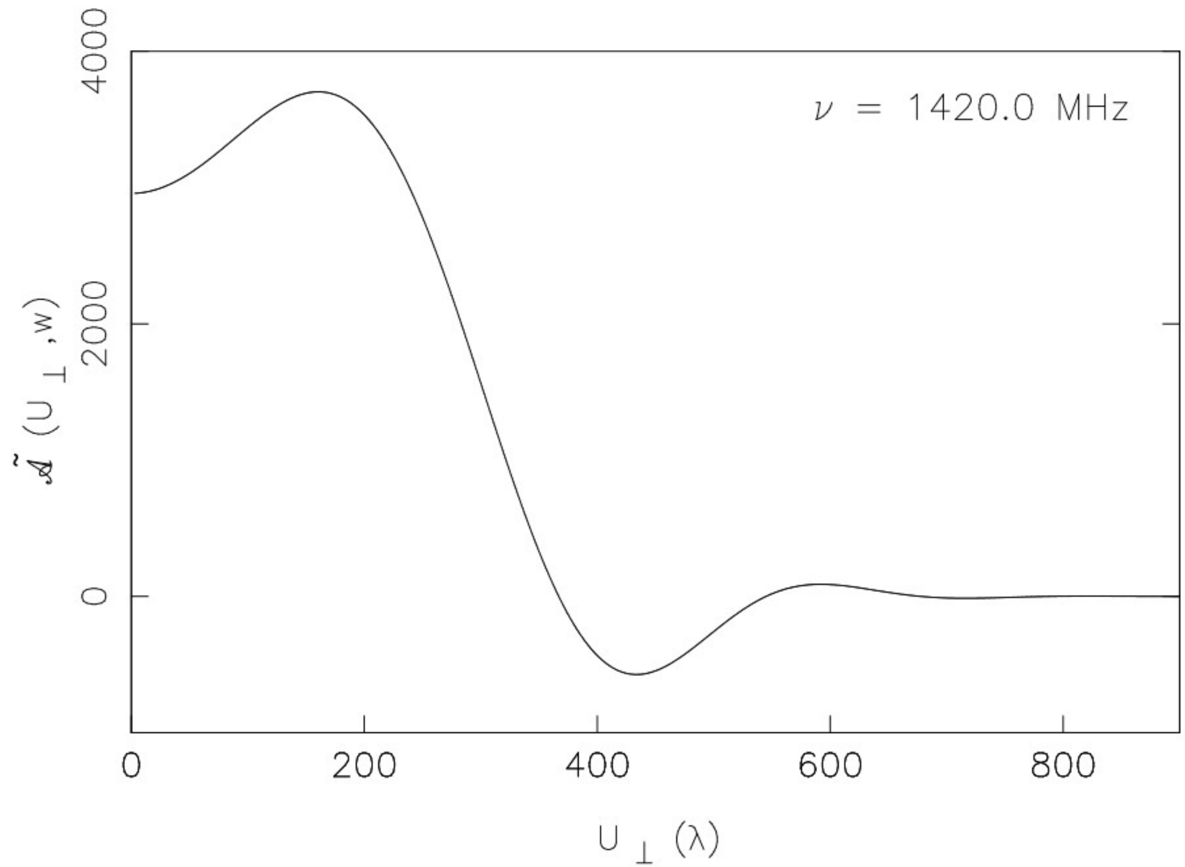


Figure 3. Modified aperture $\tilde{\mathcal{A}}(\vec{U}_\perp, w)$ plotted as a function of $\vec{U}_\perp$ for $w = 120.$ k λ at $\nu = 1420.$ MHz.

Empirical expression for $U_w \,|_{w=U_{max}}$

$$\tilde{\mathcal{A}}(\vec{U},\nu) = 4\pi \int d^2\vec{U}'_\perp \, j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{\mathcal{A}}(\vec{U}'_\perp,\nu)$$

$$U_w \,|_{w=U_{max}} = \frac{d_{max}}{\pi D}$$

Maximum baseline separation

Antenna diameter

Depends only on the array geometry, does not depend on frequency

Empirical expression for $U_w \mid_{w=U_{max}}$

$$\tilde{A}(\vec{U},\nu) = 4\pi \int d^2\vec{U}'_{\perp}\; j_0(2\pi|\vec{U}'_{\perp} - \vec{U}|)\tilde{A}(\vec{U}'_{\perp},\nu)$$

$$U_w \mid_{w=U_{max}} = \frac{d_{max}}{\pi D}$$

0.18kλ

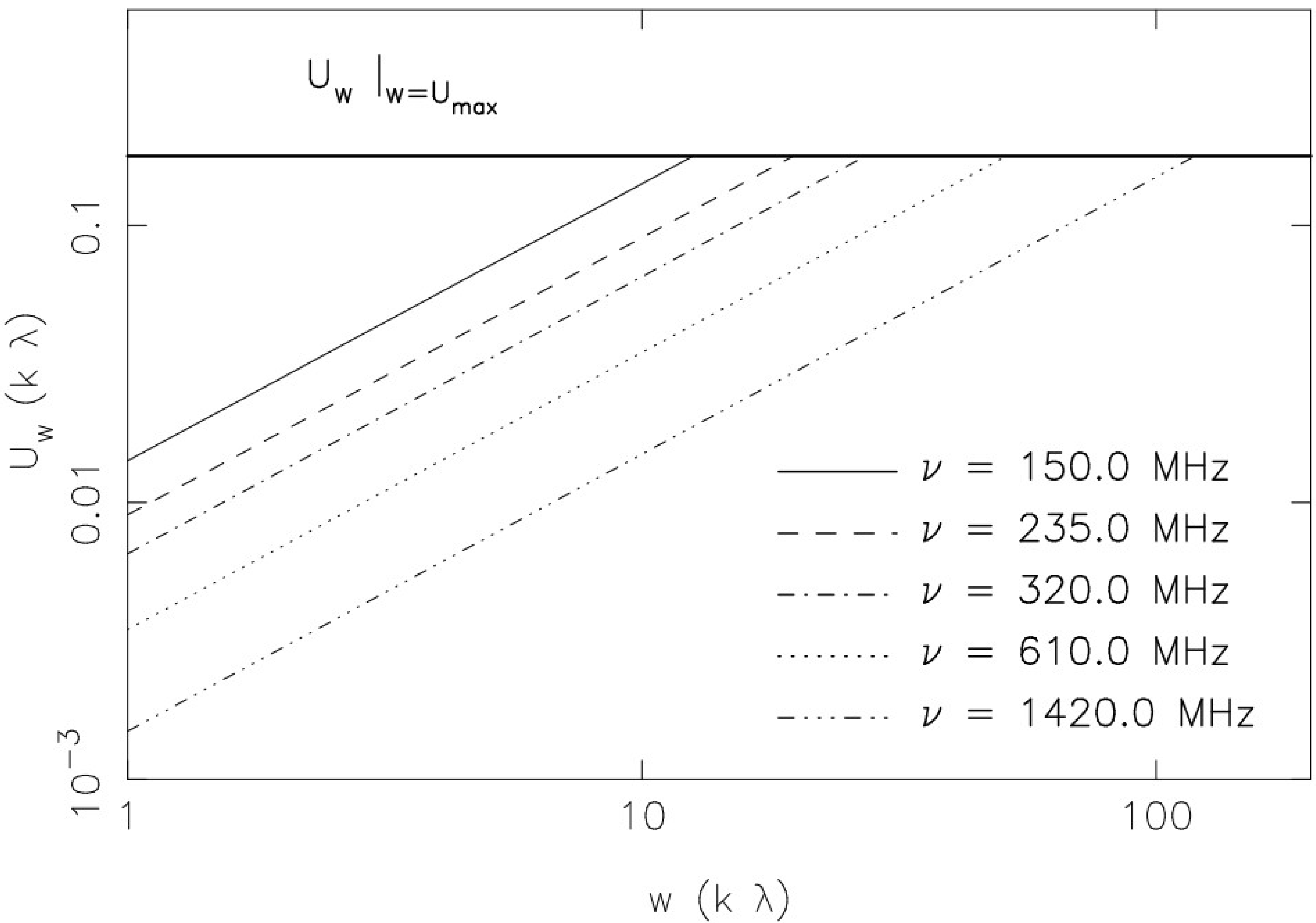
25 km

45 m

Maximum baseline separation

Antenna diameter

Depends only on the array geometry, does not depend on frequency



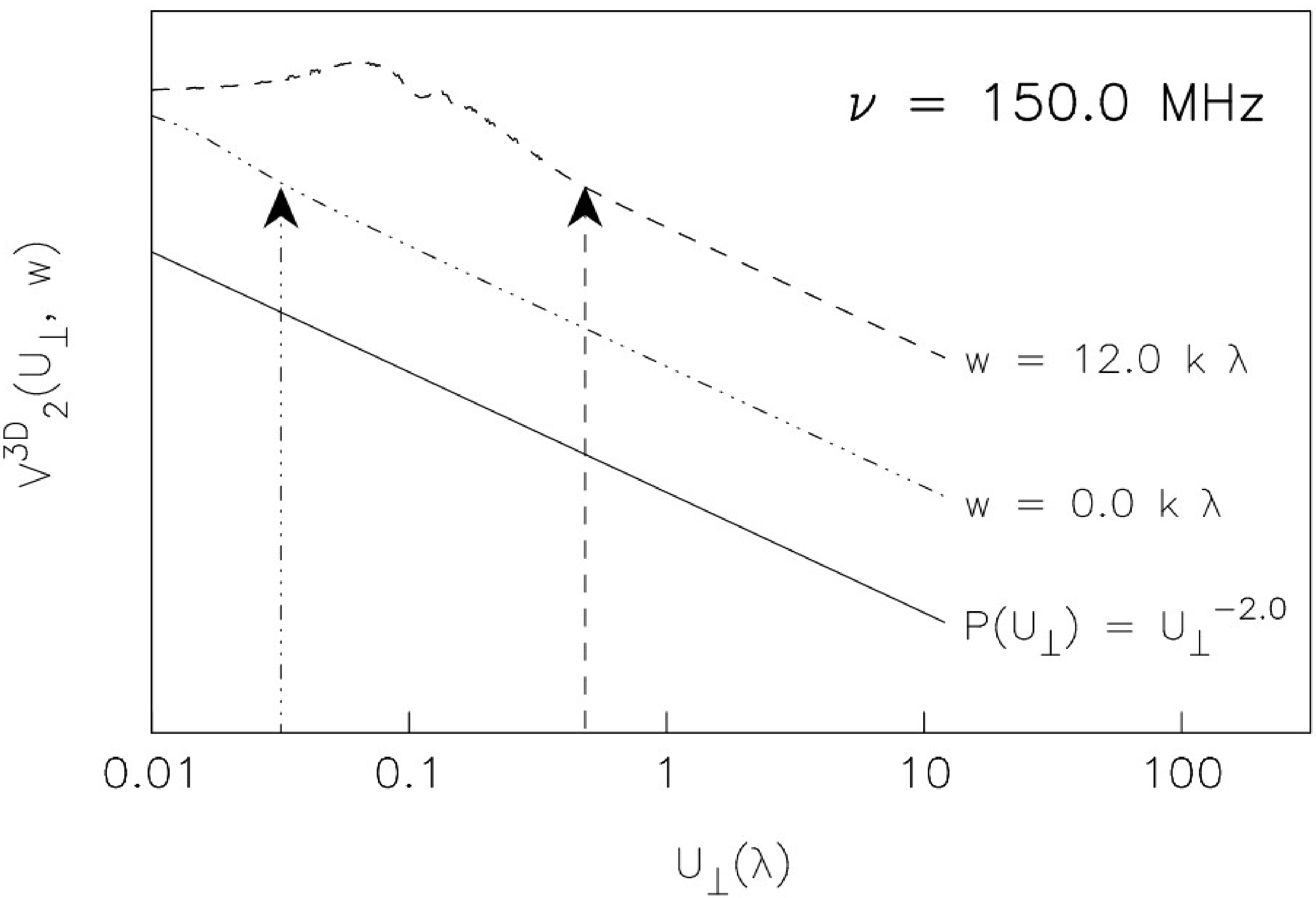
$U_w \;|_{w=U_{max}}$
0.1
0.01
10^{-3}
U_w (k λ)
1
10
100
w (k λ)
ν = 150.0 MHz
ν = 235.0 MHz
ν = 320.0 MHz
ν = 610.0 MHz
ν = 1420.0 MHz

VISIBILITY CORRELATION AND POWER SPECTRUM ESTIMATION

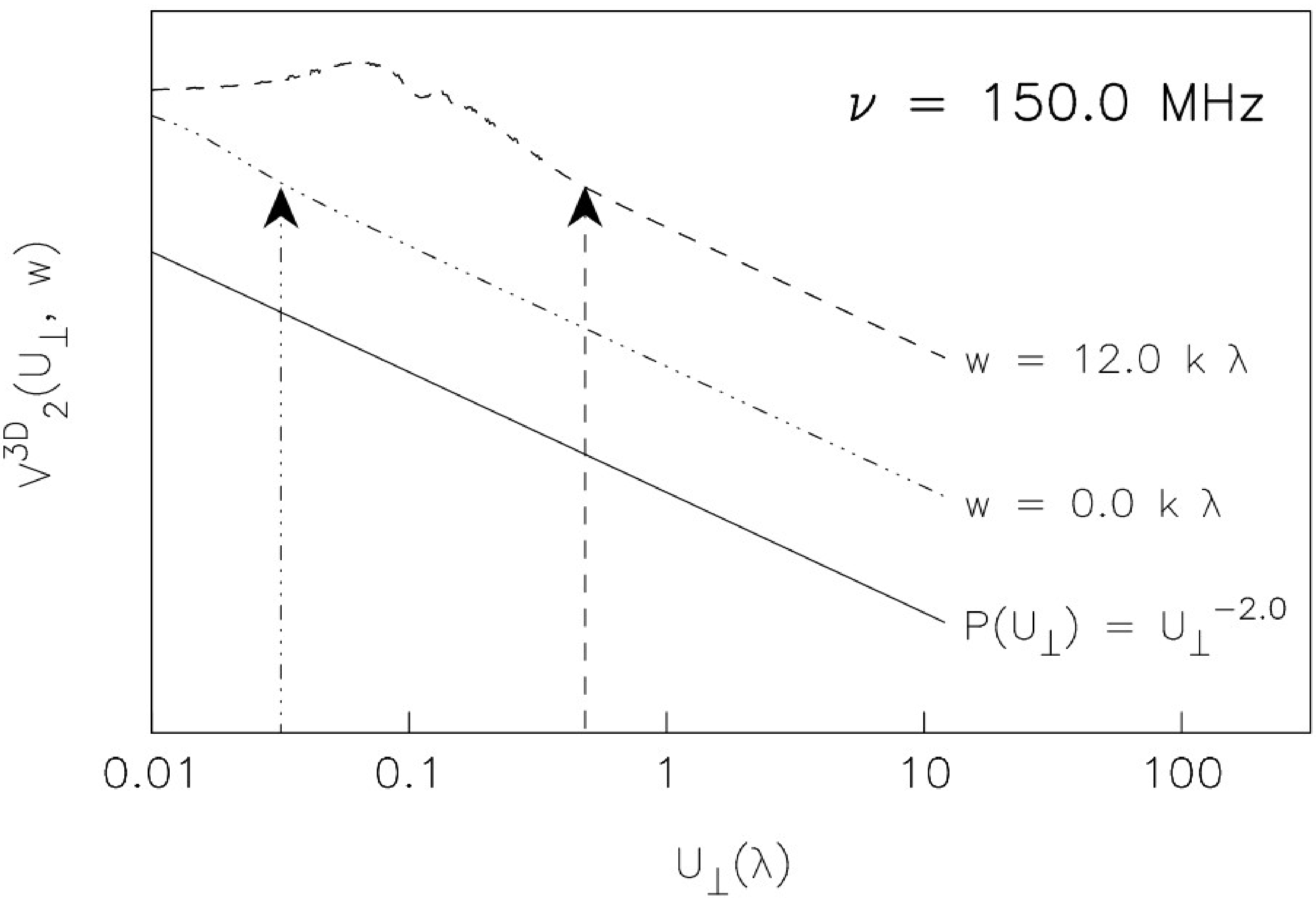
$$V_2^{2D}(U_\perp) = \int d^2\vec{U}'_\perp \left|\tilde{A}(\vec{U}_\perp - \vec{U}'_\perp)\right|^2 P(U'_\perp).$$

$$V_2^{3D}(U) = \int d^2\vec{U}'_\perp \left|\tilde{\mathcal{A}}(\vec{U} - \vec{U}'_\perp)\right|^2 P(U'_\perp)$$

$$\tilde{\mathcal{A}}(\vec{U},\nu) = 4\pi \int d^2\vec{U}'_\perp \, j_0(2\pi|\vec{U}'_\perp - \vec{U}|)\tilde{A}(\vec{U}'_\perp,\nu)$$



ν = 150.0 MHz
w = 12.0 k λ
w = 0.0 k λ
P(U⊥) = U⊥^−2.0
V^3D_2(U⊥, w)
0.01
0.1
1
10
100
U⊥(λ)



$$\mathcal{E}(U_\perp) = \int_0^{U_{max}} dw\, \rho(w)\, V_2^{3D}(U_\perp, w).$$

Freqency (M Hz)	$U_w \mid_{w=U_{max}}$ (k λ)	U_{max} (k λ
150.	0.176	12.5
235.	0.176	19.6
320.	0.176	26.7
610.	0.176	50.8
1420.	0.176	118.3

150 MHz	GMRT	VLA A	VLA B	MWA	ASKAP
D (m)	45	25	25	6	12
d (km)	25	36	10	3	6
$U_w \|_{w=U_{max}}$ (k λ)	0.18	0.46	0.13	0.16	0.16
U_{max} (k λ)	12.5	18	5	1.5	3.


Thanks